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Next-to-next-to-leading order QCD analysis of the revised CCFR data for xF_3 structure function and the higher twist contributions

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Abstract

We present the results of the next-to-next-to-leading order QCD analysis of the recently revised experimental data of the CCFR collaboration for the xF_3 structure function using the Jacobi polynomial expansion method. The effects of the higher twist contributions are included into the fits following the infrared renormalon motivated model. The special attention is paid to the checks of the predictive abilities of the infrared renormalon model and to the independent extractions of the x-dependent shape of the twist-4 contributions to the xF_3 structure function in the process of the leading order, next-to-leading order and next-to-next-to-leading order fits of the revised CCFR data. We stress that at the next-to-next-to-leading order the results for $\alpha_s(M_Z)$ turn out to be almost nonsensitive to the higher-twist terms. We obtain the following result $\alpha_s^{NNLO}(M_Z) = 0.117 \pm 0.002(stat) \pm 0.005(syst) \pm 0.003(theory)$. The comparison of the outcomes of our next-to-leading order and next-to-next-to-leading order analysis indicate that the theoretical QCD uncertainties were underestimated in the process of the next-to-leading order determination of $\alpha_s(M_Z)$, made recently by the CCFR collaboration itself.

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1. The detailed QCD studies of the behavior of the structure functions of the non-polarized deep-inelastic scattering (DIS) are still very important both from the phenomenological and theoretical points of view (see e.g. the reviews [1, 2]). The revision [3] of the experimental data of the CCFR collaboration, previously presented in Ref.[4] put on the agenda the reconsideration of the results of the fits of the old CCFR data, performed at the next-to-next-to-leading order (NNLO) of perturbative QCD both without higher twist [5] and with higher twist contributions [6] using the Jacobi polynomial expansion method. The results of application of this method for the extraction of the x-shape of the twist-4 contributions from the NNLO fits of the combined xF_3 data (besides the ones analyzed in Ref.[6]) can be found in Ref.[7]. The Jacobi polynomial approach was developed in Refs.[8]-[10] and successfully used in the process of the next-to-leading order (NLO) fits [11]-[14] and NNLO fits [15, 5, 6, 7] of different experimental data.

It should be stressed that at the NLO level the results obtained turned out to be in good agreement with the outcomes of applications of the traditional variant of the DGLAP method [16], based on the solution of the integro-differential equation (compare the results of Refs.[13, 5] and Ref.[4] in the case of neutrino-nucleon DIS and of Refs.[11, 12] with the ones of Ref.[17] in the case of the analysis of the data for the F_2 structure function, obtained by the BCDMS and SLAC collaborations). However, the Jacobi polynomial expansion method is also giving the possibility to include into the QCD fits the existing information about the NNLO approximations of the coefficient functions and anomalous dimensions of the finite number of Mellin moments of the DIS structure functions, calculated analytically in Refs.[18]- [20] and Ref.[21] respectively (for the detailed methodological description of the application of the Jacobi polynomial expansion machinery at the NNLO see Ref.[5]). Therefore, in view of the current lack of information about the NNLO corrections to the Alarelli-Parisi kernel, the Jacobi polynomial expansion method turns out to be the unique method of including selfconsistently the available NNLO perturbative QCD effects into the fits of the DIS structure functions data.

In the present work we demonstrate once more the compatibility of the DGLAP method and the Jacobi polynomial expansion approach at the NLO level and extract the values of the parameter $\Lambda^{(4)}_{\overline{MS}}$ from the revised CCFR data of Ref.[3] (hereafter called as the CCFR'97 data) with taking into account target mass corrections and twist-4 effects, which were introduced in our analysis using the concept of the infrared renormalon (IRR) model of Ref.[22]. At the next stage we include into our analysis the NNLO perturbative QCD corrections. We extract the x-shape of the twist-4 contributions to the xF_3 structure function at the leading order (LO), NLO and NNLO and compare it with the IRR model predictions of Ref.[22]. We also get the values of $\alpha_s(M_Z)$ both at the NLO and NNLO and compare them with the ones obtained by the CCFR collaboration in Ref.[3].

2. The application of the Jacobi polynomial expansion machinery at the NLO and NNLO was described in detail in Refs.[8]-[10] and Refs.[15, 5] correspondingly. Nevertheless we will introduce again some basic definitions. The NNLO QCD Q^2-evolution of the moments of the xF_3 structure function $M_n^{NS}(Q^2) = \int_0^1 x^{n-1}F_3(x,Q^2)dx$ (where $n = 2,3,4,...$) is determined by the solution of the renormalization group equation as

$$\frac{M_n^{NS}(Q^2)}{M_n^{NS}(Q_0^2)} = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right]^{\gamma^{(0)}_{NS}(n)/2\beta_0} H_n(Q_0^2,Q^2) \quad ,n \geq 2 \tag{1}$$

where

$$H_n(Q_0^2,Q^2) = \frac{[1+p(n)A_s(Q^2)+q(n)(A_s(Q^2))^2]}{[1+p(n)A_s(Q_0^2)+q(n)(A_s(Q_0^2))^2]}\frac{C^{(n)}_{NS}(A_s(Q^2))}{C^{(n)}_{NS}(A_s(Q_0^2))} \tag{2}$$

