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# Bounds on CP violating anomalous trilinear gauge couplings from $B \to K^* \ell^+ \ell^-$

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## Abstract

We study the contributions of the CP violating anomalous $WW\gamma$ interactions to $b \to s\ell^+\ell^-$. We obtain cutoff independent bounds on $\tilde{\kappa}$ and $\tilde{\lambda}$, by constructing an asymmetry for the process $B \to K^*\ell^+\ell^-$, where the B and $\bar{B}$ events are added. For $10^4$ $B \to K^*\epsilon^+\epsilon^-$ events, we obtain bounds on $\tilde{\kappa}$ that are much tighter than the recent constraints from D0.

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Two of the aspects of the Standard Model(SM) which remain to be further explored are the gauge nature of the W, Z bosons and CP violation. The vector boson self interactions are uniquely fixed by the $SU(2)_L \times U(1)$ gauge structure of the SM. However, new physics as well as higher order corrections will modify these self interactions. While tests of trilinear gauge couplings are now being done at present colliders, only weak bounds on them exist [1,2]. CP violation on the other hand is only paramerized in the SM, via the CKM matrix, and has been seen only in the neutral K mesons. Much effort is now being put into studying CP violation in the B mesons. Large number of B mesons are expected to be produced in the near future, at the new asymmetric colliders and upgraded Tevatron, which would enable study of its rare decay modes. Rare decays of B mesons, occur in the SM only through loops and are thus expected to be very sensitive to new physics. In particular the mode $b \to s\ell^+\ell^-$ has been extensively studied [3,4] for possible contributions from new physics. New physics that affects the gauge nature of the W, Z bosons, can be parametrized in terms of CP conserving as well as CP violating anomalous trilinear gauge boson couplings. The CP conserving couplings have been considered in the rare modes $b \to s\gamma$, $b \to s\ell^+\ell^-$ [5–7]. Since, the origin of CP violation is not understood, any potential source of CP violation should be pursued. Here we focus attention on the CP violating anomalous couplings, by studying the mode $B \to K^*\ell^+\ell^-$.

The couplings of W bosons to the photon and Z can be described by an effective Lagrangian, the most general form of which may be written down with the minimum requirements– that of Lorentz invariance, global $SU(2)$ and local $U(1)$ symmetry [8,9] as:

$$
\begin{aligned}
\mathcal{L}^V_{\mathit{eff}} = & - g_V \Big[ i g^V_1 \left( W^\dagger_{\alpha\beta} W^\alpha - W^{\dagger\alpha} W_{\alpha\beta} \right) V^\beta + i\kappa_V W^\dagger_\alpha W_\beta V^{\alpha\beta} \\
& + i \frac{\lambda_V}{M_W^2} W^\dagger_{\alpha\beta} W^\beta{}_\sigma V^{\sigma\alpha} + i\tilde{\kappa}_V W^\dagger_\alpha W_\beta \tilde{V}^{\alpha\beta} + i \frac{\tilde{\lambda}_V}{M_W^2} W^\dagger_{\alpha\beta} W^\beta{}_\sigma \tilde{V}^{\sigma\alpha} \\
& + g^V_4 W^\dagger_\alpha W_\beta (\partial^\alpha V^\beta + \partial^\beta V^\alpha) + g^V_5 \epsilon^{\alpha\beta\mu\nu} W^\dagger_\alpha \overleftrightarrow{\partial}_\mu W_\beta V_\nu \Big]
\end{aligned}
\tag{1}
$$

In the above equation $V$ represents the neutral gauge bosons either the photon or the Z, $V_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha$, $W_{\alpha\beta} = \partial_\alpha W_\beta - \partial_\beta W_\alpha$ and $g_V$ is the $WWV$ coupling strength in the

