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Quark mixings as a test of a new symmetry of quark Yukawa couplings

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Abstract

We assume that Yukawa couplings of up and down quarks are related by $Y_u \propto Y_d^2$ at grand unification (GU) scale. This ansatz gives rise to a symmetrical CKM matrix at the GU scale. Using three specific models as illustrative examples for the evolution down to low energies, we obtain a non-symmetric CKM matrix whose entries are in very good agreement with their measured values. Therefore, the small asymmetry of the CKM matrix at low energies may be the effect of the renormalization group evolution only.

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1 Introduction

The quark masses and mixings in the standard model (SM) are the only traces left at low energies by quark Yukawa couplings (YC) after spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry. As is known, this gauge symmetry alone is not enough to put constraints on the YC which therefore remain as arbitrary complex constants in the model. Conversely, if additional symmetries are present in YC before symmetry breaking, one can expect to observe their traces in the structure of quarks masses and mixings at low energies.

Even though the values of the YC cannot be uniquely recovered from the observable quark masses and mixings, their measured values should be used as important constrains in the search for symmetries of YC. Some proposed ansatzs for the structure of YC were mainly motivated by the observed structure of quark masses and mixings [1]. Nowadays, the presence of additional flavor symmetries in grand unified (GUT) models can be used to constrain the structure of the Yukawa matrices at the GUT energy scales.

The hierarchy observed in the low energy values of quark masses and mixings [2] can be used as a guide to search for the structure of quark YC. Unitarity is the only theoretical constraint imposed to the quark mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix. This unitarity constraint together with the observation that $|V_{cd}| \approx |V_{us}|$ [2], indicates that the CKM matrix is almost symmetric[1]. For example, using the Wolfenstein's parameterization of the CKM matrix [3], the small asymmetries can be written as

$$|V_{ij}|^2 - |V_{ji}|^2 = \Delta \sum_{k=1}^{3} \epsilon_{ijk}, \tag{1}$$

where $\Delta \approx \lambda_W^6 \sim \hat{\lambda}_W^7$ and $\lambda_W = |V_{us}| \approx 0.22$.

At the same time, the running quark masses taken at a common weak scale (the

[1]Hereafter by symmetric CKM matrix we mean $|V_{ij}| = |V_{ji}|$.

top mass in this case) exhibits the following hierarchical structure:

$$
\begin{aligned}
M_u(m_t) &= m_t(m_t) \cdot \mathrm{Diag}\left(\alpha_1^u \lambda_W^8, \alpha_2^u \lambda_W^4, 1\right) \equiv m_t(m_t)\widetilde{M}_u \\
M_d(m_t) &= m_b(m_t) \cdot \mathrm{Diag}\left(\alpha_1^d \lambda_W^4, \alpha_2^d \lambda_W^2, 1\right) \equiv m_b(m_t)\widetilde{M}_d
\end{aligned} \tag{2}
$$

where the coefficients α_i^u and α_i^d are of the order 1 (see for example [4]). Notice that $\widetilde{M}_u \approx \widetilde{M}_d^2$.

This information lead us to ask whether these properties of quark masses and mixings at low energies can be correctly reproduced by assuming that[2]

$$
Y_u \propto Y_d \cdot Y_d \tag{3}
$$

($Y_{u,d}$ are the YC of up and down quarks) is an exact property valid at the energy scales of grand unification. In this letter we adopt this hypothesis. We find that a symmetrical CKM matrix is obtained at GUT scales by assuming Eq. (3) for symmetrical Y_u and Y_d. The evolution of YC down to observable low energies, generates the correct asymmetries and absolute values for the entries of the CKM matrix.

To our knowledge, this approach is different from previous studies (see for example Refs. [5, 6]). Current approaches to the problem of the generation of quark masses and mixings, are based on the postulation of specific YC valid at GUT chosen so that the corresponding low energy data are reproduced. Usually, some textures (zeros) for entries in the mass matrices may also be assumed in order to account for additional symmetries present at GUT scales.

