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TWO-GLUON CONTRIBUTION TO THE BFKL KERNEL *

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Abstract

The differential cross section for the production of two gluons with a fixed invariant mass at high energy collision of partons is expressed in terms of the corresponding cross section for the Reggeon-Reggeon interactions. The infrared divergencies are removed with the use of the dimensional regularization. Performing the integration over relative momenta of the produced gluons we calculate two-gluon correction to the integral kernel of the BFKL equation.

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1 Introduction

The problem of calculating parton distributions in the region of small values of the Bjorken variable $x = Q^2/s$ became especially important in the last years, in connection with results of recent experiments on the deep inelastic ep scattering [1]. In the leading logarithmic approximation (LLA) of QCD, when only the terms which contain $\ln(1/x)$ for each power of the coupling constant α_s are summed, one can use the Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation [2]. It can be presented in the form of the evolution equation in $\ln(1/x)$:

$$\frac{\partial}{\partial \ln(1/x)}\mathcal{F}(x,\vec{q}_1^{\,2}) = \int d^2q_2 \mathcal{K}(\vec{q}_1,\vec{q}_2)\mathcal{F}(x,\vec{q}_2^{\,2}), \tag{1.1}$$

where the function $\mathcal{F}(x,\vec{k}^2)$ is the unintegrated gluon density, connected with the gluon distribution $g(x,Q^2)$ with squared transverse gluon momenta up to Q^2 as follows

$$xg(x,Q^2) = \int_0^{Q^2} d(\vec{k}^2)\mathcal{F}(x,\vec{k}^2). \tag{1.2}$$

In the LLA the kernel of this equation has the form [2]

$$\mathcal{K}(\vec{q}_1,\vec{q}_2) = -\frac{g^2N}{(2\pi)^3}\vec{q}_1^{\,2}\int\frac{d^2k}{\vec{k}^2(\vec{q}_1-\vec{k})^2}\delta^{(2)}(\vec{q}_1-\vec{q}_2) + \frac{g^2N}{4\pi^3}\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\ . \tag{1.3}$$

Here g is the gauge coupling constant ($\alpha_s = g^2/4\pi$), N is the number of colours ($N=3$ for QCD), and the vector sign serves for denotation of the projection of 4-momenta to the plane orthogonal to the initial particle momenta p_A, p_B . Two contributions in the kernel (1.3) have infrared singularities which cancel each other in Eq.(1.1). In the LLA it can be shown by performing an azimuthal integration in Eq.(1.1) [2]:

$$\frac{\partial}{\partial \ln(1/x)}\mathcal{F}(x,\vec{q}_1^{\,2}) =$$

$$\frac{\alpha_s N}{\pi}\int_0^\infty d(\vec{q}_2^{\,2})\left[\frac{\mathcal{F}(x,\vec{q}_2^{\,2})}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|} - \mathcal{F}(x,\vec{q}_1^{\,2})\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\left(\frac{1}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|} - \frac{1}{\sqrt{\vec{q}_1^{\,4}+4\vec{q}_2^{\,4}}}\right)\right]. \tag{1.4}$$

In the next-to-leading approximation, which will be discussed below, the cancellation of the infrared singularities is not so simple. To remove the infrared divergencies in the intermediate expressions we'll use the dimensional regularization

$$\frac{d^2k}{(2\pi)^3} \to \frac{d^{D-2}k}{(2\pi)^{D-1}}, \tag{1.5}$$

where $D = 4+2\epsilon$ is the space-time dimension.

The LLA leads to a sharp power growth of cross sections with c.m.s. energy $\sqrt{s}$. In terms of parton distributions this means a fast increase of the gluon density $g(x,Q^2)$ in small x region:

$$g(x,Q^2) \sim x^{-j_0}\ , \tag{1.6}$$

where j_0 is the j- plane position of the singularity of the t- channel partial amplitude with vacuum quantum numbers [2]:

$$j_0 = 1 + \frac{4\alpha_s}{\pi} N \ln 2, \tag{1.7}$$

with $N = 3$ for QCD. Such behaviour violates the Froissart bound $\sigma_{tot} < const(\ln s)^2$ and therefore LLA can not be applied at asymptotically small x. Nevertheless, in the region of parameters accessible for modern experiments the screening corrections are negligible [3], and we will not discuss here the unitarization problem which will appear at asymptotically large energies. From the practical point of view it seems more important to find the region of energies and momentum transfers where the BFKL approach is applicable. For this purpose we have to calculate the next-to-leading corrections to LLA. The importance of the radiative corrections is strengthened by the circumstance that the coefficient in the linear dependence of the argument of the QCD running coupling α_s on the gluon virtuality is beyond the accuracy of LLA. It diminishes considerably the predictive power of the LLA, because numerical results can be strongly modified by changing this coefficient.

