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Effective $SU(2)_L \otimes U(1)$ theory and the Higgs boson mass

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Abstract

We assume the stability of the vacuum expectation value of the Higgs boson field under radiative corrections in the context of the standard electroweak theory. We find that this theory behaves as a good effective model already at cut off energy scales as low as 0.7 TeV. This stability criterion allows to predict $m_H = 318 \pm 13$ GeV for the Higgs boson mass.

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Since the standard electroweak theory is renormalizable, the ultraviolet (quadratic and logarithmic) divergencies appearing in loop calculations can be removed by proper redefinitions of a small number of parameters (*e.g.*, masses). The renormalization program allows to get finite predictions for physical quantities except for those few parameters which require redefinitions. In this way, we renounce to give some physical content to the divergent behavior of the theory at very high energies [1].

Instead of following the renormalization program, we can consider that $SU(2)_L \otimes U(1)$ is an *effective* theory with an ultraviolet cut off at the energy scale Λ and try to trigger some information on the model itself from the behavior of divergent terms. In other words, we assume that a more complete theory than $SU(2)_L \otimes U(1)$ introduces new physical effects at high energy that cut off the ultraviolet divergencies. Below Λ, $SU(2)_L \otimes U(1)$ is supposed to describe all the electroweak processes in a satisfactory way.

In this letter we consider $SU(2)_L \otimes U(1)$ as an effective theory below Λ and assume the vanishing of the common divergent contributions (tadpoles) to the masses of all the particles. We find that the electroweak theory becomes effective even at cut off energies as small as $\Lambda \sim 0.7$ TeV. This criterion for the stability of the vacuum expectation value (*vev*) of the Higgs field under radiative corrections allows to derive a mass for the Higgs boson that differs from its asymptotic value ($\Lambda \to \infty$) $m_H = \sqrt{4(m_t^2 + m_b^2) - 2m_W^2 - m_Z^2}$ [2]- [3] by less than 0.6 % already at $\Lambda \sim 0.7$ TeV.

Let us start by considering the divergent one-loop tadpole contributions. These tadpoles give a universal contribution to self-mass corrections because the bare masses of all the particles are proportional to the vacuum expectation value v of the Higgs field. The tadpoles are gauge-dependent and are given by [4]

$$\frac{\delta v_t}{v} = \left(\frac{\alpha}{16\pi}\right) \frac{1}{m_H^2 m_W^2 \sin^2\theta_W} \Big\{ -2m_Z^4 - 4m_W^4 - \frac{3}{2} m_H^2 A_0(m_H) - 3m_Z^2 A_0(m_Z)$$

$$
\begin{aligned}
&-6m_W^2 A_0(m_W) - \frac{1}{2}m_H^2 A_0(\sqrt{\xi} m_Z) - m_H^2 A_0(\sqrt{\xi} m_W) \\
&+12[m_t^2 A_0(m_t) + m_b^2 A_0(m_b)]\Big\} \qquad (1)
\end{aligned}
$$

where m_i $(i = W, Z, H, t, b)$ denote the masses of gauge bosons, the Higgs boson and top and bottom quarks (we have neglected fermion masses other than $m_{t,b}$), ξ is the gauge parameter ($\xi = 1$ in the t'Hooft-Feynman gauge and $\xi = 0$ in the Landau gauge) and [4]

$$
A_0(m) = \begin{cases} -m^2\left(\frac{2}{\epsilon} - \gamma + \log(4\pi\mu^2) + 1 - \log m^2\right), \\ \Lambda^2 - m^2 \log\dfrac{\Lambda^2}{m^2} - m^2 \end{cases} \qquad (2)
$$

The two expressions in Eq. (2) are obtained by using the n-dimensional and corresponding cut off regularization methods, respectively.