SM with $g_\gamma = e$ and $g_Z = ec/s$, where $c^2 \equiv 1 - s^2 \equiv M_W^2/M_Z^2$. In the SM, the couplings $g_1^V$, $\kappa_V$ are unity and all others are zero. New physics may result in a modification of these couplings. The deviation from the SM, the so called anomalous couplings, need to be constrained experimentally. We shall here concentrate on the CP violating couplings $\tilde{\kappa}$ and $\tilde{\lambda}$. While many theoretical and experimental studies of anomalous triple gauge boson couplings have been done, few bounds exist on the anomalous CP violating couplings. Direct bounds have recently been obtained [1] on $\tilde{\kappa}$ and $\tilde{\lambda}$ from $p\bar{p} \to W\gamma X$ at the tevatron. Stringent indirect constraints [10] on these couplings come from electric dipole moment (EDM) of the neutron and electron. However, these constraints assume naturalness and are cutoff dependent. Hence, bounds from CP violating asymmetries that we pursue here, would be complementary to the EDM bounds. Possible contributions of the couplings $\tilde{\kappa}$ and $\tilde{\lambda}$ to processes at colliders have also been examined for LEPII [9], upgraded tevatron [11] as well as future linear colliders [12]. In the chiral lagrangian approach the dimension 6 operator with coefficient, $\tilde{\lambda}$, is ignored. However, for b penguin modes such as $b \to s\gamma$ or $b \to s\ell^+\ell^-$, unitarity of the CKM, results in cutoff independence for contributions from $\tilde{\kappa}$ and $\tilde{\lambda}$. Since both $\tilde{\kappa}$ and $\tilde{\lambda}$ contributions are finite, requiring no cutoff, we do not neglect $\tilde{\lambda}$. The constraints on $\tilde{\lambda}$ are expected to be weaker.

The effective short distance Hamiltonian relevant to the decay $b \to s\ell^+\ell^-$ [13–15] leads to the QCD corrected matrix element

$$\mathcal{M}(b \to s\ell^+\ell^-) = \frac{\alpha G_F}{\sqrt{2}\pi} v_t \{-2iC_7 m_b \frac{q^\nu}{q^2}\, \bar{s}\sigma_{\mu\nu} b_R\, \bar{\ell}\gamma^\mu \ell + C_8\, \bar{s}\gamma_\mu b_L\, \bar{\ell}\gamma^\mu\ell + C_9\, \bar{s}\gamma_\mu b_L\, \bar{\ell}\gamma^\mu\gamma_5\ell\} \quad (2)$$

where only the dominant top quark contribution to the loop is retained. $C_j$ $(j = 7, 8, 9)$ are the Wilson coefficients given in Ref. [13,14], $m_b$ is the mass of the b quark, $q^2$ is the invariant lepton mass squared, $b_{L,R} = (1 \mp \gamma_5)/2\, b$ and $v_t = V_{ts}^* V_{tb}$ is the product of the CKM matrix elements. The anomalous WWV couplings, result in a shift in the values of the short distance coefficients $C_j$'s at $\mu = M_W$,

$$C_j = C_j^{SM} + \imath\, \tilde{\kappa} C_j^{\tilde{\kappa}} + \imath\, \tilde{\lambda} C_j^{\tilde{\lambda}} \quad (3)$$

where, $C_j^{SM}$ are the SM coefficients and $C_j^{\tilde{\kappa}}$, $C_j^{\tilde{\lambda}}$ are the contributions from the anomalous couplings. These shifted values of $C_j$'s are then evolved down to the b quark scale. The anomalous $WW\gamma$ coupling coefficients have been evaluated to be

$$C_8^{\tilde{\kappa}} = C_8^{\tilde{\lambda}} = \frac{m_b^2}{M_W^2}\,\frac{x_t}{12}\left(\frac{2+37\,x_t+10\,x_t^2-x_t^3}{6\,(1-x_t)^4}+\frac{x_t\,(3+5\,x_t)\,\ln x_t}{(1-x_t)^5}\right)$$

$$C_7^{\tilde{\kappa}} = \frac{x_t}{2}\left(\frac{1}{(1-x_t)^2}+\frac{x_t\,(3-x_t)\,\ln x_t}{2(1-x_t)^3}\right) \tag{4}$$

$$C_7^{\tilde{\lambda}} = \frac{x_t}{2}\,\frac{(1-x_t^2+2\,x_t\,\ln x_t)}{2\,(1-x_t)^3}$$

