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Small-x Resummations for the Structure Functions F_2^p, F_L^p and F_2^γ [1]

J. Blümlein* and A. Vogt†

*DESY–Zeuthen, Platanenallee 6, D–15735 Zeuthen, Germany
† Institut für Theoretische Physik, Universität Würzburg, Am Hubland, D–97074 Würzburg, Germany

Abstract. The numerical effects of the known all-order leading and next-to-leading logarithmic small-x contributions to the anomalous dimensions and coefficient functions of the unpolarized singlet evolution are discussed for the structure functions $F_2^{ep}(x,Q^2)$, $F_L^{ep}(x,Q^2)$, and $F_2^{e\gamma}(x,Q^2)$.

Introduction

The evolution kernels of the deep-inelastic scattering (DIS) structure functions contain large logarithmic contributions for small Bjorken-x. The effect of resumming these terms to all orders in α_s can be consistently studied in a framework based on the renormalization group (RG) equations, which describes the mass factorization. In this framework, the evolution equations of fixed-order perturbative QCD are generalized by including the resummed small-x contributions to the respective anomalous dimensions and Wilson coefficients [1–4] beyond next-to-leading order in α_s (NLO). The numerical impact of these higher-order contributions has been investigated for the non-singlet nucleon structure functions F_2^{p-n} and $F_3^{\nu N}$ [5], g_1^{p-n} [5,6] and $g_5^{\gamma Z}$ [7]; for the polarized singlet quantity $g_{1,\mathrm{S}}^p$ [8], and for the unpolarized singlet structure functions $F_{2,\mathrm{S}}$ [9–11] and $F_{L,\mathrm{S}}^p$ [10]. $F_{2,\mathrm{S}}^p$ and $F_{L,\mathrm{S}}^p$ have been studied using different RG-based approaches as well [12].

In the present note we extend a previous account [7] by considering, besides the resummed next-to-leading logarithmic small-x (NLx) quark terms of ref. [2], also the recently derived NLx contributions $\propto N_f$ to the anomalous

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dimension γ_{gg} [3] and their impact on F_2^p. Furthermore, we briefly discuss the numerical resummation effects on the evolution of F_L^p and the photon structure function F_2^γ. Details of the calculations may be found in ref. [10].

The NLx Contributions $\propto N_f$ to γ_{gg}

These terms were calculated in ref. [3]. In the $\overline{\mathrm{MS}}$–DIS scheme they read [10]

$$\begin{aligned}\gamma_{gg,\mathrm{NL}}^{q\overline{q},\mathrm{DIS}} &= \gamma_{gg,\mathrm{NL}}^{q\overline{q},Q_0} + \frac{\beta_0}{4\pi}\,\alpha_s^2\frac{d\ln R(\alpha_s)}{d\alpha_s} + \frac{C_F}{C_A}\left[1-R(\alpha_s)\right]\gamma_{qg,\mathrm{NL}}^{Q_0} \\ &\equiv \alpha_s\sum_{k=1}^{\infty}\left[\frac{N_f}{6\pi}\left(d_{gg,k}^{q\overline{q},(1)}+\frac{C_F}{C_A}d_{gg,k}^{q\overline{q},(2)}\right)+\frac{\beta_0}{4\pi}\hat{r}_k\right]\left(\frac{\overline{\alpha}_s}{N-1}\right)^{k-1}, \end{aligned} \qquad (1)$$

with $\gamma_{gg,\mathrm{NL}}^{q\overline{q},Q_0}$ being the N_f contribution in the Q_0 scheme [13]. N denotes the usual Mellin variable, $\overline{\alpha}_s \equiv C_A\,\alpha_s/\pi$, and $R(\alpha_s)$ is defined in ref. [2]. $\gamma_{gg,\mathrm{NL}}^{q\overline{q}}$ contains terms $\propto C_F/C_A$ in both schemes, whereas the β_0-contribution originates in transformation from the Q_0 scheme to the $\overline{\mathrm{MS}}$–DIS scheme. Numerical values for the coefficients $d_{gg,k}^{q\overline{q},(1,2)}$ and $\hat{r}_k$ are given in Table 1.

