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Matching conditions and Higgs mass upper bounds reexamined

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Abstract

Matching conditions relate couplings to particle masses. We discuss the importance of one-loop matching conditions in Higgs and top-quark sector as well as the choice of the matching scale. We argue for matching scales $\mu_{0,t} \simeq m_t$ and $\mu_{0,H} \simeq \max\{m_t, M_H\}$. Using these results, the two-loop Higgs mass upper bounds are reanalyzed. Previous results for $\Lambda \approx$ few TeV are found to be too stringent, and a recent update is found to be wrong. For $\Lambda = 10^{19}$ GeV we find $M_H < 180 \pm 4 \pm 5$ GeV, the first error indicating the theoretical uncertainty, the second error reflecting the experimental uncertainty due to $m_t = 175 \pm 6$ GeV. Hence a Higgs mass of about 160–170 GeV certainly satisfies both upper and lower Higgs mass bounds for cutoff scales up to $\Lambda = 10^{19}$ GeV if $m_t = 175$ GeV. For such Higgs and top masses the renormalization group behaviour of the minimal standard model does not require new physics to set in before the Planck scale.

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I. INTRODUCTION

The Higgs sector of the minimal standard model (SM) is usually considered to be an effective theory. The possible triviality problem connected to the underlying ϕ^4 theory [1] can be avoided if new physics appears at some high energy Λ. Depending on the specific value of Λ, an upper bound on the mass M_H of the SM Higgs boson can be derived [2–4]. This upper bound is connected to an unsatisfactory high energy behaviour of the Higgs quartic self-coupling λ if M_H is large. It manifests itself in the (one-loop) Landau pole [5] when using a perturbative approach, or in large cutoff effects when performing lattice calculations [6–9].

Previous work [3,4] extensively investigated the dependence of the M_H upper bound on the top quark mass m_t. The discovery of the top quark and the steadily improving mass determination of m_t allows for an update of this work and leads to the question: Which uncertainties are remaining in the theoretical prediction of the M_H upper bound? Using the perturbative approach up to two loops, we investigate the sensitivity of the M_H upper bound with regard to various cutoff criteria, the inclusion of matching corrections, and the choice of the matching scale μ_0. We show that choosing M_Z as matching scale, taken in some earlier analyses, is unreasonable and can lead to unreliable predictions. This result is also important for higher-order calculations other than two-loop Higgs mass upper bounds.

Taking into account the results from lattice calculations [7–9], we investigate the dependence of the M_H upper bound on the cutoff condition. In the case of a low cutoff scale, $\Lambda \simeq 10^3 - 10^4$ GeV, we find the sum of theoretical uncertainties to be large, $O(200$ GeV). Earlier work underestimated these uncertainties and gave too stringent bounds. At high cutoff scale, $\Lambda \simeq 10^{15} - 10^{19}$ GeV, the theoretical uncertainties are much smaller, $O(10$ GeV). Taking the present-day value of the top quark mass, $m_t = 175 \pm 6$ GeV [10] and a cutoff scale $\Lambda = 10^{19}$ GeV, we find an upper bound $M_H < 180 \pm 4 \pm 5$ GeV, where the first error estimates the theoretical uncertainty, and the second error indicates the remaining top quark mass dependence. Comparing our results with recent vacuum stability bounds [11–13], we find that a Higgs mass of about 160–170 GeV satisfies both upper and lower Higgs mass bounds for cutoff scales up to $\Lambda = 10^{19}$ GeV if $m_t = 175$ GeV. For such Higgs and top masses the renormalization group behaviour of the minimal standard model does not require new physics to set in before the Planck scale. The recent update of Higgs mass upper bounds in [14] is wrong.

In Sec. II we review the scale dependence of the SM matching conditions. We argue that the most reasonable choice of the matching scale for the Higgs quartic coupling is $\mu_{0,H} \simeq \max\{m_t, M_H\}$, and the top quark Yukawa coupling should be fixed at $\mu_{0,t} \simeq m_t$. In particular, the use of the scale M_Z leads to unreliable results in the case of the Higgs coupling. Using these observations, we calculate the SM Higgs mass upper bounds at the two-loop level in Sec. III.

II. MATCHING CONDITIONS

We start with a detailed look at the so-called matching conditions in the SM: the relations between the physical masses and the corresponding running couplings. This part of our letter is therefore not specific to the calculation of Higgs mass upper bounds but has further applications.

The $\overline{\rm MS}$ Higgs quartic coupling, $\bar{\lambda}$, and top Yukawa coupling, $\bar{g}_t$, are related to M_H and m_t using the following matching conditions:

$$\bar{\lambda}(\mu_0) = \frac{M_H^2}{2v^2}[1+\delta_H(\mu_0)]\,, \tag{2.1}$$

$$\bar{g}_t(\mu_0) = \frac{\sqrt{2}m_t}{v}[1+\delta_t(\mu_0)]\,, \tag{2.2}$$

where $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV. The definitions of the corresponding tree level couplings are obtained by dropping the matching corrections δ, thus fixing our notation. The use of two-loop renormalization group (RG) equations in connection with $\overline{\rm MS}$ couplings requires one-loop expressions for the corrections $\delta_H(\mu_0)$ and $\delta_t(\mu_0)$; they are given in [15] and [16], respectively.