where $x_t = \frac{m_t^2}{M_W^2}$, $m_t$ is the mass of the top quark. The WWZ coefficients can be related to the corresponding $WW\gamma$ coefficients listed above, by modifying the couplings and propagator. $C_9^{\tilde{\kappa},\tilde{\lambda}}$ also contributes to the WWZ vertex, with a functional form in $x_t$, identical to that of $C_8^{\tilde{\kappa},\tilde{\lambda}}$. However, the anomalous contributions from the WWZ vertex may be neglected in comparison to that of the photon. Our results for $C_7^{\tilde{\kappa}}$ are in agreement with those of refs. [5] and [7] but we disagree with ref. [7] on $C_7^{\tilde{\lambda}}$ and on $C_8^{\tilde{\lambda}}$. $C_8^{\tilde{\kappa}}$ has been evaluated for the first time. Note that $C_8^{\tilde{\kappa}}$ turns out to be exactly equal to $C_8^{\tilde{\lambda}}$, perhaps indicating that the contact term $C_8$ is independent of derivatives in the higher dimensional operator. Another interesting feature is that $C_8$ is doubly chirally suppressed for the CP violating couplings $\tilde{\kappa}, \tilde{\lambda}$.

Previous studies [5,7] of CP violating anomalous gauge boson couplings in rare B decays, have used the mode $b \to s\gamma$, to constrain $\tilde{\kappa}$ and $\tilde{\lambda}$ separately, disallowing any possible cancellations between them. They also used only the decay rate of $b \to s\gamma$ and hence depended quadratically on these couplings. In order to achieve maximum sensitivity to CP violating couplings and to have a clear signal of CP violation, we need to estimate observables that are explicitly CP odd and must therefore depend linearly on the CP violating anomalous couplings. CP violating asymmetries have also been considered in $b \to s\gamma$, in SM and two Higgs doublet model [16–18]. However, such asymmetries rely on the presence of a large strong phase arising out of final state interactions. CP violating asymmetries in rare modes that require flavour tagging are going to be very difficult to detect. We therefore prefer to use a technique that does not need flavour or time tagging. Such an asymmetry has been considered in ref. [19]. We adopt this technique and construct asymmetries for the mode $B \to K^* \ell^+ \ell^-$ ($\ell^+\ell^-$ non resonant), to obtain simultaneous bounds on $\tilde{\kappa}$ and $\tilde{\lambda}$. The mode $B \to K^* \ell^+ \ell^-$ is theoretically clean and can provide additional information due to its richer kinematics. We present here only the case of charged B's. For neutral B's, after time integration, the asymmetries that we construct, have exactly the same form as that for the charged B's [19].

The transition matrix element for the exclusive process $B(p) \to K^*(k)\ell^+\ell^- \to K(k_1)\pi(k_2)\ell^+(q_1)\ell^-(q_2)$ can be written for each of the operators in Eq.(2) as,

$$\begin{aligned} \langle K\pi|\bar{s} i\sigma_{\mu\nu}(1 \pm \gamma_5) q^\nu b|B\rangle &= i\mathcal{A}\epsilon_{\mu\nu\alpha\beta}K^\nu k^\alpha q^\beta \pm \mathcal{B}K_\mu \pm \mathcal{C}k_\mu \,, \\ \langle K\pi|\bar{s}\gamma_\mu(1 \mp \gamma_5) b|B\rangle &= i\mathcal{D}\epsilon_{\mu\nu\alpha\beta}K^\nu k^\alpha q^\beta \pm \mathcal{E}K_\mu \pm \mathcal{F}k_\mu \,. \end{aligned} \tag{5}$$

The form-factors $\mathcal{A}, \cdots, \mathcal{F}$ are unknown functions of $q^2 = (p-k)^2$ and other dot products involving momentum, $k = k_1 + k_2$ and $K = k_1 - k_2$ and can be related to those used in [4], as given in the table. They can be similarly related to those in heavy quark effective theory (HQET). The HQET form factors cannot currently be reliably predicted over the entire dilepton mass range. The variable $\sigma$ arises due to the decay of $K^* \to K\pi$, evaluated in the zero width approximation. The current proportional to $q_\mu$ does not contribute as it couples to light leptons. In our notation $M_B, m_{K^*}, m_K$ and $m_\pi$ are the masses of the $B$, $K^*$, $K$ mesons and the pion respectively. The matrix element for the process $B \to K^*\ell^+\ell^- \to K\pi\ell^+\ell^-$ can be written as

