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# $CP$ violation in B mesons using Dalitz plot asymmetries

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## Abstract

We study $CP$ violation in $B \to K^* \ell^+ \ell^-$ using generalized Dalitz plot asymmetries in the angular distribution. These new kind of asymmetries are constructed by adding $B$ and $\bar{B}$ events, and do not require flavor or time tagging, nor is the presence of strong phases needed. Using this method one requires about $2 \times 10^8/\eta$ $B$'s to measure the $CP$ violating parameter $\eta$ in the Standard model. The two–Higgs doublet model requires only $10^7$ $B$'s to constrain parameters better than done by the electric dipole moment of the neutron.

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$B$ mesons are expected to exhibit $CP$ violation like the $K$ mesons, which is the only system where this phenomena has been observed. Much effort has thus been devoted to studying possible signals of $CP$ violation in $B$. Large numbers of $B$ mesons are expected to be produced in the future, which would enable study of its rare decay modes. However, since flavor and time tagging is difficult, except at the asymmetric $e^+e^-$ factories, the large number of $B$'s cannot be efficiently used to study $CP$ violation.

In view of this difficulty Burdman and Donoghue [1] studied the possibility of detecting $CP$ violation in $B$ decays without the need for flavor identification. Much like the Dalitz plot asymmetry [2] for $K^\pm$, Ref. [1] considers asymmetries in the hadronic three body decays of neutral $B$ mesons to flavor states that are $C$ eigenstates, as well as to states which under $C$ conjugation return to $C$ partners. A very important point realized by these authors is that if *"one searches for quantities for which CP invariance says that they should change sign when comparing B and $\bar{B}$ decays, summing B and $\bar{B}$ should produce a net null result unless CP is violated."* Since such techniques do not involve flavor identification, they do not depend on the production characteristics and can be studied using any source of $B$ mesons. These Dalitz plot asymmetries are logically distinct from the partial rate asymmetries usually considered, in the sense that they may be present even when partial rate asymmetries vanish.

In this letter we discuss such asymmetries in angular variables that require no flavor or time tagging and also *do not need strong phases to show up.* These asymmetries are independent of CKM phase of $B - \bar{B}$ mixing, and depend only on direct or indirect CP violation. We construct such asymmetries for the rare decay $B \to K^*\ell^+\ell^-$ ($\ell^+\ell^-$ non resonant), where new physics contributions are expected to show up. Our choice of mode is such that the asymmetries are free from undetermined strong phases. Another valuable feature is a clean signal which will prove to be one of the easiest to measure. Our analysis is similar in spirit to the approach of Ref. [3] where CP violation in $K_L \to \pi\pi e^+e^-$ has been considered.

The effective short distance Hamiltonian relevant to the decay $b \to s\ell^+\ell^-$ [4–6] leads to the QCD corrected matrix element

$$\mathcal{M}(b\to s\ell^+\ell^-)=\frac{\alpha G_F}{\sqrt{2}\pi}\sum_j v_j\{-2iC_7^j m_b\frac{q^\nu}{q^2}\,\bar{s}\sigma_{\mu\nu}b_R\;\bar{\ell}\gamma^\mu\ell+C_8^j\;\bar{s}\gamma_\mu b_L\;\bar{\ell}\gamma^\mu\ell+C_9^j\;\bar{s}\gamma_\mu b_L\;\bar{\ell}\gamma^\mu\gamma_5\ell\} \tag{1}$$

where the sum j is over all the flavors "$u, c, t$" in the loops, $C^j_{7,8,9}$ are the Wilson coefficients given in Ref. [4,5], $m_b$ is the mass of the b quark, $q^2$ is the invariant lepton mass squared, $b_{L,R}=(1\mp\gamma_5)/2\,b$ and $v_j=V^*_{js}V_{jb}$ is the product of the CKM matrix elements.

