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Weak Form Factors of Heavy Mesons Expanded in $1/m_Q$ in a Potential Model

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Abstract

Based on a semi-relativistic potential model proposed recently, we develop a formulation how to calculate form factors and numerically compute them for the process $\bar{B} \to D^{(*)}\ell\nu$ up to the first order in $1/m_Q$, with a heavy quark mass m_Q. It is shown that there is no contribution from the first order corrections of the wave functions to the weak form factors. The lower bound for a slope of the Isgur-Wise function at the origin is given by $-1/2$ based on our model and the numerical value of the slope of the Isgur-Wise function is calculated to be -0.820 at the zero-th order in $1/m_Q$.

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I. INTRODUCTION

The heavy quark effective theory (HQET) has brought us a very convenient tool or idea to study heavy mesons/baryons including at least one heavy quark. [1,2] Especially there have been many studies on the Isgur-Wise function [3]$\sim$ [13] but most of their calculations has used a simple-minded Gaussian form for the meson wave function or a solution to a single particle Dirac equation in a potential. Some other people used the Bethe-Salpeter solution for the meson wave function but have not developed a systematic way to calculate the higher order terms in $1/m_Q$ with a heavy quark mass, m_Q.

In the former papers [14–17], we have developed a semi-relativistic formulation how to calculate the meson masses and wave functions for $Q\bar{q}$ (heavy mesons) system with one heavy quark, Q, introducing phenomenological dynamics. That is, assuming Coulomb-like vector and confining scalar potentials to $Q\bar{q}$ bound states, we have expanded a hamiltonian then perturbatively solved Schrödinger equation in $1/m_Q$. Then fitting the smallest eigenvalues of a hamiltonian with masses of D and D^* mesons, a strong coupling α_s and other parameters included in scalar and vector potentials are determined uniquely. Using parameters obtained this way, other mass levels are calculated and compared with the experimental data for D/B mesons up to the second order of perturbation. Trying to find out the relation with HQET, the lowest degenerate eigenvalues of the Schrödinger equation gives the so-called $\bar{\Lambda}$ parameters for u, d and s light quarks. [3] Meson wave functions obtained thereby and expanded in $1/m_Q$ can be used to calculate ordinary form factors or Isgur-Wise functions and their higher order corrections in $1/m_Q$ for semileptonic weak decay processes. All the above calculations are calculated up to $1/m_Q^2$ and analyzed order by order in $1/m_Q$ to determine parameters as well as to compare with results of HQET, e.g., the parameters, λ_1, λ_2, $\bar{\Lambda}$, and the mass relations.

Here in this paper following the former study we will develop a formulation how to calculate the semi-leptonic weak form factors out of the heavy meson wave functions at the rest frame obtained in the former paper [16] in the next Section II and will give the numerical

values for the slope of the Isgur-Wise function at the origin and their forms up to the first order in $1/m_Q$ in the Section III. All the details associated to calculation of form factors are given in the Appendix.

II. FORMULATION

A. Fermi-Yang Equation

In the former papers [14–17], we have calculated mass spectrum of heavy mesons in a so-called Cornell potential model, which includes both scalar and vector potentials. Although we have needed only a stationary system, or rest frame wave function, in [16], we must now treat a moving frame of a heavy meson to calculate weak form factors. Although we can not work in a fully relativistic system as far as Hamiltonian formulation is concerned, we adopt the notations of the Nambu-Bethe-Salpeter equation as far as possible. First we define a heavy meson (X) wave function composed of a light anti-quark, $q^c(x)$, and a heavy quark, $Q(y)$, as

$$\langle 0|\, q^c_\alpha(t,\vec{x})\, Q_\beta(t,\vec{y})\, |X; P_{\boldsymbol{X}}\,\{\ell\}\rangle = \psi^\ell_{\boldsymbol{X}\,\alpha\,\beta}\left((0,\vec{x}-\vec{y})\,; P_{\boldsymbol{X}}\right)\, e^{-iP_X\cdot y}, \tag{1}$$

where α and β denote Dirac indices of four-spinors, $P_{\boldsymbol{X}\mu}$ heavy meson's four-momentum, $\{\ell\}$ a set of quantum numbers, and $P_{\boldsymbol{X}}\cdot y = P_{\boldsymbol{X}\mu}\, y^\mu$. Note that this definition of the wave function, $\psi^\ell_{\boldsymbol{X}}$, is minus sign times that of the former paper. [16] This state is an eigen state of a four-momentum operator, $\mathcal{P}_\mu$,

$$\mathcal{P}_\mu\, |X; P_{\boldsymbol{X}}\,\{\ell\}\rangle = P_{\boldsymbol{X}\mu}\, |X; P_{\boldsymbol{X}}\,\{\ell\}\rangle\,, \qquad P_{\boldsymbol{X}\,0} = \sqrt{M_{\boldsymbol{X}}^2 + \vec{P}_{\boldsymbol{X}}^{\,2}}, \tag{2}$$

with $M_{\boldsymbol{X}}$ a heavy meson mass. In this case, the Fermi-Yang equation [18] or the Schrödinger equation in a X moving frame is given by

$$H\,\psi^\ell_{\boldsymbol{X}}\left((0,\vec{x}-\vec{y})\,; P_{\boldsymbol{X}}\right) = \sqrt{M_{\boldsymbol{X}}^2 + \vec{P}_{\boldsymbol{X}}^{\,2}}\,\psi^\ell_{\boldsymbol{X}}\left((0,\vec{x}-\vec{y})\,; P_{\boldsymbol{X}}\right), \tag{3}$$

where the Hamiltonian is given by

$$H = (\vec{\alpha}_q \cdot \vec{p} + \beta_q m_q) + \left[\vec{\alpha}_Q \cdot (\vec{P}_{\boldsymbol{X}} - \vec{p}) + \beta_Q m_Q\right] + \sum_{i,j} \beta_q O_{q\,i} V_{i\,j}(\vec{x} - \vec{y}) \beta_Q O_{Q\,j}, \tag{4}$$

where $\vec{p} = -i\vec{\nabla}_{\boldsymbol{x}}$ and $\vec{P}_{\boldsymbol{X}}$ is a heavy meson momentum. In our case, Eq. (6) gives $O_{q\,i} = O_{Q\,i} = 1$ for $V_{i\,j}(\vec{x} - \vec{y}) = S(r)$ and $O_{q\,i} = \gamma_{q_\mu}$ and $O_{Q\,j} = \gamma_Q{}^\mu$ for $V_{i\,j}(\vec{x} - \vec{y}) = V(r)$ with $r = |\vec{x} - \vec{y}|$. Actually we have included a transverse part of vector potential and the total potential in Eq. (4) is given by

$$\beta_q \beta_Q \, S(r) + \left\{1 - \frac{1}{2}\left[\, \vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})\,(\vec{\alpha}_Q \cdot \vec{n})\,\right]\right\} V(r), \tag{5}$$

with

$$S(r) = \frac{r}{a^2} + b, \qquad V(r) = -\frac{4}{3}\frac{\alpha_s}{r}. \tag{6}$$

What we have done in the former paper [16] is to solve the Schrödinger equation given by Eq. (3) with $\vec{P}_{\boldsymbol{X}} = \vec{0}$ order by order in $1/m_Q$ by expanding it in $1/m_Q$, to numerically calculate the mass spectrum, i.e., eigenvalues, $M_{\boldsymbol{X}}$ in each order of perturbation, and then to relate the results with those of HQET.

B. Lorentz Boost

In order to calculate the decay rate we need to Lorentz-boost the rest frame wave function which we have obtained in the former papers. To explain the idea we take $B \to D\ell\nu_\ell$ decay process as an example and express the wave function a the moving frame in terms of the one in a rest frame.

Assume that the heavy meson is moving in the $+z$ direction with a velocity β. Then we consider two cases below of the composite particles, i.e., q^c and Q. In the following for the initial state of the heavy meson the notations, $q^c(t_{\boldsymbol{x}}, \vec{x})$ and $Q(t_{\boldsymbol{y}}, \vec{y})$, are adopted and for the final state $q^c(t_{\boldsymbol{x}'}, \vec{x}\,')$ and $Q(t_{\boldsymbol{y}'}, \vec{y}\,')$ are adopted. A relation between the constituent particles in the rest and moving frames is given by

$$\mathcal{G}\, q^c_\alpha(t_{\boldsymbol{x}}, \vec{x})\, \mathcal{G}^{-1} = G^{-1}_{\alpha\,\beta}\, q^c_\beta(t_{\boldsymbol{x}'}, \vec{x}\,'), \tag{7}$$

$$\mathcal{G}\, Q_\alpha(t_{\boldsymbol{y}}, \vec{y})\, \mathcal{G}^{-1} = G^{-1}_{\alpha\,\beta}\, Q_\beta(t_{\boldsymbol{y}'}, \vec{y}\,'), \tag{8}$$

where $\mathcal{G}$ is a Lorentz-boost operator, G is its spinor representation, and their definitions are given by

$$\mathcal{G}\,\left|X;\left(M_{\boldsymbol{X}},\vec{0}\right)\,\{\ell\}\right\rangle = |X;P_{\boldsymbol{X}}\,\{\ell\}\rangle \tag{9}$$

$$G=\cosh\frac{\varphi}{2}+\alpha^3\sinh\frac{\varphi}{2},\qquad \beta=\tanh\varphi\qquad \gamma=\frac{1}{\sqrt{1-\beta^2}}=\cosh\varphi. \tag{10}$$

We have assumed that a heavy meson is boosted in $+z$ (or $+3$) direction with a velocity β. Here $\vec{\alpha}=\gamma^0\vec{\gamma}$, β and γ being a velocity and a Lorentz factor of a heavy meson, respectively, which can be expressed in terms of one parameter, φ.