and $C^{(n)}_{NS}(A_s(Q^2)) = 1 + C^{(1)}(n)A_s(Q^2) + C^{(2)}(n)A_s^2(Q^2)$, $A_s(Q^2) = \alpha_s(Q^2)/(4\pi)$, $p(n)$, $q(n)$ are related to the coefficients of the anomalous dimension functions $\gamma^{(n)}_{NS}(A_s)$ and the QCD β-function by the equations defined in Ref.[5] and the numerical expressions of the coefficients $\gamma^{(1)}_{NS,xF_3}(n)$, $\gamma^{(2)}_{NS}(n)$, $C^{(1)}_{xF_3}(n)$, $C^{(2)}_{xF_3}(n)$ for $f = 4$ are given in our previous paper of Ref.[5].

Having at hand the QCD expressions for the Mellin moments $M_n(Q^2)$ we can reconstruct the structure function $xF_3(x,Q^2)$ using the Jacobi polynomial expansion method:

$$xF_3^{N_{max}}(x,Q^2)=x^{\alpha}(1-x)^{\beta}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)M_{j+2,xF_3}\left(Q^2\right), \tag{3}$$

where $\Theta_n^{\alpha,\beta}$ are the Jacobi polynomials and α,β are their parameters, fixed by the condition of the requirement of the minimization of the error of the reconstruction of the structure functions (see Ref.[10] for details) and $M_n(Q^2)$ is the QCD expression for the Mellin moments, corrected by the target mass contributions and twist-4 effects. Throughout this paper we will use the following expression

$$M_n(Q^2)=M_n^{NS}(Q^2)+\frac{n(n+1)}{n+2}\frac{M_{nucl.}^2}{Q^2}M_{n+2}^{NS}(Q^2)+\tilde{C}(n)M_n^{NS}(Q^2)\frac{A_2^{'}}{Q^2}+O(\frac{1}{Q^4}) \tag{4}$$

where $A_2^{'}$ is the free parameter, M_{nucl} is the mass of the nucleon, $\tilde{C}(n)=-n-4+4/n+2/(n+1)+4/(n+2)+4S_1(n)$ ($S_1(n)=\sum_{j=1}^{n-1}1/j$) is determined by the IRR model calculations of Ref.[22] (for the related studies of the IRR model predictions for the non-singlet structure functions of DIS see Ref.[23]). It is worth to mention here that the IRR model calculations are known to be complimentary to the ones, based on the dispersion approach for the definition of the running coupling, developed e.g. in the papers of Ref.[24]. Note also, that the effects of the $O(1/Q^4)$-contributions in Eq.(4) come from two sources-the target mass effects (see Ref.[5]) and the twist-6 contributions, which can be extracted from the IRR model calculations of Ref.[22]. However, the concrete fits of the experimental data demonstrate that the $O(1/Q^4)$-effects of the target mass corrections are negligibly small [5]. The similar feature was revealed while taking into account twist-6 contributions in the process of the fits of the CCFR'97 data (see the second citation in Ref.[3]). Therefore, in our concrete studies we will neglect the $O(1/Q^4)$-contributions as the whole. We will also neglect the problem of defining selfconsistently the perturbative contribution to the coefficient function of the twist-4 correction in Eq.(4), which generally speaking differs from $M_n^{NS}(Q^2)$.

One more theoretical input of our analysis is the factorization-scale dependent factor $M_n^{NS}(Q_0^2)$ in Eq.(1). Throughout this work we will use the $\overline{MS}$-factorization prescription, fixing $M_n^{NS}(Q_0^2)$ as

$$M_n^{NS}(Q_0^2)=\int_0^1 dx x^{n-2}A(Q_0^2)x^{b(Q_0^2)}(1-x)^{c(Q_0^2)}(1+\gamma(Q_0^2)x) \tag{5}$$

where Q_0^2 is the relevant factorization scale. Note also that we will take in Eq.(3) $N_{max}=10$ for performing LO and NLO fits, while our NNLO fits will be done in the case of $N_{max}=6$. At the NNLO we will use the calculated in Ref.[21] NNLO contributions $\gamma_{NS}^{(2)}(2)$, $\gamma_{NS}^{(2)}(4)$, $\gamma_{NS}^{(2)}(6)$, $\gamma_{NS}^{(2)}(8)$, $\gamma_{NS}^{(2)}(10)$ to the anomalous dimension functions and the estimated by the procedure of smooth interpolation results for $\gamma_{NS}^{(2)}(3)$, $\gamma_{NS}^{(2)}(5)$, $\gamma_{NS}^{(2)}(7)$, $\gamma_{NS}^{(2)}(9)$ [5]. It is worth to mention here that the similar procedure was previously applied in the process of the NNLO non-singlet fits of the BCDMS data in Ref.[15]. However, we will avoid the application of the extrapolation of the expression for $\gamma_{NS}^{(2)}(n)$ at $n>10$.