In the next-to-leading approximation, when all terms of the type $\alpha_s[\alpha_s \ln(1/x)]^n$ have to be summed, the form of the equation for $\mathcal{F}(x, \vec{k}^2)$ is not changed [4], so one should find corrections to the kernel $\mathcal{K}(\vec{q}_1, \vec{q}_2)$. In this approximation the kernel can be presented in the form:

$$\mathcal{K}(\vec{q}_1, \vec{q}_2) = 2\omega(t_1)\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_{real}(\vec{q}_1, \vec{q}_2) \ , \tag{1.8}$$

where $t_1 = -\vec{q}_1^{\,2}$, $\omega(t_1)$ is related with the gluon Regge trajectory $j(t_1) = 1 + \omega(t_1)$, and $\mathcal{K}_{real}(\vec{q}_1, \vec{q}_2)$ is determined by a probability of the real particle production in collision of two Reggeized gluons with momenta $q_1 = \beta p_A + q_{1\perp}$ and $-q_2 = \alpha p_B - q_{2\perp}$, where $\alpha, \beta \ll 1$ and $q_{1,2_\perp}$ are their Sudakov parameters and transverse components. More definitely,

$$\mathcal{K}_{real}(\vec{q}_1, \vec{q}_2) = \frac{1}{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \frac{1}{(N^2-1)} \sum_{i_1, i_2, f} \int d\kappa d\rho_f$$

$$\times \, \delta^{(D)}(q_1 - q_2 - \sum_{n \in \{f\}} k_n) \left[|\gamma_{i_1 i_2}^{\{f\}}(q_1, q_2)|^2 \, - |\gamma_{i_1 i_2}^{\{f\}}(q_1, q_2)|^2_{asympt} \right] , \tag{1.9}$$

where the sum is taken over the colours i_1, i_2 of the Reggeized gluons and over all types of produced particles $\{f\}$. The integration is performed over the squared invariant mass of the two Reggeons $\kappa = (q_1 - q_2)^2$ and over momenta k_n of produced particles with taking into account their phase space

$$d\rho_f = \prod_{n \in \{f\}} \frac{d^{D-1} k_n}{(2\pi)^{D-1} 2\omega_n}. \tag{1.10}$$

The quantities $\gamma_{i_1 i_2}^{\{f\}}(q_1, q_2)$ are effective amplitudes for the production of the system $\{f\}$ in the Reggeon-Reggeon collision and the subscript "*asympt*" means their values at asymptotically large κ. The subtraction in Eq.(1.9) is performed in order to escape a double counting of the region of large κ, because in this region amplitudes $\gamma_{i_1 i_2}^{\{f\}}(q_1, q_2)$ with the gluon quantum numbers in the t- channel are factorized into products of the simple effective vertices and therefore the contribution from the large κ region is taken into account by iterating the BFKL eqution in LLA. In the lowest order the Regge trajectory is [2]

$$\omega^{LLA}(t_1) = \frac{g^2 t_1}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k}{\vec{k}^2(\vec{q}_1 - \vec{k})^2} , \tag{1.11}$$

and in the Reggeon-Reggeon collision only one gluon is produced. The Reggeon-Reggeon-Gluon (RRG) vertex in the Born approximation is

$$\gamma^d_{i_1 i_2}(q_1, q_2) = g T^d_{i_2 i_1} e^*_\mu(k) C^\mu(q_2, q_1), \tag{1.12}$$

where $T^d_{i_2 i_1}$ - matrix elements of the $SU(N)$ group generators in the adjoint representation, $e(k)$ - polarization vector of the produced gluon, $k = q_1 - q_2$ is its momentum and the vector C is given below [2]

$$C(q_2, q_1) = -q_1 - q_2 + p_A \left(\frac{q_1^2}{kp_A} + 2\frac{kp_B}{p_A p_B} \right) - p_B \left(\frac{q_2^2}{kp_B} + 2\frac{kp_A}{p_A p_B} \right) . \tag{1.13}$$

