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Solar neutrinos : Eclipse effect

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Abstract

It is pointed out that the enhancement of the solar neutrino rate in a real time detector like Super–Kamioka, SNO or Borexino due to neutrino oscillations in the moon during a partial or total solar eclipse may be observable. The enhancement is calculated as a function of the neutrino parameters in the case of three flavor mixing. This enhancement if seen, can further help to determine the neutrino parameters.

I. INTRODUCTION

The sun is a copious source of neutrinos with a wide spectrum of energies and these neutrinos have been detected by terrestrial neutrino detectors, although at a rate lower than expected from theoretical calculations. A new generation of detectors [1–3] with high counting rates will soon be producing abundant data on solar neutrinos. Mixing and the consequent oscillations among the neutrinos of different flavors is generally believed to be the cause of the reduced intensity of neutrino flux detected on the earth. However, neutrino–oscillation is a complex phenomenon depending on many unknown parameters (six parameters for three flavors ν_e, ν_μ, ν_τ) and considerable amount of experimental work and ingenuity will be required before the neutrino problem is solved.

Hence it would be desirable, if apart from direct detection, we can subject the solar neutrino beam to further tests by passing it through different amounts of matter, in our attempts to learn more about the neutrinos. Nature has fortunately provided us with such opportunities: (1) Neutrinos detected at night pass through the earth. (2) Neutrinos detected during a solar eclipse pass through the moon. (3) Neutrinos detected at the far side of the earth during a solar eclipse pass through the moon and the earth. We shall call this scenario (3) a *double eclipse*. The scenario (1) has been studied in the literature rather extensively [4,5]. The purpose of the present work is to examine the scenarios (2), and (3). Two previous works [6,7] have discussed scenario (2), however both are incomplete in many respects.

The plan of the paper is as follows. The relevant astronomy is presented in Sec.II. In Sec.III we give the theory of the passage of the solar neutrinos through the moon and the earth, taking into account properly the non-adiabatic transitions occuring at the boundaries of the moon and the earth. In Sec.IV we present the numerical calculations of the neutrino detection rates during the single and double eclipses and the results. Sec.V is devoted to discussion.

II. ECLIPSES AND DOUBLE ECLIPSES

Solar neutrinos are produced within the solar core whose radius is of order 1/10 of the solar radius and we shall approximate this by a point at the centre of the sun. What is required for our purpose is that the lunar disc must cover this point at the centre of the sun and so as far as the neutrino radiation is concerned , the solar eclipse is more like an occultation of a star or a planet by the moon.

Astronomers characterize the solar eclipse by the optical coverage C which is defined as the ratio of the area of the solar disc covered by the lunar disc to the total area of the solar disc.For neutrino physics we require the distance d_M travelled by the neutrino inside the moon. Defining the fraction $x = \frac{d_M}{2R_M}$ where R_M is the lunar radius, x can be given in terms of C by the following formulae:

$$x = \sqrt{(4z(2-z)-3)} \tag{1}$$

$$C = \frac{2}{\pi}\left(\cos^{-1}(1-z) - (1-z)\sqrt{z(2-z)}\right) \tag{2}$$

Eqn(2) can be inverted to get the parameter z for a given C and this z can be substituted in eqn(1). The relationship between x and C so obtained is plotted in Fig1. When the lunar disc passes through the centre of the sun, C is 0.39 and the neutrino eclipse starts at this value of C.When the optical coverage increases above 39%, x rises sharply from zero and reaches 0.6 and 0.95 for optical coverage of 50% and 80% respectively.

For any point of observation of the usual solar eclipse (which we shall call *single eclipse*) there is a corresponding point on the other side of the earth where a double eclipse occurs. With the coordinates labeled as (latitude, longitude), the single eclipse point (α, β) is related to the double eclipse point (λ, σ) by the relations (see Fig.2):

$$\lambda = \alpha + 2\delta$$

$$\sigma = \pi - 2\,\Theta_{UT} - \beta \tag{3}$$

where,

$$\sin\delta = \sin 23.5^o \sin(\frac{2\pi t}{T_Y}), \quad (4)$$

T_Y is the length of the year , zero of time t is chosen at midnight of autumnal equinox *i.e.*Sept. 21, and Θ_{UT} is the angle corresponding to the Universal Time – UT.