To extract the x-shape of the twist-4 contributions we will simulate the xF_3 structure function by the following expression

$$xF_3(x,Q^2)=xF_3^{LT}(x,Q^2)+\frac{h(x)}{Q^2} \tag{6}$$

where the Q^2-dependence of the leading twist term $xF_3^{LT}(x,Q^2)$ in Eq.(6) is determined by perturbative QCD (including the proper treatment of the target mass corrections) and the problem of fixing the Q^2-dependence of the coefficient function of the twist-4 contribution is not considered. The constants $h(x_i)$ (one per each x-bin) are parametrizing the x-dependence of the twist-4 contribution, which in accordance with the theoretical predictions of Refs.[25, 26] in the limit $x\to 1$ can be modeled as $h(x)\to(1-x)^{c-1}$, where the parameter c is introduced in Eq.(5) and like the parameters $h(x_i)$, A,b,γ and the QCD scale parameter $\Lambda_{\overline{MS}}^{(4)}$ can be determined by fitting the concrete sets of the experimental data for the xF_3 structure function. Note, that the parameter $\Lambda_{\overline{MS}}^{(4)}$ is related to the expression for the QCD coupling constant α_s via the standard expansion in inverse powers of $ln(Q^2/\Lambda_{\overline{MS}}^{(4)\,2})$.

2. At the first stage we performed the fits of the CCFR'97 data of Ref.[3] for the cuts of these data at $x < 0.7$ using the value of the factorization point $Q_0^2 = 5\ GeV^2$ with the help of two independently written computer programs, which realize the Jacobi polynomial expansion method. The first Program has been created as the result of the works of Ref.[10], while the second Program previously found its more distinguished application in the process of the fits of Ref.[15]. The Jacobi polynomial parameters were fixed as $\alpha = 0.7$ and $\beta = 3$ like in our previous paper of Ref.[5]. Both Programs are giving the identical results, which are presented in Table 1. All fits were done in the LO, NLO and NNLO level both without twist-4 contributions and with twist-4 corrections, taken into account through the expression of Eq.(4).

In Table 2 the NLO results, obtained with taking into account the IRR-model estimates of the twist-4 contributions, are compared to the outcomes of the independent fits of the CCFR collaboration, which are presented in the second reference from Ref. [3]. Let us remind that these results were got using the DGLAP-equation. One can see that within the statistical uncertainties our NLO results agree with the results obtained by the CCFR collaboration with the help of another method and another computer program.

Looking carefully on Table 1 we arrive to the following conclusions:

- The effects of the NNLO perturbative QCD contributions turn out to be very important in the analysis of the CCFR'97 data for the xF_3 structure function. Indeed, for the different Q^2-cuts they are diminishing the values of the QCD scale parameter $\Lambda^{(4)}_{\overline{MS}}$ by over $50 - 130\ MeV$ (!) provided the twist-4 corrections are taken into account through the IRR-model of Ref.[22];

- For the low Q^2-cuts $Q^2 > 1\ GeV^2$ and $Q^2 > 5\ GeV^2$ we observe the existence of the following hierarchy $(\chi^2)_{NNLO} < (\chi^2)_{NLO} < (\chi^2)_{LO}$, which demonstrate the importance of taking into account of the effects of perturbative QCD corrections in the process of the fits of the concrete experimental data for the structure functions of DIS;

- When the effects of higher-twist corrections are included, the NNLO results for $\Lambda^{(4)}_{\overline{MS}}$ are less sensitive to the values of Q^2-cuts of the CCFR'97 experimental data than the similar NLO results;

- The inclusion of the twist-4 contributions into the fits of the CCFR'97 data results in the detectable increase of the central values of $\Lambda^{(4)}_{\overline{MS}}$ at the NLO for almost all Q^2-cuts besides the cut $Q^2 > 1\ GeV^2$ (to our point of view, the analysis of the data in the latest region can be more sensitive to the change of the value of the factorization scale $Q_0^2 = 5\ GeV^2$ say to the value $Q_0^2 = 1\ GeV^2$, which is more appropriate for fitting the data in the region of momentum transfered $Q^2 > 1\ GeV^2$);

- At the NLO level the values of the fitted parameter A_2', responsible for the inclusion of the twist-4 corrections, do not contradict the value $A_2' \approx -0.2\ GeV^2$, which was chosen in the work of Ref.[22] for the parametrization of the twist-4 corrections within the IRR model approach;

- We found that at the NNLO level the IRR-motivated expression of the twist-4 contributions influence the outcomes of the fits less vividly than at the NLO level. This feature is related to the fact, that the central values of the parameter $A_2'(HT)$ which result from the NNLO fits turn out to be rather small. Moreover, we even are unable to fix the sign of $A_2'(HT)$ at the NNLO level, since the statistical error-bars of the value of this parameter are large (!!). This observed property might reveal the feature of the partial cancellation of the IRR contribution to the twist-4 correction of the DIS structure function with the possible part of the $O(1/Q^2)$-term, which can arise from the summation of the ultraviolet renormalon effects, discussed recently e.g. in Ref.[27] and partly taken into account in our analysis through the certain sets of the perturbative NNLO corrections.