Below we consider two cases for the constituent particles, in the first case they have equal time at the rest frame, and in the second case they have equal time at the moving frame. In each case a relation of wave functions in the rest and moving frames is derived.

i) $t=t_{\boldsymbol{x}}=t_{\boldsymbol{y}}$ and $t_{\boldsymbol{x}'}\neq t_{\boldsymbol{y}'}$ (equal time at the rest frame)

$$\begin{aligned} x'^3&=x^3\cosh\varphi+t\sinh\varphi, \qquad t'_{\boldsymbol{x}}=t\cosh\varphi+x^3\sinh\varphi,\\ y'^3&=y^3\cosh\varphi+t\sinh\varphi, \qquad t'_{\boldsymbol{y}}=t\cosh\varphi+y^3\sinh\varphi. \end{aligned} \tag{11}$$

In this case an approximate relation between the rest and moving frame wave functions is given by

$$\psi^{\ell}_{\boldsymbol{X}\,\alpha\,\beta}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right)\simeq G_{\alpha\,\gamma}G_{\beta\,\delta}\,\psi^{\ell}_{\boldsymbol{X}\,\gamma\,\delta}\left(\left(0,\vec{x}_{\perp},\gamma^{-1}x^3\right);\left(M_{\boldsymbol{X}},\vec{0}\right)\right)\,e^{i\left(M_X-m_Q\right)\gamma\,\beta\,x^3}. \tag{12}$$

On the other hand

ii) $t'=t_{\boldsymbol{x}'}=t_{\boldsymbol{y}'}$ and $t_{\boldsymbol{x}}\neq t_{\boldsymbol{y}}$ (equal time at the moving frame)

$$\begin{aligned} x^3&=x'^3\cosh\varphi-t'\sinh\varphi, \qquad t_{\boldsymbol{x}}=t'\cosh\varphi-x'^3\sinh\varphi,\\ y^3&=y'^3\cosh\varphi-t'\sinh\varphi, \qquad t_{\boldsymbol{y}}=t'\cosh\varphi-y'^3\sinh\varphi. \end{aligned} \tag{13}$$

In this case an approximate relation between the rest and moving frame wave functions is given by

$$\psi^{\ell}_{\boldsymbol{X}\,\alpha\,\beta}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right)\simeq G_{\alpha\,\gamma}G_{\beta\,\delta}\,\psi^{\ell}_{\boldsymbol{X}\,\gamma\,\delta}\left(\left(0,\vec{x}_{\perp},\gamma x^3\right);\left(M_{\boldsymbol{X}},\vec{0}\right)\right)\,e^{i\left(M_X-m_Q\right)\gamma\,\beta\,x^3}. \tag{14}$$

C. Normalization

Normalization of the state, $|X; P_{\boldsymbol{X}}\,\{\ell\}\rangle$, is given by

$$\langle X; P'_{\boldsymbol{X}}\,\{\ell'\}\,|X; P_{\boldsymbol{X}}\,\{\ell\}\rangle = (2\pi)^3\, 2P_{\boldsymbol{X}\,0}\,\delta^3\left(\vec{P}_{\boldsymbol{X}} - \vec{P}'_{\boldsymbol{X}}\right)\delta_{\ell',\ell}, \tag{15}$$

or

$$\mathrm{tr}\int d^3z\,\psi^{\ell'\,\dagger}_{\boldsymbol{X}}\left((0,\vec{z})\,;P_{\boldsymbol{X}}\right)\psi^{\ell}_{\boldsymbol{X}}\left((0,\vec{z})\,;P_{\boldsymbol{X}}\right) \simeq 2P_{\boldsymbol{X}\,0}\,\delta_{\ell\,\ell'}, \tag{16}$$

whose derivation is given in the Appendix B. Actually the normalization given by Eq. (16) does not hold because of approximations which are adopted in the former subsection, and hence instead of Eq. (16) we will define the normalization at the rest frame as follows.

$$\mathrm{tr}\int d^3z\,\psi^{\ell'\,\dagger}_{\boldsymbol{X}}\left((0,\vec{z})\,;\left(M_{\boldsymbol{X}},\vec{0}\right)\right)\psi^{\ell}_{\boldsymbol{X}}\left((0,\vec{z})\,;\left(M_{\boldsymbol{X}},\vec{0}\right)\right) = 2M_{\boldsymbol{X}}\,\delta_{\ell\,\ell'}. \tag{17}$$

In order to calculate the normalization in a moving frame we have to specify the quantum numbers we are interested in. Since we would like to compute the Isgur-Wise function and its higher orders, the cases of the pseudoscalar state 0^- and vector state 1^- are calculated in the following. The wave function we have obtained in the former paper has a form,

$$U_{\boldsymbol{X}}(p)\psi^{\ell}_{\boldsymbol{X}}\left(\vec{x}\right) = \psi^{\ell}_{\boldsymbol{X}\;\mathrm{FWT}}(\vec{x}) \tag{18a}$$

$$U_{\boldsymbol{X}}(p) \equiv U_c U_{\mathrm{FWT}}(p) \qquad \psi^{\ell}_{\boldsymbol{X}}\left(\vec{x}\right) \equiv \psi^{\ell}_{\boldsymbol{X}}\left((0,\vec{x})\,;\left(M_{\boldsymbol{X}},\vec{0}\right)\right) \tag{18b}$$

$$\psi^{\ell}_{\boldsymbol{X}\;\mathrm{FWT}}(\vec{x}) = \psi^{\ell}_{\boldsymbol{X}\,0}(\vec{x}) + \psi^{\ell}_{\boldsymbol{X}\,1}(\vec{x}) + \ldots \tag{18c}$$

$$\psi^{\ell}_{\boldsymbol{X}\,0}(\vec{x}) = \Psi^{+}_{\ell} \equiv \sqrt{2M_{\boldsymbol{X}}}\;\left(\,0 \quad \Psi^{k}_{j\,m}(\vec{x})\,\right), \tag{18d}$$

where the 4×4 wave function, $\psi^{\ell}_{\boldsymbol{X}}$, is transformed just once with the Foldy-Wouthuysen-Tani transformation, U_{FWT}, acting on a heavy quark and with the charge conjugation operator, U_c, into $\psi^{\ell}_{\boldsymbol{X}\;\mathrm{FWT}}$, and its i-th order expanded in $1/m_{\boldsymbol{Q}}$ is given by $\psi^{\ell}_{\boldsymbol{X}\,i}$. Higher order corrections depend on the negative component of a heavy quark, Ψ^{-}_{ℓ}, which is given by

$$\Psi^{-}_{\ell} \equiv \sqrt{2M_{\boldsymbol{X}}}\;\left(\,\Psi^{k}_{j\,m}(\vec{x}) \quad 0\,\right). \tag{19}$$

Here $\vec{p} = \vec{p}_Q$ is an initial momentum of a heavy quark, and henceforth the color, $N_c = 3$, is omitted since the form of the wave function is the same for all colors, ℓ stands for a set of quantum numbers, j, m, and k, and the positive/negative component wave functions, $\Psi^{\pm}_{\ell}$, are given in terms of a 4×2 wave function, $\Psi^{k}_{j\,m}(\vec{x})$, as

$$\Psi^{k}_{j\,m}(\vec{x}) = \frac{1}{r} \begin{pmatrix} u_k(r) \\ -i\, v_k(r)\, (\vec{\sigma} \cdot \vec{n}) \end{pmatrix} \; y^{k}_{j\,m}(\Omega), \tag{20}$$

where $r = |\vec{x}|$, $\vec{n} = \vec{x}/r$, j is a total angular momentum of a meson, m is its z component, k is a quantum number which takes only values, $k = \pm j,\ \pm(j+1)$ and $\neq 0$. Normalization condition given by Eq. (17) can be rewritten in terms of $\psi^{\ell}_{\boldsymbol{X}\,\mathrm{FWT}}(\vec{x})$ as,

$$\mathrm{tr} \int d^3z\, {\psi^{\ell'}_{\boldsymbol{X}'\,\mathrm{FWT}}}^{\dagger}(\vec{x})\, \psi^{\ell}_{\boldsymbol{X}'\,\mathrm{FWT}}(\vec{x}) = 2M_{\boldsymbol{X}}\, \delta_{\ell\ell'}. \tag{21}$$

All the details on the wave functions are given in the Appendix C.