$$\mathcal{M}(B \to K\pi\ell^+\ell^-) = \frac{\alpha G_F}{\sqrt{2}\pi}\left\{\left(i\alpha_L\,\epsilon_{\mu\nu\alpha\beta}\,K^\nu k^\alpha q^\beta \;+\beta_L\,K_\mu + \rho_L\,k_\mu\right)\bar{\ell}\gamma_\mu\,L\,\ell + L \to R\right\}\,, \tag{6}$$

where $L, R = \dfrac{(1 \mp \gamma_5)}{2}$, $q = q_1 + q_2$ and $Q = q_1 - q_2$, and the coefficients $\alpha_{L,R}$, $\beta_{L,R}$ and $\rho_{L,R}$ are given by

$$\alpha_{R,L} \;=\; v_t\,\{\frac{(C_8 \pm C_9)}{2}\,\mathcal{D} - \frac{m_b}{q^2}\,C_7\,\mathcal{A}\} \;=\; |\mathrm{a}_{R,L}|\,exp\left(i\delta^{\alpha}_{R,L}\right)\;exp\left(i\phi^{\alpha}_{R,L}\right)$$
$$\beta_{R,L} \;=\; v_t\,\{\frac{(C_8 \pm C_9)}{2}\,\mathcal{E} - \frac{m_b}{q^2}\,C_7\,\mathcal{B}\} \;=\; |\mathrm{b}_{R,L}|\,exp\left(i\delta^{\beta}_{R,L}\right)\;exp\left(i\phi^{\beta_j}_{R,L}\right) \tag{7}$$
$$\rho_{R,L} \;=\; v_t\,\{\frac{(C_8 \pm C_9)}{2}\,\mathcal{F} - \frac{m_b}{q^2}\,C_7\,\mathcal{C}\} \;=\; |\mathrm{r}_{R,L}|\,exp\left(i\delta^{\rho}_{R,L}\right)\;exp\left(i\phi^{\rho_j}_{R,L}\right)\,.$$

In the above equation $\alpha$, $\beta$ and $\rho$ are recast in terms of $a$, $b$ and $r$ so as to identify the strong phases $\delta$ and the weak phases $\phi$. Using $CPT$ invariance, the matrix element for the decay $\bar{B} \to \bar{K}\bar{\pi}\ell^+\ell^-$ can be obtained from the $B \to K\pi\ell^+\ell^-$ by replacing $\alpha_{L,R} \to -\bar{\alpha}_{L,R}, \beta_{L,R} \to \bar{\beta}_{L,R}, \rho_{L,R} \to \bar{\rho}_{L,R}$ [20,21], where

$$\bar{\alpha}_{R,L} = |\mathrm{a}_{R,L}|\,exp\left(i\delta^{\alpha}_{R,L}\right)\;exp\left(-i\phi^{\alpha}_{R,L}\right) \tag{8}$$

and similar relations hold for $\bar{\beta}$ and $\bar{\rho}$. The matrix element mod. squared for the process $B \to K^*\ell^+\ell^- \to K\pi\ell^+\ell^-$ is worked out retaining the imaginary parts in $\alpha, \beta$,and $\rho$ to be