However, one should keep in mind the results of the recent quantum mechanical consideration of Ref.[28], which are demonstrating that the renormalons might give a noncomplete information on the structure of nonperturbative effects. In its turn this can indicate that the IRR higher-twist contribution to Eq.(4) might have non-controllable theoretical uncertainties. These interesting conclusions make very important the direct extraction of the information about the higher-twist contributions from the analysis of the CCFR'97 experimental data following the lines of the work of Ref.[6], devoted to the similar analysis of the old CCFR data.

3. In this section we follow the ideas of Ref.[17, 6, 7] and undertake the simultaneous extraction of the parameters A, b, c, γ, $\Lambda^{(4)}_{\overline{MS}}$ and the x-shape of the twist-4 corrections from the CCFR'97 xF_3 experimental data using the model of Eq.(6). In accordance with the x-bin structure of the CCFR'97 data we consider 16 bins x_i, presented in Table 3. We have taken the number of active flavours $f = 4$ in all region of momentum transfered $5\ GeV^2 \leq Q^2 \leq 199.5\ GeV^2$. The results of the LO, NLO and NNLO fits, performed in the kinematical conditions $Q_0^2 = 5\ GeV^2$ and $Q^2 > 5\ GeV^2$, are presented in Tables 3,4 and Figs.1-3. At Fig.2 we also plot the prediction for $h(x)$, obtained within the framework of the IRR model approach of Ref.[22]. Note, that in the work of Ref.[22] the concrete predictions were obtained for the twist-4 contribution, written down in the slightly different form: $xF_3^{HT}(x, Q^2) = xF_3^{LT} D(x)/Q^2$. To transform this prediction to our normalization of Eq.(6) we used the program to compute the x-shape of this power term, written down by the authors of Ref.[22][1].

Let us now describe the main conclusions, which follow from the results of Tables 3,4.

- The statistical uncertainties of the values of $\Lambda^{(4)}_{\overline{MS}}$ are increasing drastically. This property results from increasing of the number of the fitted parameters and reflects the correlation of the uncertainties of the values of the parameters A, b, c, γ and $\Lambda^{(4)}_{\overline{MS}}$ with the ones, which appear in view of the lack of the precise theoretical information about the twist-4 contributions;

- The non-normalized to the number of degrees of freedoms value of the χ^2-parameter drastically decreases and turn out to be rather stable to the inclusion of the NLO and NNLO corrections into the fits of the CCFR'97 experimental data. This property is reflecting the positive feature of the incorporation of the additional 16 parameters $h(x_i)$ into the analysis of the CCFR'97 experimental data;

- We confirm the conclusion of Sec.2 about the importance of taking into account NNLO contributions to Eq.(2) in the concrete fits of the experimental data. Indeed, the NNLO QCD corrections are decreasing both the central value of the parameter $\Lambda^{(4)}_{\overline{MS}}$ and the corresponding statistical uncertainties. Moreover, making additional fits, we found that the strong decrease of the value of the parameter $\Lambda^{(4)}_{\overline{MS}}$ at the NNLO is resulting from the incorporation of the NNLO corrections into Eq.(2), but not into the relation between $\alpha_s(Q^2)$ and $\Lambda^{(4)}_{\overline{MS}}$;

- We have checked that the results of the LO and NLO fits are almost non-sensitive to the changes of $N_{max} = 10$ in Eq.(3) to $N_{max} = 6$, considered in the process of the NNLO fits. This feature is rather welcomed from the point of view of the confirmation of the reliability of the application of the Jacobi polynomial expansion machinery;

- At the NLO and in the region of the intermediate values of x the form of the twist-4 correction $h(x)$ turns out to be in satisfactory agreement with the x-behavior of the IRR-model prediction of Ref.[22]. This property is confirming the conclusion of Ref.[29], that in the first approximation the IRR model does in fact good job as far as x-dependence of the higher-twist corrections is concerned;

- However, we also confirm the conclusion of Refs.[28, 29] that the IRR model might give the non-complete estimate of the mass scale of the higher-twist contributions. Indeed, we observe that the inclusion of the NNLO corrections into the fits of the experimental data is resulting in the less vivid x-dependence of the twist-4 correction $h(x)$ (!), which model the effects of the truncation of the perturbative QCD expressions for the related moments $M_n^{NS}(Q^2)$ and of the reconstructed structure function $xF_3^{LT}(x, Q^2)$ at the definite order of perturbation theory. Moreover, looking

[1] We are grateful to B.R. Webber for providing us this program

carefully on Table 3 we observe that in almost all x_i-bins the following tendency $|h^{cv}(x_i)|_{NNLO} \leq |h^{cv}(x_i)|_{NLO}$ is revealed, where $h^{cv}(x_i)$ denotes the central values of the corresponding twist-4 parameters. These observations are one of the main outcomes of our analysis, presented in this Section. Following the hypothesis, made in the item 6 of Sec.2, we are relating the explanation of this feature to the possible NNLO manifestation of the summed up ultraviolet renormalon contributions, discussed in detail in Ref.[27];

- In the region of the intermediate values of x the results of our extarctions of $h(x_i)$ (see Figs.1-3) do not contradict to the predicted in Ref.[25, 26] behaviour $|h(x)| \to (1-c)^{c-1}$ (where we find $3 < c \leq 4$). Note, however, that the predictions of Ref.[25, 26] were obtained in the asymptotic limit $x_i \to 1$, while our results are related to the kinematical region $0.0125 \leq x_i \leq 0.65$, which in fact is lying quite far from this asymptotic regime.

