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Field Theory approach to $K^0 - \overline{K^0}$ and $B^0 - \overline{B^0}$ systems

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Abstract

Quantum field theory provides a consistent framework to deal with unstable particles. We present here an approach based on field theory to describe the production and decay of unstable $K^0 - \overline{K^0}$ and $B^0 - \overline{B^0}$ mixed systems. The formalism is applied to compute the time evolution amplitudes of K^0 and $\overline{K^0}$ studied in DAPHNE and CPLEAR experiments. We also introduce a new set of parameters that describe CP violation in $K \to \pi\pi$ decays without recourse to isospin decomposition of the decay amplitudes.

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1 Introduction

Neutral strange and beauty pseudoscalar mesons, $K^0\overline{K^0}$ and $B^0\overline{B^0}$, are systems of two unstable mixed states of special interest for the study of weak interactions. They are particularly suited to study the phenomena of CP violation together with the oscillations in their time-dependent decay probabilities [1, 2].

The traditional description of unstable neutral kaons is based on the Wigner-Weisskopf (WW) formalism [3]. In this approach, the time evolution of decaying states is governed by a Schrödinger-like equation based on a *non-hermitian* hamiltonian [4] that allows particle decays. As a result, the diagonalizing transformations, in general, are not unitary, the corresponding eigenstates are not orthogonal and the normalization cannot be done without ambiguities.

Besides these unsatisfactory features of the WW formalism, one faces other difficulties. Projected factories of K and B mesons [5, 6] are expected to measure the CP violation and oscillation parameters to a higher accuracy than present experiments. While it is not clear whether the approximations involved in the WW formalism are valid for both the K and B systems, a consistent scheme is certainly required to compute these observables to a high degree of accuracy.

In this paper we adopt the view that the quantum mechanical behavior of a complete process involving the production and decay of unstable states can only be consistently described in the framework of quantum field theory [7]. In QFT, the S-matrix amplitude becomes the basic object that describes the properties of a physical process among particles. This amplitude is taken between *in-* and *out-* asymptotic states which are defined as non-interacting states (stable particles) existing far away the interaction region. Therefore, as a general rule, unstable particles cannot be considered as asymptotic states.

Under these conditions, unstable particles appear only as intermediate states to which we associate Green functions (propagators) to describe the propagation amplitudes from their production to their decay spacetime locations. The form of these propagators, which is consistent with special relativity and causality, determines the time evolution of the decay probabilities. Since Lorentz covariance is implicit to the field theory approach, neither boost transformations nor the choice of a specific frame [8] are required to define the time parameter in the amplitude.

In this paper we will also address some questions related to the usual treatment of CP violating parameters. As is well known, the $K^0\overline{K^0}$ (and $B^0\overline{B^0}$) system requires two parameters to account for CP violation in the propagation (*indirect*) and decay (*direct*) of neutral kaons, usually related to two theoretically defined complex parameters called respectively ε and ε' [9]. On the one hand the description based on the WW formalism is not valid beyond order ε because of the aforementioned difficulty in the normalization of non-orthogonal states. Since $\varepsilon' \sim \mathcal{O}(\varepsilon^2)$ for the $K^0\overline{K^0}$ system, it becomes necessary to establish a correct formalism [10] to account consistently for terms of order ε^2. This is all the more needed because the usual approximations for neutral kaons in the WW formalism might fail in the case of B mesons [11].

On the other hand, the reduction of observable CP violating parameters in the $K^0\overline{K^0}$ system to only two theoretical parameters ε and ε' cannot be done without assuming isospin symmetry and the factorization of strong rescattering effects [9]. These assumptions are rather strong in view of the smallness of direct CP violating effects [12, 13]. In this paper, we give up the isospin decomposition of the amplitudes, parametrizing CP violation in terms of three parameters. The first, $\hat{\epsilon}$, describes the mixing of CP eigenstates $K_1 - K_2$ (indirect CP violation), while the two other, χ_{+-} and χ_{00}, account for the CP violating 2π decays of K_2 (direct CP violation) in our approach.

The paper is organized as follows. In section 2 we discuss the diagonalization of mixed propagators in momentum space for the system of unstable neutral pseudoscalar K and B mesons. In section 3 we focus on the space-time representations of these propagators. Section 4 is devoted to the applications of our formalism to compute the time-dependent distributions of neutral kaon decays as adapted to CPLEAR and DAPHNE experiments. Our conclusions are presented in section 5.

2 Unstable particle propagator in momentum space

As previously discussed, the propagator is the basic object in the S-matrix amplitude that describes the propagation of an unstable state from its production at space-time point x to its decay at point x'. In this section we study the momentum space representation of the propagator for the neutral kaon system, which will be needed to compute the S-matrix amplitudes.