$$\begin{aligned}
|\mathcal{M}(B(p) \;&\to K(k_1)\pi(k_2)\ell^+(q_1)\ell^-(q_2))\Big|^2 = \frac{\alpha^2 G_F^2}{2\pi^2} \\
&\{\;\; 2\,\epsilon_{\mu\nu\rho\sigma}k^\mu K^\nu q^\rho Q^\sigma\,(K\cdot Q\,\mathrm{Im}(\alpha_L\,\beta_L^* + \alpha_R\,\beta_R^*) + \mathrm{Im}(\rho_R\,\beta_R^* - \rho_L\,\beta_L^*) \\
&\quad - k\cdot Q\,\mathrm{Im}(\rho_L\,\alpha_L^* + \rho_R\,\alpha_R^*)) + 2\,\mathrm{Re}(\rho_L\,\alpha_L^* - \rho_R\,\alpha_R^*) \\
&\quad \left(-k\cdot K\,q^2\,k\cdot Q + k\cdot q\,k\cdot Q\,K\cdot q + m_{K^*}^2\,q^2\,K\cdot Q - k\cdot q^2\,K\cdot Q\right) \\
&+\;\; 2\,\mathrm{Re}(\rho_L\,\beta_L^* + \rho_R\,\beta_R^*)\,\left(-k\cdot K q^2 + k\cdot q\,K\cdot q - k\cdot Q\,K\cdot Q\right) \\
&+\;\; 2\,\mathrm{Re}(\alpha_R\,\beta_R^* - \alpha_L\,\beta_L^*)\left(K^2\,q^2\,k\cdot Q - k\cdot Q\,K\cdot q^2 - k\cdot K\,q^2\,K\cdot Q\right. \\
&+\;\; \left.k\cdot q\,K\cdot q\,K\cdot Q\right) + \left(\rho_L^2 + \rho_R^2\right)\,\left(-m_{K^*}^2\,q^2 + k\cdot q^2 - k\cdot Q^2\right) \\
&+\;\; \left({\alpha_L}^2 + {\alpha_R}^2\right)\,\left(-K^2\,q^2\,k\cdot Q^2 + k\cdot Q^2\,K\cdot q^2 + 2\,k\cdot K\,q^2\,k\cdot Q\,K\cdot Q\right. \\
&\quad \left.-\,2\,k\cdot q\,k\cdot Q\,K\cdot q\,K\cdot Q - m_{K^*}^2\,q^2\,K\cdot Q^2 + k\cdot q^2\,K\cdot Q^2\right) \\
&+\;\; \left({\beta_L}^2 + {\beta_R}^2\right)\left(-K^2\,q^2 + K\cdot q^2 - K\cdot Q^2\right)\}
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
k\cdot K &= m_K^2 - m_\pi^2\,,\; q\cdot Q \;=\; 0 \\
k\cdot q &= \frac{1}{2}(M_B^2 - m_{K^*}^2 - q^2) \\
k\cdot Q &= X M_B\cos\theta_e
\end{aligned}$$

$$K\cdot q = \lambda_K\, X M_B \cos\theta_{K^*} + \frac{m_K^2 - m_\pi^2}{m_{K^*}^2}\, k\cdot q$$

$$K\cdot Q = k\cdot Q \frac{m_K^2 - m_\pi^2}{m_{K^*}^2} + \lambda_K\,(k\cdot q\cos\theta_l\cos\theta_K - \sqrt{q^2 m_{K^*}^2}\sin\theta_l\sin\theta_K\cos\varphi).$$

Here $X$ is the three momentum of the $\ell^+\ell^-$ or $K\pi$ invariant system in the B meson rest frame and is given by

$$X = \frac{(k\cdot q^2 - q^2 m_{K^*}^2)^{\frac{1}{2}}}{M_B}$$

$\lambda_K$ is related to the $K$ three momentum in the $K^*$ rest frame and is defined as

$$\lambda_K = \left(1 - \frac{(m_K + m_\pi)^2}{m_{K^*}^2}\right)^{\frac{1}{2}} \left(1 - \frac{(m_K - m_\pi)^2}{m_{K^*}^2}\right)^{\frac{1}{2}}$$

and similarly, $\lambda_e$ is related to the lepton momentum in the $\ell^+\ell^-$ rest frame and is given by,

$$\lambda_e = \sqrt{1 - \frac{4\,m_e^2}{q^2}}$$

$$\epsilon_{\mu\nu\rho\sigma} k^\mu K^\nu q^\rho Q^\sigma = -X M_B \lambda_K \sqrt{q^2 m_{K^*}^2}\sin\theta_l\sin\theta_K\sin\varphi\ ,$$

$\theta_l$ is the angle of the $\ell^-$ in the $\ell^+\ell^-$ rest frame with the $\ell^+\ell^-$ invariant direction, and $\theta_k$ is the angle of the $K$ in the $K\pi$ rest frame with the $K^*$ direction. $\varphi$ is the angle between the planes defined by $\ell^+\ell^-$ and the $K\pi$ directions. The differential decay rate is then given by

$$d\Gamma = \frac{1}{2^{14}\pi^6 M_B^2}\int |\mathcal{M}|^2 X\lambda_K\lambda_e\, dq^2 d\cos\theta_K d\cos\theta_l d\varphi\,, \tag{10}$$

assuming a narrow width approximation for the decay $K^* \to K\pi$.