The transition matrix element for the exclusive process $B(p)\to K^*(k)\ell^+\ell^-\to K(k_1)\pi(k_2)\ell^+(q_1)\ell^-(q_2)$ can be written for each of the operators in Eq.(1) as,

$$\langle K\pi|\bar{s}i\sigma_{\mu\nu}(1\pm\gamma_5)q^\nu b|B\rangle=i\mathcal{A}\epsilon_{\mu\nu\alpha\beta}K^\nu k^\alpha q^\beta\pm\mathcal{B}K_\mu\pm\mathcal{C}k_\mu\,,$$

$$\langle K\pi|\bar{s}\gamma_\mu(1\mp\gamma_5)b|B\rangle=i\mathcal{D}\epsilon_{\mu\nu\alpha\beta}K^\nu k^\alpha q^\beta\pm\mathcal{E}K_\mu\pm\mathcal{F}k_\mu\,.$$

The form-factors $\mathcal{A},\cdots,\mathcal{F}$ are unknown functions of $q^2=(p-k)^2$ and other dot products involving momentum, $k=k_1+k_2$ and $K=k_1-k_2$ and can be related to those used in [7], [8] as given in Table I. The variable $\sigma$ arises due to the decay of $K^*\to K\pi$, evaluated in the zero width approximation. The current proportional to $q_\mu$ does not contribute as it couples to light leptons. In our notation $M_B, m_{K^*}, m_K$ and $m_\pi$ are the masses of the $B$, $K^*$, $K$ mesons and the pion respectively. We first consider the case of charged B's and later generalize to neutral ones. The matrix element for the process $B\to K^*\ell^+\ell^-\to K\pi\ell^+\ell^-$ can be written as

$$\mathcal{M}(B\to K\pi\ell^+\ell^-)=\frac{\alpha G_F}{\sqrt{2}\pi}\Big\{\Big(i\alpha_L\,\epsilon_{\mu\nu\alpha\beta}\,K^\nu k^\alpha q^\beta\;\;+\beta_L\,K_\mu+\rho_L\,k_\mu\Big)\,\bar{\ell}\gamma_\mu\,L\,\ell+L\to R\Big\}\,,$$

where $L,R=\dfrac{(1\mp\gamma_5)}{2}$, $q=q_1+q_2$ and $Q=q_1-q_2$, and the coefficients $\alpha_{L,R}$, $\beta_{L,R}$ and $\rho_{L,R}$ are given by

$$\begin{aligned}
\alpha_{R,L} &= \sum_j v_j\,\{\frac{(C_8^j\pm C_9^j)}{2}\,\mathcal{D}-\frac{m_b}{q^2}\,C_7^j\,\mathcal{A}\} &=& \sum_j |\mathrm{a^j}_{R,L}|\,exp\left(i\delta^{\alpha_j}_{R,L}\right)\,exp\left(i\phi^{\alpha_j}_{R,L}\right)\\
\beta_{R,L} &= \sum_j v_j\,\{\frac{(C_8^j\pm C_9^j)}{2}\,\mathcal{E}-\frac{m_b}{q^2}\,C_7^j\,\mathcal{B}\} &=& \sum_j |\mathrm{b^j}_{R,L}|\,exp\left(i\delta^{\beta_j}_{R,L}\right)\,exp\left(i\phi^{\beta_j}_{R,L}\right)\\
\rho_{R,L} &= \sum_j v_j\,\{\frac{(C_8^j\pm C_9^j)}{2}\,\mathcal{F}-\frac{m_b}{q^2}\,C_7^j\,\mathcal{C}\} &=& \sum_j |\mathrm{r^j}_{R,L}|\,exp\left(i\delta^{\rho_j}_{R,L}\right)\,exp\left(i\phi^{\rho_j}_{R,L}\right)\,.
\end{aligned} \tag{2}$$

In the above equation $\alpha$, $\beta$ and $\rho$ are recast in terms of $a$, $b$ and $r$ so as to identify the strong phases $\delta$ and the weak phases $\phi$. Using $CPT$ invariance, the matrix element for the decay $\bar{B} \to \bar{K}\bar{\pi}\ell^+\ell^-$ can be obtained from the $B \to K\pi\ell^+\ell^-$ by replacing $\alpha_{L,R} \to -\bar{\alpha}_{L,R}, \beta_{L,R} \to \bar{\beta}_{L,R}, \rho_{L,R} \to \bar{\rho}_{L,R}$ [9,10], where

$$\bar{\alpha}_{R,L} = \sum_j |\mathrm{a^j}_{R,L}|\, exp\left(i\delta^{\alpha_j}_{R,L}\right)\, exp\left(-i\phi^{\alpha_j}_{R,L}\right) \tag{3}$$