In the case of the electroweak gauge couplings, one-loop matching corrections have also been calculated [17,18]. However, it is custom to extract the $\overline{\rm MS}$ gauge couplings directly using $\overline{\rm MS}$ definitions for experimental observables. The measured values for the $\overline{\rm MS}$ electroweak mixing angle and QED coupling fix the $\overline{\rm MS}$ electroweak couplings at the scale $\mu_0 = M_Z = 91.187$ GeV [19]:

$$\bar{\alpha}^{-1}(M_Z) = 4\pi\frac{\bar{g}^2(M_Z)+\bar{g}'^2(M_Z)}{\bar{g}^2(M_Z)\bar{g}'^2(M_Z)} = 127.90\,, \tag{2.3}$$

$$\sin^2\theta_W^{\overline{\rm MS}}(M_Z) = \frac{\bar{g}'^2(M_Z)}{\bar{g}^2(M_Z)+\bar{g}'^2(M_Z)} = 0.2315\,. \tag{2.4}$$

The $\overline{\rm MS}$ electroweak couplings are obtained as $\bar{g}(M_Z) = 0.651...$ and $\bar{g}'(M_Z) = 0.357....$

For comparison, it is nevertheless interesting to define gauge sector matching conditions in analogy to Eqs. (2.1) and (2.2), that is, using gauge boson masses and matching corrections: $\bar{g}^2(M_Z) \equiv \frac{4M_W^2}{v^2}(1+\delta_W)$ and $\bar{g}^2(M_Z)+\bar{g}'^2(M_Z) \equiv \frac{4M_Z^2}{v^2}(1+\delta_Z)$. Taking $M_W = 80.35$ GeV and M_Z as above one obtains $\delta_W \approx -0.4\%$ and $\delta_Z \approx 0.7\%$. As we will see below, the one-loop matching corrections δ_t and especially δ_H are significantly larger.

In the following we examine in detail the interesting structure of the matching corrections $\delta_H(\mu_0)$ and $\delta_t(\mu_0)$ as a function of μ_0 and M_H. For δ_H, the heavy top mass of $m_t = 175$ GeV changes drastically the original discussion[1] presented in [15], except for $M_H \gg m_t$.

Using the result of [15], the correction $\delta_H(\mu_0)$ can be rewritten in the following way:

$$\delta_H(\mu_0) = \frac{2v^2}{M_H^2}\frac{1}{32\pi^2v^4}\{h_0(\mu_0)+M_H^2h_1(\mu_0)+M_H^4h_2(\mu_0)\} \tag{2.5}$$

with

[1]The analysis of [15] is based on a value m_t=40 GeV.

$$h_0(\mu_0) = -24m_t^4 \ln\frac{\mu_0^2}{m_t^2} + 6M_Z^4 \ln\frac{\mu_0^2}{M_Z^2} + 12M_W^4 \ln\frac{\mu_0^2}{M_W^2} + c_0\,, \tag{2.6}$$

$$h_1(\mu_0) = 12m_t^2 \ln\frac{\mu_0^2}{m_t^2} - 6M_Z^2 \ln\frac{\mu_0^2}{M_Z^2} - 12M_W^2 \ln\frac{\mu_0^2}{M_W^2} + c_1\,, \tag{2.7}$$

$$h_2(\mu_0) = \frac{9}{2}\ln\frac{\mu_0^2}{M_H^2} + \frac{1}{2}\ln\frac{\mu_0^2}{M_Z^2} + \ln\frac{\mu_0^2}{M_W^2} + c_2\,. \tag{2.8}$$

The constants c_i are independent of μ_0. For m_t=175 GeV and 75 GeV $< M_H <$ 570 GeV their total contribution to δ_H is in magnitude less than 0.02, though some individual terms can exceed 0.05. Depending on the choice of μ_0, the logarithmic terms in Eqs. (2.6) – (2.8) can yield a much larger correction. In Fig. 1a we show the one-loop result of δ_H as a function of μ_0 and M_H for m_t = 175 GeV. We find that the matching correction δ_H can be in magnitude larger than 25% for various regions in the parameter space (μ_0, M_H), even exceeding 100%. Clearly the matching correction should be taken into account and the choice of the matching scale μ_0 is important: some choices are more appropriate than others.

To discuss the dependence of δ_H on μ_0 we consider its derivative:

$$\begin{aligned}\frac{d\delta_H(\mu)}{d\mu} &= \frac{1}{\mu}\frac{2v^2}{M_H^2}\frac{1}{8\pi^2v^4}\left[3M_H^4 - 3M_H^2(M_Z^2 + 2M_W^2 - 2m_t^2) + 3M_Z^4 + 6M_W^4 - 12m_t^4\right] \\ &= \frac{1}{\mu}\frac{2v^2}{M_H^2}\beta_\lambda\,,\end{aligned} \tag{2.9}$$

where β_λ is the one-loop beta function of the coupling λ expressed in terms of the different physical masses rather than in terms of the various $\overline{\rm MS}$ couplings (which is consistent at one-loop order). For m_t = 175 GeV, β_λ equals zero if $M_H \simeq$ 208 GeV. Taking M_H to be different from this value, β_λ quickly becomes large. If $M_H \ll$ 208 GeV, the m_t^4 contribution dominates and $\beta_\lambda \ll 0$; if $M_H \gg$ 208 GeV, the M_H^4 contribution causes $\beta_\lambda \gg 0$. Correspondingly, the magnitude of δ_H is insensitive to the choice of μ_0 only for a small range of M_H values, see Fig. 1.