In a similar way, we can compute the corrections to the neutrino mass (we assume a non-vanishing value for the neutrino mass). We get

$$
\begin{aligned}
\frac{\delta m_\nu}{m_\nu} &= \left(\frac{\alpha}{16\pi}\right)\frac{1}{m_W^2 \sin^2\theta_W}\left\{\frac{3}{2}m_W^2 + B_0(0,0,\sqrt{\xi} m_W)\right. \\
&\quad \left. +\frac{3}{4}m_Z^2 + \frac{1}{2}B_0(0,0,\sqrt{\xi} m_Z)\right\} + \frac{\delta v_t}{v} \qquad (3)
\end{aligned}
$$

where [4]

$$
B_0(0,0,m) = \begin{cases} -m^2\left(\frac{2}{\epsilon} - \gamma + \log(4\pi\mu^2) + 1 - \log m^2\right), \\ -m^2 \log\dfrac{\Lambda^2}{m^2} - m^2, \end{cases} \qquad (4)
$$

respectively, using the n-dimensional and cut off methods. Eq. (3) is in agreement with corresponding results in Ref. [5] where finite terms of the form m^2 and m^4 are neglected.

As it can be easily checked from the previous expressions, $\delta m_\nu/m_\nu$ is explicitly gauge-independent. This means that if we write the tadpoles in the Landau gauge we obtain

$$
\left.\frac{\delta v_t}{v}\right|_{\xi=0} = \frac{\delta m_\nu}{m_\nu} - \frac{3}{4}\left(\frac{\alpha}{16\pi}\right)\frac{1}{m_W^2 \sin^2\theta_W}\{2m_W^2 + m_Z^2\}, \qquad (5)
$$

i.e., this quantity is $SU(2)_L \otimes U(1)$ gauge-invariant. As is well known, the tadpoles can be computed from an effective potential only in the Landau gauge [6].

Now let us assume that tadpole contributions in the Landau gauge vanish, namely $\delta v_t = 0$. According to Eq. (5), $\delta m_\nu/m_\nu$ becomes negligible small ($\sim 1.6 \times 10^{-3}$), *i.e.* the radiative corrections to the small neutrino mass are very small. Since tadpole radiative corrections affect the masses of all the particles in an universal way, the *vev* stability under radiative corrections ($\delta v_t = 0$) implies that the masses of all the particles in the effective model scale as v_{class}, the classical *vev* of the Higgs boson field.

In figure 1, we plot the Higgs boson mass as a function of Λ when we impose the condition $\delta v_t = 0$ in the Landau gauge. The three different curves correspond to $m_t = 169,\ 175$ and 181 GeV and we have used $m_W = 80.35$ GeV, $m_Z = 91.187$ GeV and $m_b = 4.5$ GeV [7]. When $\Lambda > 0.7$ TeV, we observe that the mass of the Higgs boson is approximately given by the relation

$$m_H^2 = 4(m_t^2 + m_b^2) - 2m_W^2 - m_Z^2 \tag{6}$$

$$\approx 318 \pm 13 \text{ GeV}. \tag{7}$$

Eq. (6) corresponds to the condition for the cancellation of quadratic divergencies [2]-[3] in the self-masses of *all* the particles in the standard electroweak theory.

If we have chosen $\delta m_\nu/m_\nu = 0$ instead of $\delta v_t/v = 0$, the plots obtained would have been almost identical for $\Lambda \geq 0.7$ TeV because finite terms in Eq. (5) are negligible above these cut off energies.

The present approach to the effective standard theory by requiring the stability of the Higgs *vev* is different from the approaches based on the effective Higgs potential (see for instance [8, 9]). Actually, the condition for the minimum of the one-loop effective potential (in the Landau gauge) requires that the sum of tree-level and one-loop tadpole contributions vanishes identically. In our case, those contributions vanish individually because we require that the Higgs *vev*, which defines the minimum of the classical potential, remains stable after including its one-loop corrections.

It is interesting to note that our predicted value for m_H is somewhat higher than the one predicted in the version of the effective electroweak theory presented in [9] where the Higgs potential is reduced to its quartic term. Actually, the mass of the Higgs boson as defined from effective Higgs potentials corresponds to a mass parameter obtained from one-particle irreducible diagrams evaluated at zero external momenta. In contradistinction, our m_H corresponds to the physical mass taking part in loop corrections.

In conclusion, by assuming the *vev* stability, *i.e.* the vanishing of tadpole radiative corrections we find that the standard electroweak theory behaves as a good effective model with a relatively small cut off energy scale. The mass obtained for the Higgs boson departs by less than 0.6 % from the value obtained by requiring cancellation of quadratic divergencies [2]-[3] already at $\Lambda \sim 0.7$ TeV. Thus, no fine-tunning of physical masses [1] are required to suppress the quadratic divergent corrections to self-masses in the effective version of the standard electroweak theory.