Therefore in this approximation

$$\mathcal{K}^{LLA}_{real}(\vec{q}_1, \vec{q}_2) = \frac{1}{2(2\pi)^{D-1}(N^2-1)\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \sum_{i_1, i_2, d, \lambda} |\gamma^d_{i_1 i_2}(q_1, q_2)|^2 =$$

$$\frac{g^2 N}{(2\pi)^{D-1}} \frac{2}{(\vec{q}_1 - \vec{q}_2)^2} . \tag{1.14}$$

Here λ is a helicity of the produced gluon. Substituting Eqs.(1.11), (1.14) in Eq.(1.8) and putting $D = 4$ we obtain Eq.(1.3).

The next-to-leading corrections to the kernel are expressed [4] in terms of the two-loop contribution $\omega^{(2)}(t)$ to the gluon Regge trajectory $\omega(t)$, one-loop correction to the RRG vertex $\gamma^d_{i_1 i_2}(q_1, q_2)$, and contributions from two-gluon and quark-antiquark production in the quasi-multi-Regge kinematics (QMRK), which, in turn, are expressed in terms of the Reggeon-Reggeon-Gluon-Gluon (RRGG) and Reggeon-Reggeon-Quark-anti-Quark (RR$Q\bar{Q}$) amplitudes. Corrections to the RRG vertex and to the Reggeized gluon trajectory were calculated in Refs. [5] and [6] correspondingly. The real emission of the quark-antiquark pair was considered in Refs. [7]. Investigations of the contributions from two-gluon production in the QMRK were started in Ref. [4], where the two-gluon production amplitude in the QMRK, and, correspondingly, the effective RRGG amplitude, was found. The next step was done in Ref. [8], where the two-gluon and quark-antiquark production amplitudes in the QMRK were simplified using helicity representation, the corresponding next-to-leading contributions to the BFKL kernel were expressed in terms of the integrals from the squares of these helicity amplitudes over relative transverse and longitudinal momenta of produced particles, all infrared divergencies were extracted in an explicit form and cancellation of the infrared divergencies between real and virtual contributions of quark-antiquark pairs to the kernel was shown. In general case such cancellation was demonstrated in Ref. [9].

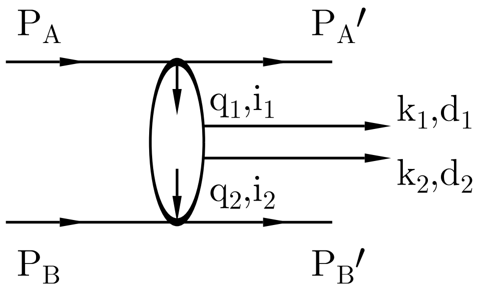

Figure 1:

In this paper we calculate the totally differential cross section of two-gluon production in the QMRK and express it in terms of the Reggeon-Reggeon $\rightarrow$ gluon-gluon cross section. Performing the integration over relative transverse and longitudinal momenta of the produced gluons, we find the two-gluon correction to the BFKL kernel.

The outline of the paper is the following. In Section 2 we remind the amplitude of two-gluon production in the QMRK in gluon-gluon collisions. In Section 3 we express the cross-section of this process through the Reggeon-Reggeon $\rightarrow$ gluon-gluon cross section and present the totally differential distribution for the produced gluons. The two-gluon correction to the BFKL kernel is presented in Section 4. Details of calculations are given in Appendices A - C.

2 The amplitude of two-gluon production in QMRK

Let us consider the production of two gluons with their fixed invariant mass in the QMRK. Since in this kinematical region production amplitudes have a factorized form [4], we will, essentially without a loss of generality, confine ourselves for simplicity to the case when only one pair of gluons is produced in gluon-gluon collisions. This process is shown schematically at Fig.1 where denotations for all momenta which are used below and colour indices of produced gluons are presented.

