Integral Relations for Twist 2 and Twist 3 Contributions to Polarized Structure Functions

Johannes Blümlein* and Nikolai Kochelev*,†

*DESY – Zeuthen, Platanenallee 6, D – 15735 Zeuthen, Germany
† Bogoliubov Laboratory of Theoretical Physics, JINR, RU–141980 Dubna, Russia

Abstract. We discuss the relations between the twist 2 and twist 3 contributions to polarized deep-inelastic scattering structure functions both for neutral and charged current interactions which are predicted by the operator product expansion in lowest order in QCD.

INTRODUCTION

In the case of polarized deep inelastic scattering the cross sections depend on (up to) three unpolarized $F_i(x,Q^2)_{i=1}^3$ and five polarized structure functions $g_j(x,Q^2)_{j=1}^5$ in the limit of vanishing fermion masses. The lowest twist contributions are those of twist 2 for the structure functions $F_i(x,Q^2)$ and $g_{1,4,5}(x,Q^2)$. The structure functions $g_2(x,Q^2)$ and $g_3(x,Q^2)$ contain as well twist 3 terms [1,2]. In this note we give a summary on the relations between the twist 2 and twist 3 contributions to the different structure functions in lowest order in QCD. We also comment on a sum rule which has recently been derived in ref. [3].

TWIST 2

For the twist 2 contributions to the structure functions one may seek a partonic interpretation. However, in lowest order in QCD only two generic parton combinations exist to describe the polarized structure functions :

$$g_1(x,Q^2) \propto \Delta q(x,Q^2) + \Delta \overline{q}(x,Q^2) \quad \text{and} \quad g_5(x,Q^2) \propto \Delta q(x,Q^2) - \Delta \overline{q}(x,Q^2). \tag{1}$$

The remaining three structure functions are therefore related to this basis by three linear operators. Two of the correponding relations have been known for several years already, the DICUS relation [4]

$$g_4^i(x,Q^2) = 2x g_5^i(x,Q^2), \tag{2}$$

and the WANDZURA-WILCZEK relation [5]

$$g_2^i(x,Q^2) = -g_1^i(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^i(y,Q^2). \tag{3}$$

The third relation has only recently been derived in ref. [6]

$$g_3{}^i(x,Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4{}^i(y,Q^2). \tag{4}$$

Eqs. (2–4) can either be obtained analyzing the polarized structure functions by means of the operator product expansion [6,2] or in applying the covariant parton model [7], cf. [6][1]. In the operator product expansion they result from equating different expression for the matrix element a_n of the symmetric part of the quark operators in lowest order QCD, see e.g. [2][2]. These relations between the different contributions to the longitudinal and transverse spin projections of the hadronic tensor are illustrated in Figure 1.

$$\begin{array}{lclclc}
 & & & & \text{Dicus} & \\
 & & & & \swarrow \quad\quad \searrow & \\
W^{||}_{\mu\nu} & = & i\varepsilon_{\mu\nu\alpha\beta}\dfrac{q_\alpha P_\beta}{\nu} g_1(x) & + & \dfrac{P_\mu P_\nu}{\nu} g_4(x) & -g_{\mu\nu} g_5(x) \\
 & & \uparrow & & \uparrow & \\
 & & \text{Wandzura} - \text{Wilczek} & & \text{ref. [6]} & \\
 & & \downarrow & & \downarrow & \\
W^{\perp}_{\mu\nu} & = & i\varepsilon_{\mu\nu\alpha\beta}\dfrac{q_\alpha S^\perp_\beta}{\nu}[g_1(x)+g_2(x)] & + & \dfrac{P_\mu S^\perp_\nu + P_\nu S^\perp_\mu}{2\nu} g_3(x) & \\
 & & \overbrace{\Delta q \quad + \quad \Delta\overline{q}} & | & & \overbrace{\Delta q \quad - \quad \Delta\overline{q}}
\end{array}$$

Figure 1 : Relations between the twist 2 contributions of the polarized structure functions

Whereas the corresponding parts of $W^{||}_{\mu\nu}$ and $W^{\perp}_{\mu\nu}$ are connected by integral relations, those acting in either part are just multiplications by a factor. For the valence parts this holds as well for the two contributions to $W^{\perp}_{\mu\nu}$.

---

1) A derivation of eq. (3) using the latter method was given in refs. [8] before.

2) The lowest moments of $g_i(x)|_{i=1}^{5}$ were studied in ref. [9] and agree with the corresponding relations derived directly from eq. (2–4).