and similar relations hold for $\bar{\beta}$ and $\bar{\rho}$. The matrix element mod. squared for the process $B \to K^*\ell^+\ell^- \to K\pi\ell^+\ell^-$ is worked out retaining the imaginary parts in $\alpha, \beta$ and $\rho$ and presented in Table II. We define $X$ as the three momentum of the $\ell^+\ell^-$ or $K\pi$ invariant system in the B meson rest frame and $\lambda_K(\lambda_e)$ is related to the $K(e)$ three momentum in the $K^*(e^+e^-)$ rest frame . $\theta_e(\theta_K)$ is the angle of the $e^-(K)$ in the $e^+e^-(K\pi)$ rest frame with the $e^+e^-(K^*)$ invariant direction. $\varphi$ is the angle between the planes defined by $e^+e^-$ and the $K\pi$ directions. Our choice of variables and the general treatment presented so far resembles the formalism developed for the $K_{\ell 4}$ decays [11]. The essential difference is, that while the latter was aimed at obtaining the $\pi - \pi$ phase shifts *i.e.* strong phases, our interest here is in the $CP$ violating weak phases. The differential decay rate is then given by

$$d\Gamma = \frac{1}{2^{14}\pi^6 M_B^2}\int |\mathcal{M}|^2 X\lambda_K\lambda_e\, dq^2 d\cos\theta_K d\cos\theta_e d\varphi\,,$$

assuming a narrow width approximation for the decay $K^* \to K\pi$.

It can easily be seen from Table II that, the only terms proportional to $\sin(\varphi)$ or $\sin(2\varphi)$ are those that depend on the imaginary parts of the $\alpha$, $\beta$ or $\rho$. For instance only the coefficient of $\mathrm{Im}(\alpha_L\beta_L^* + \alpha_R\beta_R^*)$ is proportional to $\sin(2\varphi)$. Hence we can isolate this term by considering the following asymmetry in terms of the differential decay rates of the B meson with respect to $\varphi$,

$$A_1 = \frac{1}{\Gamma}\left(\int_0^{\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\pi} + \int_{\pi}^{\frac{3\pi}{2}} - \int_{\frac{3\pi}{2}}^{2\pi}\right)\frac{d\Gamma}{d\varphi}d\varphi\,. \tag{4}$$

The imaginary part in the term under consideration can be due to either a strong phase or a weak phase. An astute reader will, however, have realized that such $CP$ violating asymmetries can be obtained not by considering the difference of differential rates for $B$ and $\bar{B}$ , but the sum of these rates. It follows trivially from eqn.(2 and 3) that the asymmetry for $B(\bar{B}$ ) is

$$A_1(\bar{A}_1) \propto \pm \sum_{j,k}\{|a_L^j||b_L^k| \sin((\delta_L^{jk}) \pm (\phi_L^{jk})) + L \rightarrow R\}$$

where $\delta_L^{jk} \equiv (\delta_L^{\alpha_j} - \delta_L^{\beta_k})$ and $\phi_L^{jk} \equiv (\phi_L^{\alpha_j} - \delta_L^{\beta_k})$. The sum of the two asymmetries $\mathsf{A}_1^{CP} = A_1 + \bar{A}_1$ becomes

$$\mathsf{A}_1^{CP} \propto \sum_{j,k}\{|a_L^j||b_L^k| \cos(\delta_L^{jk}) \sin(\phi_L^{jk}) + L \rightarrow R\}\,, \tag{5}$$

which is *nonzero if and only if there is CP violation represented by non-zero phases* $\phi$ [1,9]. For $B \to K^*\ell^+\ell^-$, $\delta$ can arise either from the quark in the penguin loop going on shell, *i.e.* $q^2 \geq 4\,m_q^2$, or from electromagnetic final state interactions which are negligible and ignored. The case of quark on shell is taken care of in evaluating the coefficients $C_{7,8,9}^j$ and included in our analysis. It is also possible to construct a different asymmetry that isolates another combination of the imaginary terms. Such an asymmetry [12] considers the difference distribution of the same hemisphere and opposite hemisphere events, and can be defined by

$$A_2 = \frac{1}{\Gamma}(\int_0^{\pi} - \int_{\pi}^{2\pi})d\varphi \int_D d\cos\theta_e \int_D d\cos\theta_K \tilde{\Gamma} \tag{6}$$

where $\int_D \equiv \int_{-1}^{0} - \int_0^1$ and $\tilde{\Gamma} = \dfrac{d\Gamma}{d\cos\theta_e d\cos\theta_K d\varphi}$.