During a double eclipse, the neutrinos travel through the earth in addition. The distance d_E travelled by the neutrino inside the earth along the chord between the points (α, β) and (λ, σ) as a function of time t, is given by

$$d_E = 2R_E(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos(\frac{2\pi t}{T_D})) \quad (5)$$

where R_E is the radius of the earth and T_D is the length of the day. This is the same distance that needs to be calculated in the study of the day–night effect and a plot of this distance as a function of t is given in our earlier paper [5].

Present and upcoming high statistics neutrino detectors expect to collect a few solar neutrino events every hour. As discussed in Secs. III A and III B, single and double eclipse can lead to enhancements of rates by upto two and a half times. Even with such large enhancements during the eclipse the signal may not exceed statistical errors, since each solar eclipse lasts only for a few hours. However they occur fairly often. As many as 32 solar eclipses are listed to occur during the 14 year period 1996 through 2010. Global maps and charts are available [8] for location and duration of both the umbral and penumbral coverage.

The eclipses during the 2 year period 1997 through 1999 are the following:

1997 March 9 – Total Solar Eclipse

1997 Oct 12 – Partial Solar Eclipse

1998 Feb 26 – Total Solar Eclipse

1998 Aug 22 – Annular Solar Eclipse

1999 Feb 16 – Annular Solar Eclipse

1999 Aug 11 – Total Solar Eclipse

We have analyzed the five total/annular eclipses. We have not examined the partial solar eclipse of 1997 October 12, since we have not so far been able to procure the data for this eclipse from the references cited in [8]. It is important to remark here that although partial solar eclipses are not so useful to astronomers they are nevertheless relevant for neutrino physics as long as C is above 0.39.

A study of the global maps of the five total/annular eclipses listed above shows that only three of them can occur as a single or a double eclipse at any of the three existing detector sites Kamioka, Sudbury and Gran Sasso with coordinates $(36.4^o N, 140^o E)$, $(46.5^o N, 81^o W)$ and $(42.5^o N, 13.5^o E)$ respectively. Approximate estimates are presented below for the duration and optical coverage of the relevant detectors at these sites:

1997 March 9 – This eclipse will be a single eclipse for the Super–Kamioka, with an approximate duration of two and a half hours and a maximum optical coverage of just over 60%. In addition, there will be a double eclipse at the Gran–Sasso, the site of Borexino, at almost the same time as at Kamioka and for the same duration, with a 70% optical coverage.

1998 Feb 26 – Though Sudbury will have an optical eclipse the coverage is less than 39% and so no neutrino–eclipse occurs. However, a double eclipse occurs at Gran–Sasso with a maximum coverage of 70% to 80%. This corresponds to the single eclipse at $(31^o N, 45^o W)$.

1999 Aug 11 – This eclipse will provide one of the best opportunities, as a single eclipse with 90% optical coverage and almost 3 hour duration at Gran-Sasso. In addition Super–Kamioka site will also get a double eclipse with 90% to 100% optical coverage, corresponding to the single eclipse at approximately $(36^o N, 40^o E)$

III. THEORY

A. Regeneration in the moon

We now describe a straightforward way of obtaining the neutrino regeneration effect in the moon by using a model of moon of constant density (3.33gms/cc). Let a neutrino of flavor α be produced at time $t = t_0$ in the core of the sun. Its state vector is

$$|\Psi_\alpha(t_0)\rangle = |\nu_\alpha\rangle = \sum_i U^S_{\alpha i}|\nu^S_i\rangle. \tag{6}$$

where $|\nu^S_i\rangle$ are the matter dependent mass eigenstates with mass eigenvalues μ^S_i and $U^S_{\alpha i}$ are the matrix elements of the matter dependent mixing matrix in the core of the sun. We use Greek index α to denote the three flavors e, μ,τ and Latin index i to denote the mass eigenstates i = 1,2,3. The neutrino propagates in the sun adiabatically upto t_R (the resonance point), makes non-adiabatic transitions at t_R, propagates adiabatically upto t_1 (the edge of the sun) and propagates as a free particle upto t_2 when it enters the moon. So the state vector at t_2 is

