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Time–of–night variation of solar neutrinos

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Abstract

We investigate the time–of–night variation of solar neutrino rate which will be of relevance to Super–Kamioka and Sudbury neutrino detectors in the framework of oscillations among the three flavors. An analytical method of computing the regeneration in the earth is presented. If day-night effect is seen, we show how the study of the time–of–night variation will allow the determination of the neutrino parameters.

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It has been known for quite some time that an asymmetry between night rate and day rate for the real time solar neutrino detectors is an unambiguous signal for neutrino mixing and oscillations. Conversely the absence of such an effect can put constraints on the neutrino masses and mixing angles. Although no day–night asymmetry outside the error–bars was seen at the Kamioka detector [1,2] the high statistics detectors like Super-Kamioka [3], SNO [4]and Borexino [5] will be much more effective in investigating this effect. If there is a day-night asymmetry, then the profile of the asymmetry as a function of the time during night is a very sensitive function of the neutrino parameters. The counting rates in these detectors are expected to be high enough for the study of this time–of–night variation. The aim of this note is to focus attention on this aspect of the day–night effect [6], in view of the fact that Super–Kamioka has already started functioning and SNO is expected to do so soon.

The neutrino samples different amounts of matter in the earth during a single night and also during a year. The distance d travelled by the neutrino inside the earth during night, as a function of time t, is given by

$$d = 2R(\sin\phi_l \sin\delta + \cos\phi_l \cos\delta \cos(\frac{2\pi t}{T_D})) \tag{1}$$

where

$$\sin\delta = \sin 23.5^o \sin(\frac{2\pi t}{T_Y}), \tag{2}$$

R is the radius of the earth, ϕ_l is the latitude of the location of the detector, T_D is the length of the day, T_Y is the length of the year and zero of t is chosen at midnight on autumnal equinox. Thus assuming that the neutrino parameters are in a suitable range, neutrino data collected as a function of d contain an enormous amount of information on neutrino oscillations, which in principle can be analyzed to yield the neutrino parameters.

The time variation of $x = (\frac{d}{2R})$ during the night and year are illustrated in Fig.1. If the data collected during successive nights are accumulated at the corresponding $x-$ bins, the calculation of neutrino rates per unit bin will require the function $f(x)$ defined as the time duration per unit interval of x and plotted in Fig.2.

We now describe a straightforward way of obtaining the neutrino regeneration effect in a model of earth of constant density (5.52gms/cc). Neutrinos that travel upto $x < 0.84$ will pass only through the mantle. Hence, our single slab model of the earth is quite accurate for most of the range of x.

Let a neutrino of flavor ν_α be produced at time $t = t_0$ in the core of the sun. It's state vector is

$$|\Psi_\alpha(t_0)\rangle = |\nu_\alpha\rangle = \sum_i U^S_{\alpha i} |\nu^S_i\rangle. \tag{3}$$

where $|\nu^S_i\rangle$ are the matter dependent mass eigenstates with mass eigenvalues μ^S_i and $U^S_{\alpha i}$ are the matrix elements of the matter dependent mixing matrix in the core of the sun. We use Greek index α to denote the three flavors e, μ,τ and Latin index i to denote the mass eigenstates i = 1,2,3. The neutrino propagates in the sun adiabatically upto t_R (the resonance point), makes nonadiabatic transitions at t_R, propagates adiabatically upto t_1 (the edge of the sun) and propagates as a free particle upto t_2 when it enters the earth. So the state vector at t_2 is

$$|\Psi_\alpha(t_2)\rangle = \sum_j \sum_i |\nu_j\rangle exp\left(-i\varepsilon_j(t_2 - t_1)\right) exp\left(-i\int_{t_R}^{t_1} \varepsilon^S_j(t)dt\right) M^S_{ji} exp\left(-i\int_{t_0}^{t_R} \varepsilon^S_i(t)dt\right) U^S_{\alpha i}. \tag{4}$$

where $\varepsilon^S_i(t)(\equiv E + (\mu^S_i(t))^2/2E)$ are the matter dependent energy eigenvalues in the sun, ε_i and $|\nu_i\rangle$ are the energy eigenvalues and the corresponding eigenstates in vacuum and M^S_{ji} is the probability amplitude for the non adiabatic transition $i \to j$. We multiply the right hand side of eq(4) by $\sum_k |\nu^E_k\rangle\langle\nu^E_k|$ (= 1) where $|\nu^E_k\rangle$ (i =1,2,3) is the complete set of matter dependent mass eigenstates inside the earth. The neutrino propagates upto the the other end of the earth at t_3, and the final state vector at t_3 is

$$|\Psi_\alpha(t_3)\rangle = \sum_k \sum_j \sum_i |\nu^E_k\rangle exp\left(-i\varepsilon^E_k(t_3 - t_2)\right) \langle\nu^E_k|\nu_j\rangle exp\left(-i\varepsilon_j(t_2 - t_1) - i\int_{t_R}^{t_1} \varepsilon^S_j(t)dt\right) \tag{5}$$

$$\times M^S_{ji} exp\left(-i\int_{t_0}^{t_R} \varepsilon^S_i(t)dt\right) U^S_{\alpha i}$$

$$= \sum_k \sum_j \sum_i |\nu^E_k\rangle M^E_{kj} M^S_{ji} U^S_{\alpha i} exp\left(-i\Phi_{ijk}\right) \tag{6}$$

where

$$\Phi_{ijk} = \varepsilon_k^E(t_3 - t_2) + \varepsilon_j(t_2 - t_1) + \int_{t_R}^{t_1} \varepsilon_j^S(t)dt + \int_{t_0}^{t_R} \varepsilon_i^S(t)dt. \tag{7}$$