The description of the evolution of an unstable particle requires non perturbative information to be introduced in the bare propagator. The full propagator is obtained from a Dyson summation of self-energy graphs. Since the weak interaction couples the flavor states K^0 and $\overline{K^0}$, the renormalized propagator for these two unstable particles is a non diagonal 2×2 matrix [14]. By imposing the CPT symmetry, we can parametrize the inverse propagator for unstable kaons of four-momentum p as follows

$$i\, D^{-1}(p^2) = \begin{pmatrix} d & a+b \\ a-b & d \end{pmatrix} \tag{1}$$

where

$$d \equiv p^2 - m_0^2 - i\, \mathcal{I}m\Pi_{00}(p^2), \tag{2a}$$

$$a + b \equiv -\Pi_{0\overline{0}}(p^2), \tag{2b}$$

$$a - b \equiv -\Pi_{\overline{0}0}(p^2), \tag{2c}$$

where m_0 is the renormalized mass and $-i\Pi_{\alpha\beta}(p^2)$ with $\alpha, \beta = 0, \overline{0}$ are the renormalized complex self-energies of the neutral kaon system in an obvious notation. Remark that the non diagonal terms depend on the phase convention chosen for the kaons.

We define the CP eigenbasis as

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} \equiv S \begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} \tag{3}$$

where $S = S^{-1}$ and $\mathrm{CP}|K^0\rangle = |\overline{K^0}\rangle$. The corresponding inverse propagator is

$$i\,\overline{D}^{-1}(p^2) \;\equiv\; S\, iD^{-1}(p^2) S^{-1} \;=\; \begin{pmatrix} d+a & -b \\ b & d-a \end{pmatrix}. \tag{4}$$

Now, if we introduce the complex parameter $\hat{\varepsilon}$ as

$$\frac{\hat{\varepsilon}}{1+\hat{\varepsilon}^2} \;\equiv\; \frac{b}{2a} \;, \tag{5}$$

we can diagonalize the inverse propagator as follows

$$i\,\overline{D}^{-1}(p^2) \;=\; \frac{1}{1-\hat{\varepsilon}^2} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} d+a\,\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} & 0 \\ 0 & d-a\,\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \end{pmatrix} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{6}$$

Therefore, the physical basis of neutral kaons consists of two states $K_{L,S}$ of definite masses $m_{L,S}$ and decay widths $\Gamma_{L,S}$, such that

$$d_S \equiv p^2 - m_S^2 + im_S\Gamma_S = d + a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \tag{7a}$$

$$d_L \equiv p^2 - m_L^2 + im_L\Gamma_L = d - a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \;, \tag{7b}$$

The constant width approximation will be justified in section 3. Consistency between Eqs.(2a) and (7) demands to approximate $\mathcal{I}m\Pi_{00}(p^2)$ by a constant term $-m_0\Gamma_0$. The propagator $\overline{D}(p^2)$ now reads

$$-i\,\overline{D}(p^2) = \frac{1}{1-\hat{\varepsilon}^2} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} d_S^{-1} & 0 \\ 0 & d_L^{-1} \end{pmatrix} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{8}$$

As already anticipated, the diagonalization of the non-hermitian matrix given in Eq.(4) involves a non-unitary matrix. In order to provide a link with the usual formalism, we can obtain a proper orthogonal and normalized physical basis if we define independent *ket* (in-) and *bra* (out-) states, respectively, as left-hand and right-hand eigenvectors of the inverse propagator [4] :

$$\begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} \tag{9}$$

and

$$\begin{pmatrix} \langle K_S| \\ \langle K_L| \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} \langle K_1| \\ \langle K_2| \end{pmatrix} . \tag{10}$$

Notice that for an arbitrary $\hat{\varepsilon}$ bra states do not correspond to hermitian conjugate of ket states.

The quantities $m_{S,L}, \Gamma_{S,L}$ can be measured experimentally, while the parameters a, b, m_0 and Γ_0 can be in principle computed from the theory. The relationships between these two sets of parameters are

$$a = \frac{1}{2}\left(\frac{1+\hat{\varepsilon}^2}{1-\hat{\varepsilon}^2}\right)\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\} \ , \tag{11a}$$

$$m_0^2 - im_0\Gamma_0 = \frac{1}{2}\{m_L^2 + m_S^2 - i(m_L\Gamma_L + m_S\Gamma_S)\} \ , \tag{11b}$$

$$b = \frac{\hat{\varepsilon}}{1-\hat{\varepsilon}^2}\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\}. \tag{11c}$$

3 Space-time evolution of resonance propagators

In this section we are interested in the time dependent properties of the propagation of unstable particles for the purposes of studying CP violation and the time oscillations in the kaon system. We shall therefore focus on the properties of the unstable state propagator in configuration space.