$$|\Psi_\alpha(t_2)\rangle = \sum_{j,i}|\nu_j\rangle exp\left(-i\varepsilon_j(t_2-t_1)\right) exp\left(-i\int_{t_R}^{t_1}\varepsilon^S_j(t)dt\right) M^S_{ji} exp\left(-i\int_{t_0}^{t_R}\varepsilon^S_i(t)dt\right) U^S_{\alpha i}. \tag{7}$$

where $\varepsilon^S_i(t)(\equiv E + (\mu^S_i(t))^2/2E)$ are the matter dependent energy eigenvalues in the sun, ε_i and $|\nu_i\rangle$ are the energy eigenvalues and the corresponding eigenstates in vacuum and M^S_{ji} is the probability amplitude for the non-adiabatic transition $i \to j$. We multiply the right hand side of eq.(7) by $\sum_k |\nu^M_k\rangle\langle\nu^M_k|$ (= 1)where $|\nu^M_k\rangle$ (i =1,2,3) is the complete set of matter dependent mass eigenstates inside the moon. The neutrino propagates upto the the other end of the moon at t_3, and the state vector at t_3 is

$$\begin{aligned}|\Psi_\alpha(t_3)\rangle &= \sum_{k,j,i}|\nu^M_k\rangle exp\left(-i\varepsilon^M_k(t_3-t_2)\right)\langle\nu^M_k|\nu_j\rangle exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1}\varepsilon^S_j(t)dt\right) \\ &\quad\times M^S_{ji} exp\left(-i\int_{t_0}^{t_R}\varepsilon^S_i(t)dt\right) U^S_{\alpha i} \\ &= \sum_{k,j,i}|\nu^M_k\rangle exp\left(-i\varepsilon^M_k(t_3-t_2)\right) M^M_{kj}\, exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1}\varepsilon^S_j(t)dt\right) \\ &\quad\times M^S_{ji} exp\left(-i\int_{t_0}^{t_R}\varepsilon^S_i(t)dt\right) U^S_{\alpha i}\end{aligned} \tag{8}$$

We have introduced the probability amplitude M^M_{kj} for non-adiabatic transitions $j \rightarrow k$ due to the abrupt change in density when the neutrino enters the moon . It is given by

$$M^M_{kj} = \langle \nu^M_k | \nu_j \rangle = \sum_\gamma \langle \nu^M_k | \nu_\gamma \rangle \langle \nu_\gamma | \nu_j \rangle = \sum_\gamma U^M_{\gamma k} U^*_{\gamma j} \tag{9}$$

where $U_{\gamma j}$ is the mixing matrix in vacuum . We multiply the right hand side of eq.(8) by $\sum_l |\nu_l\rangle\langle\nu_l|$ where $|\nu_l\rangle$ (l =1,2,3) is the complete set of vacuum mass eigenstates. The neutrino leaves the other end of the moon at $t = t_3$ and propagates upto the surface of the earth ,which it reaches at t_4 .So the state vector at t_4 is

$$\begin{aligned} |\Psi_\alpha(t_4)\rangle = & \sum_{k,j,i,l} |\nu_l\rangle exp\left(-i\varepsilon^M_k(t_3-t_2)\right) \langle \nu_l|\nu^M_k\rangle M^M_{kj}\, exp\left(-i\varepsilon_j(t_2-t_1) - i\int_{t_R}^{t_1} \varepsilon^S_j(t)dt\right) \\ & \times M^S_{ji} exp\left(-i\int_{t_0}^{t_R} \varepsilon^S_i(t)dt\right) U^S_{\alpha i} exp\left(-i\varepsilon_l(t_4-t_3)\right) \\ = & \sum_{k,j,i,l} |\nu_l\rangle M^M_{kj} M^{M*}_{kl} M^S_{ji} U^S_{\alpha i} exp\left(-i\Phi_{ijkl}\right) \end{aligned} \tag{10}$$

where

$$\Phi_{ijkl} = \varepsilon^M_k(t_3-t_2) + \varepsilon_l(t_4-t_3) + \varepsilon_j(t_2-t_1) + \int_{t_R}^{t_1} \varepsilon^S_j(t)dt + \int_{t_0}^{t_R} \varepsilon^S_i(t)dt \tag{11}$$