In order to calculate normalization of the wave function at a moving frame and/or matrix elements of the semi-leptonic decay processes, we need to develop a formulation of their calculations in terms of the rest frame wave functions. Let us assume that the physical quantity is already given in terms of the rest frame wave functions and rewrite them in terms of the FWT-transformed ones as

$$\begin{aligned} &\int d^3x\, (\mathcal{O}_q)_{\alpha\,\alpha'} (\mathcal{O}_Q)_{\beta\,\beta'}\, {A'}^{*}_{\alpha\,\gamma} {B'}^{*}_{\beta\,\delta} {\psi^{\ell'}_{\boldsymbol{X}'}}^{*}{}_{\gamma\,\delta} A_{\alpha'\,\gamma'} B_{\beta\,\delta'} \psi^{\ell}_{\boldsymbol{X}\,\gamma'\,\delta'} = \int d^3x\, \mathrm{tr} \left(\psi^{\ell'\,\dagger}_{\boldsymbol{X}'} {A'}^{\dagger} \mathcal{O}_q A \psi^{\ell}_{\boldsymbol{X}} B^{\mathrm{T}} \mathcal{O}^{\mathrm{T}}_Q {B'}^{*} \right) \\ &= \int d^3x\, \mathrm{tr} \left({\psi^{\ell'\,\dagger}_{\boldsymbol{X}'\,\mathrm{FWT}}} {A'}^{\dagger} \mathcal{O}_q A \psi^{\ell}_{\boldsymbol{X}\,\mathrm{FWT}} U^{-1}_{\boldsymbol{X}}(p_Q) B^{\mathrm{T}} \mathcal{O}^{\mathrm{T}}_Q {B'}^{*}\, U_{\boldsymbol{X}'}(p'_Q) \right), \end{aligned} \tag{22}$$

where $(\mathcal{O}_q)_{\alpha\,\alpha'}$, $A'_{\alpha\,\gamma}$, and $A_{\alpha\,\gamma'}$ act on light quarks while $(\mathcal{O}_Q)_{\beta\,\beta'}$, $B'_{\beta\,\delta}$ and $B_{\beta\,\delta'}$ on heavy quarks, and use has been made of,

$$\begin{aligned} &\psi^{\ell}_{\boldsymbol{X}} = U^{-1}_{\boldsymbol{X}}(p_Q) \otimes \psi^{\ell}_{\boldsymbol{X}\,\mathrm{FWT}}(p_Q) = \psi^{\ell}_{\boldsymbol{X}\,\mathrm{FWT}}\, U^{-1}_{\boldsymbol{X}}(p_Q), \qquad \psi^{\ell}_{\boldsymbol{X}'} = \psi^{\ell}_{\boldsymbol{X}'\,\mathrm{FWT}}\, U^{-1}_{\boldsymbol{X}}(p'_Q), \\ &U^{-1}_{\boldsymbol{X}}(p_Q) B^{\mathrm{T}} \mathcal{O}^{\mathrm{T}}_Q {B'}^{*}\, U_{\boldsymbol{X}'}(p'_Q) = U^{-1}_{\boldsymbol{X}\,\mathrm{FWT}}(p_Q) U^{-1}_{c} B^{\mathrm{T}} \mathcal{O}^{\mathrm{T}}_Q {B'}^{*}\, U_c U_{\boldsymbol{X}'\,\mathrm{FWT}}(p'_Q). \end{aligned} \tag{23}$$

In the course of obtaining relativistic results out of the rest frame wave functions, we calculate normalization of the wave function in a moving frame in two extreme cases, i) $t =$

$t_{\boldsymbol{x}} = t_{\boldsymbol{y}}$ and ii) $t' = t'_{\boldsymbol{x}} = t'_{\boldsymbol{y}}$. The FWT-transformed rest frame wave function is given by, up to the first order in $1/m_{\boldsymbol{Q}}$,

$$\psi^{\ell}_{\boldsymbol{X}'\,\mathrm{FWT}}(0^-) = \Psi^+_{-1} + c^{-1,\,1}_{1-}\Psi^-_1 + O\left(1/m^2_{\boldsymbol{Q}}\right), \tag{24}$$

$$\psi^{\ell}_{\boldsymbol{X}'\,\mathrm{FWT}}(1^-) = \Psi^+_{-1} + c^{-1,\,2}_{1+}\Psi^+_2 + c^{-1,\,1}_{1-}\Psi^-_1 + c^{-1,\,-2}_{1-}\Psi^-_{-2} + O\left(1/m^2_{\boldsymbol{Q}}\right), \tag{25}$$

where only the value of a k quantum number is written as a subscript of $\Psi^{\pm}_{\ell}$. The total angular momentum, j, though not given explicitly, should be the same on both sides of equations and the coefficients, $c^{k,k'}_{1\pm}$, are given in [16] whose explicit expressions are not necessarily known here since as we shall show that the higher order corrections do not contribute to the physical quantities in this order, $O(1/m_{\boldsymbol{Q}})$. Now we calculate normalization up to the first order in $1/m_{\boldsymbol{Q}}$ below.

i) $t = t_{\boldsymbol{x}} = t_{\boldsymbol{y}}$ and $t_{\boldsymbol{x}'} \neq t_{\boldsymbol{y}'}$ (equal time at the rest frame)

Using Eqs. (12, 21, 22) one obtains, up to the first order in $1/m_{\boldsymbol{Q}}$ and both for 0^- and 1^- states given by Eqs. (24, 25),

$$\mathrm{tr}\int d^3x\; \psi^{\ell'\,\dagger}_{\boldsymbol{X}}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right)\psi^{\ell}_{\boldsymbol{X}}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right) = 2M_{\boldsymbol{X}}\gamma^3. \tag{26}$$

ii) $t' = t_{\boldsymbol{x}'} = t_{\boldsymbol{y}'}$ and $t_{\boldsymbol{x}} \neq t_{\boldsymbol{y}}$ (equal time at the moving frame)

Similarly using Eq. (14, 21, 22) one obtains,

$$\mathrm{tr}\int d^3x\; \psi^{\ell'\,\dagger}_{\boldsymbol{X}}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right)\psi^{\ell}_{\boldsymbol{X}}\left((0,\vec{x})\,;P_{\boldsymbol{X}}\right) = 2M_{\boldsymbol{X}}\gamma. \tag{27}$$

D. Matrix Elements

The light anti-quark is treated as a spectator. That is, it does not interact with other particles except for gluons represented by potentials in our model. The heavy quark has a weak vertex and its current is in general given by

$$j(t,\vec{x}) = Q^{\dagger}_{\boldsymbol{X}'\alpha}(t,\vec{x})\,\mathcal{O}_{\alpha\beta}\,Q_{\boldsymbol{X}\beta}(t,\vec{x}), \tag{28}$$

where $\mathcal{O}$ is some 4×4 matrix. By inserting the number operator of the anti-quark and by assuming the vacuum dominance among intermediate states, we obtain the formula for the matrix element of the above current, Eq. (28), between heavy mesons as follows.

$$\begin{aligned}
&\langle X'; P_{X'}\,\{\ell'\}|\, j(t,\vec{x})\,|X; P_X\,\{\ell\}\rangle \\
&\quad \simeq \mathrm{tr}\int d^3y\, \psi^{\ell'\,\dagger}_{X'}\left((0,\vec{y}-\vec{x})\,; P_{X'}\right)\left(\mathcal{O}\otimes\psi^{\ell}_{X}\left((0,\vec{y}-\vec{x})\,; P_X\right)\right)\, e^{-i(P_X-P_{X'})\cdot x},
\end{aligned} \tag{29}$$

or

$$\langle X'; P_{X'}\,\{\ell'\}|\, j(0,\vec{0})\,|X; P_X\,\{\ell\}\rangle \simeq \mathrm{tr}\int d^3x\, \psi^{\ell'\,\dagger}_{X'}\left((0,\vec{x})\,; P_{X'}\right)\, \psi^{\ell}_{X}\left((0,\vec{x})\,; P_X\right)\, \mathcal{O}^{\mathrm{T}}. \tag{30}$$

Now we would like to evaluate the matrix element of the process, $\bar{B}\to D^{(*)}\ell\nu_\ell$, in two initial cases, i.e., $\bar{B}$ rest frame and the Breit frame (center of mass). In each case we apply the former two cases of the constituent particles, i.e., i) $t = t_x = t_y$ and $t_{x'} \neq t_{y'}$ and ii) $t' = t_{x'} = t_{y'}$ and $t_x \neq t_y$. As has been mentioned in the former subsection the normalization condition given by Eq. (16) for a moving frame does not hold because of approximations adopted in this paper to obtain the wave function in a moving frame from the one in a rest frame. Instead we adopt Eq. (21) as a first principle. We need to compensate these differences between the expected one, Eq. (16), and the approximated one that is given below by redefining form factors as follows. Matrix elements of the numerators are calculated below using the results of the fomer subsection, II B.

$$\left\langle D^{(*)}\right|\, j(0,\vec{0})\,\left|\bar{B}\right\rangle = \frac{\sqrt{2P_{\bar{B}\,0}2P_{D^{(*)}\,0}}\,\mathrm{tr}\int d^3x\, \psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,; P_{D^{(*)}}\right)\, \psi^{\ell}_{\bar{B}}((0,\vec{x})\,; P_{\bar{B}})\mathcal{O}^{\mathrm{T}}}{\left(\mathrm{tr}\int d^3x\, \left|\psi^{\ell'}_{D^{(*)}}((0,\vec{x})\,; P_{D^{(*)}})\right|^2\, \mathrm{tr}\int d^3x\, \left|\psi^{\ell}_{\bar{B}}((0,\vec{x})\,; P_{\bar{B}})\right|^2\right)^{1/2}}, \tag{31}$$

whose denominators are calculated in the former subsection II C and given by Eqs. (26, 27).