It can easily be seen from eq.(9) that, the only terms proportional to $\sin(\varphi)$ or $\sin(2\varphi)$ are those that depend on the imaginary parts of the $\alpha$, $\beta$ or $\rho$. For instance only the coefficient of $\mathrm{Im}(\alpha_L\beta_L^* + \alpha_R\beta_R^*)$ is proportional to $\sin(2\varphi)$. Hence we can isolate this term by considering the following asymmetric width in terms of the differential decay rates of the B meson with respect to $\varphi$,

$$\Delta\Gamma_1 = \left(\int_0^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\pi} + \int_{\pi}^{\frac{3\pi}{2}} - \int_{\frac{3\pi}{2}}^{2\pi}\right)\frac{d\Gamma}{d\varphi}d\varphi\,. \tag{11}$$

The imaginary part in the term under consideration can be due to either a strong phase or a weak phase. Such $CP$ violating asymmetries can be obtained not by considering the difference of differential rates for $B$ and $\bar{B}$ , but the sum of these rates. It follows trivially from eqn.(7 and 8) that the asymmetric width for $B(\bar{B}\ )$ is

$$\Delta\Gamma_1(\Delta\bar{\Gamma}_1) \propto \pm \sum_{j,k}\{|a_L^j||b_L^k| \sin((\delta_L^{jk}) \pm (\phi_L^{jk})) + L \to R\} \tag{12}$$

where $\delta_L^{jk} \equiv (\delta_L^{\alpha_j} - \delta_L^{\beta_k})$ and $\phi_L^{jk} \equiv (\phi_L^{\alpha_j} - \delta_L^{\beta_k})$. We define the asymmetry $\mathsf{A}_1$ as the sum of the asymmetric widths, normalized to the total widths, $\mathsf{A}_1 = \frac{\Delta\Gamma_1+\Delta\bar{\Gamma}_1}{\Gamma+\bar{\Gamma}}$ and from eq.(12), we have,

$$\mathsf{A}_1 \propto \sum_{j,k}\{|a_L^j||b_L^k| \cos(\delta_L^{jk})\, \sin(\phi_L^{jk}) + L \to R\}\,, \tag{13}$$

which is *nonzero if and only if there is CP violation represented by non-zero phases* $\phi$ [20,22]. $\delta$ can arise from electromagnetic final state interactions, which are negligible and ignored. For top quark in the penguin loop, the case of $\delta$ being nonzero due to intermediate quark on shell, does not arise.

It is also possible to construct a different asymmetry that isolates another combination of the imaginary terms. Such an asymmetry [23] considers the difference distribution of the same hemisphere and opposite hemisphere events, and the asymmeric width in this case can be defined by

$$\Delta\Gamma_2 = (\int_0^{\pi} - \int_{\pi}^{2\pi}) d\varphi \int_D d\cos\theta_l \int_D d\cos\theta_K \tilde{\Gamma} \tag{14}$$

where $\int_D \equiv \int_{-1}^{0} - \int_0^1$ and $\tilde{\Gamma} = \dfrac{d\Gamma}{d\cos\theta_l d\cos\theta_K d\varphi}$. Analogous to $\mathsf{A}_1$, we define the asymmetry $\mathsf{A}_2$, $\mathsf{A}_2 = \dfrac{\Delta\Gamma_2 + \Delta\bar{\Gamma}_2}{\Gamma + \bar{\Gamma}}$. The asymmetries $\mathsf{A}_1$ and $\mathsf{A}_2$ are evaluated to be,

$$\mathsf{A}_1 = -2\,x\,\Delta \int dq^2\, \mathsf{C}\,(a_0\, V - A_0\, g)\, X^2\,,\ \mathsf{A}_2 = x \int dq^2\, \mathsf{C}\, \mathsf{F} \frac{1}{m_{K^*}\sqrt{q^2}}\, X^2\,. \tag{15}$$

where $x = \dfrac{v_t^2\, \alpha^2 G_F^2\, m_b}{24\,\pi^6 M_B(\Gamma + \bar{\Gamma})(M_B + m_{K^*})}$, $\mathsf{C} = (\tilde{\kappa} C_7^{\tilde{\kappa}} + \tilde{\lambda} C_7^{\tilde{\lambda}}) C_8^{SM}$, $\Delta = M_B^2 - m_{K^*}^2$ and $\mathsf{F} = \left\{2X^2 M_B^2(a_+\, V - A_+\, g) + (a_0\, V - A_0\, g)\, \Delta\, k\cdot q\right\}$. Note that these asymmeties are independent of $C_9^{SM}$. Since, $C_8^{\tilde{\kappa},\tilde{\lambda}}$ terms are suppressed wrt $C_7^{\tilde{\kappa},\tilde{\lambda}}$, for all our estimations, we ignore them.