k	$d_{gg,k}^{q\overline{q},(1)}$	$d_{gg,k}^{q\overline{q},(2)}$	$\hat{r}_k$
1	–1.000000000 E+0	0.000000000 E+0	0.000000000 E+0
2	–3.833333333 E+0	0.000000000 E+0	0.000000000 E+0
3	–2.299510376 E+0	0.000000000 E+0	0.000000000 E+0
4	–5.065605818 E+0	3.205485075 E+0	9.616455224 E+0
5	–3.523670351 E+1	8.568702514 E+0	–3.246969702 E+0
6	–3.218245315 E+1	1.835447655 E+1	2.281241061 E+1
7	–1.060268680 E+2	8.632838009 E+1	1.654162989 E+2
8	–4.853159484 E+2	1.924088636 E+2	–2.469139930 E+0
9	–5.806186371 E+2	4.962344972 E+2	7.458249428 E+2
10	–2.176371931 E+3	1.794742819 E+3	2.784859262 E+3
11	–7.553679737 E+3	4.023320193 E+3	1.505001272 E+3
12	–1.158215080 E+4	1.136559381 E+4	1.818320928 E+4
13	–4.328579102 E+4	3.589638820 E+4	4.899274185 E+5
14	–1.269309428 E+5	8.412529889 E+4	6.109247725 E+5
15	–2.392549581 E+5	2.456097133 E+5	3.984470167 E+5
16	–8.469557573 E+5	7.168572021 E+6	9.205515787 E+5
17	–2.262541206 E+6	1.764587230 E+6	1.783326920 E+6
18	–4.974873276 E+6	5.167844173 E+6	8.347774614 E+6
19	–1.648990863 E+7	1.443009883 E+7	1.842662795 E+7
20	–4.222994214 E+7	3.702246358 E+7	4.535538189 E+7

Table 1: Numerical values of the expansion coefficients for $\gamma_{gg,\mathrm{NL}}^{q\overline{q},\mathrm{DIS}}$ in eq. (1).

Less Singular Small-x Contributions to γ

The small-x resummed anomalous dimension matrix $\hat{\gamma}^{\,\mathrm{res}}$ does not comply with the energy-momentum sum rule for the parton densities. Several prescriptions have been imposed for restoring this sum rule beyond NLO [9–11], e.g.,

$$\begin{array}{ll} \mathrm{A}: & \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s) \to \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s) - \hat{\gamma}^{\,\mathrm{res}}(0,\alpha_s) \\ \mathrm{B}: & \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s) \to \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s)\,(1-n) \\ \mathrm{D}: & \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s) \to \hat{\gamma}^{\,\mathrm{res}}(n,\alpha_s)\,(1-2n+n^3)\ . \end{array} \tag{2}$$

The difference between the results obtained with these prescriptions allows for a rough estimate of the possible effect of the presently unknown higher-order terms less singular at small-x ($n \equiv N-1 \to 0$).

The Resummed Evolution of F_2^{ep} and F_L^{ep}

The numerical effect of the known small-x resummations on the behavior of the proton structure functions F_2 and F_L is illustrated in Fig. 1. For both the NLO and the resummed calculations, the MRS(A′) DIS-scheme parton densities have been employed as initial distributions at $Q_0^2 = 4\ \mathrm{GeV}^2$, together with $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 231$ MeV [14]. They behave like $xg, xq \sim x^{-0.17}$ at small x, with the quark part rather directly constrained by present HERA F_2 data.

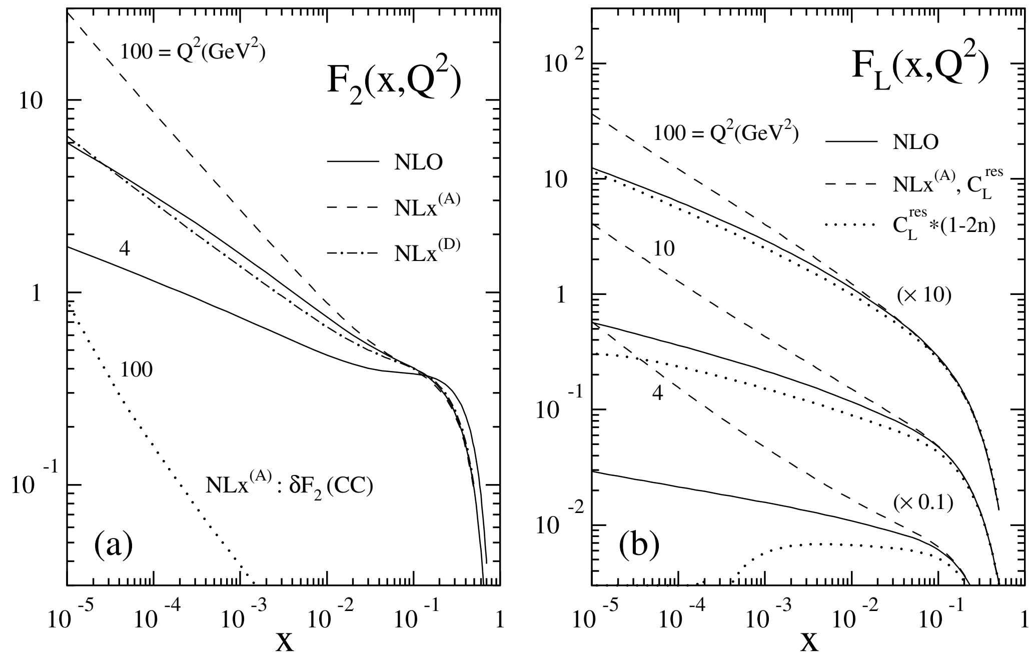

Figure 1: The resummed small-x evolutions of the proton structure functions F_2 and F_L compared to the NLO results. The dotted curve in the F_2 part represents the contribution of $\gamma^{q\bar{q},\,\mathrm{DIS}}_{gg,\,\mathrm{NL}}$ only. The possible impact of (presently unknown) less singular higher-order terms is indicated, cf. eq. (2) and the discussion in the text.