1) $\bar{B}$ rest frame

1-i) $t = t_x = t_y$ for $D^{(*)}$ meson

In this case the matrix element is given by, within our approximations,

$$\begin{aligned}
&\mathrm{tr}\int d^3x\, \psi^{\ell'\,\dagger}_{D^{(*)}}\left((0,\vec{x})\,; P_{D^{(*)}}\right)\, \psi^{\ell}_{\bar{B}}((0,\vec{x})\,; P_{\bar{B}})\mathcal{O}^{\mathrm{T}} \\
&= \int d^3x\, G^*_{\gamma\,\delta}G^*_{\alpha\,\epsilon}\,\psi^{\ell'\,\,*}_{D^{(*)}\,\delta\,\epsilon}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right)\,;\left(M_D,\vec{0}\right)\right)\,\mathcal{O}_{\alpha\,\beta}\,\psi^{\ell}_{\bar{B}\gamma\,\beta}\left(\vec{x}\right)e^{-i(M_D-m_c)\gamma\,\beta\,x^3},
\end{aligned} \tag{32}$$

1-ii) $t' = t_{x'} = t_{y'}$ for $D^{(*)}$ meson

In this case the matrix element is given by,

$$\begin{aligned}
&\mathrm{tr} \int d^3x \; \psi^{\ell' \, \dagger}_{D^{(*)}} \left((0,\vec{x}) \, ; P_{D^{(*)}}\right) \; \psi^{\ell}_{\bar{B}}((0,\vec{x}) \, ; P_{\bar{B}}) \mathcal{O}^{\mathrm{T}} \\
&= \int \; d^3x \, G^{*}_{\gamma \, \delta} G^{*}_{\alpha \, \epsilon} \, \psi^{\ell'}_{D^{(*)} \, \delta \, \epsilon}{}^{*} \left(\left(0, \vec{x}_{\perp}, \gamma x^3\right) ; \left(M_D, \vec{0}\right)\right) \mathcal{O}_{\alpha \, \beta} \, \psi^{\ell}_{\bar{B} \gamma \, \beta} \left(\vec{x}\right) e^{-i(M_D - m_c)\gamma \, \beta \, x^3} ,
\end{aligned} \tag{33}$$

2) Breit frame (center of mass of $\bar{B}$ and $D^{(*)}$)

2-i) $t = t_{x} = t_{y}$ for $\bar{B}$ and $D^{(*)}$ mesons

In this case the matrix element is given by, within our approximations,

$$\begin{aligned}
&\mathrm{tr} \int d^3x \; \psi^{\ell' \, \dagger}_{D^{(*)}} \left((0,\vec{x}) \, ; P_{D^{(*)}}\right) \; \psi^{\ell}_{\bar{B}}((0,\vec{x}) \, ; P_{\bar{B}}) \mathcal{O}^{\mathrm{T}} \\
&= \gamma \; \int \; d^3x \, \psi^{\ell'}_{D^{(*)} \, \gamma \, \alpha}{}^{*} \left(\vec{x}\right) \left(G^{\dagger} \mathcal{O} G^{-1} \right)_{\alpha \, \beta} \psi^{\ell}_{\bar{B} \gamma \, \beta} \left(\vec{x}\right) e^{-\frac{i}{2}(M_D - m_c + M_B - m_b)\gamma^2 \, \beta \, x^3} .
\end{aligned} \tag{34}$$

2-ii) $t' = t_{x'} = t_{y'}$ for $\bar{B}$ and $D^{(*)}$ mesons

In this case the matrix element is given by,

$$\begin{aligned}
&\mathrm{tr} \int d^3x \; \psi^{\ell' \, \dagger}_{D^{(*)}} \left((0,\vec{x}) \, ; P_{D^{(*)}}\right) \; \psi^{\ell}_{\bar{B}}((0,\vec{x}) \, ; P_{\bar{B}}) \mathcal{O}^{\mathrm{T}} \\
&= \gamma^{-1} \; \int \; d^3x \, \psi^{\ell'}_{D^{(*)} \, \gamma \, \alpha}{}^{*} \left(\vec{x}\right) \left(G^{\dagger} \mathcal{O} G^{-1} \right)_{\alpha \, \beta} \psi^{\ell}_{\bar{B} \gamma \, \beta} \left(\vec{x}\right) e^{-\frac{i}{2}(M_D - m_c + M_B - m_b)\beta \, x^3} .
\end{aligned} \tag{35}$$

III. ISGUR-WISE FUNCTION

Using the results derived in the former section we can now calculate semi-leptonic weak form factors including the Isgur-Wise function. In this case the currents are given by

$$j_{\mu} = c^{\dagger}(t,\vec{x}) \, \gamma^0 \gamma_{\mu} \, b \, (t,\vec{x}) \tag{36}$$

$$j^5_{\mu} = c^{\dagger}(t,\vec{x}) \, \gamma^0 \gamma_{\mu} \gamma_5 \, b \, (t,\vec{x}) \tag{37}$$

Define the six independent semi-leptonic weak form factors as [3]

$$\langle D|\, j_\mu(0,\vec{0})\,\Big|\bar{B}\Big\rangle \;=\sqrt{M_B M_D}\,(\xi_+(\omega)(v_{\bar{B}}+v_D)_\mu+\xi_-(\omega)(v_{\bar{B}}-v_D)_\mu)\,, \tag{38a}$$

$$\langle D^*|\, j_\mu(0,\vec{0})\,\Big|\bar{B}\Big\rangle \;=i\,\sqrt{M_B M_{D^*}}\xi_V(\omega)\;\epsilon_{\mu\nu\rho\sigma}v^\rho_{D^*}\;v^\sigma_{\bar{B}}, \tag{38b}$$

$$\begin{aligned}\langle D^*|\, j^5_\mu(0,\vec{0})\,\Big|\bar{B}\Big\rangle \;&=\sqrt{M_B M_{D^*}}\,\Big((1+\omega)\xi_{A_1}(\omega)\;\epsilon^*_\mu\\ &\quad -\xi_{A_2}(\omega)\;(\epsilon^*\cdot v_{\bar{B}})\,v_{\bar{B}\,\mu}-\xi_{A_3}(\omega)\;(\epsilon^*\cdot v_{D^*})\,v_{D^*\,\mu}\Big)\,,\end{aligned} \tag{38c}$$

with

$$v^\mu_X=P^\mu_X/M_X,\quad \text{for } X=\bar{B},\ D^{(*)}\qquad \omega=v_B\cdot v_D. \tag{39}$$

A. Zeroth Order

After some calculations, all the above form factors are proportional to the Isgur-Wise function, $\xi(\omega)$, and are given by

$$\xi_+(\omega)=\xi_V(\omega)=\xi_{A_1}(\omega)=\xi_{A_3}(\omega)=\xi(\omega), \tag{40a}$$

$$\xi_-(\omega)=\xi_{A_2}(\omega)=0, \tag{40b}$$

Below we will show briefly how to calculate $\xi_+(\omega)=\xi(\omega)$ from Eq. (38a) since the rests are similarly obtained. All the details are given in the Appendix E. Define the Isgur-Wise function as

$$\begin{aligned}\langle D|\, j_0(0,\vec{0})\,\Big|\bar{B}\Big\rangle \;&=\frac{\sqrt{2P_{\bar{B}\,0}2P_{D\,0}}\,\mathrm{tr}\int d^3x\;\psi^{\ell'\,\dagger}_{D\,0}\,((0,\vec{x})\,;P_D)\;\psi^{\ell}_{\bar{B}\,0}\,((0,\vec{x})\,;P_{\bar{B}})}{\sqrt{\mathrm{tr}\int d^3x\;\Big|\psi^{\ell'}_{D\,0}((0,\vec{x})\,;P_D)\Big|^2\;\mathrm{tr}\int d^3x\;\Big|\psi^{\ell}_{\bar{B}\,0}((0,\vec{x})\,;P_{\bar{B}})\Big|^2}}\\ &=\sqrt{M_B M_D}\xi(\omega)(v_{\bar{B}}+v_D)_0.\end{aligned} \tag{41}$$

The expression, Eq. (41), can be further reduced in two cases of $\bar{B}$ frames as follows.

1) $\bar{B}$ rest frame

$$\xi(\omega)=\frac{\sqrt{2\omega}\,\mathrm{tr}\int d^3x\;\psi^{\ell'\,\dagger}_{D\,0}\,((0,\vec{x})\,;P_D)\,\psi^{\ell}_{\bar{B}\,0}\,(\vec{x})}{\sqrt{M_B}(1+\omega)\sqrt{\mathrm{tr}\int d^3x\;\Big|\psi^{\ell'\,\dagger}_{D\,0}\,((0,\vec{x})\,;P_D)\Big|^2}}, \tag{42}$$

with relations given by

$$P_{\bar{B}\,0} = M_{B}, \qquad P_{D\,0} = M_{D} v_{D\,0} = M_{D}\omega, \qquad v_{\bar{B}\,0} = 1,$$
$$v_{D\,0} = v_{B} \cdot v_{D} = \gamma = \frac{1}{\sqrt{1-\beta^2}} = \omega, \qquad \beta = \frac{\sqrt{\omega^2-1}}{\omega}. \tag{43}$$

2) Breit frame

$$\xi(\omega) = \frac{\mathrm{tr} \int d^3x \; \psi^{\ell' \,\dagger}_{D\,0}\left((0,\vec{x})\,;P_{D}\right) \psi^{\ell}_{\bar{B}\,0}\left(\vec{x}\right)}{\sqrt{\mathrm{tr} \int d^3x \left|\psi^{\ell' \,\dagger}_{D\,0}\left((0,\vec{x})\,;P_{D}\right)\right|^2 \frac{1}{2}\,\mathrm{tr} \int d^3x \;\left|\psi^{\ell}_{\bar{B}\,0}\left(\vec{x}\right)\right|^2}}, \tag{44}$$

with relations given by

$$P_{\bar{B}\,0} = M_{B}\gamma, \qquad P_{D\,0} = M_{D}\gamma, \qquad v_{\bar{B}\,0} = v_{D\,0} = \gamma = \frac{1}{\sqrt{1-(\beta/2)^2}},$$
$$\gamma = \sqrt{\frac{\omega+1}{2}}, \qquad \frac{\beta}{2} = \sqrt{\frac{\omega-1}{\omega+1}}. \tag{45}$$

Now we will calculate this Isgur-Wise function in four cases below.

1-i) $\bar{B}$ rest frame and $t = t_{x} = t_{y}$

$$\xi(\omega) = \frac{1}{\omega} - \frac{1}{6}\beta^2\omega\tilde{E}^2_{D}\left\langle r^2\right\rangle + \frac{\beta^2}{4} + \; O(\beta^4), \tag{46}$$

where

$$\left\langle r^2\right\rangle \equiv \int d^3x\, \psi^{\ell}_{X\,0}{}^{\dagger}(\vec{x}) r^2 \psi^{\ell}_{X\,0}(\vec{x}), \qquad \text{for } X = D, \text{ or } \bar{B}, \tag{47}$$

either of which gives the same result since this is the lowest order calculation.