A different view based on the notion of *natural* mass matrices, was introduced recently by Peccei and Wang [4]. Their main idea is to derive the GU textures of quark YC by evolving to high energies the observed values of quark masses and mixings. Their

[2]Eq. (3) means that Y_u is a quadratic function of Y_d. The most general form of such a relation is $(Y_u)_{ij} = A_{ijklmn}(Y_d)_{kl}(Y_d)_{mn}$ where A_{ijklmn} are complex constants. These constants depend on the choice of the quark basis. Later we give a precise definition of our basis in which Eq. (3) has a form $Y_u = CY_d^2$ with Y_u and Y_d being symmetric.

naturalness condition is based on the requirement that none of the small observables at low energies are derived by the approximate cancellation of large quantities. As a result they have obtained several possible scenarios for the GU scale YC.

The properties given in Eqs. (1) and (3) will be the basis of our further discussion. Therefore, we implicitly assume that the bottom and top quarks play a distinctive role in setting the hierarchical scales for the remaining lighter quarks. Notice that hierarchical structures in terms of λ_W for YC, compatible with mass and flavor mixing matrices as Eqs. (1) and (2), have been previously derived [7].

2 Yukawa ansatz and quark mixing at GUT scales

Since there is not a manifest symmetry at low energies that forces the CKM matrix to be symmetric, we shall consider an ansatz for the YC that gives rise to a symmetric CKM matrix at GU scales. It will be shown that the small asymmetries of this matrix at low energy scales are generated by the effect of the renormalization group evolution.

According to the motivation given in the introduction, we propose the following ansatz

$$Y_u = C\ Y_d^2, \tag{4}$$

valid at GUT scales, where C is some proportionality constant. This relation (and *a priori* any relation of the form $\widetilde{M}_u = \widetilde{M}_d^n$) forces the mass matrices to be diagonalized by the same transformation.

As is known, without loss of generality, the YC matrices can be chosen as hermitian. Furthermore, we shall assume that $Y_{u,\ d}$ can be rendered real and symmetric by the following phase transformation:

$$U_L \to PU_L, \quad D_L \to P^\dagger D_L \tag{5}$$

where $P = \mathrm{Diag}(e^{-i\frac{\pi}{3}},\ e^{i\frac{\pi}{3}},\ 1)$ and U_L (D_L) denotes the triplet of left-handed up (down) quarks.

Upon this transformation only the weak charged current and the Yukawa couplings are affected. The charged current flavor matrix becomes

$$D = \mathrm{Diag}\left(e^{-2\pi i/3}, \quad e^{2\pi i/3}, \quad 1\right) \tag{6}$$

which may be therefore interpreted as the zeroth order CKM matrix.

We choose the YC at the GU scale to be[3] (only the leading powers in λ are displayed)

$$\frac{v}{m_b\sqrt{2}}\, Y_d \sim \begin{pmatrix} \alpha_{11}\lambda^4 & \dfrac{\alpha_{22}\lambda^3}{\sqrt{3}} & \dfrac{A\lambda^3}{3} \\ \dfrac{\alpha_{22}\lambda^3}{\sqrt{3}} & \alpha_{22}\lambda^2 & \dfrac{A\lambda^2}{\sqrt{3}} \\ \dfrac{A\lambda^3}{3} & \dfrac{A\lambda^2}{\sqrt{3}} & 1 \end{pmatrix}, \tag{7}$$

where v is the vacuum expectation value of the Higgs field. The corresponding structure of Y_u is determined by Eq. (4). α_{ij} are real numerical constants of $\mathcal{O}(1)$ which specific values are not essential for our discussion. Actually, the structure of the CKM matrix will emerge mainly from the hierarchy of the third row and column of $Y_{u,d}$. Eq. (7) coincides with one of the two possible hierarchical forms of YC, compatible with the structure of the CKM matrix at low energies, derived in Ref. [7]. The form in Eq. (7) was chosen because it is also compatible with the parameterization given in Ref. [8].