We will use the Sudakov parametrization for produced gluon momenta

$$k_i = \beta_i p_A + \alpha_i p_B + k_{i\perp} \ , \quad s\alpha_i\beta_i = -k_{i\perp}^2, \tag{2.1}$$

where $s = (p_A + p_B)^2$ is the squared total c.m.s. energy, which is assumed to increase infinitely, and we take into account the reality condition for produced gluons. Then the QMRK means that all $k_{i\perp}$ are fixed (i.e. don't grow at large s) and

$$\beta_{A'} \gg \beta_1 \sim \beta_2 \gg \beta_{B'}, \qquad \alpha_{A'} \ll \alpha_1 \sim \alpha_2 \ll \alpha_{B'}. \tag{2.2}$$

The squared invariant mass of the produced gluons $\kappa = (q_1 - q_2)^2 = (k_1 + k_2)^2$ in the QMRK has the same order of magnitude as $|k_{i\perp}^2|$.

The amplitude of the considered process was calculated in Ref. [4] and has the following factorized form:

$$A_{2\to 4} = 2sg^2 \delta_{\lambda_{A'},\lambda_A} T^{i_1}_{A'A} \frac{1}{q_{1\perp}^2} \gamma^{d_1 d_2}_{i_1 i_2}(q_1, q_2) \frac{1}{q_{2\perp}^2} T^{i_2}_{B'B} \delta_{\lambda_{B'},\lambda_B}, \tag{2.3}$$

where λ_i are helicities of the corresponding particles. The effective RRGG amplitude $\gamma^{d_1 d_2}_{i_1 i_2}(q_1, q_2)$ is given by the expression:

$$\gamma^{d_1 d_2}_{i_1 i_2}(q_1, q_2) = g^2\, e^*_{\alpha_1}(k_1)\, e^*_{\alpha_2}(k_2) \left[T^{d_1}_{i_1 j}\, T^{d_2}_{j i_2}\, A^{\alpha_1 \alpha_2}(k_1, k_2) + T^{d_2}_{i_1 j}\, T^{d_1}_{j i_2}\, A^{\alpha_2 \alpha_1}(k_2, k_1) \right], \tag{2.4}$$

where tensor $A^{\alpha_1\alpha_2}(k_1, k_2)$ was obtained in Ref. [4]. It satisfies the transversality condition to the corresponding k_i:

$$k_1^{\alpha_1} A_{\alpha_1 \alpha_2}(k_1, k_2) \;=\; k_2^{\alpha_2} A_{\alpha_1 \alpha_2}(k_1, k_2) \;=0. \tag{2.5}$$

Due to relations (2.5) two terms in (2.4) are invariant under the independent gauge transformations of the polarization vectors $e^{\alpha_i}(k_i) \to e^{\alpha_i}(k_i) + c k_i^{\alpha_i}$, which permits us to choose a convenient gauge for each of produced gluons without introducing the Faddeev-Popov ghosts. Following [8], we'll use for each of produced gluons its own light-cone gauge:

$$e_\alpha(k_1) k_1^\alpha = e_\alpha(k_1) p_A^\alpha = 0\,, \;\; e_\alpha(k_2) k_2^\alpha = e_\alpha(k_2) p_B^\alpha = 0. \tag{2.6}$$

Then the polarization vectors can be presented in the form

$$e(k_1) = e_\perp(k_1) - \frac{(k_1 e_\perp(k_1))}{k_1 p_A} p_A, \;\; e(k_2) = e_\perp(k_2) - \frac{(k_2 e_\perp(k_2))}{k_2 p_B} p_B, \tag{2.7}$$

and their convolutions with the tensor $A_{\alpha_1\alpha_2}(k_1, k_2)$ can be expressed in terms of the tensor $c_{\alpha_1\alpha_2}(k_1, k_2)$, which has only transverse components and is defined in the following way:

$$e^*_{\alpha_1}(k_1)\, e^*_{\alpha_2}(k_2)\, A^{\alpha_1\alpha_2}(k_1, k_2) \equiv 4 e^*_{\perp\alpha_1}(k_1)\, e^*_{\perp\alpha_2}(k_2)\, c^{\alpha_1\alpha_2}(k_1, k_2). \tag{2.8}$$