1-ii) B rest frame and $t' = t'_{x} = t'_{y}$

$$\xi(\omega) = 1 - \frac{1}{6}\beta^2\omega\tilde{E}^2_{D}\left\langle r^2\right\rangle - \frac{\beta^2}{4} + \; O(\beta^4), \tag{48}$$

2-i) Breit frame and $t = t_{x} = t_{y}$

$$\xi(\omega) = \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^2(\tilde{E}_{B} + \tilde{E}_{D})^2\left\langle r^2\right\rangle + O(\beta^4). \tag{49}$$

2-ii) Breit frame and $t' = t'_{\boldsymbol{x}} = t'_{\boldsymbol{y}}$

$$\xi(\omega) = \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^{-2}(\tilde{E}_{\boldsymbol{B}} + \tilde{E}_{\boldsymbol{D}})^2 \left\langle r^2 \right\rangle + O(\beta^4). \tag{50}$$

Here in the above defined are

$$\tilde{E}_{\boldsymbol{D}} = E_{\boldsymbol{D}} - m_c, \qquad \tilde{E}_{\boldsymbol{B}} = E_{\boldsymbol{B}} - m_b. \tag{51}$$

All these expressions obtained above have the same form up to the first order in ω in the vicinity of $\omega = 1$ as given by

$$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle\right)(\omega - 1)\,, \qquad \bar{\Lambda} = \tilde{E}_{\boldsymbol{D}} = \tilde{E}_{\boldsymbol{B}}, \tag{52}$$

that is,

$$\xi(1) = 1, \qquad \xi'(1) = -\frac{1}{2} - \frac{1}{3}\bar{\Lambda}^2 \left\langle r^2 \right\rangle. \tag{53}$$

This form for $\xi(\omega)$ given by Eq. (52) is derived from our semi-relativistic formulation and coincides with those derived from other considerations [8,11,13] but we believe that our derivation is the most general derivation and does not depend on any specific model. This form gives the lower bound for $\xi'(1)$ as $-1/2$ or the upper bound for $-\xi'(1)$ as $1/2$ as shown in the other papers. [8,11,13] All the other form factors, $\xi_-(\omega)$, $\xi_{\boldsymbol{V}}(\omega)$, and $\xi_{\boldsymbol{A}_i}(\omega)$ for $i = 1 \sim 3$ are similarly calculated and given by Eqs. (40).

Using the values of parameters obtained in the former paper [16] to calculate mass spectra of heavy mesons, we can calculate $\xi'(1)$ at the first and second order in $1/m_{\boldsymbol{Q}}$ as

$$\xi'(1) = -0.820, \tag{54}$$

where we have used the values, $\bar{\Lambda} = 0.412$ GeV and $\langle r^2 \rangle = 5.663$ GeV^{-2}. The several values of $\xi'(1)$ in references are listed in [11].

B. First order

First order corrections to the form factors defined by Eq. (38) are straightforwardly calculated, which is cumbersome work to do. Hence here we do not repeat the similar

calculations to the former subsection III A and give only the final results below. We have calculated all the four cases, 1-i) $\sim$ 2-ii), which show that there are no contributions from the first order corrections of the wave functions to the form factors. However only one case (2-ii): Breit frame with $t' = t_{x'} = t_{y'}$) gives the results which coincide with those of [3]. Although we try to obtain relativistically invariant results from the rest frame knowledge by considering four different Lorentz-boost frames, the first order corrections in all cases but one are not invariant. Hence the following results are just a conjecture from our model but we believe that it is very plausible results since they coincide with general relativistic results at least with the kinetic terms and since they come from a semi-relativistic potential model. Brief derivations of these form factors are given in the Appendix E 2.

Six form factors are given by, including zeroth order terms for completeness,

$$\xi_+(\omega) = \xi(\omega), \tag{55a}$$

$$\xi_-(\omega) = -\frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\xi(\omega), \tag{55b}$$

$$\xi_V(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c} + \frac{1}{m_b}\right)\right]\xi(\omega), \tag{55c}$$

$$\xi_{A_1}(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c} + \frac{1}{m_b}\right)\right]\xi(\omega), \tag{55d}$$

$$\xi_{A_2}(\omega) = -\frac{\bar{\Lambda}}{\omega+1}\frac{1}{m_c}\xi(\omega), \tag{55e}$$

$$\xi_{A_3}(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\left(\frac{\omega-1}{\omega+1}\frac{1}{m_c} + \frac{1}{m_b}\right)\right]\xi(\omega), \tag{55f}$$

which should be compared with those of [3],

$$\xi_+(\omega) = \left[1 + \left(\frac{1}{m_c} + \frac{1}{m_b}\right)\rho_1(\omega)\right]\xi(\omega), \tag{56a}$$

$$\xi_-(\omega) = \left[-\frac{\bar{\Lambda}}{2} + \rho_4(\omega)\right]\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\xi(\omega), \tag{56b}$$

$$\xi_V(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c} + \frac{1}{m_b}\right) + \frac{1}{m_c}\rho_2(\omega) + \frac{1}{m_b}\left(\rho_1(\omega) - \rho_4(\omega)\right)\right]\xi(\omega), \tag{56c}$$

$$\xi_{A_1}(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c} + \frac{1}{m_b}\right) + \frac{1}{m_c}\rho_2(\omega) + \frac{1}{m_b}\left(\rho_1(\omega) - \frac{\omega-1}{\omega+1}\rho_4(\omega)\right)\right]\xi(\omega), \tag{56d}$$

$$\xi_{A_2}(\omega) = \frac{1}{\omega+1}\frac{1}{m_c}\left[-\bar{\Lambda} + (\omega+1)\rho_3(\omega) - \rho_4(\omega)\right]\xi(\omega), \tag{56e}$$

$$\xi_{A_3}(\omega) = \left[1 + \frac{\bar{\Lambda}}{2}\left(\frac{\omega-1}{\omega+1}\frac{1}{m_c} + \frac{1}{m_b}\right) + \frac{1}{m_c}\left(\rho_2(\omega) - \rho_3(\omega) - \frac{1}{\omega+1}\rho_4(\omega)\right)\right.$$
$$\left. + \frac{1}{m_b}\left(\rho_1(\omega) - \rho_4(\omega)\right)\right]\xi(\omega). \tag{56f}$$

From comparison of Eqs. (55, 56), we conclude that all $\rho_i(\omega) = 0$ ($i = 1 \sim 4$), i.e., there are no $1/m_Q$ corrections but kinetic terms found by Luke, Neubert, and Rieckert [3].

IV. SUMMARY AND DISCUSSIONS

In this paper, we have calculated the semi-leptonic weak form factors up to the first order in $1/m_Q$ following the study done before [14–17] and using the results obtained in those papers. What we have found in this paper is

1. The Isgur-Wise function has the following form up to the first order in $1/m_Q$ and in $(\omega - 1)$.

$$\xi(\omega) = 1 - \left(\frac{1}{2} + \frac{1}{3}\bar{\Lambda}^2\left\langle r^2\right\rangle\right)(\omega - 1)\,, \qquad \bar{\Lambda} = \tilde{E}_D = \tilde{E}_B,$$

and hence

$$\xi(1) = 1, \qquad \xi'(1) = -\frac{1}{2} - \frac{1}{3}\bar{\Lambda}^2\left\langle r^2\right\rangle.$$

2. The first order corrections are given only by kinetic terms and hence no contributions from the corrections of the wave functions are found . They are explicitly given by Eqs. (55). That is, in the terminology of Neubert and Rieckert in [3],

$$\rho_i(\omega) = 0 \quad \text{for } i = 1 \sim 4.$$

We have used the rest frame wave functions to obtain the relativistic results, and to do this the four different Lorentz boost frames are adopted to check the validity of our results. The same form is obtained for the Isgur-Wise function for all four cases up to the first order in $1/m_Q$ and in $(\omega - 1)$, however their first order corrections in $1/m_Q$ are not the same for

all four cases. Only the case, written as 2-ii) in the main text, i.e., in the Breit frame with $t' = t_{x'} = t_{y'}$ both for $\bar{B}$ and $D^{(*)}$, gives consistent results with the relativistic ones given by [3].