The resummation effects on $F_2(x,Q^2)$ at small x are displayed in Fig. 1 (a). Note the huge effect arising from the NLx quark anomalous dimensions [2] and its large uncertainty due to unknown less singular terms. The impact of $\gamma_{gg,\mathrm{NL}}^{q\overline{q}}$ [3] is displayed separately. It amounts to less than 3% over the full x-range shown. It will be interesting to see to which extent the forthcoming complete NLx anomalous dimensions [15] will modify these results.

The longitudinal structure function $F_L(x,Q^2)$ is considered in Fig. 1 (b). Obviously substantial contributions can also be expected from subleading small-x terms in the coefficient functions C_L. In fact, these uncertainties are large. Thus both for the small-x resummed contributions to anomalous dimensions and coefficient functions further subleading terms need to be calculated. Further insight into the interplay of leading and less singular terms in N may also be gained from the structure of the fixed-order anomalous dimensions and coefficient functions. Besides the known NLO result, particularly the yet unknown 3–loop anomalous dimensions are of interest here.

The Resummation of the Small-x Contributions to F_2^γ

The evolution of the photon structure functions is, at the lowest order in α_{em} considered here, governed by an inhomogeneous generalization of the hadronic evolution equations. At the present resummation accuracy [1,2] the additional anomalous dimensions $\gamma_{q\gamma}$ and $\gamma_{g\gamma}$ do not receive any non-vanishing higher-order small-x contributions [10]. Hence the resummation effect on the photon-specific inhomogeneous solution originates solely from the resummed homogeneous evolution operator.

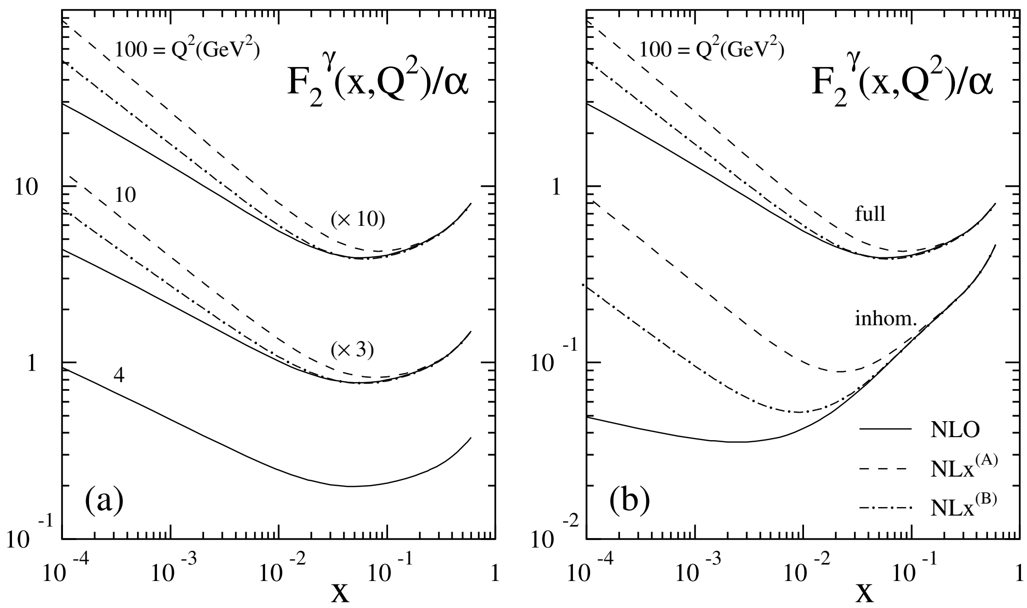

Figure 2: The small-x evolution of the photon structure function F_2^γ in NLO and using the NLx resummed anomalous dimensions.

The resummed evolution of the structure function F_2^γ is compared to the NLO results in Fig. 2. The NLO GRV parametrization has been used for the initial distributions at $Q_0^2 = 4$ GeV^2, together with $\Lambda^{(4)}_{\overline{\rm MS}} = 200$ MeV [16]. The overall small-x behavior, presented in Fig. 2 (a), is rather similar to the hadronic case, due to the dominance of the homogeneous solution. Note, however, the significantly enhanced resummation effect in the inhomogeneous solution separately shown in Fig. 2 (b). This behavior is dominated by the convolution of the resummed hadronic evolution operator with the leading-order photon-quark anomalous dimension, which, unlike the hadronic initial distributions, is large for $x \to 1$.

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