We have used the fact that the the probability amplitude for non-adiabatic transitions $k \rightarrow l$ due to the abrupt change in density when the neutrino leaves the moon is

$$\langle \nu_l | \nu^M_k \rangle = M^{M*}_{kl} \tag{12}$$

The probability of detecting a neutrino of flavor β at t_4 is

$$|\langle \nu_\beta | \Psi_\alpha(t_4)\rangle|^2 = \sum U^*_{\beta l} U_{\beta l'} M^M_{kj} M^{M*}_{k'j'} M^{M*}_{kl} M^M_{k'l'} M^S_{ji} M^{S*}_{j'i'} U^S_{\alpha i} U^{S*}_{\alpha i'} exp\left(-i(\Phi_{ijkl} - \Phi_{i'j'k'l'}\right) \tag{13}$$

where the summation is over the set of indices $i, j, k, l, i', j', k', l'$ Averaging over t_R leads to $\delta_{ii'}\delta_{jj'}$ and this results in the desired incoherent mixture of mass eigenstates of neutrinos reaching the surface of the moon at t_2. Calling this averaged probability as $P^M_{\alpha\beta}$ (the probability for a neutrino produced in the sun as ν_α to be detected as ν_β in the earth after passing through the moon), we can write the result as

$$P^M_{\alpha\beta} = \sum_j P^S(\alpha\rightarrow j)P^M(j\rightarrow\beta) \tag{14}$$

where

$$P^S(\alpha\rightarrow j) = \sum_i |M^S_{ji}|^2|U^S_{\alpha i}|^2 \tag{15}$$

$$P^M(j\rightarrow\beta) = \sum_{l,k,l',k'} U^*_{\beta l}U_{\beta l'}M^M_{kj}M^{M*}_{k'j}M^{M*}_{kl}M^M_{k'l'}exp\left(-i(\varepsilon^M_k-\varepsilon^M_{k'})d_M - i(\varepsilon_l-\varepsilon_{l'})r\right) \tag{16}$$

where we have replaced $(t_3 - t_2)$ by d_M, the distance travelled by the neutrino inside the moon, and $(t_4 - t_3)$ by r the distance travelled by the neutrino from the moon to the earth. If there is no moon , we put $d_M = 0$, so that $P^M(j\rightarrow\beta)$ becomes $|U_{\beta j}|^2$ and so eqn.(14) reduces to the usual [9,12] averaged probability for ν_α produced in the sun to be detected as ν_β in the earth :

$$P^O_{\alpha\beta} = \sum_{i,j} |U_{\beta j}|^2|M^S_{ji}|^2|U^S_{\alpha i}|^2. \tag{17}$$

B. Regeneration during double eclipse.

We use a model of earth of constant density (5.52gms/cc). We start with $\Psi_\alpha(t_4)$ given by eq.(10) and multiply the right hand side by $\sum_p |\nu^E_p\rangle\langle\nu^E_p|$ $(=1)$ where $|\nu^E_p\rangle$ $(i = 1,2,3)$ is a complete set of mass eigenstates inside the earth. The neutrino enters the earth at time t $= t_4$ and is detected at time $t = t_5$ inside the earth.The state vector at time $t= t_5$ is

$$|\Psi_\alpha(t_5)\rangle = \sum_{k,j,i,l,p} |\nu_p\rangle M^E_{pl}M^M_{kj}M^{M*}_{kl}M^S_{ji}U^S_{\alpha i}\exp\left(-i\Phi_{ijklp}\right) \tag{18}$$