APPENDIX A: FERMI-YANG EQUATION

The Fermi-Yang equation given by Eq. (3) is derived by calculating

$$\langle X'; P_{X'}\,\{\ell'\}|\,\mathcal{H}\,|X; P_X\,\{\ell\}\rangle = \langle X; P_X\,\{\ell\}|\,\mathcal{P}^0\,|X; P_X\,\{\ell\}\rangle\,, \tag{A1}$$

with $\mathcal{P}'$ being the 0-th component of the four-momentum $\mathcal{P}^\mu$, and by varying in terms of $\psi^{\ell'*}_{X'}$. Here the Hamiltonian density is given by

$$\begin{aligned}\mathcal{H} =& \int d^3x\,\{q^{c\dagger}(\vec{x})(\vec{\alpha}_q\cdot\vec{p}_q+\beta_q m_q)q^c(\vec{x}) + Q^\dagger(\vec{x})(\vec{\alpha}_Q\cdot\vec{p}_Q+\beta_Q m_Q)Q(\vec{x})\}\\ &+\int d^3x d^3y\; q^{c\dagger}(\vec{x})\beta_q O_{iq} q^c(\vec{x}) V_{ij}(\vec{x}-\vec{y}) Q^\dagger(\vec{y})\beta_Q O_{jQ} Q(\vec{y}).\end{aligned} \tag{A2}$$

The lhs of Eq. (A1) can be approximated as

$$\begin{aligned}&\langle X'; P_{X'}\,\{\ell'\}|\,\mathcal{H}\,|X; P_X\,\{\ell\}\rangle\\ &= \langle X; P_X\,\{\ell\}|\int d^3x\int d^3y q_\alpha^{c\,\dagger}(t,\vec{x}) q_\alpha^c(t,\vec{x}) Q^\dagger_\beta(t,\vec{y}) Q_\beta(t,\vec{y})\mathcal{H}\,|X; P_X\,\{\ell\}\rangle\\ &\simeq \int d^3x\int d^3y\,\langle X'; P_{X'}\,\{\ell'\}|\,Q^\dagger_\beta(t,\vec{y}) q_\alpha^{c\,\dagger}(t,\vec{x})\,|0\rangle\,\langle 0|\,q_\alpha^c(t,\vec{x}) Q_\beta(t,\vec{y})\mathcal{H}\,|X; P_X\,\{\ell\}\rangle\\ &\simeq \int d^3x\int d^3y \psi^{\ell'\,*}_{X'\,\alpha\beta}\left((0,\vec{x}-\vec{y})\,; P_{X'}\right) H_{\alpha\gamma,\beta\delta}\psi^{\ell}_{X\gamma\delta}\left((0,\vec{x}-\vec{y})\,; P_X\right) e^{-i(P_X-P_{X'})\cdot y},\end{aligned} \tag{A3}$$

where the number operators for q^c and Q have been inserted and the vacuum dominance has been used among the intermediate states. The rhs of Eq. (A1) can be given by

$$\langle X'; P_{X'}\,\{\ell'\}|\,\mathcal{P}^0\,|X; P_X\,\{\ell\}\rangle = \sqrt{M_X^2+\vec{P}_X^2}\,\langle X'; P_{X'}\,\{\ell'\}|\,X; P_X\,\{\ell\}\rangle\,, \tag{A4}$$

thus we obtain the Fermi-Yang equation, Eq. (3).

APPENDIX B: NORMALIZATION

Derivation of Eq. (16) is also similar to the former subsection A. Setting $\mathcal{H} = 1$ in Eq. (A3), then we obtain

$$
\begin{aligned}
&\langle X; P_{X'}\,\{\ell'\}|\; X; P_X\,\{\ell\}\rangle \\
&\simeq \int d^3x \int d^3y \psi^{\ell'\,*}_{X\,\alpha\,\beta}\left((0,\vec{x}-\vec{y})\,; P_{X'}\right)\,\psi^{\ell}_{X\,\alpha\,\beta}\left((0,\vec{x}-\vec{y})\,; P_X\right)\,e^{-i(P_X-P_{X'})\cdot y} \\
&= (2\pi)^3\;\delta\left(\vec{P}_{X'}-\vec{P}_X\right)\mathrm{tr}\int d^3z\,\psi^{\ell'\,\dagger}_X\left((0,\vec{z})\,; P_X\right)\psi^{\ell}_X\left((0,\vec{z})\,; P_X\right). \qquad \text{(B1)}
\end{aligned}
$$

The lhs is given by

$$
\langle X; P'_X\,\{\ell'\}\;|X; P_X\,\{\ell\}\rangle = (2\pi)^3\,2P_{X\,0}\,\delta^3\left(\vec{P}_X-\vec{P}'_X\right)\delta_{\ell',\ell}, \qquad \text{(B2)}
$$

hence

$$
\mathrm{tr}\int d^3z\,\psi^{\ell'\,\dagger}_X\left((0,\vec{z})\,; P_X\right)\psi^{\ell}_X\left((0,\vec{z})\,; P_X\right) \simeq 2P_{X\,0}\,\delta_{\ell\,\ell'} \qquad \text{(B3)}
$$

holds.

We have to calculate the normalization of the wave function in the moving frame in two cases, i) $t = t_x = t_y$ and ii) $t' = t'_x = t'_y$.

i) $t = t_x = t_y$ and $t_{x'} \neq t_{y'}$ (equal time at the rest frame)

Using Eq. (12) one obtains,

$$
\begin{aligned}
&\int d^3x\;\psi^{\ell'\,*}_{X\,\alpha\beta}\left((0,\vec{x})\,; P_X\right)\psi^{\ell}_{X\;\alpha\beta}\left((0,\vec{x})\,; P_X\right) = \gamma\int d^3x\;\psi^{X\;\ell'\,*}_{\alpha\beta}\left(\vec{x}\right)\left(G^2\right)_{\alpha\gamma}\left(G^2\right)_{\beta\delta}\psi^{\ell}_{X\;\gamma\delta}\left(\vec{x}\right) \\
&= \gamma\,\mathrm{tr}\int d^3x\;\psi^{\ell'\,\dagger}_X\left(\vec{x}\right)G^2\psi^{\ell}_X\left(\vec{x}\right)G^{2\,T} = 2M\gamma^3. \qquad \text{(B4)}
\end{aligned}
$$

ii) $t' = t_{x'} = t_{y'}$ and $t_x \neq t_y$ (equal time at the moving frame)

Using Eq. (14) one obtains,

$$
\begin{aligned}
&\int d^3x\;\psi^{\ell'\,*}_{X\,\alpha\beta}\left((0,\vec{x})\,; P_X\right)\psi^{\ell}_{X\;\alpha\beta}\left((0,\vec{x})\,; P_X\right) = \gamma^{-1}\int d^3x\;\psi^{\ell'\,*}_{X\,\alpha\beta}\left(\vec{x}\right)\left(G^2\right)_{\alpha\gamma}\left(G^2\right)_{\beta\delta}\psi^{\ell}_{X\;\gamma\delta}\left(\vec{x}\right) \\
&= \gamma^{-1}\,\mathrm{tr}\int d^3x\;\psi^{\ell'\,\dagger}_X\left(\vec{x}\right)G^2\psi^{\ell}_X\left(\vec{x}\right)G^{2\,T} = 2M\gamma. \qquad \text{(B5)}
\end{aligned}
$$

Here use has been made of,

$$
\begin{aligned}
&\frac{1}{2}\mathrm{tr}\left[\begin{pmatrix}0 & u_{-1}(r)\\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}^{\dagger} G^2\begin{pmatrix}0 & u_{-1}(r)\\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}G^{2T}\right] \\
&= \frac{\gamma^2}{2}\,\mathrm{tr}\left[\begin{pmatrix}0 & 0\\ u_{-1}(r) & i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}\begin{pmatrix}1 & \sigma^3\beta\\ \sigma^3\beta & 1\end{pmatrix}\begin{pmatrix}0 & u_{-1}(r)\\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}\begin{pmatrix}1 & \sigma^3\beta\\ \sigma^3\beta & 1\end{pmatrix}\right] \\
&= \gamma^2\left(u^2_{-1}+v^2_{-1}\right). \qquad \text{(B6)}
\end{aligned}
$$

APPENDIX C: WAVE FUNCTIONS

The Foldy-Wouthuysen-Tani transformation and charge conjugation is defined as

$$U_{\rm FWT}\,(p) = \exp\left(W\,(p)\;\vec{\gamma}_Q\cdot\vec{\hat{p}}\right) = \cos W + \vec{\gamma}_Q\cdot\vec{\hat{p}}\,\sin W, \tag{C1a}$$

$$\vec{\hat{p}} = \frac{\vec{p}}{p},\quad \tan W\,(p) = \frac{p}{m_Q + E},\quad E = \sqrt{\vec{p}^{\,2} + m_Q^2}, \tag{C1b}$$

$$U_c = i\,\gamma_Q^0\gamma_Q^2 = -U_c^{-1}. \tag{C1c}$$

Here $\vec{p} = \vec{p}_Q$ is an initial momentum of the heavy quark and $U_{\rm FWT}\,(p)$ operates on heavy quarks, henceforth the color, $N_c = 3$, is neglected since the form of the wave function is the same for all colors, ℓ stands for a set of quantum numbers, j, m, and k, and

$$\Psi^k_{j\,m}(\vec{x}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\,v_k(r)\,(\vec{\sigma}\cdot\vec{n}) \end{pmatrix}\; y^k_{j\,m}(\Omega), \tag{C2}$$

where $r = |\vec{x}|$, $\vec{n} = \vec{x}/r$, j is a total angular momentum of a meson, m is its z component, k is a quantum number which takes only values, $k = \pm j,\ \pm(j+1)$ and $\neq 0$, whose operator form is given by $-\beta_q\left(\vec{\Sigma}_q\cdot\vec{\ell}+1\right)$. The scalar functions, $u_k(r)$ and $v_k(r)$ are polynomials of a radial variable r and satisfy

$$\int dr\left(u_k^2(r) + v_k^2(r)\right) = 1. \tag{C3}$$

$y^k_{j\,m}(\Omega)$ are functions of angles and spinors of a total angular momentum, $\vec{j} = \vec{\ell} + \vec{s}_q + \vec{s}_Q$ with $\vec{\ell} = -\vec{r}\times\nabla$, whose first few explicit forms are given by

$$\begin{aligned} y^{-1}_{00} &= \frac{1}{\sqrt{4\pi}},\quad y^{-1}_{1m} = \frac{1}{\sqrt{4\pi}}\sigma^m, \\ y^2_{1m} &= \frac{-1}{4\pi}\frac{3}{\sqrt{6}}\left(n^i n^m - \frac{1}{3}\delta^{i\,m}\right)\sigma^i, \end{aligned} \tag{C4}$$

and satisfy some relation and the normalization condition as

$$y^{-k}_{j\,m} = -\,(\vec{\sigma}\cdot\vec{n})\,y^k_{j\,m}, \tag{C5}$$

$$\frac{1}{2}{\rm tr}\left(\int d\Omega\; {y^{k'}_{j'\,m'}}^{\dagger}(\Omega)\; y^k_{j\,m}(\Omega)\right) = \delta^{k\,k'}\delta_{j\,j'}\delta_{m\,m'}. \tag{C6}$$

The relative phases among $y^k_{j\,m}$ given by Eq. (C4) are determined so that they give the correct relative phases among form factors which are determined by Eq. (38a$\sim$38c). From now on whenever the trace of $y^k_{j\,m}$ occurs it is understood to be in a sense of Eq. (C6).