Let us first consider the propagator for a stable spin zero particle :

$$\Delta_F(x'-x) \ = \ i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip.(x'-x)}}{p^2-m^2+i\varepsilon}. \tag{12}$$

The time dependence will become manifest in the amplitude if we put this expression into another form showing a separate time evolution for the particle and the antiparticle. A contour integration in the complex p^0 plane gives

$$\begin{aligned}\Delta_F(x'-x) &= \int \frac{d^3p}{(2\pi)^3}\, \frac{e^{i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E}\,\theta(t'-t) \\ &+ \int \frac{d^3p}{(2\pi)^3}\, \frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{iE(t'-t)}}{2E}\,\theta(t-t') \end{aligned} \tag{13}$$

with $E \equiv \sqrt{\vec{p}^{\,2}+m^2}$.

Depending of the specific process, the first (second) term in Eq. (13) will survive in the time-dependent amplitude and will describe a particle (antiparticle) propagating forward in time.

Let us now consider the propagator of a spin zero resonance. The Dyson summation of self-energy graphs leads to the following renormalized propagator in momentum space representation :

$$\frac{i}{p^2-m^2-i\,\mathcal{I}m\Pi(p^2)}\ , \tag{14}$$

where $-i\,\Pi(p^2)$ is the renormalized self-energy whose absorptive part vanishes under a threshold p^2_{th} in the case of only one decay channel.

In order to justify the constant width approximation used in Eqs. (7), let us consider the 2π contribution to the kaon self-energy. A direct computation of $\mathcal{I}m\Pi(p^2)$, for a kaon of squared four-momentum s, gives

$$\mathcal{I}m\Pi(s) = \frac{-(g_1^2+g_2^2/2)}{8\pi}\left(\frac{s-s_{th}}{s}\right)^{1/2}\theta(s-s_{th}) \tag{15}$$

where $s_{th} = 4m_\pi^2$ (with the approximation $m_{\pi^+} = m_{\pi^0}$) and where g_1, g_2 are the effective couplings for $K^0 \to \pi^+\pi^-$, $\pi^0\pi^0$ respectively.

Remarking that cutting rules give $\mathcal{I}m\Pi(s=m^2) = -m\Gamma$ where Γ is the particle width in the center-of-mass frame and m the kaon mass, we can write the above expression as

$$\mathcal{I}m\Pi(p^2) = \frac{-m^2}{\sqrt{s}}\left(\frac{s-s_{th}}{m^2-s_{th}}\right)^{1/2}\Gamma\;\theta(s-s_{th}). \tag{16}$$

The influence of the kaon width in the propagator is only felt for $\sqrt{s}$ values near the kaon mass. Therefore the propagator can be greatly simplified by neglecting all but the first term in an expansion of $\mathcal{I}m\Pi(s)$ around $s = m^2$. More precisely,

$$\mathcal{I}m\Pi(s) = -m\Gamma \, \left(1 + \mathcal{O}\left(\frac{x\,\Gamma}{m - s_{th}}\right)\right) \tag{17}$$

for $m - x\Gamma \leq \sqrt{s} \leq m + x\Gamma$, with x an arbitrary number such that $x\Gamma/m << 1$ and with $s_{th} << m$.

Since $\Gamma_S/(m_S - 2m_\pi) \sim O(10^{-14})$ [2], the form of the propagator with a constant width

$$\frac{i}{p^2 - m^2 + im\Gamma\,\theta(p^2 - p_{th}^2)} \quad , \tag{18}$$

turns out to be an extremely good approximation for the renormalized propagator.

Therefore, the space-time representation of the spin zero propagator for the unstable particle can be written as

$$\Delta_R(x' - x) \;=\; i\int \frac{d^4p}{(2\pi)^4}\, \frac{e^{-ip.(x'-x)}}{p^2 - m^2 + im\Gamma\,\theta(p^2 - p_{th}^2)}. \tag{19}$$

Similarly as done above for the stable particle propagator, we would like to express explicitly the time dependence of $\Delta_R(x' - x)$. It becomes convenient to separate the propagator into two pieces :

$$\Delta_R(x' - x) = \Delta_R^{(1)}(x' - x) + \Delta_R^{(2)}(x' - x) \tag{20}$$

with

$$\begin{aligned} \Delta_R^{(1)}(x' - x) \;&=\; i\int \frac{d^4p}{(2\pi)^4}\, \frac{e^{-ip.(x'-x)}}{p^2 - m^2 + im\Gamma} \\ \Delta_R^{(2)}(x' - x) \;&=\; i\int \frac{d^3p}{(2\pi)^3} \int_{-p_{th}^0}^{p_{th}^0} \frac{dp^0}{2\pi} e^{-ip.(x'-x)} \left\{\frac{1}{p^2 - m^2} - \frac{1}{p^2 - m^2 + im\Gamma}\right\} \end{aligned} \tag{21}$$

where $p_{th}^0 = \sqrt{\vec{p}^{\,2} + p_{th}^2}$.