- The results of the fits demonstrate that the twist-4 contributions can have the sign-alternating character, discussed for the first time in the process of the fits of the DIS experimental data in Ref.[30], where another phenomenological approach was used for modeling higher-twist contributions. It should be stressed, that our picture of the x-behaviour of $h(x)$ is more rich. Indeed, while the considerations of Ref.[30] indicate the existence of only one zero in the values of $h(x)$, we see that $h(x)$ can have two zeros: the first one at small values of x and another one in the region of larger values of x (see Fig.1-Fig.3).

4. In Ref.[3] the final result for the parameter $\Lambda_{\overline{MS}}^{(4)}$, extracted from the xF_3 CCFR'97 data, was presented for the case of the fits, made within the kinematical conditions $Q_0^2 = 5\ GeV^2$ and $Q^2 > 5\ GeV^2$. It was obtained with the help of the newly proposed non-conventional procedure of the treatment of the systematic uncertainties, called as the "global systematic fit". It is based on the incorporation of the systematic variations of the structure functions as the part of the fit and on the redefinition of the χ^2-parameter. The application of this procedure results in the substantial reduction of the value of the newly defined χ^2-parameter and of the combined statistical and systematic error bars. However, in order to get the feeling, whether the theoretical error of the NLO value of $\Lambda_{\overline{MS}}^{(4)}$ was estimated in the process of the work of Ref.[3] correctly, we will compare the outcomes of our NLO and NNLO studies to the following CCFR'97 result, obtained in the second work from Ref.[3]:

$$NLO: \quad \Lambda_{\overline{MS}}^{(4)} = 387 \pm 42(stat) \pm 93(syst) \pm 17(HT) \pm 70(scale) \quad MeV \tag{7}$$

where the statistical and systematic uncertainties were determined by the conventional way and the higher-twist error was estimated by varying the IRR model parameter A_2' in the following interval $(0, -0.2\ GeV^2)$ with the central value $A_2' = A_2'(DW)/2 = -0.1\ GeV^2$ [2].

Notice, that in Eq.(7) the scale-scheme uncertainties, which in part reflect the sensitivity of the outcomes of the fits to the non-taken into account higher order QCD corrections, were *ad hoc* taken from the estimate $\Delta\alpha_s(M_Z) = \pm 0.004$, made in the process of the NLO studies of Ref.[17], aimed to the NLO analysis of the BCDMS F_2 structure function data of the μN DIS. Evolving now the result of Eq.(7) through the threshold of the production of the b-quark with the help of the derived in Ref.[31] relations with the choice $m_b \approx 4.6\ GeV$, we get the following value of $\alpha_s(M_Z)$:

$$NLO: \quad \alpha_s(M_Z) = 0.122 \pm 0.002(stat) \pm 0.005(syst) \pm 0.001(HT) \pm 0.004(scale) \quad . \tag{8}$$

which corresponds to analysis of the CCFR'97 xF_3 experimental data, performed in the second work of Ref.[3] with the separate determination of the statistical and systematic experimental uncertainties.

Since we are able to incorporate in our fits the effects of the NNLO corrections and extract the value of A_2'– parameter (see Sec.2), we can get the feeling on the validity of the estimates of the theoretical uncertainties in Eq.(7) and Eq.(8). Indeed, for $Q_0^2 = 5\ GeV^2$ and $Q^2 > 5\ GeV^2$ our NLO

[2] The abbreviation $A_2'(DW)$ corresponds to the exact parametrization of the twist-4 contribution through the IRR-model of Ref.[22]

and NNLO results, which follow from the ones of Table 1, read

$$\begin{aligned} NLO\ \ HT\ of\ Ref.[22]: \ \Lambda^{(4)}_{\overline{MS}} &= 371 \pm 31(stat) \pm 93(syst)\ MeV \\ NNLO\ \ HT\ of\ Ref.[22]: \ \Lambda^{(4)}_{\overline{MS}} &= 293 \pm 29(stat) \pm 93(syst)\ MeV \end{aligned} \tag{9}$$

where the systematic uncertainties are taken from the original analysis of Ref.[3] (which we are unable neither to check nor to improve) and the statistical uncertainties in Eq.(9) are taking into account the deviation of the twist-4 parameter A_2' in Eq.(4) from its canonical value of Ref.[22].

One can see that the inclusion of the NNLO QCD corrections into the fits of the CCFR'97 experimental data decreases the central value of the parameter $\Lambda^{(4)}_{\overline{MS}}$ by over 80 MeV. This amount is comparable with the estimate of the scale-scheme dependence uncertainty in Eq.(7). In its turn, this fact means that the theoretical uncertainties in the NLO results of Eq.(7) and Eq.(8) are underestimated.