The leading order pseudoscalar state (0^-) corresponds to $\ell = (k=-1,\ j=m=0)$ and hence we have the wave function,

$$\psi^{\ell}_{\boldsymbol{X}\,0}\,(\vec{x}) = \sqrt{2M_{\boldsymbol{X}}}\ (\,0 \quad \Psi^{-1}_{0\,0}(\vec{x})\,) = \sqrt{\frac{2M_{\boldsymbol{X}}}{4\pi}}\frac{1}{r}\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}, \tag{C7}$$

with $X = D$ or B. On the other hand the leading order vector state (1^-) has a set of quantum number, $\ell = (k=-1,\ j=1)$, and is given by

$$\psi^{\ell}_{\boldsymbol{X}\,0}\,(\vec{x}) = \sqrt{2M_{\boldsymbol{X}}}\ (\,0 \quad \Psi^{-1}_{1\,m}(\vec{x})\,\epsilon^m) = \sqrt{\frac{2M_{\boldsymbol{X}}}{4\pi}}\frac{1}{r}\begin{pmatrix} 0 & u_{-1}(r) \\ 0 & -i(\vec{\sigma}\cdot\vec{n})v_{-1}(r)\end{pmatrix}(\vec{\sigma}\cdot\epsilon)\,, \tag{C8}$$

with ϵ^m being a polarization vector and $X = D^*$ or B^*.

APPENDIX D: MATRIX ELEMENTS

The light anti-quark is treated as a spectator. That is, it does not interact with other particles except for gluons represented by potentials in our model. The heavy quark has a weak vertex and its current is in general given by

$$j(\vec{x},t) = Q^{\dagger}_{\boldsymbol{X'}\,\alpha}(t,\vec{x})\,\mathcal{O}_{\alpha\,\beta}\,Q_{\boldsymbol{X}\,\beta}(t,\vec{x}), \tag{D1}$$

where $\mathcal{O}$ is some four by four matrix. By inserting the number operator of the anti-quark and by assuming the vacuum dominance among intermediate states, we obtain the formula for the matrix element of the above current, Eq. (D1), between heavy mesons as follows.

$$\begin{aligned}
&\langle X';P_{\boldsymbol{X'}},\{\ell'\}|\,j(t,\vec{x})\,|X;P_{\boldsymbol{X}}\,\{\ell\}\rangle \\
&= \mathcal{O}_{\alpha\,\beta}\,\langle X';P_{\boldsymbol{X'}}\,\{\ell'\}|\,Q^{\dagger}_{\boldsymbol{X'}\,\alpha}(t,\vec{x})\int d^3y\,q^{c\,\dagger}_{\gamma}(t,\vec{y})\,q^{c}_{\gamma}(t,\vec{y})\,Q_{\boldsymbol{X}\,\beta}(t,\vec{x})\ |X;P_{\boldsymbol{X}}\,\{\ell\}\rangle \\
&\simeq \int d^3y\,\mathcal{O}_{\alpha\,\beta}\ \langle X';P_{\boldsymbol{X'}}\,\{\ell'\}|\,Q^{\dagger}_{\boldsymbol{X'}\,\alpha}(t,\vec{x})q^{c\,\dagger}_{\gamma}(t,\vec{y})\,|0\rangle\,\langle 0|\,,q^{c}_{\gamma}(t,\vec{y})\,Q_{\boldsymbol{X}\,\beta}(t,\vec{x})\ |X;P_{\boldsymbol{X}}\,\{\ell\}\rangle \\
&= \mathrm{tr}\int d^3y\,\psi^{\ell'\,\dagger}_{\boldsymbol{X'}}\,(\vec{y}-\vec{x};P_{\boldsymbol{X'}})\ \mathcal{O}\,\psi^{\ell}_{\boldsymbol{X}}\,(\vec{y}-\vec{x};P_{\boldsymbol{X}})\ e^{-i(P_X-P_{X'})\cdot x},
\end{aligned} \tag{D2}$$

or

$$\langle X';P_{X'}\{\ell'\}|\,j(0,\vec{0})\,|X;P_X\{\ell\}\rangle = \mathrm{tr}\int d^3x\,\psi^{\ell'\,\dagger}_{X'}(\vec{x};P_{X'})\,\mathcal{O}\,\psi^{\ell}_{X}(\vec{x};P_X)\,, \tag{D3}$$

where in Eq. (D2) we have assumed the vacuum dominance among intermediate states to make some approximations.

APPENDIX E: ISGUR-WISE FUNCTION

1. First Order

Now we will show the derivations of how to calculate the Isgur-Wise function given in the main text in four cases below.

1-i) B rest frame and $t = t_x = t_y$

$$\begin{aligned}
\xi(\omega) &= \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B}}\frac{\mathrm{tr}\int d^3x\;\psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x});P_D)\,\psi^{\ell}_{B\,0}((0,\vec{x});P)}{\sqrt{\mathrm{tr}\int d^3x\;\left|\psi^{\ell'}_{D\,0}((0,\vec{x});P_D)\right|^2}}\\
&= \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B 2M_D\gamma^3}}\int d^3x\;G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right)\\
&\qquad\times e^{-i(M_D-m_c)\gamma V x^3}\,\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{x})\\
&= \frac{2\sqrt{\omega}}{2\bar{M}(1+\omega)\gamma^{3/2}}\int d^3x\;\left\{\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})+(\gamma^{-1}-1)x^3\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})\right\}\\
&\qquad\times G_{\alpha\gamma}G_{\beta\delta}\psi^{\ell}_{B\,0\,\gamma\delta}(\vec{x})\,e^{-i(M_D-m_c)\gamma V x^3}+\;O(\beta^4)\\
&= \frac{1}{\omega}-\frac{1}{6}\beta^2\omega\tilde{E}^2_D\left\langle r^2\right\rangle-\frac{\beta^2}{2}\frac{1}{2\bar{M}}\int d^3x\;x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})\right)\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{x})+\;O(\beta^4)\\
&= \frac{1}{\omega}-\frac{1}{6}\beta^2\omega\tilde{E}^2_D\left\langle r^2\right\rangle+\frac{\beta^2}{4}+\;O(\beta^4),
\end{aligned} \tag{E1}$$

where we have used

$$\begin{aligned}
&\int d^3x\;x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})\right)\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{x})\\
&= 2\bar{M}\int d^3x\;\frac{x^3}{2}\mathrm{tr}\;\left[\frac{\partial}{\partial x^3}\left\{\frac{1}{r}\begin{pmatrix}0 & 0\\ u_{-1} & i(\vec{n}\cdot\vec{\sigma})\,v_{-1}\end{pmatrix}\right\}\frac{1}{r}\begin{pmatrix}0 & u_{-1}\\ 0 & -i(\vec{n}\cdot\vec{\sigma})v_{-1}\end{pmatrix}\right]\\
&= 2\bar{M}\int d^3x\;\frac{x^3}{2}\mathrm{tr}\;\left[\left\{\frac{\partial}{\partial x^3}\left(\frac{u_{-1}}{r}\right)\right\}\frac{u_{-1}}{r}+\left\{\frac{\partial}{\partial x^3}(\vec{n}\cdot\vec{\sigma})\,\frac{v_{-1}}{r}\right\}(\vec{n}\cdot\vec{\sigma})\;\frac{v_{-1}}{r}\right]
\end{aligned}$$

$$= 2\bar{M}\int d^3x\ \frac{x^3}{4}\frac{\partial}{\partial x^3}\mathrm{tr}\ \left[\left(\frac{u_{-1}}{r}\right)^2 + (\vec{n}\cdot\vec{\sigma})^2\left(\frac{v_{-1}}{r}\right)^2\right] = -\bar{M}\int_0^\infty dr\ \left(u_{-1}^2 + v_{-1}^2\right)$$
$$= -\bar{M}, \tag{E2}$$

where $\bar{M} = \sqrt{M_B M_D}$.