We have introduced the probability amplitude M^E_{kj} for non adiabatic transitions $j \to k$ due to the abrupt change in density when the neutrino enters the earth . It is given by

$$M^E_{kj} = \langle \nu_k^E | \nu_j \rangle = \sum_\gamma \langle \nu_k^E | \nu_\gamma \rangle \langle \nu_\gamma | \nu_j \rangle = \sum_\gamma U^E_{\gamma k} U^*_{\gamma j} \tag{8}$$

where $U_{\gamma j}$ is the mixing matrix in vacuum .

The probability of detecting a neutrino of flavor β at t_3 is

$$|\langle \nu_\beta | \Psi_\alpha(t_3) \rangle|^2 = \sum_k \sum_j \sum_i \sum_{k'} \sum_{j'} \sum_{i'} U^{E*}_{\beta k} U^E_{\beta k'} M^E_{kj} M^{E*}_{k'j'} M^S_{ji} M^{S*}_{j'i'} U^S_{\alpha i} U^{S*}_{\alpha i'} exp\left(-i(\Phi_{ijk} - \Phi_{i'j'k'}\right) \tag{9}$$

Averaging over t_R leads to $\delta_{ii'}\delta_{jj'}$ and this results in the desired incoherent mixture of mass eigenstates of neutrinos reaching the surface of the earth at t_2. Calling this averaged probability as $P^N_{\alpha\beta}$ (the probabilty for a neutrino produced in the sun as ν_α to be detected as ν_β in the earth at night), we can write the result as

$$P^N_{\alpha\beta} = \sum_j P^S(\alpha \to j) P^E(j \to \beta) \tag{10}$$

where

$$P^S(\alpha \to j) = \sum_i |M^S_{ji}|^2 |U^S_{\alpha i}|^2 \tag{11}$$

$$P^E(j \to \beta) = \sum_k \sum_{k'} U^{E*}_{\beta k} U^E_{\beta k'} M^E_{kj} M^{E*}_{k'j} exp\left(-i(\varepsilon^E_k - \varepsilon^E_{k'})d\right) \tag{12}$$

where we have replaced $(t_3 - t_2)$ by d, the distance travelled by the neutrino inside the earth. For two flavors, eqn.(10) reduces to the formula of Mikheyev and Smirnov [7]. For the daytime, put $d = 0$ so that $P^E(j \to \beta)$ becomes $|U_{\beta j}|^2$ and so eqn.(10) reduces to the usual [8,9] transition probability in the day:

$$P^D_{\alpha\beta} = \sum_i \sum_j |U_{\beta j}|^2 |M^S_{ji}|^2 |U^S_{\alpha i}|^2. \tag{13}$$

We parametrize the mixing matrix U in vacuum as $U = U^{23}(\psi)U^{13}(\phi)U^{12}(\omega)$ where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the ith and the jth mass eigenstates with the mixing angle θ_{ij}, neglecting CP violation. In the solar neutrino problem ψ drops out [10,11] The mass differences in vacuum are defined as $\delta_{21} = \mu_2^2 - \mu_1^2$ and $\delta_{31} = \mu_3^2 - \mu_1^2$. It has been shown [9,12] that the simultaneous solution of both the solar and the atmospheric neutrino problems requires

$$\delta_{31} \gg \delta_{21} \tag{14}$$

and under this condition δ_{31} also drops out. The rediagonalization of the mass matrix in the presence of matter (in solar core or earth) under condition (14) leads to the following results [9]

$$\tan 2\omega_m = \frac{\delta_{21}\sin 2\omega}{\delta_{21}\cos 2\omega - A\cos^2\phi} \tag{15}$$

$$\sin\phi_m = \sin\phi \tag{16}$$

$$\delta_{21}^m = \delta_{21}\cos 2(\omega - \omega_m) - A\cos^2\phi\cos 2\omega_m \tag{17}$$

where A is the Wolfenstein term $A = 2\sqrt{2}\ G_F\ N_e\, E$ (N_e is the number density of electrons and E is the neutrino energy) We note that $\delta_{31} \gg A^S, A^E$. In eqs.(15)–(17) the subscript "m" stands for matter. Under the condition $\delta_{31} \gg A \approx \delta_{21}$ we need the non adiabatic transition probability $|M_{ij}^S|^2$ for i,j =1,2 only and this is taken to be the modified Landau–Zener jump probability for an exponentially varying solar density [11].