For neutral $B$ mesons the asymmetries are even more interesting, as it is here, that flavor tagging not being needed, is a real advantage. In addition we also find that the time integrated asymmetries are independent of the parameters describing the oscillation, and reduce to the asymmetries for charged $B$'s. The time evolution of $B$ mesons is given by

$$\begin{aligned}|B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{1}{\xi}g_-(t)|\bar{B}^0\rangle \\ |\bar{B}^0(t)\rangle &= g_+(t)|\bar{B}^0\rangle + \xi g_-(t)|B^0\rangle\end{aligned} \tag{7}$$

with $g_\pm = \exp\{-(\frac{\Gamma_B}{2} - i\, M_B)t\}(\cos\frac{\Delta M\, t}{2}, i\sin\frac{\Delta M\, t}{2})$ and $\xi = \frac{p}{q}$. $M_B(\Gamma_B)$ and $\Delta M(\Delta\Gamma)$ are the average and the difference of the masses (widths) of the two mass eigenstates $B_H$ and $B_L$ respectively. Hence for $B^0$ mesons we have

$$\mathcal{M}(B^0(t) \to K\pi\ell^+\ell^-) = \frac{\alpha G_F}{\sqrt{2}\pi}\left\{\left(i\alpha^0_L\, \epsilon_{\mu\nu\alpha\beta}\, K^\nu k^\alpha q^\beta \;\; +\beta^0_L\, K_\mu + \rho^0_L\, k_\mu\right)\bar{\ell}\gamma_\mu\, L\,\ell + L \to R\right\} ,$$

with an analogous relation for $\bar{B}^0 \to \bar{K}\bar{\pi}\ell^+\ell^-$ written by replacing $\alpha^0_{L,R} \to \bar{\alpha}^0_{L,R}, \beta^0_{L,R} \to \bar{\beta}^0_{L,R}, \rho^0_{L,R} \to \bar{\rho}^0_{L,R}$, where

$$\begin{aligned}
\alpha^0 &= g_+\, \alpha - \xi^{-1}\, g_-\, \bar{\alpha}\,, \;\; \overline{\alpha^0} = \xi\, g_-\, \alpha - g_+\, \bar{\alpha} \\
\beta^0 &= g_+\, \beta + \xi^{-1}\, g_-\, \bar{\beta}\,, \;\; \overline{\beta^0} = \xi\, g_-\, \beta + g_+\, \bar{\beta} \\
\rho^0 &= g_+\, \rho + \xi^{-1}\, g_-\, \bar{\rho}\,, \;\; \overline{\rho^0} = \xi\, g_-\, \rho + g_+\, \bar{\rho}\,.
\end{aligned} \tag{8}$$

Here we have suppressed the subscripts $L, R$, and a summation over both is implied, in what follows.

By adding differential decay rates for $B^0$ and $\bar{B}^0$, one can construct an asymmetry of the type $\mathsf{A}_1^{CP}$, which shall be proportional to $\alpha^0\beta^{0*} + \overline{\alpha^0\beta^{0*}}$, and given by

$$\begin{aligned}
\mathsf{A}_1^{0\,CP}(t) \propto\; & e^{-\Gamma_B t}\left\{\mathrm{Im}\left(\alpha\,\beta^* - \bar{\alpha}\bar{\beta}^*\right)\,\cos^2\frac{\Delta M t}{2} + \mathrm{Im}\left(|\xi|^2\,\alpha\,\beta^* - \left|\xi^{-1}\right|^2\bar{\alpha}\bar{\beta}^*\right)\,\sin^2\frac{\Delta M t}{2}\right. \\
- & \left.\frac{i}{2}\mathrm{Im}\left(\left[\xi^* - \xi^{-1}\right]\bar{\alpha}\beta^* + \left[\xi - (\xi^*)^{-1}\right]\alpha\bar{\beta}^*\right)\sin(\Delta M t)\right\}\,.
\end{aligned} \tag{9}$$