1-ii) B rest frame and $t' = t'_{\boldsymbol{x}} = t'_{\boldsymbol{y}}$

$$\xi(\omega) = \frac{2\sqrt{\omega}}{(1+\omega)\sqrt{2M_B 2M_D\gamma}}\int\ d^3x\ G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma x^3\right);\left(M_D,\vec{0}\right)\right)$$
$$\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{x})\times e^{-i(M_D-m_c)\gamma\beta x^3}$$
$$= \frac{2\sqrt{\omega}}{2\bar{M}(1+\omega)\gamma^{1/2}}\int d^3x\ \left\{\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x}) + (\gamma-1)\,x^3\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})\right\}$$
$$\times G_{\alpha\gamma}G_{\beta\delta}\psi^{\ell}_{B\,0\,\gamma\delta}(\vec{x})\,e^{-i(M_D-m_c)\gamma\beta x^3} + O(\beta^4)$$
$$= 1 - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle + \frac{\beta^2}{2}\frac{1}{2\bar{M}}\int d^3x\ x^3\left(\frac{\partial}{\partial x^3}\psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{x})\right)\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{x}) + O(\beta^4)$$
$$= 1 - \frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle - \frac{\beta^2}{4} + O(\beta^4), \tag{E3}$$

2-i) Breit frame and $t = t_{\boldsymbol{x}} = t_{\boldsymbol{y}}$

$$\xi(\omega) = \frac{\mathrm{tr}\int d^3x\ \psi^{\ell'\,\dagger}_{D\,0}((0,\vec{x});P_D)\,\psi^{\ell}_{B\,0}((0,\vec{x});P_B)}{\sqrt{\mathrm{tr}\int d^3x\ \left|\psi^{\ell'}_{D\,0}((0,\vec{x});P_D)\right|^2\ \mathrm{tr}\int d^3x\ \left|\psi^{\ell}_{B\,0}((0,\vec{x});P_B)\right|^2}}$$
$$= \frac{1}{\sqrt{2M_B\gamma^3 2M_D\gamma^3}}\int\ d^3x\ G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right)e^{-i(M_D-m_c)\gamma\frac{\beta}{2}x^3}$$
$$\times G^{-1}_{\alpha\epsilon}G^{-1}_{\beta\tau}\psi^{\ell}_{B\,0\,\epsilon\tau}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_B,\vec{0}\right)\right)e^{-i(M_B-m_b)\gamma\frac{\beta}{2}x^3}$$
$$= \frac{\gamma}{2\bar{M}\gamma^3}\int d^3z\ \psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{z})\,\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{z})\,e^{-i(\tilde{E}_B+\tilde{E}_D)\gamma^2\frac{\beta}{2}z^3}$$
$$= \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^2(\tilde{E}_B+\tilde{E}_D)^2\left\langle r^2\right\rangle + O(\beta^4). \tag{E4}$$

2-ii) Breit frame and $t' = t'_{\boldsymbol{x}} = t'_{\boldsymbol{y}}$

$$\xi(\omega) = \frac{1}{\sqrt{2M_B\gamma 2M_D\gamma}}\int\ d^3x\ G^*_{\alpha\gamma}G^*_{\beta\delta}\psi^{\ell'\,*}_{D\,0\,\gamma\delta}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_D,\vec{0}\right)\right)$$
$$\times G^{-1}_{\alpha\epsilon}G^{-1}_{\beta\tau}\psi^{\ell}_{B\,0\,\epsilon\tau}\left(\left(0,\vec{x}_\perp,\gamma^{-1}x^3\right);\left(M_B,\vec{0}\right)\right)e^{-i(\tilde{E}_B+\tilde{E}_D)\gamma\frac{\beta}{2}x^3}$$
$$= \frac{\gamma^{-1}}{2\bar{M}\gamma}\int d^3z\ \psi^{\ell'\,*}_{D\,0\,\alpha\beta}(\vec{z})\,\psi^{\ell}_{B\,0\,\alpha\beta}(\vec{z})\,e^{-i(\tilde{E}_B+\tilde{E}_D)\frac{\beta}{2}z^3}$$
$$= \gamma^{-2} - \frac{1}{6}(\frac{\beta}{2})^2\gamma^{-2}(\tilde{E}_B+\tilde{E}_D)^2\left\langle r^2\right\rangle + O(\beta^4). \tag{E5}$$

2. Second Order

Here in this Appendix we will show how to calculate the first order corrections to the form factors by using Eqs. (22$\sim$ 25, 27) and equations in the Appendix C.

$$\begin{aligned}\xi_+(\omega) &= \frac{1}{2\bar{M}\gamma^2}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}}^{\boldsymbol{\ell}}(\vec{z})U_{\boldsymbol{B}}^{-1}G^{-1}G^{*}U_{\boldsymbol{D}}\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= \frac{1}{2\bar{M}\gamma^2}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}\,0}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}\,0}^{\boldsymbol{\ell}}(\vec{z})\left(1+\frac{1}{2}\left[\frac{1}{m_c}\vec{\gamma}\cdot\vec{p}_c-\frac{1}{m_b}\vec{\gamma}\cdot\vec{p}_b\right]\right)\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= \xi(\omega), \end{aligned} \tag{E6}$$

where $U_{\boldsymbol{X}}$ for $X=\bar{B}$ and/or D are defined by Eqs. (18, C1) and $\bar{M}=\sqrt{M_{\boldsymbol{B}}M_{\boldsymbol{D}}}$.

$$\begin{aligned}\xi_-(\omega) &= \frac{1}{2\bar{M}\gamma^2\frac{\beta}{2}}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}}^{\boldsymbol{\ell}}(\vec{z})U_{\boldsymbol{B}}^{-1}G^{-1}\alpha^{3\,\mathrm{T}}G^{*}U_{\boldsymbol{D}}\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= -\frac{1}{2\bar{M}\gamma^2}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}\,0}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}\,0}^{\boldsymbol{\ell}}(\vec{z})\left(\alpha^3+\frac{1}{2}\left[\frac{1}{m_c}p_c^3+\frac{1}{m_b}p_b^3\right]\gamma^0\right)\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= -\frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\xi(\omega). \end{aligned} \tag{E7}$$

$$\begin{aligned}\xi_{\boldsymbol{V}}(\omega) &= \frac{1}{2\bar{M}^*\gamma^2\frac{\beta}{2}(-i\epsilon^{2\,*})}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}^*}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}}^{\boldsymbol{\ell}}(\vec{z})U_{\boldsymbol{B}}^{-1}G^{-1}\alpha^{1\,\mathrm{T}}G^{*}U_{\boldsymbol{D}^*}\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= -\frac{1}{2\bar{M}^*\gamma\frac{\beta}{2}(-i\epsilon^{2\,*})}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}^*\,0}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}\,0}^{\boldsymbol{\ell}}(\vec{z})\left(\alpha^1+i\frac{\beta}{2}\Sigma^2+\frac{i}{2}\left[\frac{1}{m_c}p_c^3-\frac{1}{m_b}p_b^3\right]\gamma^0\Sigma^2\right)\right)\\ &\quad\times e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= \left[1+\frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\xi(\omega), \end{aligned} \tag{E8}$$

where $\vec{\Sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$, $\vec{\epsilon}$ is a static polarization vector for D^*, and $\bar{M}^*=\sqrt{M_{\boldsymbol{B}}M_{\boldsymbol{D}^*}}$.

$$\begin{aligned}\xi_{\boldsymbol{A}_1}(\omega) &= \frac{1}{2\bar{M}^*\gamma^2\epsilon^{1\,*}}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}^*}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}}^{\boldsymbol{\ell}}(\vec{z})U_{\boldsymbol{B}}^{-1}G^{-1}\gamma_5^{\mathrm{T}}\alpha^{1\,\mathrm{T}}G^{*}U_{\boldsymbol{D}}\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= \frac{1}{2\bar{M}^*\gamma^2\epsilon^{1\,*}}\int d^3z \mathrm{tr}\left(\psi_{\boldsymbol{D}^*\,0}^{\boldsymbol{\ell}\,\dagger}(\vec{z})\psi_{\bar{\boldsymbol{B}}\,0}^{\boldsymbol{\ell}}(\vec{z})\left(\alpha^1+i\frac{\beta}{2}\Sigma^2+\frac{1}{2}\left[\frac{1}{m_c}p_c-\frac{1}{m_b}p_b\right]\gamma^0\Sigma^1\right)\right)\\ &\quad\times e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3}\\ &= \left[1+\frac{\bar{\Lambda}}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\right]\xi(\omega). \end{aligned} \tag{E9}$$

$$
\begin{aligned}
-\xi_{A_1}(\omega)+\xi_{A_2}(\omega)+\xi_{A_3}(\omega) &= \frac{1}{2\bar{M}^*\gamma^4\frac{\beta}{2}\epsilon^{3*}}\int d^3z \mathrm{tr}\left(\psi^{\ell}_{D^*}{}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}\gamma_5^{\mathrm{T}}G^*U_{D^*}\right)\, e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\bar{M}^*\gamma^4\frac{\beta}{2}\epsilon^{3*}}\int d^3z \mathrm{tr}\left(\psi^{\ell}_{D^*\,0}{}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}\,0}(\vec{z})\gamma_5\left(1+\frac{1}{2}\left[\frac{1}{m_c}p_c^3+\frac{1}{m_b}p_b^3\right]\right)\right) e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3} \\
&= \frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\xi(\omega). \qquad \text{(E10)}
\end{aligned}
$$

$$
\begin{aligned}
\xi_{A_1}(\omega)+\frac{\omega-1}{\omega+1}\left(\xi_{A_2}(\omega)-\xi_{A_3}(\omega)\right) &= \frac{1}{2\bar{M}^*\gamma^4\epsilon^{3*}}\int d^3z \mathrm{tr}\left(\psi^{\ell}_{D^*}{}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}}(\vec{z})U_B^{-1}G^{-1}\gamma_5^{\mathrm{T}}\alpha^{3\,\mathrm{T}}G^*U_{D^*}\right) \\
&\quad\times e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3} \\
&= -\frac{1}{2\bar{M}^*\gamma\epsilon^{3*}}\int d^3z \mathrm{tr}\left(\psi^{\ell}_{D^*\,0}{}^{\dagger}(\vec{z})\psi^{\ell}_{\bar{B}\,0}(\vec{z})\left(\Sigma^3-\frac{1}{2}\left[\frac{1}{m_c}p_c^3-\frac{1}{m_b}p_b^3\right]\gamma_5\gamma^0\right)\right) e^{-2i\bar{\Lambda}\frac{\beta}{2}z^3} \\
&= \left(1-\frac{\omega-1}{\omega+1}\right)\xi(\omega). \qquad \text{(E11)}
\end{aligned}
$$

Even though the first order corrections to the wave functions are present as given by Eqs. (24, 25), their contributions vanish because of matrix structures after taking a trace over indices. From the above equations one can easily reproduce our results given by Eqs. (55).

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