The matrix $\hat{A}$ that diagonalizes Y_u and Y_d is orthogonal and has the form

$$\hat{A} = \begin{pmatrix} n_1 & -\dfrac{\lambda}{\sqrt{3}}(1+\dfrac{A^2\lambda^4}{3})n_2 & 0 \\ \dfrac{\lambda}{\sqrt{3}}n_1 & n_2 & -\dfrac{A\lambda^2}{\sqrt{3}}n_3 \\ \dfrac{A\lambda^3}{3}n_1 & \dfrac{A\lambda^2}{\sqrt{3}}n_2 & n_3 \end{pmatrix}, \tag{8}$$

where $\frac{1}{n_1} = \sqrt{1+\frac{|\lambda|^2}{3}+\frac{|A|^2|\lambda|^6}{9}}$, $\frac{1}{n_2} = \sqrt{1+\frac{|\lambda|^2}{3}\left|1+\frac{A^2\lambda^4}{3}\right|^2+\frac{|A|^2|\lambda|^4}{3}}$ and $\frac{1}{n_3} = \sqrt{1+\frac{|A|^2|\lambda|^4}{3}}$.

Thus, the corresponding CKM matrix at the GU scale is given by

$$V_{ckm}(GU) = \hat{A}D\hat{A}^T. \tag{9}$$

[3] λ is a hierarchical parameter at the GU scale which does not necessarily coincide with λ_W.

This matrix contains only two parameters and can be shown that is symmetric.

An explicit computation shows that the entries of Eq. (9) exhibit the Wolfenstein's hierarchy of the CKM matrix with fixed values of ρ and η. Moreover the matrix (9) already includes a CP violating piece and the Jarlskog's parameter

$$J(GU) = \mathrm{Im}\,(V_{11}V_{22}V_{12}^{*}V_{21}^{*}) = \frac{A^2\lambda^6}{2\sqrt{3}} \approx (0.9 \sim 1.8)\cdot 10^{-5}, \tag{10}$$

exhibits the right order of magnitude (see the values of A and λ in Table 1).

The quark mixing matrix of Eq. (9) looks very similar to the representation obtained from the eigenvalues and eigenvectors of the CKM matrix (see Eqs. (8) and (9) in [8])

$$V_{ckm} = \mathcal{A}D\mathcal{A}^{\dagger}, \tag{11}$$

where D is the same as in Eq. (6) and is built from the eigenvalues of the CKM matrix.

The unitary matrix $\mathcal{A}$ is build out of the corresponding eigenvectors and depends on 4 parameters. It can be shown that by setting $\beta_3 = 0$ in Eqs. (8) and (9) in Ref. [8], the matrix $\mathcal{A}$ has a similar structure as $\hat{A}$. Thus, the representation proposed in [8] provides a natural framework to describe our ansatz.

The transformation given in Eq. (5) has two consequences. On the one hand, it allows to chose the Yukawa matrices as real and symmetric. On the other hand, it allows to separate the physical CP violating phase from the hierarchical structure of the CKM matrix. Therefore, the CP violating phase can be ascribed to the non-perturbative D matrix whereas the hierarchical structure of the CKM matrix would arise from the real YC.

3 Quark mixing matrix at low energy scales

Now we perform the RG evolution of the matrices Y_u and Y_d from the GU energy to the scale of m_t and derive the corresponding CKM matrix. The RG equations become modified by the phase transformation given in Eq. (5) and the evolution of the matrices Y_u and Y_d is different. In order to illustrate this evolution, we consider the one loop RG equations for YC in the SM, its two-Higgs doublet (DHM) and minimal supersymmetric (MSSM) extensions [9].