Natural choices of μ_0 in Eq. (2.1) are the various masses appearing in the logarithms in Eqs. (2.6)–(2.8): M_H, m_t or M_Z. Since the impact of the choice of μ_0 is connected to the value of M_H, we consider three cases:

(1) $\underline{M_H \ll m_t\ (M_H \simeq \text{70-100 GeV})}$: This is the range where $\beta_\lambda \ll 0$ due to the dominant m_t^4 term in Eq. (2.9). Such a large contribution to β_λ is possible for low values of M_H because there is no symmetry in the scalar sector which imposes β_λ to go to 0 for $\lambda \to 0$. This is in contrast to the beta functions of the gauge and Yukawa sectors. Consequently, the coefficients of the logarithmic terms in δ_H can be large for small values of M_H, actually going to infinity as $M_H \to 0$. (In contrast, the coefficients in the matching corrections of the non-scalar sectors vanish or approach a finite constant if the corresponding particle mass goes to zero). Indeed, the m_t^4 term in β_λ gives rise to the large coefficient m_t^4/M_H^2 which multiplies $\ln(m_t^2/\mu_0^2)$. (The overall factor $1/M_H^2$ is present because δ_H is the ratio of the loop contribution to the lowest order contribution to $\bar\lambda$, the latter being proportional to M_H^2). Consequently if M_H is small then δ_H is small only if μ_0 is chosen close to m_t, not M_H. For example, if M_H = 70 GeV and m_t = 175 GeV, we find $\delta_H(M_H) \simeq 80\%$ whereas $\delta_H(m_t) \simeq -0.7\%$. The dominance of the $\ln(\mu_0^2/m_t^2)$ term indicates that the top mass scale

is the correct scale of reference for low values of M_H. Interestingly, even if the top one-loop correction to δ_H is large, perturbation theory is still applicable : δ_H is formally the product of a series in powers of g_t and λ, with an overall factor $1/M_H^2$. The higher-order terms contributing to δ_H are expected to be small in the same way in which the two-loop term of β_λ [20,21] is smaller than the one-loop contribution to β_λ.[2]
(2) $\underline{M_H \simeq 0.8\text{-}1.7\ m_t}$: Taking $m_t = 175$ GeV, the function β_λ features a zero in this Higgs mass range, indicating that both Higgs and top-quark contributions have similar weight. Both $\mu_0 = m_t$ and $\mu_0 = M_H$ are acceptable choices. In fact, we find the Higgs matching corrections to satisfy $|\delta_H| < 5\%$ for a large range of μ_0 around $\mu_0 \simeq M_H \simeq m_t$. This property remains true if the top quark mass has a value somewhat different from $m_t = 175$ GeV. Choosing $\mu_0 = \max\{m_t, M_H\}$, a variation of 160 GeV $< m_t <$ 190 GeV results in $-1.1\% < \delta_H(m_t) < -1.0\%$ if $M_H = 140$ GeV, and $2.4\% < \delta_H(M_H) < 3.6\%$ if $M_H = 300$ GeV.
(3) $\underline{M_H \gg m_t}$.: Such a value of M_H causes a large and positive value of β_λ. The leading logarithmic contribution to δ_H is the $M_H^2 \ln(\mu_0^2/M_H^2)$ term which can be suppressed choosing $\mu_0 \simeq M_H$. Yet the other terms, including $\ln(\mu_0^2/m_t^2)$, are viable for $\mu_0 = M_H$. For example, $M_H = 570$ GeV results in $\delta_H(M_H) \simeq 20\%$. For larger M_H the matching correction approaches the heavy-Higgs result

$$\delta_H = \frac{M_H^2}{32\pi^2 v^2}\left(12 \ln \frac{\mu_0^2}{M_H^2} + 25 - 3\pi\sqrt{3}\right) . \tag{2.10}$$

A possible choice, used in [15], would be $\mu_0 \simeq 0.7 M_H$ such that $\delta_H(\mu_0) \simeq 0$. This approach, however, fails at two loops since the two-loop heavy-Higgs terms are sizeable [23]. Adding these two-loop contributions to the full one-loop result of δ_H, we show the resulting μ_0 dependence in Fig. 1b. A satisfactory perturbative behaviour is obtained for $\mu_0 = M_H$ if $M_H < O(800$ GeV). The choices $\mu_0 = m_t$ or M_Z are inappropriate since they lead to unreliable perturbative predictions for even smaller values of M_H. In particular, the choice $\mu_0 = M_Z$ leads to $\delta_H < -1.0$ for $M_H > 690$ GeV, resulting in an unphysical *negative* $\overline{\text{MS}}$ Higgs coupling.
Summarizing our results for the three different Higgs-mass scenarios described above, we find the scale $\mu_0 \simeq \max\{m_t, M_H\}$ to be the appropriate Higgs matching scale for $m_t \simeq 175$ GeV. The calculation of the M_H upper bound (see Sect. II) is an example how physical quantities are sensitive to the choice of μ_0.

Next we consider the matching correction $\delta_t(\mu_0)$ entering Eq. (2.2). It has been given at one loop in [16], with the dominant QCD correction given earlier in [24] and the Yukawa corrections in [25]. The result can be written as

$$\delta_t(\mu_0) = \left(-4\frac{\alpha_s}{4\pi} - \frac{4}{3}\frac{\alpha}{4\pi} + \frac{9}{4}\frac{m_t^2}{16\pi^2 v^2}\right) \ln \frac{\mu_0^2}{m_t^2} \ + \ c_t \, , \tag{2.11}$$

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[2]The simultaneous largeness and perturbativity of the top quark contribution in the scalar sector could be the origin of the symmetry breaking of SU(2) $\times$ U(1). A recent model [22] using this approach yields $M_H \simeq$ 80-100 GeV.

where c_t is independent of μ_0 and can be evaluated using the results in [16].[3] Taking $\alpha_s(M_Z) = 0.118$ [26], we find $-0.052 < c_t < -0.042$ for top quark mass of 150 GeV $< m_t <$ 200 GeV and Higgs mass of 50 GeV $< M_H <$ 600 GeV. The correction due to c_t is therefore in magnitude larger than the sum of the μ_0-independent contributions c_i to δ_H. The largeness of c_t is mostly due to the QCD correction. In contrast, there is no one-loop QCD correction contributing to δ_H.