TWIST 3

For the twist 3 contributions to the structure functions $g_2(x,Q^2)$ and $g_3(x,Q^2)$, which emerge in the different neutral and charged current reactions, the operator product expansion [2] implies the relation :

$$\int_0^1 dx x^n \{4g_5 - \frac{n+1}{x} g_3\}^{\nu n - \nu p} = \frac{12(n-1)}{n} \int_0^1 dx x^n \{n g_1 + (n+1) g_2\}^{\gamma p - \gamma n}, n = 2,4 \ldots . \quad (5)$$

It results from equations between differences of the matrix elements d_n of the non-symmetric part of the quark operators in lowest order QCD. Since one may express the twist 3 contributions to g_2 and g_3 by

$$g_2^{\mathrm{III}}(x,Q^2) = g_2(x,Q^2) + g_1(x,Q^2) - \int_x^1 \frac{dy}{y} g_1(y,Q^2), \quad (6)$$

$$g_3^{\mathrm{III}}(x,Q^2) = g_3(x,Q^2) - 4x \int_x^1 \frac{dy}{y} g_5(y,Q^2), \quad (7)$$

the analytic continuation of eqs. (6,7) in n can be rewritten by

$$g_3^{\mathrm{III},\nu n - \nu p}(x,Q^2) = 12 \left[x g_2^{\mathrm{III}}(x,Q^2) - \int_x^1 dy g_2^{\mathrm{III}}(y,Q^2) \right]^{\gamma p - \gamma n} \quad (8)$$

as a relation between twist 3 contributions *only*.

Recently, a sum rule for the valence part of the structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$

$$\int_0^1 dx x (g_1^V(x) + 2 g_2^V(x)) = 0 \quad (9)$$

was discussed in ref. [3][3]. We would like to investigate the relation of eq. (9) to the operator product expansion. Here one first meets the problem that the valence parts $g_1^V(x)$ and $g_2^V(x)$ cannot be isolated for electromagnetic interactions from the complete structure functions and a formulation of eq. (9) with the help of the local operator product expansion is thus not straightforward. On the other hand, one may consider

$$\int_0^1 dx x^n (g_1^-(x,Q^2) + 2g_2^-(x,Q^2)) = \sum_q \frac{((g_V^q)^2 + (g_A^q)^2)(n d_n^{-q} - (n-1) a_n^{-q})}{4(n+1)}, \quad n = 1,3 \ldots , \quad (10)$$

[3] This sum rule was first derived in ref. [10] for a specific flavor combination.

which results from eqs. (61,62), ref. [2], in the charged current case. It is easily seen that the left-hand-side of eq. (10) includes only valence quark contributions and one may even rewrite eq. (10) for individual quark flavors separately, denoting the valence parts of the corresponding matrix elements by a_n^{Vq} and d_n^{Vq}, respectively. For the first moment one obtains

$$\int_0^1 dx x(g_1^{Vq}(x,Q^2) + 2g_2^{Vq}(x,Q^2)) = \frac{e_q^2}{8} d_1^{Vq}. \tag{11}$$

The right-hand-side of eq. (11) vanishes in the case of massless quarks, because

$$\begin{aligned} d_1^{Vq}(S^\beta P^\mu - S^\mu P^\beta) &= m_q \langle PS|\bar{q} i \gamma_5 \sigma^{\beta\mu} q|PS\rangle \\ &\equiv \frac{m_q}{M}(P^\beta S^\mu - P^\mu S^\beta) \int_0^1 dx (h_1^q(x) - \bar{h}_1^q(x)) \ . \end{aligned} \tag{12}$$

For the latter equation, see [11]. $h_1^q(x)$, $\bar{h}_1^q(x)$ are the quark and antiquark transversity functions, respectively, which can be measured in the Drell-Yan process. The right-hand-side of eq. (12) vanishes in the limit $m_q \rightarrow 0$, which yields $d_1^{Vq} = 0$. Due to this, eq. (12) is not described by the operator product expansion, but is formally consistent with it. This is similar to the case of the Burkhardt–Cottingham sum rule [12],

$$\int_0^1 dx g_2^i(x,Q^2) = 0, \tag{13}$$

REFERENCES

1. M.A. Ahmed and G.G. Ross, *Phys. Lett.* **B56** 385 (1975); *Nucl. Phys.* **B111** 441 (1976).
2. J. Blümlein and N. Kochelev, DESY 96–175, *hep-ph/9612318*, *Nucl. Phys.* **B** (1997) in print.
3. A.V. Efremov, O.V. Teryaev, and E. Leader, *Phys. Rev.* **D55** 4307 (1997).
4. D.A. Dicus, *Phys. Rev.* **D5** 1367 (1972).
5. S. Wandzura and F. Wilczek, *Phys. Lett.* **B72** 195 (1977).
6. J. Blümlein and N. Kochelev, *Phys. Lett.* **B381** 296 (1996).
7. P.V. Landshoff and J.C. Polkinghorne, *Phys. Rep.* **C5** 1 (1972), and references therein.
8. J.D. Jackson, G.G. Ross and R.G. Roberts, *Phys. Lett.* **B226** 159 (1989); R.G. Roberts and G.G. Ross, *Phys. Lett.* **B373** 235 (1996).
9. M. Maul, B. Ehrensperger, E. Stein, and A. Schäfer, *Z. Phys.* **A356** (1997) 443.
10. A.S. Joshipura and P. Roy, *Ann. Phys. (NY)* **104** (1977) 460.
11. R.L. Jaffe, *hep-ph/9603422*.
12. H. Burkhardt and W.N. Cottingham, *Ann. Phys. (NY)* **56** 453 (1970).