According to this definition we have [8]:

$$\begin{aligned} c^{\mu\nu}(k_1, k_2) = {} & \frac{(q_1 - k_1)^\mu_\perp (q_1 - k_1)^\nu_\perp}{t} - \frac{(q_1 - k_1)^\mu_\perp}{\kappa} \left(k_1 - \frac{x}{1-x} k_2 \right)^\nu_\perp + \left(k_2 - \frac{x k_{2\perp}^2}{(1-x) k_{1\perp}^2} k_1 \right)^\mu_\perp \\ & \times \frac{(q_1 - k_1)^\nu_\perp}{\kappa} - \frac{k_{1\perp}^\mu k_{1\perp}^\nu}{\kappa} \frac{x t_2}{k_{1\perp}^2} - \frac{k_{2\perp}^\mu k_{2\perp}^\nu}{\kappa} \frac{x t_1}{Z} - \frac{k_{1\perp}^\mu k_{2\perp}^\nu}{\kappa} \left(1 - \frac{x t}{(1-x) k_{1\perp}^2} \right) + \frac{k_{2\perp}^\mu k_{1\perp}^\nu}{\kappa} \\ & - \frac{1}{2} g^{\mu\nu}_{\perp\perp} \left(1 + \frac{t}{\kappa} - \frac{(1-x) k_{1\perp}^2}{x t} + \frac{x k_{2\perp}^2}{(1-x)\kappa} - \frac{(1-x) k_{1\perp}^2}{x \kappa} - \frac{x t_1 k_{2\perp}^2}{\kappa Z} - \frac{x t_2}{\kappa} \right). \end{aligned} \tag{2.9}$$

We use denotations (because the Bjorken variable will not appear in the following, using denotation x for other value can not be misleading)

$$x = \frac{\beta_1}{\beta} = 1 - \frac{\beta_2}{\beta}, \;\; \beta = \beta_1 + \beta_2,$$

$$t=(q_1-k_1)^2=\frac{1}{x}\left((k_1-xq_1)_\perp^2+x(1-x){q_1}_\perp^2\right),$$

$$\kappa=(k_1+k_2)^2=-\frac{((1-x)k_1-xk_2)_\perp^2}{x(1-x)},\;\; Z=(1-x){k_1}_\perp^2+x{k_2}_\perp^2,$$

$$t_1={q_1}_\perp^2,\quad t_2={q_2}_\perp^2,\quad \Delta_\perp=(q_1-q_2)_\perp=(k_1+k_2)_\perp, \tag{2.10}$$

and $g_{\perp\perp}^{\mu\nu}$ is the metric tensor in the subspace orthogonal to p_A, p_B:

$$g_{\perp\perp}^{\mu\nu}=g^{\mu\nu}-\frac{p_A^\mu p_B^\nu+p_B^\mu p_A^\nu}{(p_Ap_B)}. \tag{2.11}$$

It is important to note that, since we use different gauges for two produced gluons, the second term in (2.4) can not be obtained by the simple substitution $k_1\leftrightarrow k_2$ from (2.8). Instead, we have [8], [10]

$$e_{\alpha_1}^*(k_1)\,e_{\alpha_2}^*(k_2)\,A^{\alpha_2\alpha_1}(k_2,k_1)\equiv 4e_{\perp\alpha_1}^*(k_1)\,e_{\perp\alpha_2}^*(k_2)\,\Omega^{\alpha_1\beta_1}(k_1)\Omega^{\alpha_2\beta_2}(k_2)c_{\beta_2\beta_1}(k_2,k_1), \tag{2.12}$$

where

$$\Omega^{\alpha\beta}(k)=g_{\perp\perp}^{\alpha\beta}-2\frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2}. \tag{2.13}$$

The tensor $\Omega^{\alpha\beta}(k)$ serves for the transformation from the gauge $(e(k)p_A)=0$ to the gauge $(e(k)p_B)=0$ and vise versa.