Since LEP I provides the result for α_s at scale M_Z, it seems plausible to use a matching scale $\mu_0 = M_Z$. This yields $\delta_t(M_Z) \simeq O(-2\%)$ as can be seen in Fig. 2. Looking at the logarithm appearing in Eq. (2.11), however, the adequate choice is $\mu_0 \simeq m_t$: no other particle mass enters the μ_0-dependent logarithms. With this choice we immediately obtain $\delta_t(m_t) = c_t = O(-5\%)$. Here the difference in taking $\alpha_s(M_Z)$ *vs.* $\alpha_s(m_t)$ amounts to higher-order corrections which are suppressed.

The present-day experimental result of $m_t = 175 \pm 6$ GeV [10] leads to $\pm 3.4\%$ uncertainty in the tree-level result of g_t. Comparing with the results above, we find the one-loop matching correction δ_t to be of equal importance. This concludes our review of the matching conditions.

III. HIGGS MASS UPPER BOUNDS

The triviality problem of the SM is completely fixed by the beta functions of the theory. The functions β_i for all SM couplings have been calculated in the $\overline{\text{MS}}$ scheme up to two loops [20,21,27]. At the one-loop level, a heavy Higgs particle gives rise to a positive function β_λ, causing the running Higgs quartic coupling $\lambda(\mu)$ to permanently increase as μ increases. At some value $\mu = \Lambda_L$, the position of the one-loop Landau pole [5], the Higgs running coupling becomes infinite: perturbation theory has ceased to be meaningful long before.

At the two-loop level, a heavy Higgs mass causes $\lambda(\mu)$ to approach an ultraviolet (metastable) fixpoint. This fixpoint is almost entirely determined by the leading Higgs coupling contributions to β_λ at two loops:

$$\beta_\lambda = 24\frac{\lambda^2}{(16\pi^2)^2} - 312\frac{\lambda^3}{(16\pi^2)^3}\,. \tag{3.1}$$

The resulting fixpoint value, corresponding to $\beta_\lambda = 0$, is

$$\lambda_{\text{FP}} = 12.1...\ . \tag{3.2}$$

Increasing the scale μ even further, the growing value of the running top quark coupling can no longer be neglected and changes β_λ, hence modifying the above fixpoint behaviour. Since perturbation theory is already meaningless even before $\lambda(\mu)$ reaches λ_{FP}, we are not concerned about the details of the $\lambda(\mu)$ behaviour beyond the metastable fixpoint.

At three loops, only the leading contribution to β_λ is known [23,28,29]. It causes the running Higgs coupling to again have a Landau singularity. Since the complete set of three-loop SM beta functions and the corresponding two-loop matching conditions are not yet available, we restrict our present analysis to two-loop beta functions.

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[3] There is a misprint in Table I of [16]: the term 6.90×10^{-3} should have the opposite sign.

To obtain M_H upper bounds from the RG evolution of $\lambda(\mu)$ to some embedding scale $\mu = \Lambda$, one has to choose a cutoff value for $\lambda(\Lambda)$. We denote this cutoff condition by $\lambda_c(\Lambda)$. At one loop, the standard choice is to require that $\lambda(\mu)$ avoids the Landau singularity for $\mu < \Lambda$. This corresponds to $\lambda_c(\Lambda) = \infty$. At two loops, the running Higgs coupling remains finite and $\lambda(\mu) \to \lambda_{\rm FP}$ as μ increases. The perturbative approximation, however, fails long before reaching the fixpoint. Therefore we examine two different two-loop cutoff conditions:

$$\lambda_c(\Lambda) = \lambda_{\rm FP}/4 \qquad \text{and} \qquad \lambda_c(\Lambda) = \lambda_{\rm FP}/2\,. \tag{3.3}$$

The first choice corresponds to a two-loop correction of 25% to the one-loop beta function β_λ, see Eq. (3.1). Perturbation theory is expected to be reliable for such a value of $\lambda(\Lambda)$ [30]. The second choice causes a 50% correction, and its value is comparable with upper bounds on $\lambda(\Lambda)$ which can be obtained from lattice calculations [7–9]. In addition, it is also relatively close to the upper bound of the perturbative regime [30].

Choosing four different embedding scales, $\Lambda = 10^3$, 10^6, 10^{10}, and 10^{16} GeV, we give in Fig. 3 the different values of $\lambda(M_Z)$ which lead to the corresponding cutoff conditions $\lambda_c(\Lambda)$ when evolving all SM couplings from M_Z to Λ. The one-loop result in Fig. 3 with $\lambda_c(\Lambda) = \infty$ agrees with the result obtained by Lindner [3] when setting $\lambda(M_Z) = M_H^2/2v^2$ and $g_t(M_Z) = \sqrt{2}m_t/v$, and taking into account the updated experimental input for the various couplings at $\mu_0 = M_Z$.[4] The recent results in [14] which question the Lindner results at all scales Λ are incorrect.[5]

Taking instead the value $\lambda_c(\Lambda) = \lambda_{\rm FP}/4$ at one loop, we find a value of $\lambda(M_Z)$ for which perturbation theory is definitely reliable when evolving all SM couplings to Λ. For $\Lambda = 10^{16}$ GeV the one-loop perturbative upper bound on $\lambda(M_Z)$ is only slightly less than the nonperturbative value obtained using the Landau pole criterion, indicating the insensitivity of the upper bound to the cutoff condition. For $\Lambda = 10^3$ GeV, however, the perturbative upper bound is about 50% less than the Landau-pole bound, a sign for a strong dependence on the cutoff condition $\lambda_c(\Lambda)$.

Going to two loops, the perturbative bound corresponding to $\lambda_c(\Lambda) = \lambda_{\rm FP}/4$ differs from the corresponding one-loop result by less than 12%: perturbation theory indeed seems applicable. The maximal upper bound as modelled by $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$ gives upper bounds on $\lambda(M_Z)$ which are of the order of the one-loop Landau pole bounds.