We will consider the RG equations in the leading order of the powers of λ. In this approximation the matrix Y_u remains hermitian upon the RG evolution and the diagonalizing matrix is constant. The evolution of the Y_d matrix is not hermitian so we find it convenient to consider the RG equations for $H_d = Y_d Y_d^\dagger$ instead of Y_d. In the phase convention defined by Eq. (5) these equations read[4] (for completeness we also write equations for $H_u = Y_u Y_u^\dagger$)

$$16\pi^2 \frac{dH_y}{dt} = 2(T_y - G_y)H_y + \{S_y, H_y\}, \quad y = u, d \tag{12}$$

The curly braces in Eq. (12) denote an anticommutator. G_y and T_y are defined as in Ref. [9], while S_u and S_d become affected by the phase transformation as

$$S_u = \frac{3}{2}(bH_u + cDH_dD^*), \quad S_d = \frac{3}{2}(bH_d + cD^*H_uD).$$

The coefficients of powers of λ in the entries of $H_{u,\,d}$ satisfy the approximate equations

$$16\pi^2 \frac{d[H_u]_{ij}}{dt} = 2(3[H_u]_{33} - G_u)[H_u]_{ij} + 3b[H_u]_{i3}[H_u]_{3j} \tag{13}$$

$$16\pi^2 \frac{d[H_d]_{ij}}{dt} = 2(3a[H_u]_{33} - G_d)[H_d]_{ij} + \frac{3}{2}c[D^*H_uD]_{i3}[H_d]_{3j} + \frac{3}{2}c[H_d]_{i3}[D^*H_uD]_{3j} \tag{14}$$

where $a = (0, 1, 0)$, $b = (2, 1, 1)$ and $c = (\frac{2}{3}, -1, \frac{1}{3})$ for the MSSM, SM and DHM models, respectively. Eqs. (13) and (14) can be solved analytically [10] with the initial conditions that follow from Eqs. (7) and (4).

[4]The leptonic contributions in the *r.h.s.* of Eqs. (12) are negligible (see ref. [4]). In the case of DHM and MSSM models, equal magnitudes are assumed for the vacuum expectation values of the two Higgs doublets. We neglect threshold effects and expect our results to be stable against their influence under the same assumptions that ref. [5].

The unitary matrix that diagonalizes H_d at the top mass scale becomes[5]

$$\hat{A}_d = \begin{pmatrix} 1 & -\frac{(\alpha\tilde{\lambda})^*}{\sqrt{3}} & \frac{(\tilde{A}\tilde{\lambda}^3)^*}{3}(\alpha^* - 1) \\ \frac{\alpha\tilde{\lambda}}{\sqrt{3}} & 1 & -\frac{\tilde{A}^*\left(\tilde{\lambda}^*\right)^2}{\sqrt{3}} \\ \frac{\tilde{A}\tilde{\lambda}^3}{3} & \frac{\tilde{A}\tilde{\lambda}^2}{\sqrt{3}} & 1 \end{pmatrix} \tag{15}$$

where the values of $\tilde{\lambda}$, $\tilde{A}$ and α, derived from the RG equations, are

$$\tilde{\lambda} = \lambda\left(m_t\right) = \frac{\left(1 + Re^{-2i\pi/3}\right)}{\left(1 + Re^{2i\pi/3}\right)}\lambda, \quad \tilde{A} = A\left(m_t\right) = \frac{\bar{a}\left(1 + Re^{2i\pi/3}\right)^3}{\left(1 + Re^{-2i\pi/3}\right)^2}A$$

$$\alpha = \frac{1}{\tilde{\lambda}}\left(\frac{[H_d]_{12} - [H_d]_{13}[H_d]^*_{23}}{[H_d]_{22} - |[H_d]_{23}|^2}\right)^* \tag{16}$$

Here R and $\bar{a}$ are numerical factors that depend on the specific model. Their values are given by $R = (-0.127,\ 0.259,\ -0.074)$ and $\bar{a} = (1.145,\ 0.794,\ 1.080)$ for the MSSM, SM and DHM models, respectively. The parameter α depends on the value of α_{22} in Eq. (7).

Using the matrices $\hat{A}$ and $\hat{A}_d$ the CKM matrix at the scale of m_t becomes

$$\hat{V}_{ckm}(m_t) = \hat{A}D\hat{A}_d^\dagger. \tag{17}$$

We can easily check that the CKM matrix in Eq. (17) is not symmetric.