Using the condition $\Gamma/(m-\sqrt{p_{th}^2}) << 1$, we can show that

$$\Delta_R^{(2)}(x'-x) \sim \mathcal{O}\left(\frac{\Gamma}{m-\sqrt{p_{th}^2}}\right),$$

which allows to write

$$\Delta_R(x'-x) = i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip(x'-x)}}{p^2-m^2+im\Gamma}\left(1+\mathcal{O}\left(\frac{\Gamma}{m-\sqrt{p_{th}^2}}\right)\right). \tag{22}$$

In order to made explicit the time dependence of the unstable propagator let us use the following pole decomposition

$$p^2-m^2+i\Gamma m = \left(p_0-E+\frac{i\Gamma m}{2E}\right)\left(p_0+E-\frac{i\Gamma m}{2E}\right)\left(1+\mathcal{O}\left(\frac{\Gamma^2}{m^2}\right)\right) \tag{23}$$

where $E=\sqrt{\vec{p}^{\,2}+m^2}$.

Therefore, by neglecting very small terms of order 10^{-14}, the contour integral in the complex p^0 plane with the poles located at $\pm(E-im\Gamma/2E)$ gives

$$\begin{aligned}\Delta_R(x'-x) &= \int \frac{d^3p}{(2\pi)^3}\,\frac{e^{i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E}\;e^{-\frac{\Gamma}{2}\frac{m}{E}(t'-t)}\theta(t'-t)\\ &+\int \frac{d^3p}{(2\pi)^3}\frac{e^{i\vec{p}.(\vec{x}-\vec{x}')}e^{-iE(t-t')}}{2E}e^{-\frac{\Gamma}{2}\frac{m}{E}(t-t')}\theta(t-t').\end{aligned} \tag{24}$$

The interpretation is similar to the one for the stable particle, except for the decay constant Γ which expresses the unstability of the particle and antiparticle. The case of $K^0\overline{K^0}$ system considered in this paper is more involved, because the propagator is a 2×2 matrix. This problem is circumvented by performing the diagonalization (see Eq. (8)) before doing the contour integration.

Notice that $\tau = t'-t$ is the time elapsed between the production and decay locations of the resonance. Note also that, contrary to non-relativistic approaches, the factor m/E naturally appears in the exponential decay factor. Therefore, no boost transformations are required to relate the *proper* time to the time parameter of a moving particle. Of course, the exponential decay takes its usual form $e^{-\Gamma\tau/2}$ [1] in the rest frame of the resonance.

4 Applications

In this section we compute the full S-matrix amplitudes for the production and decay of neutral kaons as studied in CPLEAR and DAPHNE experiments. Then, we derive the time evolution of these transition amplitudes and introduce the CP violation parameters intrinsic to our description.

4.1 CPLEAR experiment

At the CPLEAR experiment [15], K^0 and $\overline{K^0}$ are produced at point x in the strong interaction annihilation of $p\bar{p}$, and subsequently decay at point x' to $\pi^+\pi^-$ by the effects of weak interactions. The production mechanisms of K^0 and $\overline{K^0}$ are $p\bar{p} \to K^0 K^- \pi^+$, $\overline{K^0} K^+ \pi^-$, thus neutral kaons can be tagged by identifying the accompanying charged kaon [15]. After their production, K^0 (or $\overline{K^0}$) oscillates between its two components K_L and K_S before decaying to the 2π final states. We are interested in the description of the time evolution of the full decay amplitude and its interference phenomena. It is interesting to note that despite the fact that charged kaons and pions have similar lifetimes as K_L, they can be treated as asymptotic particles in the present case.

In order to relate the different S-matrix amplitudes, let us first consider the production mechanism of $K^0\overline{K^0}$. Since strong interactions conserve strangeness, we have

$$\mathcal{M}(p\overline{p} \to K^-\pi^+\overline{K^0}) \;=\; \mathcal{M}(p\overline{p} \to K^+\pi^-K^0) = 0, \tag{25}$$

which, according to equation (3), implies

$$\mathcal{M}(p\overline{p} \to K^-\pi^+K_1) \;=\; \mathcal{M}(p\overline{p} \to K^-\pi^+K_2) \;\equiv\; A \tag{26a}$$

$$\mathcal{M}(p\overline{p} \to K^+\pi^-K_1) \;=\; -\mathcal{M}(p\overline{p} \to K^+\pi^-K_2) \;\equiv\; B. \tag{26b}$$