We conclude that our two-loop cutoff conditions are suitable for representing two scenarios: $\lambda_c(\Lambda) = \lambda_{\rm FP}/4$ corresponds to a perturbatively reliable Higgs sector at embedding scale Λ, and the condition $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$ is at the verge of being nonperturbative.

The procedure for obtaining an M_H upper bound from the bound on $\lambda(M_Z)$ is as follows. The couplings $\lambda(M_Z)$ and $g_t(M_Z)$ in Fig. 3 are $\overline{\rm MS}$ couplings at $\mu = M_Z$. The $\overline{\rm MS}$ gauge couplings are fixed at M_Z using Eqs. (2.3) and (2.4), and $\alpha_s(M_Z) = 0.118$. The matching

[4]In the case of $\Lambda = 10^3$ GeV, for which Lindner [3] only gives a qualitative estimate, we find a slightly higher upper bound on $\lambda(M_Z)$.

[5]For large scale Λ, the errors in [14] seem to be partially connected to the errorneous use of 10^n instead of e^n in all equations and figures where Λ is specified. This replacement, however, still does not correct all their results.

scale for the top quark coupling is taken according to our previous discussion (Sect. I) as $\mu_{0,t} \equiv m_t$, and we take $m_t = 175$ GeV. (Taking $\mu_{0,t} = M_Z$ has little effect on the final numerical results.) The matching scale for the Higgs coupling is chosen to be $\mu_{0,H} \equiv M_H$ as argued above. With these settings, we evolve $\bar{\lambda}(M_Z)$ and all other SM couplings from M_Z to some value $\mu_{0,H}$ such that Eq. (2.1) is solved for some value M_H with $\mu_{0,H} = M_H$. Subsequent evolution to $\mu_{0,t}$ checks the top quark matching condition, Eq. (2.2), using $m_t = 175$ GeV and the value of M_H found in the previous step. If the top quark matching condition is not satisfied, we iterate our procedure, starting at scale M_Z with a different value of $\bar{g}_t(M_Z)$. Eventually, we find a final solution for M_H which is consistent with both matching conditions. To investigate the importance of the one-loop matching corrections, we repeat the above procedure taking the matching corrections δ_H and δ_t to be zero.

In Fig. 4 we show the resulting two-loop upper bound on M_H with and without the use of matching corrections, fixing the cutoff condition as $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$. Using the choice $\mu_{0,H} = M_H$, the comparison of the solid line (with matching corrections) and long-dashed line (without matching corrections) allows for a conservative estimate of higher order corrections. We find that the difference of the two results can exceed 100 GeV at small embedding scale Λ, but reduces to less than about 6 GeV at large scale.

In addition to the preferred choice $\mu_{0,H} = M_H$, we also give results when using $\mu_{0,H} = M_Z$. For large embedding scale Λ (resulting in small values of M_H), the two different choices of $\mu_{0,H}$ give similar results. For small scale Λ, the difference is significant (Fig. 4, dotted line). This was already anticipated in a one-loop study of pure ϕ^4 theory which underlies the SM Higgs sector [31]. However, the inclusion of matching corrections (short-dashed curve) shows that the scale choice $\mu_{0,H} = M_Z$ is completely inadequate for large values of M_H as indicated by the largeness of the corrections compared to the choice $\mu_{0,H} = M_H$. Even more strikingly, values $\Lambda < 2 \times 10^4$ GeV (which lead to bounds $\lambda(M_Z) > 1.2$ when using $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$) have no solution in M_H which satisfy the $\overline{\rm MS}$ matching condition. This is due to the fact that the choice $\mu_{0,H} = M_Z$ restricts the $\overline{\rm MS}$ coupling to a maximal value $\bar{\lambda}(M_Z) = 1.2$ which is obtained for $M_H \approx 495$ GeV. We will only consider the $\mu_{0,H} = M_H$ results when determining the final M_H upper bounds.

The results of Fig. 4 can also be compared with the two-loop results of [4]. There no matching corrections have been included, and $\mu_{0,H} = M_Z$ is used. The cutoff-condition $\lambda_c(\Lambda)$ is determined as a turning point in the two-loop calculation rather than a fixed value. This procedure yields larger two-loop values of $\lambda_c(\Lambda)$ than used here. The resulting M_H bounds are therefore larger than our corresponding result with $\mu_{0,H} = M_Z$ and no matching corrections, but with $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$.

In Fig. 5 we analyse the dependence of the upper M_H bound on m_t. Varying m_t in the range 150–200 GeV, the bound on M_H changes less than 40 GeV for the largest embedding scale considered, $\Lambda = 10^{19}$ GeV. The latest experimental result [10], $m_t = 175 \pm 6$ GeV, reduces this uncertainty to less than 5 GeV at the 1σ level. For embedding scales $\Lambda < 10^{10}$ GeV the uncertainty due to m_t can then entirely be neglected compared to the theoretical uncertainties connected to the cutoff condition and higher-order corrections. The uncertainty in the QCD coupling, $\alpha_s(M_Z) = 0.118 \pm 0.003$ [26], causes a shift of less than 1 GeV in the M_H upper bound, with the maximal effect at $\Lambda = 10^{19}$ GeV.