For $\xi = e^{2i\beta}$ one can easily see that this asymmetry has the form similar to eqn.(5), and indeed is the same after time integration. One should note that no $\beta$ dependence survives in the asymmetry, where $\beta$ is the CKM phase of the $B - \bar{B}$ mixing diagram. Hence, the asymmetries under consideration in $B \to K^*\ell^+\ell^-$ are insensitive to mixing induced $CP$ violation and measure only direct $CP$ violation. However, if $\xi = \frac{1+\epsilon}{1-\epsilon}$ the asymmetry involves both $\mathrm{Re}(\epsilon)$ and the phases $\phi$:

$$\begin{aligned}
\mathsf{A}_1^{0\,CP}(t) \propto \sum_{j,k} e^{-\Gamma_B t}|a^j||b^k|\Big\{& 2\cos(\delta^{jk})\,\sin(\phi^{jk}) + 8\,\mathrm{Re}(\epsilon)\,\sin(\delta^{jk})\,\cos(\phi^{jk})\,\sin^2\frac{\Delta M t}{2} \\
& + 4\,\mathrm{Re}(\epsilon)\,\cos(\delta^{jk})\,\cos(\phi^{jk})\,\sin(\Delta M t)\Big\}\,.
\end{aligned}$$

For the $B_d$ system, $\mathrm{Re}(\epsilon)$ is expected to be $10^{-3}$. The measured upper limit is $\mathrm{Re}(\epsilon) < 0.045$ at 90% C.L. [13]. Hence in estimating the number of $B_d$ mesons required to detect $CP$ violation we assume $\mathrm{Re}(\epsilon) = 0$.

$CP$ violating asymmetries in $b \to (s,d)\gamma\,,(s,d)\ell^+\ell^-$ have been considered [14–16] recently, for both Standard model (SM) as well as the two–Higgs doublet model (2HDM). However all these discussions require flavor tagging and in most cases rely on the presence of large strong phases arising out of final state interactions. Since very large numbers of $B$'s are required to detect $CP$ violation in these modes (several hundreds of times more than that possible at asymmetric $B$-factories), a technique that does not require flavor or time tagging would clearly be beneficial. We construct $CP$ violating asymmetries of the type $\mathsf{A}_1^{CP}$ and $\mathsf{A}_2^{CP}$ for SM as well as 2HDM. Here we present only an estimate of the number of B mesons required to observe a $CP$ violating asymmetry. We choose to present our results in this form rather than numbers for asymmetries, so as to minimize the dependence of our results on non CP violating parameters of the models. Details of our numerical work will be presented elsewhere. In estimating numbers we have ignored statistical and systematic errors, which will be a part of any experiment measuring such asymmetries. The experimental procedure to observe $CP$ violation will assume that there is a sample with *equal numbers* of $B$ and $\bar{B}$. It would be imperative that the cuts imposed are such as to minimize inherent asymmetry in the collected samples of $B$ and $\bar{B}$. We use the form factors from the quark model (QM) of Ref. [7], since in heavy quark effective theory (HQET) they cannot currently be reliably predicted over the entire dilepton mass range. However, use of HQET will be possible once $B \to \rho\ell\nu$ data is available. At the time these asymmetries are experimentally studied this data should presumably be available and HQET would be choice for form factors. It has recently been shown [17] that contrary to held beliefs, if an interplay of weak and strong phases of two different amplitudes in addition to $B^0 - \overline{B^0}$ mixing are considered, it is possible to detect $CP$ violation at symmetric $e^+e^-$ colliders. *We have shown that this is possible even without the presence of strong phases.* All arguments presented here are equally applicable to the processes $B \to \rho\ell^+\ell^- \to \pi\pi\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^- \to KK\ell^+\ell^-$ .

In SM the $CP$ violating asymmetries $\mathsf{A}_1^{CP}$ and $\mathsf{A}_2^{CP}$ take the form

$$\mathsf{A}_1^{CP} = 2\,x\,A^2\lambda^6\eta\,\Delta \int dq^2\,\mathsf{C}\,(a_0\,V - A_0\,g)\,X^2\,,\ \mathsf{A}_2^{CP} = \,x\,A^2\lambda^6\eta \int dq^2\,\mathsf{C}\,\mathsf{F}\frac{1}{\sqrt{q^2}}X^2\,. \quad (10)$$