Assuming CPT invariance we obtain

$$\mathcal{M}(p\overline{p} \to K^-\pi^+K^0) \;=\; \mathcal{M}(p\overline{p} \to K^+\pi^-\overline{K^0}) \;\equiv\; C. \tag{26c}$$

Collecting all these constraints, we get

$$A \;=\; B \;=\; \frac{C}{\sqrt{2}}. \tag{26d}$$

Now, let us first consider the complete process for the production of a K^0 decaying into $\pi^+\pi^-$

$$p(q) + \overline{p}(q') \to K^-(k) + \pi^+(k') + K^0(p) \to K^-(k) + \pi^+(k') + \pi^+(p_1) + \pi^-(p_2)\ . \tag{27}$$

The full amplitude corresponding to this process can be written (the subscript $+-$ refers to the charges of the two pions from K^0 decay):

$$\begin{aligned} T_{+-} = \quad & \int d^4x\ d^4x'\ e^{i(p_1+p_2).x'} \Big(\mathcal{M}(K_1 \to \pi^+\pi^-),\ \mathcal{M}(K_2 \to \pi^+\pi^-)\Big) \\ & \times \Delta_R^{K_1K_2}(x'-x) \begin{pmatrix} \mathcal{M}(K^0 \to K_1) \\ \mathcal{M}(K^0 \to K_2) \end{pmatrix} \mathcal{M}(p\overline{p} \to K^-\pi^+K^0)\ e^{i(k+k'-q-q').x} \end{aligned} \tag{28}$$

where $\Delta_R^{K_1K_2}(x'-x)$ is the propagator matrix for the coupled $K_1 - K_2$ system in configuration space.

With the help of equations (8) and (26), this gives

$$\begin{aligned} T_{+-} \;=\; & \int d^4x\ d^4x'\ e^{i(p_1+p_2).x'} \Big(\mathcal{M}(K_1 \to \pi^+\pi^-),\ \mathcal{M}(K_2 \to \pi^+\pi^-)\Big) \\ & \times i \int \frac{d^4p}{(2\pi)^4}\ e^{-ip.(x'-x)} \frac{1}{1-\hat{\varepsilon}^2} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} d_S^{-1}(p) & 0 \\ 0 & d_L^{-1}(p) \end{pmatrix} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix} \\ & \times \begin{pmatrix} \mathcal{M}(K^0 \to K_1) \\ \mathcal{M}(K^0 \to K_2) \end{pmatrix} \sqrt{2}\ A\ e^{i(k+k'-q-q').x} \end{aligned} \tag{29}$$

Inserting equations (3) and (7), we obtain

$$\begin{aligned} T_{+-} \;=\; & i \int d^4x\ d^4x'\ \frac{d^4p}{(2\pi)^4}\ e^{i(p_1+p_2-p).x'}\ e^{i(k+k'-q-q'+p).x}\ \frac{A}{1+\hat{\varepsilon}} \\ & \times \Bigg\{ [\mathcal{M}(K_1 \to \pi^+\pi^-) + \hat{\varepsilon}\ \mathcal{M}(K_2 \to \pi^+\pi^-)]\ \frac{1}{p^2 - m_S^2 + i m_S \Gamma_S} \\ & \quad + [\hat{\varepsilon}\ \mathcal{M}(K_1 \to \pi^+\pi^-) + \mathcal{M}(K_2 \to \pi^+\pi^-)]\ \frac{1}{p^2 - m_L^2 + i m_L \Gamma_L} \Bigg\}. \end{aligned} \tag{30}$$

An expression depending on the four-momentum is obtained by performing all the integrals :

$$\begin{aligned} T_{+-} &= i\,(2\pi)^4\,\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{A}{1+\hat{\varepsilon}}\,\mathcal{M}(K_1\to\pi^+\pi^-) \\ &\times \left\{\frac{1+\chi_{+-}\hat{\varepsilon}}{(p_1+p_2)^2-m_S^2+im_S\Gamma_S}+\frac{\hat{\varepsilon}+\chi_{+-}}{(p_1+p_2)^2-m_L^2+im_L\Gamma_L}\right\} \end{aligned} \tag{31}$$

where

$$\chi_{+-}\equiv\frac{\mathcal{M}(K_2\to\pi^+\pi^-)}{\mathcal{M}(K_1\to\pi^+\pi^-)}\quad, \tag{32}$$

is the parameter describing direct CP violation in our approach.