Let us remind again, that the amplitude (2.3) with RRGG amplitude defined in (2.4), (2.7)-(2.13) is valid not only in the QMRK, where $\beta_1\sim\beta_2$, but also in the multi-Regge kinematics (MRK), when $\beta_1\gg\beta_2$ (or $\beta_2\gg\beta_1$). This kinematics corresponds to the case of a large invariant mass of the produced gluons, in which the RRGG amplitude factorizes into a product of two RRG vertices. When $\beta_1\gg\beta_2$ we have

$$\gamma_{i_1i_2}^{d_1d_2}(q_1,q_2)=\gamma_{i_1j}^{d_1}(q_1,q_1-k_1)\frac{1}{(q_1-k_1)_\perp^2}\gamma_{ji_2}^{d_2}(q_1-k_1,q_2). \tag{2.14}$$

3 The Reggeon-Reggeon → gluon-gluon cross-section

The cross section of the process shown at Fig.1 averaged over colours and spin projections of initial particles and summed over quantum numbers of the final gluons can be presented in the form:

$$d\sigma_{2\to4}=\frac{4N^2\alpha_s^2}{(N^2-1)(2\pi)^{2D-4}}\frac{d^{D-2}q_{1\perp}d^{D-2}q_{2\perp}}{({q_1}_\perp^2{q_2}_\perp^2)^2}\frac{d\beta}{\beta}\kappa d\kappa d\sigma_{RRGG}. \tag{3.1}$$

Here we introduced the RRGG cross section defined as:

$$d\sigma_{RRGG}=\frac{1}{2!}\frac{1}{(N^2-1)2\kappa}\sum_{i_1,i_2,d_1,d_2,\lambda_1\lambda_2}|\gamma_{i_1i_2}^{d_1d_2}(q_1,q_2)|^2(2\pi)^D\delta^{(D)}(q_1-q_2-k_1-k_2)d\rho_2, \tag{3.2}$$

where $d\rho_2$ is the two-particle phase space volume, given by (1.10), and the factor $1/2!$ takes into account the gluon identity.

Note, that for the two-gluon correction to the kernel $\mathcal{K}(\vec{q}_1, \vec{q}_2)$ in (1.9) it is implied that the identity of the final particles is taken into account by an appropriate limitation on their phase space. Now this correction can be expressed as the following integral from the RRGG cross section (3.2) over the squared invariant mass of the Reggeons:

$$\mathcal{K}_{RRGG}(\vec{q}_1, \vec{q}_2) = \int \frac{\kappa d\kappa}{q_{1\perp}^2 q_{2\perp}^2 (2\pi)^D}[d\sigma_{RRGG} - d\sigma_{RRGG}^{asympt}], \tag{3.3}$$

without any limitations on the integration region.

In terms of variables $x, k_{i\perp}$ we obtain from definition (3.2), using Eqs.(2.4), (2.8), (2.12) and Ref. [8]:

$$\kappa d\kappa d\sigma_{RRGG} = \frac{2g^4N^2}{(2\pi)^{D-2}} \frac{dx}{x(1-x)} d^{D-2}k_{1\perp}$$
$$\times \Big[(c^{\alpha_1\alpha_2}(k_1,k_2))^2 + (c^{\alpha_1\alpha_2}(k_2,k_1))^2 + c^{\alpha_1\alpha_2}(k_1,k_2)\Omega_{\alpha_1\beta_1}(k_1)\Omega_{\alpha_2\beta_2}(k_2)c^{\beta_2\beta_1}(k_2,k_1)\Big]. \tag{3.4}$$

Here $c^{\alpha_1\alpha_2}(k_1,k_2)$ is given by (2.9), $c^{\alpha_1\alpha_2}(k_2,k_1)$ is obtained by the substitution ($x \leftrightarrow 1-x$, $k_{1\perp} \leftrightarrow k_{2\perp}$), $\Omega_{\alpha\beta}(k)$ is given by (2.13) and we have used the relations:

$$T^{d_1}_{i_1j}T^{d_2}_{ji_2}T^{d_1}_{i_1k}T^{d_2}_{ki_2} = (N^2-1)N^2, \quad T^{d_1}_{i_1j}T^{d_2}_{ji_2}T^{d_2}_{i_1k}T^{d_1}_{ki_2} = (N^2-1)\frac{N^2}{2}. \tag{3.5}$$

Since all vectors entering in Eqs.(3.4), (2.9) are transverse, we'll omit from now the transversality sign.