In summary, we have discussed the uncertainties in the M_H upper bound due to the choice of the cutoff condition (Fig. 3), the importance of one-loop matching corrections and the choice of the matching scale $\mu_{0,H}$ (Fig. 4), and the top-quark mass dependence (Fig. 5). Fixing the top quark mass to be 175 GeV, using two-loop beta functions and appropriately choosing the matching scale to be $\mu_{0,H} = M_H$, we find the sum of all theoretical uncertainties to be represented by the upper solid area indicated in Fig. 6. They are obtained by choosing $\mu_{0,H} = M_H$ and using matching conditions with and without one-loop matching corrections. The cutoff condition is varied between $\lambda_c(\Lambda) = \lambda_{\rm FP}/4$ and $\lambda_{\rm FP}/2$. The lower edge of the solid area indicates a value of M_H for which perturbation theory is certainly reliable up to scale Λ; in particular, the triviality problem of the standard model is clearly avoided for such values of Λ and M_H. The upper edge of the solid area can be used to estimate the scale $\Lambda(M_H)$ at which the standard model ceases to be meaningful as an effective theory. Although the perturbative approach does not allow for extraction of absolute upper bounds, the consideration of lattice calculations in ϕ^4 theory seems to reinforce or even tighten the upper bounds presented here [7,9,8,30]. For low values of Λ, the one-loop Landau pole bounds of [3] are found to be near the perturbative lower edge of the upper solid area in Fig. 6. The additional experimental uncertainty due to the top quark mass is represented by the cross-hatched area in Fig. 6, generously varying the top-quark mass from 150 GeV to 200 GeV. The present-day 1σ result of $m_t = 175 \pm 6$ GeV is sufficient to make it the smallest source of error except for large values of the embedding scale Λ. In particular, we find:

$$M_H < 180 \pm 4 \pm 5 \text{ GeV} \qquad \text{if} \qquad \Lambda = 10^{19} \text{ GeV}, \tag{3.4}$$

the first error indicating the theoretical uncertainty, the second error reflecting the m_t dependence.[6]

For comparison, we also give the lower bounds on M_H from stability conditions on the SM Higgs effective potential. At large scale Λ, the stability bound is well approximated by requiring the Higgs running coupling to remain positive: $\lambda(\Lambda) > 0$. Such an analysis has been carried out at the two-loop level including matching corrections [11], and they agree within the theoretical errors with a more careful treatment of the one-loop effective potential [12]. The discrepancy at scales $\Lambda < 10$ TeV has been resolved recently [13], and we use the latter results. Fixing $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.118$ we show the lower bound in Fig. 6 (lower solid area), with the solid area indicating the theoretical uncertainty. At large Λ, the theoretical error is estimated by using $\mu_{0,H} = m_t$ and comparing the results with and without matching corrections, and at low Λ the theoretical error is ± 5 GeV according to [13]. The variation $m_t = 175 \pm 25$ GeV yields a much larger uncertainty in the M_H lower bound than in the M_H upper bound and is not shown.

Looking at Fig. 6 we see that a Higgs mass of about 160–170 GeV certainly satisfies both upper and lower Higgs mass bounds for cutoff scales up to $\Lambda = 10^{19}$ GeV if $m_t = 175$ GeV. For such Higgs and top masses the renormalization group behaviour of the minimal standard model does not require new physics to set in before the Planck scale.

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[6]The very recent result [32] of $M_H < 174$ GeV for $\Lambda = 10^{19}$ GeV is lower than our lowest result due to the use of the smaller cutoff condition $\lambda_c(\Lambda) = 5/3 < \lambda_{\rm FP}/4 \approx 3$ (our notation).

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FIGURES

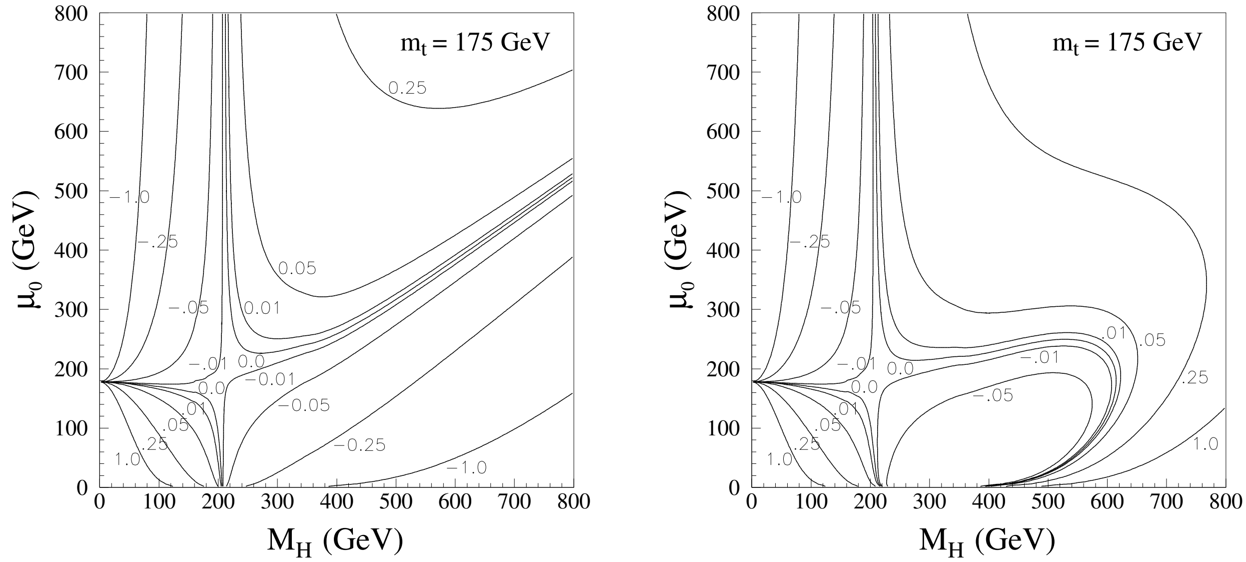

FIG. 1. (a) Values of μ_0 and M_H for which the one-loop Higgs matching correction $\delta_H(\mu_0)$, Eq. (2.1), equals the values indicated next to the various contour lines. The top quark mass is taken to be 175 GeV. (b) Same plot, but the leading two-loop heavy-Higgs corrections [23] have been added.