Another possibility is to keep the time dependence, proceeding as in section 3, on the diagonalized propagator of Eq.(30). In that case, the amplitude of an originally pure K^0 state decaying into $\pi^+\pi^-$ reads

$$\begin{aligned} T_{+-} &= (2\pi)^4\,\delta^{(4)}(q+q'-k-k'-p_1-p_2)\,\frac{A}{1+\hat{\varepsilon}}\,\mathcal{M}(K_1\to\pi^+\pi^-) \\ &\times\Bigg\{(1+\chi_{+-}\hat{\varepsilon})\int\frac{dt}{2E_S}\left[e^{-i(E_S-E)t}\,e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\,\theta(t)+e^{i(E_S-E)t}\,e^{\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\,\theta(-t)\right] \\ &\quad+(\hat{\varepsilon}+\chi_{+-})\int\frac{dt}{2E_L}\left[e^{-i(E_L-E)t}\,e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\,\theta(t)+e^{i(E_L-E)t}\,e^{\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\,\theta(-t)\right]\Bigg\} \end{aligned} \tag{33}$$

where $E=p_1^0+p_2^0$ is the total energy of the $\pi^+\pi^-$ system and $E_{S,L}=\sqrt{(\vec{p}_1+\vec{p}_2)^2+m_{S,L}^2}$.

The time t in eq. (33) has been defined as the time elapsed from the production to the decay locations of K^0. Thus, the transition amplitude $\mathcal{T}(t)$ describing the time evolution of the system for $t>0$ is given by the integrand proportional to $\theta(t)$ in eq. (33), namely

$$\begin{aligned} \mathcal{T}_{+-}(t) &= (2\pi)^4\,\delta^{(4)}(q+q'-k-k'-p_1-p_2)\,\frac{A}{1+\hat{\varepsilon}}\,\mathcal{M}(K_1\to\pi^+\pi^-)\,e^{iEt} \\ &\times\left\{\frac{1+\chi_{+-}\hat{\varepsilon}}{2E_S}\,e^{-iE_St}\,e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}+\frac{\hat{\varepsilon}+\chi_{+-}}{2E_L}\,e^{-iE_Lt}\,e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\right\}\;. \end{aligned} \tag{34}$$

Let us now consider the analogous process where a pure $\overline{K^0}$ state is initially produced and then decay to $\pi^+\pi^-$, *i.e.* $p\bar{p}\to K^+\pi^-\overline{K^0}\to K^+\pi^-\pi^+\pi^-$. Following the same procedure as in the case of K^0 production and decay, we compute the following expression for the time evolution of $\overline{K^0}$ decays

$$\begin{aligned}\overline{\mathcal{T}_{+-}}(t) &= (2\pi)^4\,\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{A}{1-\hat{\varepsilon}}\,\mathcal{M}(K_1\to\pi^+\pi^-)\,e^{iEt}\\ &\times\left\{\frac{1+\chi_{+-}\hat{\varepsilon}}{2E_S}\,e^{-iE_St}\,e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}-\frac{\hat{\varepsilon}+\chi_{+-}}{2E_L}\,e^{-iE_Lt}\,e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\right\}\,. \qquad (35)\end{aligned}$$

Let us notice that if we were interested in the $\pi^0\pi^0$ decay mode of neutral kaons, we would have to replace in Eqs. (34) and (35) $\mathcal{M}(K_1\to\pi^+\pi^-)$ by $\mathcal{M}(K_1\to\pi^0\pi^0)$ and χ_{+-} by χ_{00} where

$$\chi_{00}\equiv\frac{\mathcal{M}(K_2\to\pi^0\pi^0)}{\mathcal{M}(K_1\to\pi^0\pi^0)}\,. \qquad (36)$$

Using Eqs. (34) and (35), we can express the measurable ratio of CP-violating to CP-conserving decay amplitudes of K_L, K_S states in terms of the CP-violating parameters proper to our approach:

$$\eta^{+-}\equiv\frac{\mathcal{M}(K_L\to\pi^+\pi^-)}{\mathcal{M}(K_S\to\pi^+\pi^-)}=\frac{\hat{\varepsilon}+\chi_{+-}}{1+\chi_{+-}\hat{\varepsilon}}\,, \qquad (37)$$

and

$$\eta^{00}\equiv\frac{\mathcal{M}(K_L\to\pi^0\pi^0)}{\mathcal{M}(K_S\to\pi^0\pi^0)}=\frac{\hat{\varepsilon}+\chi_{00}}{1+\chi_{00}\hat{\varepsilon}}\,. \qquad (38)$$

As is well known, the parameters η^{+-} and η^{00} are commonly used to express the violation of CP in the two pion decays of K_L (see for example pages 422-425 in [2]). Note that the above relations between measurable quantities and the parameters that quantify direct and indirect violation of CP, are derived *without* relying on assumptions based on isospin symmetry, contrary to the relations obtained for the η parameters in terms of the usual parameters ε and ε'. Furthermore, it can be explicitly shown that the parameters η^{+-} and η^{00} are independent of the phase convention chosen for K^0, $\overline{K^0}$, which is not the case for $\hat{\varepsilon}$ and $\chi_{+-,\,00}$.