For a rate asymmetry A to provide a $n$ standard deviation signal of CP violation, we require that $A = n/\sqrt{N}$, where N is the total number of events in the channel. Thus for $\mathsf{A}_2$, at $1\sigma$ ($1.64\ \sigma$) level we obtain the bounds, $|\tilde{\kappa}| < 0.34$ (0.92) for $\tilde{\lambda} = 0$ and $|\tilde{\lambda}| < 1.02$ (2.75) for $\tilde{\kappa} = 0$, using 10,000 ($B \to K^* e^+ e^-$) events. This corresponds to 2x$10^9$ B's, however, by adding the ($B \to K^* \mu^+ \mu^-$) mode, $10^9$ B's will be required. Much weaker bounds are obtained from $\mathsf{A}_1$.

However, we can improve the statistical significance, by looking at the $q^2$ distributions, shown in fig.1. We bin the asymmetry $\mathsf{A}_2$ ($\mathsf{A}_1$ being small is ignored, and we shall drop the subscript 2 hereafter) and use a $\chi^2$ fit to obtain bounds on the parameters $\tilde{\kappa}$ and $\tilde{\lambda}$. The asymmetry as a function of $q^2$ is given by

$$A(q^2) = \frac{(\int_0^{\pi} - \int_{\pi}^{2\pi}) d\varphi \int_D d\cos\theta_l \int_D d\cos\theta_K\ \Gamma_{sum}(q^2)}{(\int_0^{\pi} + \int_{\pi}^{2\pi}) d\varphi \int_{-1}^{1} d\cos\theta_l \int_{-1}^{1} d\cos\theta_K\ \Gamma_{sum}(q^2)} \tag{16}$$

where $\Gamma_{sum}(q^2) = \frac{d(\Gamma + \bar{\Gamma})}{dq^2 d\cos\theta_l d\cos\theta_K d\varphi}$, $d(\Gamma + \bar{\Gamma})$ is the sum of the differential widths of $B$ and $\bar{B}$. The average value of asymmetry in the $i^{th}$ bin is then

$$A_i = \frac{N_B}{\Gamma_B\ N_i} \int_{q^2_{min}}^{q^2_{max}} dq^2 A(q^2) \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \tag{17}$$

where, $N_B$ is the total number of B's, $\Gamma_B$ is the total B width and $q^2_{min}$,$q^2_{max}$ are the minimum and maximum $q^2$ values in the bin; the numer of events in the $i^{th}$ bin are,

$$N_i = \frac{N_B}{\Gamma_B} \int_{q^2_{min}}^{q^2_{max}} dq^2 \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \tag{18}$$

The value of the asymmetry, in the $i^{th}$ $q^2$ bin, coming from $\tilde{\kappa}$ and $\tilde{\lambda}$ contributions, is of the form

$$A_i = \frac{a_i \tilde{\kappa} + b_i \tilde{\lambda}}{X_i}. \tag{19}$$

In $X_i$, terms quadratic in $\tilde{\kappa}$ and $\tilde{\lambda}$ are ignored, which is a reasonable approximation as long as $\tilde{\kappa}$, $\tilde{\lambda}$ are small. The observed asymmetry $A_i^{obs}$ in the $i^{th}$ bin is assumed to be chosen from a Gaussian distribution with mean equal to the theoretical asymmetry $A_i^{th}$ and variance $\sigma_i^2$. Then, the method of least squares gives

$$\chi^2 = \sum_{i=1}^{nbin} \left( \frac{A_i^{obs} - A_i^{th}}{\sigma_i} \right)^2 \tag{20}$$