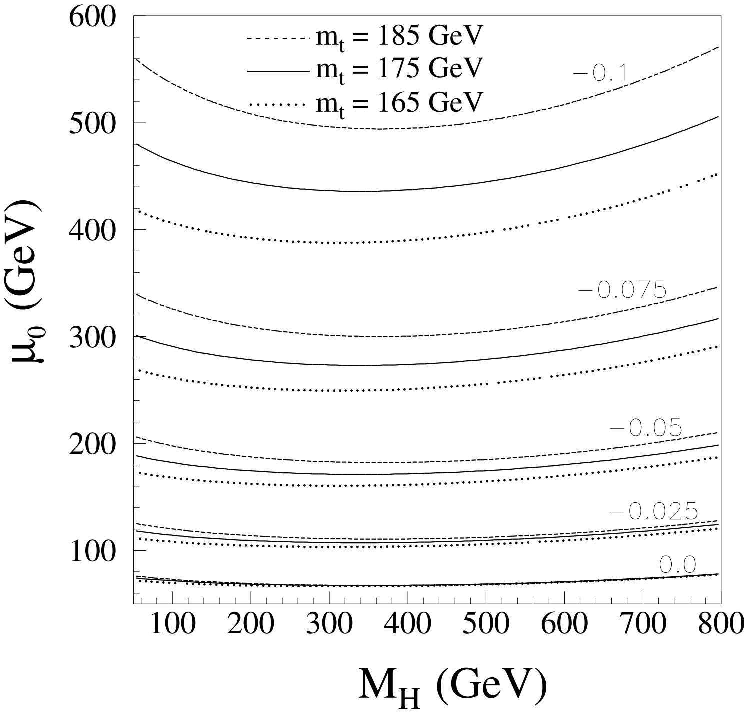

FIG. 2. Values of μ_0 and M_H for which top-quark matching correction $\delta_t(\mu_0)$, Eq. (2.2), equals the values indicated next to the various contour lines. Results are shown using $m_t = 165$ GeV (dotted), 175 GeV (solid), and 185 GeV (dashed).

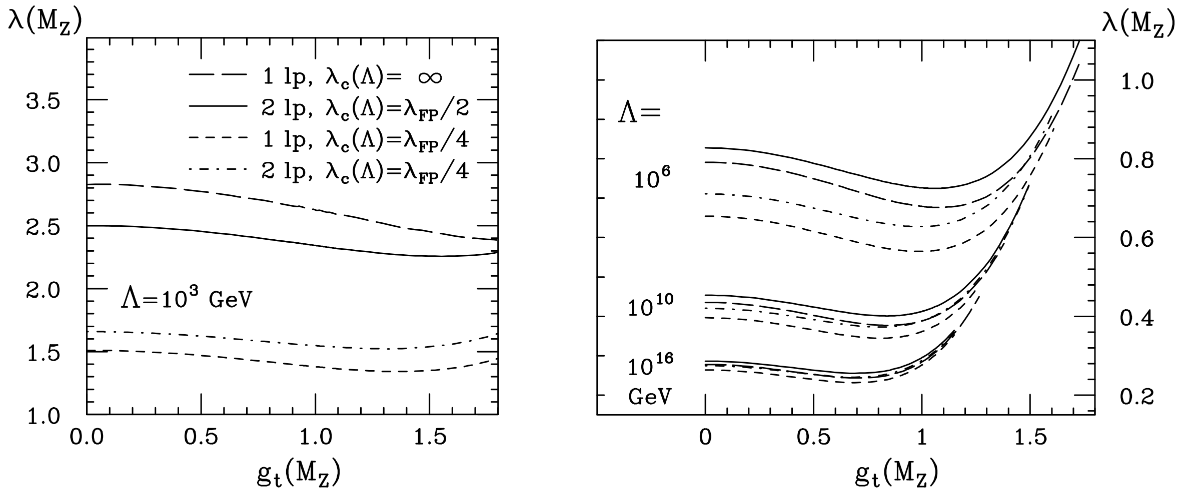

FIG. 3. Choosing either one-loop or two-loop RG evolution and various cutoff conditions $\lambda_c(\Lambda)$, the maximally allowed value of $\lambda(M_Z)$ is given as a function of $g_t(M_Z)$. The cutoff condition $\lambda_c(\Lambda)$ is imposed at scales $\Lambda = 10^3$ GeV (left plot) and $\Lambda = 10^6, 10^{10}, 10^{16}$ GeV (right plot).