Using the isospin symmetry and the Wu-Yang phase convention [9], we observe that $\epsilon=\hat{\varepsilon}$ (see Ref. [2], p.102) and the parameters $\chi_{+-,\,00}$ are expected to be very small so that terms of $O(\chi_{ij}\varepsilon)$ can be neglected in the above equations. In that limit, we obtain

$$\begin{aligned} \chi_{+-} &= \epsilon' \\ \chi_{00} &= -2\varepsilon'. \end{aligned}$$

Finally, let us mention that Eqs. (34) and (35) reduce to the current expressions for the time evolution used in the analysis of the CPLEAR collaboration [15], when we choose the center of mass frame ($\vec{p}_1 + \vec{p}_2 = \vec{0}$) of the two pion produced in $K^0 - \overline{K^0}$ decays.

4.2 Neutral kaon production at DAPHNE

In this section we consider the oscillations of the pair of neutral kaons produced in e^+e^- annihilations at DAPHNE [5]. The results obtained in the present formalism for the $K^0 - \overline{K^0}$ system can be straightforwardly generalized to describe the same phenomena in pair production of neutral B mesons in the $\Upsilon(4s)$ region [6].

Neutral and charged kaons will be copiously produced ($\sim 10^9$ pairs $K^0\overline{K^0}$/year) in e^+e^- collisions operating at a center of mass energy around the mass of the $\phi(1020)$ meson [5]. The ϕ mesons produced in e^+e^- annihilations decay at point x into $K^0\overline{K^0}$ pairs, and subsequently each neutral kaon oscillates between its $K_L - K_S$ components before decaying to final states $f_1(p)$ and $f_2(p')$ at spacetime points y and z :

$$\phi(q) \to K^0\overline{K^0} \to f_1(p) f_2(p') \tag{39}$$

where $q,\ p$ and p' are the corresponding four-momenta.

Since each final state can be produced by either K^0 or $\overline{K^0}$, we must add coherently the two amplitudes arising from the exchange of K^0 and $\overline{K^0}$ as intermediate states. Taking into account the charge conjugation properties of the electromagnetic current, the pair of neutral kaons is found to be in an antisymmetric state [16]. Thus, the relative sign of the two contributions to $\phi \to f_1 f_2$ decays must be negative. The

S-matrix amplitude for the process indicated in Eq. (39) is

$$\begin{aligned}
T_{f_1 f_2} &= \int d^4x\, d^4y\, d^4z\, e^{ip.y+ip'.z} \mathcal{M}(\phi \to K^0\overline{K^0})\, e^{-iq.x} \\
&\quad \times \Bigg[\Bigg\{ (\mathcal{M}(K_1 \to f_1),\ \mathcal{M}(K_2 \to f_1))\, \Delta_R^{K_1K_2}(y-x) \begin{pmatrix} \mathcal{M}(K^0 \to K_1) \\ \mathcal{M}(K^0 \to K_2) \end{pmatrix} \Bigg\} \\
&\qquad \times \Bigg\{ (\mathcal{M}(K_1 \to f_2),\ \mathcal{M}(K_2 \to f_2))\, \Delta_R^{K_1K_2}(z-x) \begin{pmatrix} \mathcal{M}(\overline{K^0} \to K_1) \\ \mathcal{M}(\overline{K^0} \to K_2) \end{pmatrix} \Bigg\} \\
&\qquad - \Big(\text{same expression with } K^0 \leftrightarrow \overline{K^0}\ \Big) \Bigg] .
\end{aligned} \tag{40}$$

Let us define $\mathcal{M}_{ij} \equiv \mathcal{M}(K_i \to f_j)$. With the help of Eqs. (3) and (8), we can reexpress the previous amplitude as

$$\begin{aligned}
T_{f_1 f_2} &= \int d^4x\, d^4y\, d^4z\, e^{ip.y+ip'.z-iq.x} \int \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4}\, e^{-ik.(y-x)-ik'.(z-x)}\, \frac{(-1)}{2(1-\hat{\varepsilon}^2)} \\
&\quad \times \Bigg\{ \Big(\frac{\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21}}{d_S(k)} + \frac{\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21}}{d_L(k)} \Big) \Big(\frac{\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{22}}{d_S(k')} - \frac{\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22}}{d_L(k')} \Big) \\
&\qquad - \Big(\frac{\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21}}{d_S(k)} - \frac{\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21}}{d_L(k)} \Big) \Big(\frac{\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{22}}{d_S(k')} + \frac{\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22}}{d_L(k')} \Big) \Bigg\} \\
&\quad \times \mathcal{M}(\phi \to K^0\overline{K^0}) \\
&= -(2\pi)^4\, \delta^{(4)}(q-p-p')\, \mathcal{M}(\phi \to K^0\overline{K^0})\, \frac{1}{1-\hat{\varepsilon}^2} \\
&\quad \times \Bigg\{ - \frac{\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21}}{p^2-m_S^2+im_S\Gamma_S}\, \frac{\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22}}{p'^2-m_L^2+im_L\Gamma_L} \\
&\qquad + \frac{\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21}}{p^2-m_L^2+im_L\Gamma_L}\, \frac{\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{22}}{p'^2-m_S^2+im_S\Gamma_S} \Bigg\} .
\end{aligned} \tag{41}$$