Indeed, it was already demonstrated in the second work from Ref.[3] that the outcomes of the NLO fits of the CCFR'97 experimental data for the xF_3 structure function turn out to be sensitive to the upper value of the x-cut. The variation of this cut from $x < 0.7$ to $x < 0.6$ and to $x < 0.5$ (which is equivalent to neglecting the high Q^2-data for the xF_3 structure function) is decreasing the central value of the CCFR'97 NLO result for $\Lambda^{(4)}_{\overline{MS}}$ by over 32 MeV and 40 MeV correspondingly[3]. Moreover, for the cut $x < 0.7$ there is still the place for other theoretical uncertainties (besides the controlled by us effects of the NNLO corrections), which appear due to the factorization scale dependence, the unknown N^3LO contributions and the sensitivity to nuclear effects. Since the CCFR'97 data for the $(xF_3)^{Fe}$ structure function were not corrected by the latter ones, we made the fits, analogous to the ones described in Sec.3, but with modeling the ratio $R_{Fe} = F_3^{Fe}/F_3^N$ by the ratio $R_D = F_3^D/F_3^N$, which was fixed by the approach, considered in Ref.[32]. These fits also result in the effect of the decreasing of the value of the parameter $\Lambda^{(4)}_{\overline{MS}}$, but by less considerable amount of over 15 MeV. It is highly desirable to check this conclusion by using some other approach for modeling the value of R_{Fe}, say the one, developed in Ref.[33] in the case of the analysis of the F_2 structure functions data.

Evolving now the results of Eq.(9) through the thresholds of the production of the b-quark and substituting the values of the parameter $\Lambda^{(5)}_{\overline{MS}}$ into the inverse-log expression for $\alpha_s(M_Z)$, we get

$$\begin{aligned} NLO\ \ HT\ of\ Ref.[22]: \ \alpha_s(M_Z) &= 0.121 \pm 0.002(stat) \pm 0.005(syst) \pm 0.006(theory) \\ NNLO\ \ HT\ of\ Ref.[22]: \ \alpha_s(M_Z) &= 0.117 \pm 0.002(stat) \pm 0.005(syst) \pm 0.003(theory)\ . \end{aligned} \tag{10}$$

The theoretical error bars of the NLO and NNLO values of $\alpha_s(M_Z)$ in Eq.(10) are taking into account all discussed uncertainties plus the arbitrariness of application of the procedure of passing threshold of the production of b-quark, which following the considerations of Ref.[14] we estimate as $\Delta\alpha_s(M_Z) \pm 0.001$.

The NNLO results from Eq.(10) turns out to be in agreement with the following NNLO value of $\alpha_s(M_Z)$: $\alpha_s(M_Z) = 0.115 \pm 0.001(stat) \pm 0.005(syst) \pm 0.003(twist) \pm 0.0005(scheme)$, which was extracted in Ref.[34] from the comparison of the old CCFR data for the Gross-Llewellyn Smith sum rule at the reference point $Q^2 = 3\ GeV^2$ [35] with the results of the perturbative NLO and NNLO calculations of Ref.[36], supplemented with the information about the concrete values of the twist-4 corrections, obtained in Ref.[37]. The optimistic scheme-dependence error in the NNLO result of Ref.[34] is reflecting the uncertainty due to the unknown effects of the higher N^3LO perturbative QCD corrections, estimated later on in the works of Ref.[38]. It should be stressed, however, that in view of the changes of the CCFR νN data the experimental analysis of Ref.[35] should be updated.

Let us also mention, that the discussed in Sec.3 fits of the CCFR'97 data, which have the aim to extract the x-shape of the twist-4 contributions and to verify the predictions of the IRR-model, give as the outcomes the comparable values of $\Lambda^{(4)}_{\overline{MS}}$ and thus $\alpha_s(M_Z)$. Indeed, when transformed through the thresholds of the production of the b-quark, the NLO and NNLO results of Table 4

$$\begin{aligned} NLO\ \ HT\ free: \ \Lambda^{(4)}_{\overline{MS}} &= 428 \pm 158(exp)\ MeV \\ NNLO\ \ HT\ free: \ \Lambda^{(4)}_{\overline{MS}} &= 264 \pm 85(exp)\ MeV \end{aligned} \tag{11}$$

[3] Note, that for the cut $x < 0.6$ we confirmed this decreasing tendency.

are equivalent to the following values of $\alpha_s(M_Z)$:

$$\begin{aligned} NLO\ \ HT\ free: \ \ \alpha_s(M_Z) &= 0.124 \pm 0.007(exp) \pm 0.010(theory) \\ NNLO\ \ HT\ free: \ \ \alpha_s(M_Z) &= 0.115 \pm 0.006(exp) \pm 0.003(theory)\ \ , \end{aligned} \tag{12}$$

where the experimental uncertainties are estimated by the statistical errors (which are increased due to the inclusion of the additional 16 twist-4 parameters $h(x_i)$ into the fits), the NLO theoretical errors are fixed by the difference between the NLO and NNLO results and the remaining NNLO theoretical ambiguities are determined by the uncertainties, taken by us into account in the process of estimates of the theoretical ambuguities of the NNLO value of $\alpha_s(M_Z)$, presented in Eq.(10).