The neutrino detection rates for a Super–Kamioka type of detector is given by

$$R = \int \phi(E)\,\sigma(E) P_{ee} dE + \frac{1}{6}(\int (\phi(E)\sigma(E)(1 - P_{ee})dE) \tag{18}$$

where the second term is the neutral current contribution and $\phi(E)$ is the solar neutrino flux as a function of the neutrino energy E and $\sigma(E)$ is the cross section from neutrino electron scattering and we integrate from $5MeV$ onwards. The cross section is taken from [13] and the flux from [14]. The rates for the night and day R_N and R_D are calculated using P_{ee}^N and P_{ee}^D respectively. We define the day–night asymmetry ratio as

$$\frac{N-D}{N+D} \equiv \frac{(R_N - R_D)}{(R_N + R_D)}. \tag{19}$$

We can multiply R_N and R_D by the function $f(x)$ displayed in Fig.2 to get the rates per unit interval in x. Note that $f(x)$ cancels in the asymmetry ratio calculated theoretically. However, the experimentalists have to weight the day rate R_D with $f(x)$ before comparing their data with our theoretical curves.

In Fig.3 we have plotted $(N-D)/(N+D)$ as a function of x, the fractional distance travelled by the neutrino inside the earth for various values of the neutrino parameters δ_{21}, ω and ϕ. As is to be expected an oscillation pattern is seen in the asymmetry with respect to x. These oscillations can be characterized by the wavelength, the amplitude, and the location of the maximum and the minimum of the asymmetry.

Different values of ω, δ_{21} and ϕ have clearly distinguishable characteristics. Some gross features which will enable us to specify approximate allowed domains for ω, δ_{21} and $\phi$are the following:

- The wavelengths are larger for small ω, and are of the order of the earth diameter. For large ω, a few oscillation lengths sit inside the earth diameter.

- As δ_{21} increases the amplitude decreases.

- As ϕ increases, the asymmetry at any x decreases.

- From the data divided into bins of $\Delta x = 0.1$ the first four bins should be enough to determine ω, by determining the number of peaks and valleys present. For $x < 0.2$ there is a very clear discrimination between the small ω and large ω, irrespective of δ_{21} and ϕ.

- The difference between the asymmetry at the first peak and the valleys normalized to the value at the peak is larger for smaller δ_{21}.

- The overall asymmetries are smaller for nonzero ϕ throughout the range of x. In addition the oscillation lengths are larger than those for $\phi = 0$. A combination of these two features will clearly indicate if one has a genuine three flavour oscillation, with ϕ being appreciable.

The reason for ϕ decreasing the asymmetry is that a non zero ϕ means $\nu_e \leftrightarrow \nu_\tau$ mixing, i.e a fraction of the solar neutrinos come out of the sun as ν_τ. These ν_τ cannot reconvert back to ν_e inside earth because the Wolfenstein term for earth is much smaller than δ_{31}, so the "13" mixing angle ϕ is not affected by matter (see eqn.(16)). So a non zero ϕ dilutes the night regeneration effect. Another important point is that if one has zero ω and non–zero ϕ then day–night asymmetry will be very small but one will have an appreciable day suppression due to oscillation in the sun, *i.e.* just the absence of day–night effect does not rule out oscillations. This emphasises the need for the three flavor analysis of the day–night effect.

To estimate the statistical error in$(N-D)/(N+D)$, we first note that it is approximately $(\Delta N^2 + \Delta D^2)^{1/2}/(N+D)$. The 11,000 events per year in the Super–Kamioka detector [3] can be divided into 5500 each for day and night. Assuming that the experimentalists use 10 x bins during the duration of the night one gets $\Delta N \approx 23$ and $\Delta D \approx 7$ and so the statistical error in $(N-D)/(N+D)$ is ≈ 0.02. Since the typical value of $(N-D)/(N+D)$ shown in Fig.3 are much larger than this, we expect that the experimental investigation of the time–of–night effect will yield important information, provided the neutrino parameters lie in the chosen range.

After this work was completed, we came to know of a related work: E. Lisi and D. Montanino, hep-ph/9702343.

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FIGURES

FIG. 1. The fractional distance x travelled by the neutrino inside the earth during night is plotted against time for the latitude (36.4^o) of the Super–Kamioka detector. The lower figure gives the envelope of the 365 maxima during the year . As examples of the actual curves, those for a few nights during three specific seasons of the year are shown in the upper figure.

FIG. 2. The function $f(x)$ which is defined as the time duration (in hours) per unit interval of x, is plotted for various latitudes: Super–Kamioka (36.4^o), Borexino (42.45^o), SNO (46.5^o) , equator (0.0^o) , pole (90^o).

FIG. 3. $(N-D)/(N+D)$ as a function of x, the fractional distance travelled by the neutrino, for typical values of ω, δ_{21} (in eV^2) and ϕ. For $\phi = 45.0^o$, all the curves almost collapse to the x–axis.