As in the previous subsection, the time evolution of the amplitude is obtained through the insertion of the explicit time-dependent propagator into the amplitude (41). The result is

$$T_{f_1 f_2} = \int dt\, dt' \Big(\mathcal{T}(t,t')\, \theta(t)\, \theta(t') + \text{ other terms} \sim \theta(-t) \text{ or } \theta(-t') \Big) \tag{42}$$

where t and t' are the times taken by unstable kaons to propagate from the common production point (x) up to their disintegration into f_1 at point y and f_2 at point z, respectively.

Thus, the explicit time evolution of the decaying amplitude is given by

$$\begin{aligned}
\mathcal{T}(t,t') &= (2\pi)^4 \delta^{(4)}(q-p-p')\, e^{ip^0 t + ip^{0'} t'}\, \frac{1}{4E_S E_L}\, \mathcal{M}(\phi \to K^0\overline{K^0})\, \frac{1}{1-\hat{\varepsilon}^2} \\
&\Big\{ -(\mathcal{M}_{11} + \hat{\varepsilon}\mathcal{M}_{21})(\hat{\varepsilon}\mathcal{M}_{12} + \mathcal{M}_{22})\, e^{-iE_S(p)t - \frac{1}{2}\Gamma_S \frac{m_S}{E_S} t}\, e^{-iE_L(p')t' - \frac{1}{2}\Gamma_L \frac{m_L}{E_L} t'} \\
&\quad + (\hat{\varepsilon}\mathcal{M}_{11} + \mathcal{M}_{21})(\mathcal{M}_{12} + \hat{\varepsilon}\mathcal{M}_{22})\, e^{-iE_S(p')t' - \frac{1}{2}\Gamma_S \frac{m_S}{E_S} t'}\, e^{-iE_L(p)t - \frac{1}{2}\Gamma_L \frac{m_L}{E_L} t} \Big\}
\end{aligned} \tag{43}$$

where $E_{S,L}(p) \equiv \sqrt{\vec{p}^{\,2} + m_{S,L}^2}$.

As we have already pointed out in the case of the CPLEAR experiment, no boost transformations are required to adequate the time evolution of the decay amplitude to a given reference frame. Observe that, due to the initial antisymmetrisation of the $K^0\overline{K^0}$ system, $\mathcal{T}(t,t) = 0$ if $f_1 = f_2$ and $p = p'$ as noted in Ref. [16].

5 Discussion and Conclusions

In this paper we have discussed a formalism based on quantum field theory which describes a system of two unstable coupled pseudoscalar particles. Properties related to the production and decay of these unstable states are consistently incorporated into the relativistic S-matrix amplitude which is the physically meaningfull object for a given process [7]. Therefore, this formalism does not exhibit the limitations intrinsic to the Wigner-Weisskopf approximation or non-relativistic approaches that we have discussed in the introduction of this paper.

We have applied this formalism to describe the time evolution of the K^0 and $\overline{K^0}$ decay amplitudes in CPLEAR and DAPHNE experiments. Since Lorentz covariance is implicit to the field theory approach, our results are valid in any reference frame contrary to the results obtained in other approaches which require boost transformations to relate the time parameters defined in rest and moving frames. Let us mention that other papers, which depart from the WW approximation, have appeared recently [17, 18, 19, 20]; however, they all present limitations mainly related to the introduction of a proper time parameter which is intrinsic to non-relativistic treatments. Notice also that an interference term showing time oscillations will appear in the decay probabilities obtained from Eqs. (34), (35) and (43). This term can be converted to a evolution in space by using the classical formula $t = (E/|\vec{p}|)|\vec{x}|$, which applies only to particles observed in the detector and not to (off-shell) unstable K_L and K_S states.

Finally, we would like also to stress that the present formalism allows to define direct ($\chi_{+-,\ 00}$) and indirect ($\hat{\varepsilon}$) CP-violating parameters for K decays without relying on assumptions based on isospin symmetry. This is particularly suitable because the factorization of strong rescattering effects may not be fully justified for the study of CP violation in B decays.

Let us remark that the present formalism can be extended straightforwardly to treat the isospin violation in the pion electromagnetic form factor in the $\rho - \omega$ resonance region [21].

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