It should be stressed, that our results of Eqs.(10),(12) do not contradict to the NLO value $\alpha_s(M_Z) = 0.113 \pm 0.003(exp) \pm 0.004(theory)$, obtained in the process of the NLO analysis of the BCDMS and SLAC data for the F_2 structure function of μN and eN DIS [17]. However, there are still the number of questions to be answered in future. The first, theoretical one, is related to the necessity of the inclusion into the analysis of the BCDMS and SLAC data the NNLO perturbative QCD corrections to the singlet part of the F_2 structure function (we consider the results of the NNLO non-singlet fits of Ref.[15] still non-complete, since in the process of these fits only the data for $x > 0.35$ was analyzed without taking into account target mass corrections). The second, pure experimental, question is related to the problem whether the scientific community will accept the proposed in Ref.[3] new approach to treating experimental uncertainties. Note, that the accepted at present conventional way of their definitions is leading to the conclusion, that the combination of the traditionally defined statistical and systematic uncertainties of the CCFR'97 experimental data is not too small. Keeping this in mind, we agree with the conclusions of the analysis of the experimental data for the Bjorken polarized sum rule [39, 40] that in order to extract the more precise value of $\alpha_s(M_Z)$ from the characteristics of DIS with taking into account both the higher-order perturbative QCD corrections and higher-twist effects it is necessary to have more precise experimental data. This conclusion is supporting the detailed investigation of the third problem, revealed in the process of the analysis of the CCFR'97 experimental data of Ref.[3]. It is related to the necessity of more detailed investigation of the possible discrepancies between the CCFR'97 F_2 νN data in the region of small x ($x \approx 0.01 - 0.07$) and the similar experimental data for the F_2 structure functions of the DIS of charged leptons on nucleons, measured by the BCDMS, NMC and SLAC collaborations. Among the possible ways of attacking this experimental problem are the possible more detailed studies of νAl DIS data of the IHEP-JINR neutrino detector (for the already available preliminary results see Ref.[41]) and the continuation of the careful measurements of the structure functions of νFe DIS at Fermilab Tevatron, which is the main goal of the NuTeV collaboration.

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$Q^2 >$	$\Lambda_{\overline{MS}}^{(4)}$ (MeV)	A	b	c	γ	$A_2^{'}$(HT)	χ^2/nep
1							
LO	277±31	4.65±0.14	0.69±0.01	3.90±0.04	1.82±0.13	–	146.0/110
	367±43	3.95±0.40	0.63±0.03	3.87±0.05	2.48±0.47	-0.25±0.04	114.4/110
NLO	322±27	3.67±0.72	0.62±0.05	3.76±0.10	2.32±0.94	–	114.1/110
	322±23	3.61±0.11	0.61±0.01	3.76±0.04	2.39±0.12	-0.07±0.04	111.4/110
NNLO	275±24	4.01±0.34	0.64±0.02	3.61±0.06	1.56±0.33	–	104.9/110
	276±25	4.00±0.34	0.64±0.02	3.61±0.06	1.56±0.32	0.01±0.04	104.9/110
5							
LO	266±37	5.13±0.46	0.72±0.03	3.87±0.05	1.42±0.33	–	113.2/86
	436±56	4.73±0.39	0.68±0.03	3.79±0.05	1.82±0.31	-0.33±0.06	82.8/86
NLO	341±41	4.05±0.38	0.65±0.03	3.71±0.06	1.96±0.36	–	87.1/86
	371±31	3.94±0.12	0.64±0.01	3.69±0.04	2.05±0.12	-0.12±0.05	82.4/86
NNLO	293±29	4.25±0.38	0.66±0.03	3.56±0.07	1.33±0.33	–	78.4/86
	293±29	4.25±0.37	0.66±0.03	3.56±0.07	1.33±0.32	-0.01±0.05	78.4/86
10							
LO	287±37	6.52±0.20	0.79±0.01	3.73±0.04	0.60±0.10	–	77.7/63
	531±74	6.48±0.65	0.79±0.04	3.68±0.05	0.94±0.26	-0.52±0.11	58.0/63
NLO	350±36	5.29±0.21	0.74±0.01	3.64±0.06	1.06±0.17	–	64.4/63
	439±55	4.92±1.06	0.73±0.07	3.61±0.08	1.42±0.75	-0.24±0.10	58.6/63
NNLO	308±34	4.65±0.53	0.70±0.04	3.52±0.08	1.06±0.38	–	58.7/63
	313±37	4.64±0.52	0.70±0.04	3.52±0.08	1.07±0.38	-0.03±0.09	58.6/63
15							
LO	319±41	7.58±0.24	0.84±0.01	3.61±0.05	0.16±0.09	–	58.8/50
	531±57	7.76±0.26	0.86±0.02	3.61±0.04	0.45±0.09	-0.57±0.13	50.2/50
NLO	365±40	6.28±0.21	0.80±0.02	3.60±0.04	0.58±0.09	–	53.0/50
	441±44	5.98±0.19	0.80±0.02	3.57±0.05	0.87±0.10	-0.25±0.13	50.9/50
NNLO	314±37	4.68±0.75	0.70±0.05	3.53±0.08	1.08±0.51	–	50.9/50
	308±45	4.69±0.72	0.70±0.05	3.53±0.08	1.07±0.50	0.03±0.14	50.8/50

Table 1. The results of the fits of the xF_3 (νFe) revised data of the CCFR collaboration. The values of Q^2 and $A_2^{'}(HT)$ are given in GeV^2. The statistical errors are presented. (χ^2 is normalized to the number of experimental points (nep).)