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References

[1] See for example: M. Chanowitz, *Ann. Rev. Nucl. Part. Sci.* **38** (1988) 323.

[2] R. Decker and J. Pestieau, preprint UCL-IPT-79-19; DESY Workshop, October 22-24, 1979; *Mod. Phys. Lett.* **A7** (1992) 3773; G. López Castro and J. Pestieau, *Mod. Phys. Lett* **A10** (1995) 1155.

[3] M. Veltman, *Acta Phys. Polonica* **B12** (1981) 437.

[4] J. Fleischer and F. Jegerlehner, *Phys. Rev.* **D23** (1981) 2001.

[5] I.-Hsiu Lee and S. D. Drell, in: *M.A.B. Bég Memorial Volume*, Eds. A. Ali and P. Hoodbhoy, (World Scientific,Singapore, 1991) p.13.

[6] S. Weinberg, *Phys. Rev.* **D7** (1973) 2887.

[7] R. M. Barnett *et al.*, *Particle Data Group*, *Phys. Rev.* **D54** (1996) Part I.

[8] J. Fatelo, J.-M. Gérard, T. Hambye and J. Weyers, *Phys. Rev. Lett.* **74** (1995) 492.

[9] T. Hambye, *Phys. Lett.* **B371** (1996) 87.

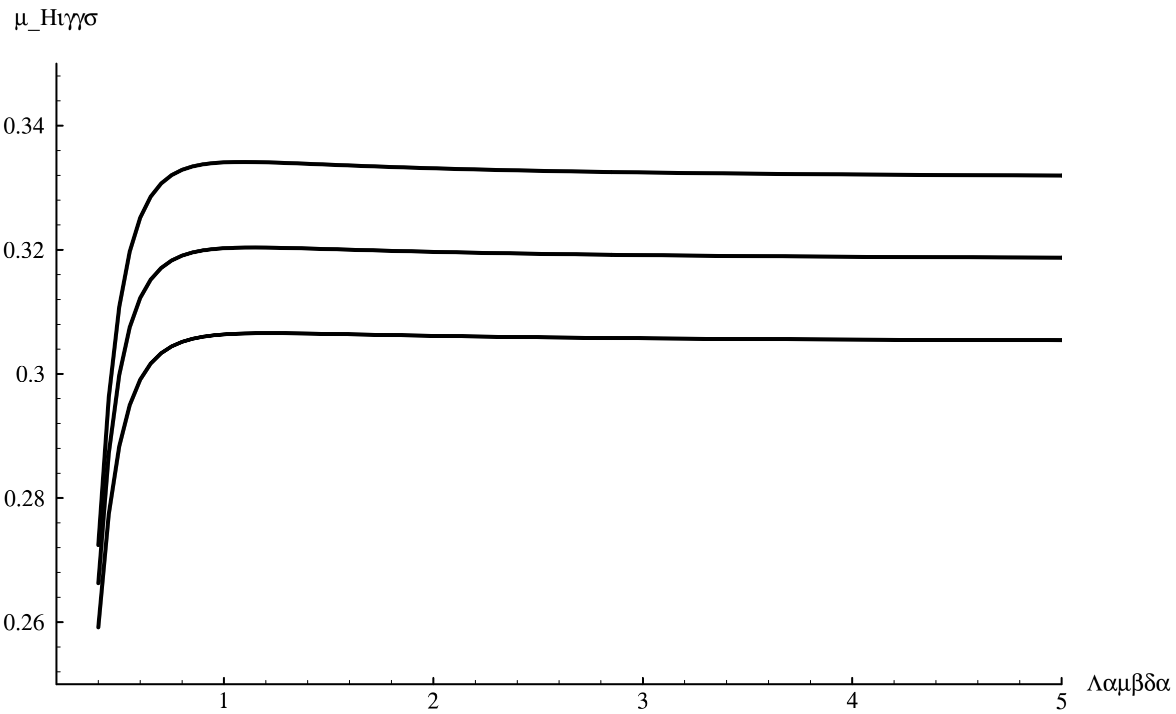

Figure 1: Mass of the Higgs boson as a function of Λ by imposing $\delta v_t = 0$. The plotted curves (from bottom to top) correspond to $m_t = 169,\ 175$ and 181 GeV, respectively. Units in both axis are TeV.