where we have introduced the probability amplitude M^E_{pl} for non adiabatic transitions $l\rightarrow p$ due to the abrupt change in density when the neutrino enters the earth . It is given by

$$M^E_{pl} = \langle\nu^E_p|\nu_l\rangle = \sum_\sigma U^E_{\sigma p}U^*_{\sigma l} \tag{19}$$

and

$$\Phi_{ijklp} = \varepsilon^E_p(t_5-t_4) + \varepsilon^M_k(t_3-t_2) + \varepsilon_l(t_4-t_3) + \varepsilon_j(t_2-t_1) + \int_{t_R}^{t_1}\varepsilon^S_j(t)dt + \int_{t_0}^{t_R}\varepsilon^S_i(t)dt \tag{20}$$

The probability of detecting a neutrino of flavor β at t_5 is

$$|\langle\nu_\beta|\Psi_\alpha(t_5)\rangle|^2 = \sum U^{E*}_{\beta p} U^{E}_{\beta p'} M^{E}_{pl} M^{E*}_{p'l'} M^{M}_{kj} M^{M*}_{k'j'} M^{M*}_{kl} M^{M}_{k'l'} M^{S}_{ji} M^{S*}_{j'i'} U^{S}_{\alpha i} U^{S*}_{\alpha i'}$$
$$\times \exp\left(-i(\Phi_{ijklp} - \Phi_{i'j'k'l'p'}\right) \tag{21}$$

where the summation is over the set of indices $i, j, k, l, p, i', j', k', l'p'$ Again averaging over t_R and calling this averaged probability as $P^{ME}_{\alpha\beta}$ (the probability for a neutrino produced in the sun as ν_α to be detected as ν_β in the earth after passing through the moon and the earth), we can write the result as

$$P^{ME}_{\alpha\beta} = \sum_j P^S(\alpha \to j) P^{ME}(j \to \beta) \tag{22}$$

where

$$P^{ME}(j \to \beta) = \sum_{l,k,p,l',k',p'} U^{E*}_{\beta p} U^{E}_{\beta p'} M^{E}_{pl} M^{E*}_{p'l'} M^{M}_{kj} M^{M*}_{k'j} M^{M*}_{kl} M^{M}_{k'l'}$$
$$\times exp\left(-i(\varepsilon^E_p - \varepsilon^E_{p'})d_E - i(\varepsilon^M_k - \varepsilon^M_{k'})d_M - i(\varepsilon_l - \varepsilon_{l'})r\right) \tag{23}$$

where we have replaced $(t_5 - t_4)$ by d_E, the distance travelled by the neutrino inside the earth, $(t_3 - t_2)$ by d_M, the distance travelled by the neutrino inside the moon and $(t_4 - t_3)$ by r the distance travelled by the neutrino from the moon to the earth.

For the sake of completeness, we state that if we put $d_M = 0$ in eqn(23) we get the regeneration in earth alone:

$$P^E_{\alpha\beta} = \sum_j P^S(\alpha \to j) P^E(j \to \beta) \tag{24}$$

where

$$P^E(j \to \beta) = \sum_{k,k'} U^{E*}_{\beta k} U^{E}_{\beta k'} M^{E}_{kj} M^{E*}_{k'j} exp\left(-i(\varepsilon^E_k - \varepsilon^E_{k'})d_E\right) \tag{25}$$

Eqns(24) and (25) have been used to study the day–night effect [5].

It is important to note that the factorization of probabilities seen in eqs(14),(22) and (24) is valid only for mass eigenstates in the intermediate state. An equivalent statement of this result is that the density matrix is diagonal only in the mass-eigenstate representation and not in the flavour representation.