Q^2 cut (GeV^2)	x cut	Order	$\Lambda_{\overline{MS}}^{(4)}$ (MeV)	χ^2/nep
$Q^2 > 5$	$x < 0.7$	NLO	371 ± 30	82.4/86
	CCFR result		387 ± 42	81.8/86
$Q^2 > 10$	$x < 0.7$	NLO	439 ± 55	58.6/63
	CCFR result		422 ± 47	58.0/63
$Q^2 > 15$	$x < 0.7$	NLO	441 ± 44	50.9/50
	CCFR result		433 ± 55	50.0/50

Table 2. The comparison of the results of our NLO fits with the revised results provided by the CCFR collaboration in Ref.[3]

	LO	NLO	NNLO
χ^2/nep	66.2/86	65.6/86	65.7/86
A	5.44 $\pm$ 1.74	3.70 $\pm$ 1.56	4.54 $\pm$ 0.88
b	0.74 $\pm$ 0.10	0.66 $\pm$ 0.11	0.69 $\pm$ 0.06
c	4.00 $\pm$ 0.18	3.78 $\pm$ 0.21	3.72 $\pm$ 0.19
γ	1.72 $\pm$ 1.25	2.86 $\pm$ 1.72	1.43 $\pm$ 0.69
x_i	$h(x_i)$ $[GeV^2]$		
0.0125	0.206 $\pm$ 0.321	0.213 $\pm$ 0.332	0.170 $\pm$ 0.302
0.0175	0.061 $\pm$ 0.268	0.091 $\pm$ 0.289	0.030 $\pm$ 0.241
0.025	0.146 $\pm$ 0.204	0.220 $\pm$ 0.241	0.136 $\pm$ 0.178
0.035	-0.021 $\pm$ 0.185	0.114 $\pm$ 0.240	0.008 $\pm$ 0.179
0.050	0.031 $\pm$ 0.142	0.245 $\pm$ 0.230	0.124 $\pm$ 0.164
0.070	-0.145 $\pm$ 0.127	0.138 $\pm$ 0.233	0.033 $\pm$ 0.165
0.090	-0.177 $\pm$ 0.125	0.139 $\pm$ 0.225	0.076 $\pm$ 0.166
0.110	-0.340 $\pm$ 0.126	-0.015 $\pm$ 0.205	-0.026 $\pm$ 0.166
0.140	-0.404 $\pm$ 0.114	-0.092 $\pm$ 0.147	-0.027 $\pm$ 0.141
0.180	-0.350 $\pm$ 0.164	-0.077 $\pm$ 0.122	0.054 $\pm$ 0.127
0.225	-0.554 $\pm$ 0.237	-0.348 $\pm$ 0.170	-0.167 $\pm$ 0.135
0.275	-0.563 $\pm$ 0.334	-0.462 $\pm$ 0.272	-0.196 $\pm$ 0.193
0.350	-0.314 $\pm$ 0.418	-0.368 $\pm$ 0.371	0.070 $\pm$ 0.200
0.450	-0.117 $\pm$ 0.415	-0.266 $\pm$ 0.401	0.183 $\pm$ 0.213
0.550	0.087 $\pm$ 0.333	-0.109 $\pm$ 0.352	0.097 $\pm$ 0.234
0.650	0.377 $\pm$ 0.215	0.221 $\pm$ 0.244	0.259 $\pm$ 0.188

Table 3. The results of the LO, NLO and NNLO QCD fits of the CCFR'97 xF_3 structure functions data for the values of twist-4 contributions $h(x)$ and for the parameters A, b, c, γ with the corresponding statistical errors.

	$h(x) \neq 0$ free		$h(x) \neq 0$ of Ref.[22]		$h(x) = 0$	
	χ^2/nep	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	χ^2/nep	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	χ^2/nep	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)
LO	66.2/86	338 $\pm$ 169	82.8/86	436 $\pm$ 56	113.3/86	266 $\pm$ 37
NLO	65.6/86	428 $\pm$ 158	82.4/86	371 $\pm$ 31	87.1/86	341 $\pm$ 41
NNLO	65.7/86	264 $\pm$ 85	78.4/86	293 $\pm$ 29	78.4/86	293 $\pm$ 29

Table 4. The comparison of the results of the LO, NLO and NNLO fits. of the CCFR'97 xF_3 structure functions data, performed with different assumptions of the values of the twist-4 parameter $h(x)$.

Figure captions.

Fig.1 The results of the LO extraction of the x-shape of the twist-4 contribution $h(x)$.

Fig.2 The results of the NLO extraction of the x-shape of the twist-4 contribution $h(x)$. For the comparison, the IRR-model prediction of Ref.[22], obtained using the NLO MRS parametrization, is also depicted.

Fig.3 The results of the NNLO extraction of the x-shape of the twist-4 contribution $h(x)$.