The statistical errors are taken to be $\delta N = \sqrt{N}$, $\delta A = \sqrt{1/N}$. We evaluate the difference, $\Delta\chi^2 = \chi^2_{SM} - \chi^2_{min}$, where $\chi^2_{SM}$ is the $\chi^2$ corresponding to the SM values of the parameters i.e. (0,0) and $\chi^2_{min}$ is evaluated from the values $(\tilde{\kappa}, \tilde{\lambda})$ that minimize $\chi^2$. The linear dependence on the parameters $\tilde{\kappa}, \tilde{\lambda}$ results in the simple form,

$$\Delta\chi^2 = \sum_{i=1}^{nbin} \frac{\left(\frac{\tilde{\kappa} a_i}{X_i}\right)^2 + \left(\frac{\tilde{\lambda} b_i}{X_i}\right)^2 + \frac{2\tilde{\lambda}\tilde{\kappa} a_i b_i}{X_i^2}}{\sigma_i^2} \tag{21}$$

Now, $\Delta\chi^2 = n^2$ will give the $n$-standard deviation bounds for $\tilde{\kappa}$ and $\tilde{\lambda}$.

For 20 bins, for 10,000 ($B \to K^* e^+ e^-$) events, we obtain the improved individual bounds,

$$\begin{aligned} |\tilde{\kappa}| &< 0.25(0.42) \quad \text{at} \quad 68(90)\%\ C.L. \\ |\tilde{\lambda}| &< 0.76(1.25) \quad \text{at} \quad 68(90)\%\ C.L. \end{aligned} \tag{22}$$

The combined $\tilde{\kappa}$ and $\tilde{\lambda}$ bounds are shown in fig. 2

To conclude, we have obtained bounds on the CP violating anomalous couplings $\tilde{\kappa}$ and $\tilde{\lambda}$. These bounds are obtained from an asymmetry for the process $B \to K^* \ell^+ \ell^- \to K\pi \ell^+ \ell^-$, constructed by adding B and $\bar{B}$ events. No strong phases, nor any flavour/time tagging are required for this technique. Although, EDM of neutron places strong bounds on the couplings $\tilde{\kappa}$ and $\tilde{\lambda}$, the CP odd asymmetries studied here, depend on a different combination of these anomalous couplings and thus therefore provide useful additional information. $\tilde{\lambda}$ being the coefficient of a higher dimensional operator, is expected to be better constrained by high energy collider events than from rare B decays. The recent results of D0 provide good bounds on $\tilde{\lambda}$, but our bounds on $\tilde{\kappa}$ are much tighter.

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## TABLES

TABLE I. Relations between the form factors used in this paper, a quark model (QM) that reproduces heavy quark limit . $W_\mu = (K_\mu - \zeta\, k_\mu)$, $\sigma^2 = 96\pi^2/(m_{K^*}^2 \lambda_K{}^3)$, $\zeta = \dfrac{k\cdot K}{m_{K^*}^2}$, $\lambda_K$ and $\Delta$ are defined in the text.

| | QM [4] |
|---|---|
| $\mathcal{A}$ | $-2\, g\, \sigma$ |
| $\mathcal{B}$ | $a_0\, \Delta\, \sigma$ |
| $\mathcal{C}$ | $2\, a_+\, W\cdot q\, \sigma - \zeta\, \mathcal{B}$ |
| $\mathcal{D}$ | $-2\, \dfrac{V}{M_B + m_{K^*}}\, \sigma$ |
| $\mathcal{E}$ | $\dfrac{A_0}{M_B + m_{K^*}}\, \Delta\, \sigma$ |
| $\mathcal{F}$ | $2\, \dfrac{A_+}{M_B + m_{K^*}}\, W\cdot q\, \sigma - \zeta\, \mathcal{E}$ |

## FIGURES

FIG. 1. (a) The asymmetry $A_2(q^2)$ as a function of $q^2$. The solid line is for $\tilde{\kappa} = 1(\tilde{\lambda} = 0)$ and the dashed for $\tilde{\lambda} = 1(\tilde{\kappa} = 0)$. (b) The differential branching fraction as a function of $q^2$.

FIG. 2. Limits on the CP-violating anomalous coupling parameters $\tilde{\kappa}$ and $\tilde{\lambda}$. The region inside the solid (dashed) lines represent the 68(90)% CL limits.