C. Three flavor mixing parameters

We parameterize the mixing matrix U in vacuum as $U = U^{23}(\psi)U^{13}(\phi)U^{12}(\omega)$ where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the i^{th} and the j^{th} mass eigenstates with the mixing angle θ_{ij}, neglecting CP violation. In the solar neutrino problem ψ drops out [10,11] The mass differences in vacuum are defined as $\delta_{21} = \mu_2^2 - \mu_1^2$ and $\delta_{31} = \mu_3^2 - \mu_1^2$. It has been shown [12,13] that the simultaneous solution of both the solar and the atmospheric neutrino problems requires

$$\delta_{31} \gg \delta_{21} \tag{26}$$

and under this condition δ_{31} also drops out. The rediagonalization of the mass matrix in the presence of matter (in solar core, moon or earth) under condition (26) leads to the following results [12]

$$\tan 2\omega_m = \frac{\delta_{21} \sin 2\omega}{\delta_{21} \cos 2\omega - A \cos^2 \phi} \tag{27}$$

$$\sin \phi_m = \sin \phi \tag{28}$$

$$\delta_{21}^m = \delta_{21} \cos 2(\omega - \omega_m) - A \cos^2 \phi \cos 2\omega_m \tag{29}$$

where A is the Wolfenstein term $A = 2\sqrt{2}\ G_F\ N_e\, E$ (N_e is the number density of electrons and E is the neutrino energy) . We note that $\delta_{31} \gg A^S, A^M, A^E$. In eqs.(27)–(29) the subscript "m" stands for matter. Under the condition $\delta_{31} \gg A \approx \delta_{21}$ we need the non adiabatic transition probability $|M_{ij}^S|^2$ for i,j =1,2 only and this is taken to be the modified Landau–Zener jump probability for an exponentially varying solar density [11].

IV. CALCULATIONS AND RESULTS

The neutrino detection rates for a Super–Kamoika type of detector is given by

$$R = \int \phi(E)\, \sigma(E) P_{ee} dE + \frac{1}{6}(\int (\phi(E)\sigma(E)(1 - P_{ee})dE) \tag{30}$$

where the second term is the neutral current contribution and $\phi(E)$ is the solar neutrino flux as a function of the neutrino energy E and $\sigma(E)$ is the cross section from neutrino electron scattering and we integrate from $5MeV$ onwards. The cross section is taken from [14] and the flux from [15]. The rates for a single eclipse, double eclipse and without eclipse (at day–time) R_M, R_{ME} and R_O are calculated using P_{ee}^M, P_{ee}^{ME} and P_{ee}^O from eqns.(14),(22) and (17) respectively. We define the enhancement factors F and G for a single and double eclipse respectively:

$$F = \frac{R_M - R_O}{R_O} \tag{31}$$

$$G = \frac{R_{ME} - R_O}{R_O}. \tag{32}$$

It is easy to see that F and G have to be less than 5 and this theoretical maximum value occurs when $P_{ee}^O = 0$ and P_{ee}^M and P_{ee}^{ME} are put 1. If one imposes the constraint that the observed [16] neutrino rate is 0.51 ± 0.07 of the prediction of the standard solar model, the maximum possible enhancement is reduced to about 1.40 (at 90% C.L.).

We calculate the enhancement factors F and G for various values of the neutrino parameters, ω, δ_{21}, and ϕ. We show the results as contour plots in the δ_{21}–ω plane for different values of ϕ. Figs.3 and 4 show the F–contours for $\phi = 0^o$ and $\phi = 30^o$ respectively. For each ϕ we show the contours for different distances of travel of the neutrino through the moon. Fig. 5 shows the G–contours for $\phi = 0$ for the maximum distance of travel of the neutrino inside the moon and the earth. The main features of the results are as follows:

- As the distance travelled by the neutrino inside the moon increases one can see an appreciable increase in the enhancement factor F. It increases from less than 10% to about almost 100% when the neutrino travels the whole diameter of the moon in the case of two flavor mixing i.e $\phi = 0$.

- Large ($> 40\%$) values of F occur for ω between 20^o and 30^o and $\delta_{21} \sim 10^{-6} eV^2$ This is true even for nonzero ϕ.

- The effect of a non zero "13" mixing angle ϕ, is to dilute the enhancement factor F for all values of distance travelled through the moon.(In fact for $\phi \approx 45^o$, F is practically zero and so we have not plotted this case.) This is because a non zero ϕ means $\nu_e \leftrightarrow \nu_\tau$ oscillations, and matter cannot reconvert ν_τ back to ν_e, because the "13" mixing angle ϕ is not affected by matter.