where $x = \dfrac{\alpha^2 G_F^2\, \sigma^2\, m_b\, m_{K^*}\, \lambda_K^3}{2^{10}\, 9\pi^8 M_B(\Gamma+\bar\Gamma)(M_B+m_{K^*})}$, $\mathsf{C} = \mathrm{Re}(C_7^t\, C_8^{c*} - C_7^{c*}\, C_8^t + C_7^u\, C_8^{t*} - C_7^{t*}\, C_8^u + C_7^c\, C_8^{u*} - C_7^{u*}\, C_8^c)$, and $\mathsf{F} = \{2X^2 M_B^2(a_+ V - A_+ g) + (a_0 V - A_0 g)\,\Delta k\cdot q\}$. We estimate the number of $B$ mesons required to see $CP$ violation as $\mathsf{N}_1^{CP} \equiv (\mathsf{A}_1^{CP} \tau_B\, \Gamma)^{-1} \approx 1\times 10^{10}$ and $\mathsf{N}_2^{CP} \approx 2\times 10^8$. Clearly asymmetry $\mathsf{A}_2^{CP}$ is more sensitive as should be expected.

In 2HDM CP violation arises from a relative phase between the two Higgs vacuum expectation values. Even if $\eta = 0$ there is $CP$ violation in 2HDM. Only contribution from the top intermediate state in the loop is enough to generate $CP$ violation. For 2HDM each of the coefficients [5] $C_7$, $C_8$ and $C_9$ get extra contribution above those in the standard model. However, only $C_7$ gets contribution from a complex phase, arising from the Higgs vacuum expectation values, giving

$$C_7 = C_7^{SM} + |\xi_t|^2\, \tilde{C_7^H} + \mathrm{Re}(\xi_t \xi_b)\, C_7^H + i\,\mathrm{Im}(\xi_t \xi_b)\, C_7^H\ .$$

The resultant asymmetries being given by equations identical to SM except that $\mathsf{C} = \mathrm{Re}\Big(C_7^t\, C_8^{t*}\Big)$ and $\lambda^2\eta \to \mathrm{Im}(\xi_t\xi_b)$. Using the constraint from electric dipole moment (*e.d.m.*) of the neutron, the upper limits on $\mathrm{Im}(\xi_t\xi_b)$ range from 0.3 –10 [15] due to large uncertainities in the hadronic form factors. We refer the reader to Ref. [15] and refrain from details here. It is found that using $\mathsf{A}_2^{CP}$ we need $\approx 7\times 10^6$ $B$'s to constrain $CP$ violating parameters of the 2HDM better than done by *e.d.m.* of the neutron.

To conclude we have studied $CP$ violating Dalitz plot asymmetries in angular variables. We consider such asymmetries for the process $B \to K^*\ell^+\ell^- \to K\pi\ell^+\ell^-$ . These asymmetries are constructed by adding $B$ and $\bar B$ events, and do not require flavor or time tagging, nor do they depend on unknown strong phases. These asymmetries provide a clean signal for $CP$ violation that will prove easy to measure. Several experiments [13] have already measured some angular distributions in a related process $B \to K^* J/\psi$. $B \to K^*\ell^+\ell^-$ is likely to be seen soon, and one should either be able to see $CP$ violation or establish better better bounds on models of $CP$ violation like the 2HDM. Using the methods discussed here it should be possible to measure $\eta$ in planned future colliders.

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## TABLES

TABLE I. Relations between the form factors used in this paper, a quark model (QM) that reproduces heavy quark limit and heavy quark effective theory (HQET). $W_\mu = (K_\mu - \zeta\, k_\mu)$, $\sigma^2 = 96\pi^2/(m_{K^*}^2 \lambda_K{}^3)$, $\lambda_K$, $\Delta$ and $\zeta$ are defined in Table II.

| | QM [7] | HQET [8] |
|---|---|---|
| $\mathcal{A}$ | $-2\, g\, \sigma$ | $(A+B)\, \sigma$ |
| $\mathcal{B}$ | $a_0\, \Delta\, \sigma$ | $-(\frac{A+B}{2}\, \Delta + \frac{A-B}{2}\, q^2)\, \sigma$ |
| $\mathcal{C}$ | $2\, a_+\, W \cdot q\, \sigma - \zeta\, \mathcal{B}$ | $2\, W \cdot q\, (\frac{A+B}{2} + \frac{C}{2} q^2)\, \sigma + \zeta\, \mathcal{B}$ |
| $\mathcal{D}$ | $-2\, \frac{V}{M_B + m_{K^*}}\, \sigma$ | $2\, g\, \sigma$ |
| $\mathcal{E}$ | $\frac{A_0}{M_B + m_{K^*}}\, \Delta\, \sigma$ | $-f\, \sigma$ |
| $\mathcal{F}$ | $2\, \frac{A_+}{M_B + m_{K^*}}\, W \cdot q\, \sigma - \zeta\, \mathcal{E}$ | $-2\, a_+\, W.q\, \sigma - \zeta\, \mathcal{E}$ |