In what follows, the parameters A and λ defined at the GU scale are adjusted so as to reproduce the CKM matrix at low energies. We set $\alpha_{22} = 1$, but its value does not change appreciably if we let it be a free parameter. The experimental data for the CKM matrix[6] are taken from Ref. [2] and our results obtained from Eq. (17) is given in Table 1. The results of our ansatz are in excellent agreement with experimental data.

[5]Only the leading powers of $\tilde{\lambda}$ are displayed.

[6]To be exact one should take the values of the CKM matrix elements at the m_t scale. Since the change in $|V_{ij}|$ is very small between 1 GeV and m_t [11] we can take into account these running effects by increasing the errors bars.

The asymmetries of the CKM matrix, $\Delta \approx \lambda^5 \sim \lambda^6 \approx \lambda_W^6 \sim \lambda_W^7$, are of the expected order of magnitude. Although good fits are obtained in all cases considered here, observe that only the values of $|V_{td}|$ and J clearly distinguishes among the different models.

4 Discussion of the results and conclusions

We have presented a new kind of pattern for the up and down quark mass matrices that generates a CKM matrix in good agreement with the experimental data (see Table 1). Our method is based on the existence of a simple relation between YC of up and down quarks at the GU scale (see Eq. (4)). We have also checked that none of the known schemes with textures fulfills our relation Eq. (4). This means that our proposed pattern offers a new possibility of the generation of the CKM matrix compatible with the observed hierarchy.

The motivation of our ansatz for the YC, Eqs. (4) and (7), originates from the hierarchy of quark masses at low energies, Eq. (2). In this paper we do not advocate for a specific flavor symmetry at the origin of this simple ansatz, but rather we are interested in its phenomenological consistency.

Our method weakly depends on the specific values of the quark masses. Actually, our results depend only on the hierarchies among mass ratios for quarks of a given electric charge. Thus, the form of the CKM matrix is mainly determined by the third row and column of the Y_u and Y_d matrices, which suggests that these matrices may have a common dynamical origin in a perturbative structure around heavy quarks [7].

In summary, the ansatz $Y_u = CY_d^2$ for real and symmetric YC leads to a symmetric CKM matrix at GUT scales. There is a CP violating phase in the CKM matrix which is associated to the eigenvalues of the CKM matrix. The RG evolution generates a CKM matrix at weak scales in good agreement with experimental data. The results for the CKM matrix at low energies are very stable against input data at the GUT scale because the hierarchy is compatible with one derived in Ref. [7]. Therefore, we consider that the flavor symmetry behind this new ansatz may be a very important piece of information about the physics at the basis of the standard model.

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	Experiment	MSSM	SM	DHM
χ^2		2.92	2.97	4.94
$A \pm \Delta A$		1.480±0.014	2.113±0.040	1.027±0.029
$\lambda \pm \Delta\lambda$		0.155±0.006	0.156±0.004	0.189±0.009
$\lvert V_{ud}\rvert$	0.9736±0.0010	0.97511	0.97512	0.97510
$\lvert V_{us}\rvert$	0.2205±0.0018	0.22165	0.22164	0.22168
$\lvert V_{cd}\rvert$	0.224±0.016	0.22160	0.22154	0.22166
$\lvert V_{ub}\rvert$	–	0.00364	0.00366	0.00433
$\lvert V_{td}\rvert$	–	0.00730	0.00810	0.00635
$\frac{\lvert V_{ub}\rvert}{\lvert V_{cb}\rvert}$	0.08±0.02	0.0892	0.0898	0.1088
$\lvert V_{cb}\rvert$	0.041±0.0030	0.0408	0.0408	0.0398
J		$1.78\,10^{-5}$	$2.46\,10^{-5}$	$2.02\,10^{-5}$

Table 1: Results for CKM matrix elements in the SM and its DHM and MSSM extensions. The fitted values of A and λ are at the GU scale while $|V_{ij}|$ and J correspond to their low energy values.