- If large enhancement F is seen for values of $x \leq 0.6$, it immediately signals a very small value of ϕ. On the other hand, if no enhancement is seen for small x but there is enhancement only for $x \geq 0.8$ it signals an appreciable value of ϕ.

- For a double eclipse there are considerable enhancements even for small values of ω. There is enhancement throughout the range of ω from small angles till about 40^o. In fact the regions of largest enhancement ($> 100\%$) are for ω between 5^o to 20^o.

- The region of maximum enhancement factor G is centered around a value of δ_{21} which is a little above $10^{-6} eV^2$, this being the value for maximum in F. This can be traced to the fact : $A^E > A^M$. However sizeable enhancement occurs over a wide range of δ_{21}.

- If enhancement is not seen, then certain regions of the neutrino parameter space can be excluded.If no enhancement is seen for single eclipse, a panel of ω between $5 - 25^o$ and $\delta_{21} \approx 2 \times 10^{-7} - 2 \times 10^{-6} eV^2$ for $\phi = 0$ can be ruled out. If it is not seen for a double eclipse, a larger region can be ruled out.

V. DISCUSSION

We have studied the effect on the solar neutrinos of their passage through the moon as well as the moon together with the earth. Although the numerical results presented in the paper cover only a representative sample of the set of various parameters, our analytical expressions can be used for more extensive calculations depending on the requirement. Also one can go beyond the hierarchy : $\delta_{31} >> \delta_{21}$.

We now offer a few concluding remarks:

- Together with the day-night effect, the eclipse effects provide us with the tools for studying solar neutrinos, in a way independent of the uncertainties of the solar models.
- If the neutrino mass differences are really very small ($\delta_{21} < 10^{-5} eV^2$) there is no way of pinning down the neutrino parameters other than using the astronomical objects such as the moon or the earth for the "long-base-line experiments".
- It is important to stress that even the demonstrated absence of any eclipse effect would provide us with definitive information on neutrino physics.
- Accumulation of data over many eclipses may be needed for good statistics. In any future planning of detector sites, this may be kept in mind.
- It appears that Nature has chosen the neutrino parameters in such a way that the sun affects the propagation of solar neutrinos. It may be hoped that Nature has similarly chosen "lucky" parameters so that the moon and the earth too can affect the neutrinos!
- Finally, we stress the novelty of the whole phenomenon, and urge the experimentalists to look for and study the eclipse effects in an unbiased manner. They may even discover some surprises, not predicted by our calculations!

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FIGURES

FIG. 1. The fractional distance travelled by the neutrino inside the moon $x(=\dfrac{d_M}{2R_M})$ is plotted against the optical coverage C of the solar eclipse.

FIG. 2. Geometry relating the double eclipse point (λ, σ) to the single eclipse point (α, β). (a) Section of the earth passing through (α, β) and perpendicular to the ecliptic. (b) Section passing through (α, β) and parallel to the equator.

FIG. 3. Contour plots of the enhancement factor for single eclipse $F(=\dfrac{R_M - R_O}{R_O})$ in the $\omega - \delta_{21}$ plane for $\phi = 0^o$ and for four values of x (x = 0.4, 0.6, 0.8 and 1.0). The enhancement factor (regarded as a percentage) increases by 10% for every adjacent ring , as we move inwards towards the centre of the plot.

FIG. 4. Contour plots of the enhancement factor for single eclipse $F(=\dfrac{R_M - R_O}{R_O})$ in the $\omega - \delta_{21}$ plane for $\phi = 30^o$ and x = 0.6, 0.8 and 1.0. The enhancement factor (regarded as a percentage) increases by 10% for every adjacent ring , as we move inwards towards the centre of the plot.

FIG. 5. Contour plot of the enhancement factor for double eclipse $G(=\dfrac{R_{ME} - R_O}{R_O})$ for $\phi =$ 0 and x= 1.0. The distance travelled by the neutrino inside the earth is also taken to be the full earth diameter. The enhancement factor increases by 20% as we move inwards.

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