TABLE II. The matrix element mod. squared for B$\to K\pi\ell^+\ell^-$.

$$|\mathcal{M}|^2 = \frac{\alpha^2 G_F^2}{2\pi^2} \Big( 2\epsilon_{\mu\nu\rho\sigma} k^\mu K^\nu q^\rho Q^\sigma \Big( K\cdot Q\, \mathrm{Im}(\alpha_L \beta_L^* + \alpha_R \beta_R^*) + \mathrm{Im}(\rho_L\, \beta_L^* - \rho_R\, \beta_R^*) - k\cdot Q\, \mathrm{Im}(\rho_L\, \alpha_L^* + \rho_R\, \alpha_R^*) \Big)$$

$$+2\,\mathrm{Re}(\rho_R\, \alpha_R^* - \rho_L\, \alpha_L^*) \big( -k\cdot K\, q^2\, k\cdot Q + k\cdot q\, k\cdot Q\, K\cdot q + m_{K^*}^2\, q^2\, K\cdot Q - k\cdot q^2\, K\cdot Q \big) + 2\,\mathrm{Re}(\rho_L\, \beta_L^* + \rho_R\, \beta_R^*)\, (k\cdot q K\cdot q$$

$$-k\cdot K q^2 - k\cdot Q K\cdot Q \big) + 2\,\mathrm{Re}(\alpha_L\, \beta_L^* - \alpha_R\, \beta_R^*) \big( K^2\, q^2\, k\cdot Q - k\cdot Q\, K\cdot q^2 - k\cdot K\, q^2\, K\cdot Q + k\cdot q\, K\cdot q\, K\cdot Q \big) + \big(\rho_L^2 + \rho_R^2\big)$$

$$\big( -m_{K^*}^2\, q^2 + k\cdot q^2 - k\cdot Q^2 \big) + \big( \beta_L{}^2 + \beta_R{}^2 \big) \big( -K^2\, q^2 + K\cdot q^2 - K\cdot Q^2 \big) + \big( \alpha_L{}^2 + \alpha_R{}^2 \big) \big( -K^2\, q^2\, k\cdot q^2 + k\cdot Q^2\, K\cdot q^2$$

$$+2\, k\cdot K\, q^2\, k\cdot Q\, K\cdot Q - m_{K^*}^2\, q^2\, K\cdot Q^2 + k\cdot q^2\, K\cdot Q^2 - 2\, k\cdot q\, k\cdot Q\, K\cdot q\, K\cdot Q \big) \Big),$$

$$k\cdot K = m_K^2 - m_\pi^2\,,\ k\cdot q = \frac{(\Delta - q^2)}{2},\ \Delta = (M_B^2 - m_{K^*}^2)\,,\ k\cdot Q = X M_B \cos\theta_e\,,\ X = \frac{(k\cdot q^2 - q^2 m_{K^*}^2)^{\frac{1}{2}}}{M_B},\ \lambda_e = \sqrt{1 - \frac{4\, m_e^2}{q^2}}\,,$$

$$K\cdot Q = \lambda_K (k\cdot q \cos\theta_e \cos\theta_K - \sqrt{q^2} m_{K^*} \sin\theta_e \sin\theta_K \cos\varphi) + k\cdot q\, \zeta,\ \zeta = \frac{k\cdot K}{m_{K^*}^2}\,, K\cdot q = \lambda_K X M_B \cos\theta_K + k\cdot q\, z,\ q\cdot Q = 0$$

$$\lambda_K = \left( 1 - \frac{(m_K + m_\pi)^2}{m_{K^*}^2} \right)^{\frac{1}{2}} \left( 1 - \frac{(m_K - m_\pi)^2}{m_{K^*}^2} \right)^{\frac{1}{2}},\ \epsilon_{\mu\nu\rho\sigma} k^\mu K^\nu q^\rho Q^\sigma = -X M_B \lambda_K \sqrt{q^2} m_{K^*} \sin\theta_e \sin\theta_K \sin\varphi\,.$$