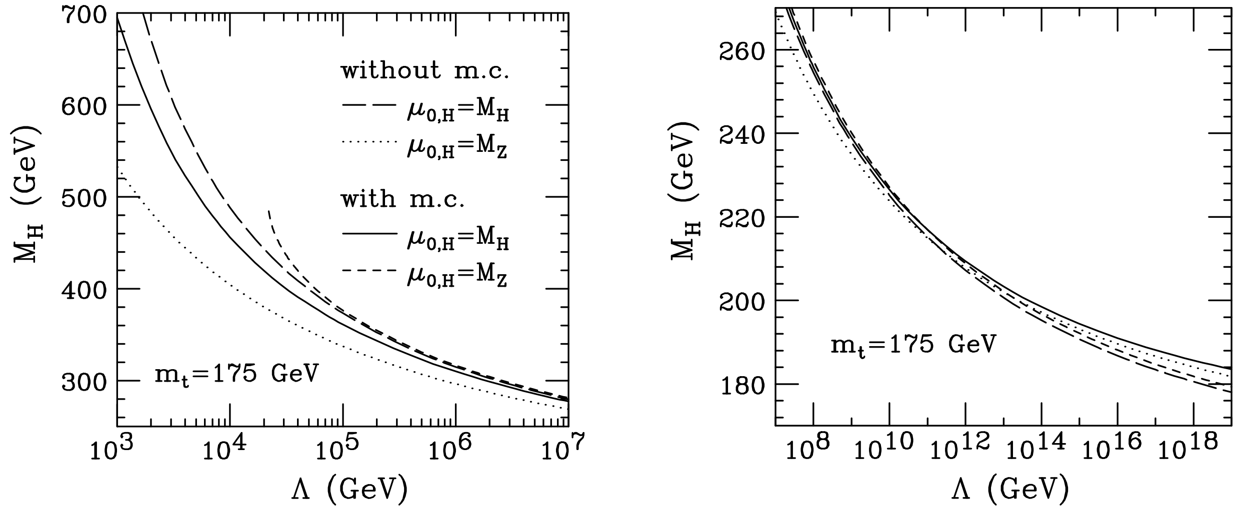

FIG. 4. Choosing two-loop RG evolution and cutoff condition $\lambda_c(\Lambda) = \lambda_{\rm FP}/2$, the upper bound on M_H is calculated. The running Higgs and Yukawa couplings, $\lambda(\mu)$ and $g_t(\mu)$, are fixed by the physical masses M_H and m_t using matching conditions with and without one-loop matching corrections. In addition, the Higgs matching scale is varied to be $\mu_{0,H} = M_H$ and M_Z. The top-quark mass is fixed at $m_t = 175$ GeV, and $\mu_{0,t} = m_t$. The left plot shows the result for small values of Λ, the right plot extends up to values of $\Lambda = 10^{19}$ GeV.

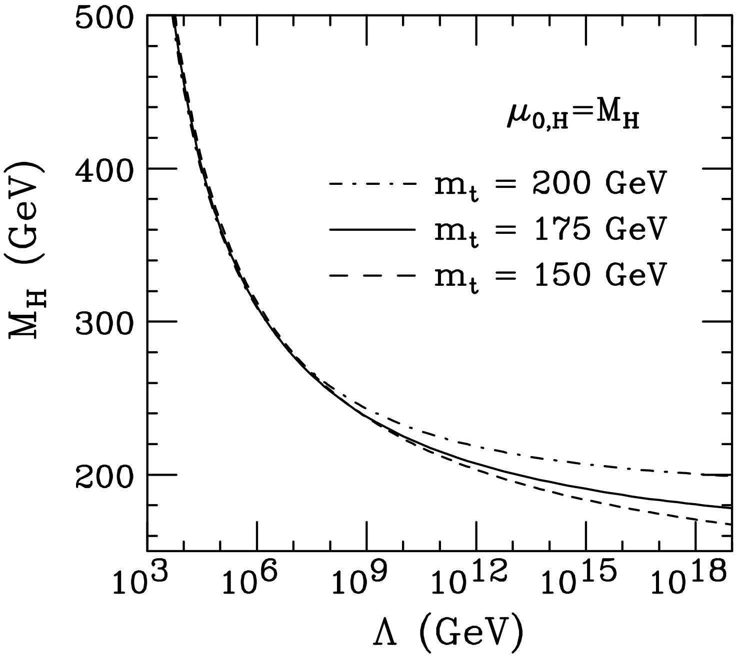

FIG. 5. The dependence of the upper M_H bound on the top-quark mass. The $\overline{\mathrm{MS}}$ matching conditions with $\mu_{0,H} = M_H$ and $\mu_{0,t} = m_t$ are used in connection with two-loop RG evolution and cutoff condition $\lambda_c(\Lambda) = \lambda_{\mathrm{FP}}/2$. For low values of the embedding scale Λ, the M_H upper bound is insensitive to the exact value of m_t. For large embedding scales there is a larger m_t dependence. Without matching corrections (not shown), the top mass dependence is qualitatively the same.

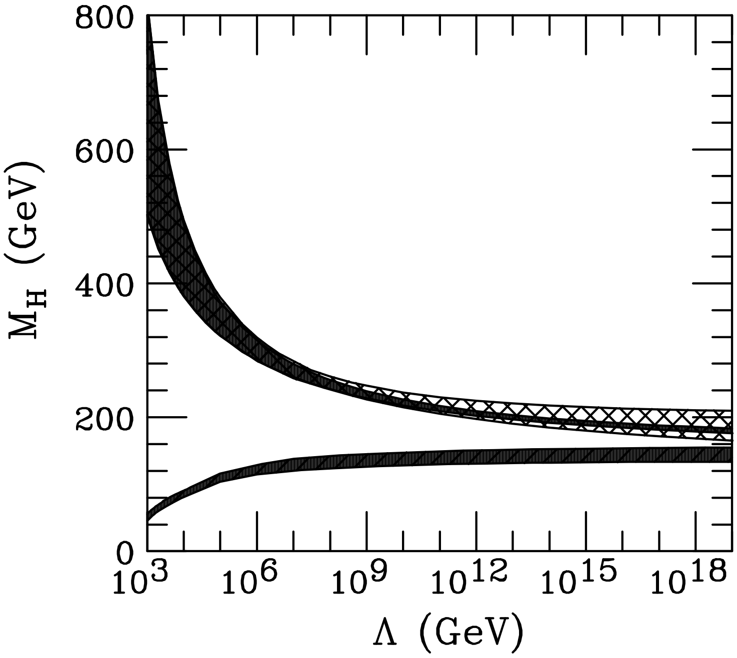

FIG. 6. Summary of the uncertainties connected to the bounds on M_H. The upper solid area indicates the sum of theoretical uncertainties in the M_H upper bound when keeping $m_t = 175$ GeV fixed. The cross-hatched area shows the additional uncertainty when varying m_t from 150 to 200 GeV. The upper edge corresponds to Higgs masses for which the SM Higgs sector ceases to be meaningful at scale Λ (see text), and the lower edge indicates a value of M_H for which perturbation theory is certainly expected to be reliable at scale Λ. The lower solid area represents the theoretical uncertaintites in the M_H lower bounds derived from stability requirements [11–13] using $m_t = 175$ GeV and $\alpha_s = 0.118$.