The calculation of convolutions of the tensors in Eq.(3.4) is rather tedious, though the result is comparatively simple. The main steps of this procedure are given in the Appendix A. Using the results presented there, we obtain:

$$\kappa d\kappa d\sigma_{RRGG} = \frac{g^4N^2}{(2\pi)^{D-2}} \frac{dx d^{D-2}k_1}{x(1-x)} \Bigg[\Bigg(\frac{1}{\kappa t}\Big[2xq_1^2q_2^2 - \left(\frac{1+\epsilon}{2}\right)(1-x)\left((q_1^2-2xq_1\Delta)^2+(q_1^2)^2\right) +$$
$$+(1+\epsilon)(1-x)^2q_1^2(q_1^2-2k_1q_1)\Big] + 4(1+\epsilon)(1-x)(q_1^2-x\Delta^2-2k_1q_2)\frac{(k_1q_1)^2}{\kappa t^2} + 2(1+\epsilon)(q_1^2-x\Delta^2-$$
$$-2k_1q_2)\frac{(q_1(k_1-x\Delta))^2}{\kappa^2 t} - 2(1+\epsilon)(1-x)(q_2^2+x\Delta^2+2k_1q_2)\frac{(k_1q_1)^2}{\kappa t\tilde{t}} - \frac{2xq_1^2}{\kappa Z t}\Bigg[\Bigg(\frac{q_2^2}{2} - (1-x)$$
$$\times(q_1^2-2k_1q_1)\Bigg)^2 - \left(\frac{1-\epsilon}{2}\right)(1-x)^2(q_1^2-2k_1q_1)^2\Bigg] + \frac{1}{2t\tilde{t}}\Big[-2q_1^2q_2^2 + \left(\frac{1+\epsilon}{2}\right)x(1-x)\Big((q_1^2)^2 +$$
$$+(\Delta^2-q_2^2)^2\Big)\Big] + \frac{x(1-x)q_1^2}{2\kappa Z}\Big[(1+\epsilon)(1-2x(1-x))q_1^2 - 2\epsilon x(1-x)(\Delta^2-q_2^2)\Big] + \left(\frac{x(1-x)q_1^2}{Z}\right)^2$$
$$\times\left[\left(\frac{1+\epsilon}{2}\right) - (3+2\epsilon)x(1-x)\right] + (1+\epsilon)(1-4x)\left(\frac{(1-x)q_1^2}{t}\right)^2 + \frac{(1-x)q_1^2q_2^2(2k_1q_1-q_1^2)}{2tk_1^2k_2^2} -$$
$$-\frac{x(q_1^2)^2q_2^2}{2\kappa t k_1^2} + \frac{(q_1^2)^2(q_2^2)^2}{4t\tilde{t}k_1^2k_2^2} - \frac{xq_1^2q_2^2}{2(1-x)\kappa k_1^2} + \frac{x^2q_1^2q_2^2}{2Zk_1^2} + \frac{q_1^2q_2^2}{2k_1^2k_2^2} + \Big\{ -4(1+\epsilon)(1-x)^2q_1^2$$

$$\times\frac{(q_1(k_1-xq_1))}{t^2}+(1+\epsilon)(1-2x)x(1-x)q_1^2\frac{(q_1(k_1-x\Delta))}{\kappa Z}-2(1+\epsilon)(1-2x)(x(1-x)q_1^2)^2$$
$$\times\frac{(\Delta(k_1-x\Delta))}{\kappa Z^2}+\left(\left(\frac{x(1-x)q_1^2}{\kappa Z}\right)^2\Delta^\mu\Delta^\nu-\frac{2x(1-x)q_1^2}{\kappa^2 Z}\Delta^\mu q_1^\nu\right)\Big(2(1+\epsilon)(k_1-x\Delta)_\mu$$
$$\times(k_1-x\Delta)_\nu-(k_1-x\Delta)^2 g_{\mu\nu}\Big)\Big\}\Big)+\Big(x\leftrightarrow 1-x, k_1\leftrightarrow k_2\Big)\Big], \tag{3.6}$$

where we introduced the new definition

$$\tilde{t}=t(x\leftrightarrow 1-x,\quad k_1\leftrightarrow k_2)=\frac{1}{1-x}\left((k_2-(1-x)q_1)^2+x(1-x)q_1^2\right)=\frac{1}{1-x}$$
$$\times\left((k_1+(q_2-xq_1))^2+x(1-x)q_1^2\right)=q_1^2+q_2^2-\kappa-t=q_1^2+q_2^2-\Delta^2+\frac{Z}{x(1-x)}-t. \tag{3.7}$$

Note, that the terms in the curly brackets in (3.6) vanish after their integration over k_1. As for others, let us denote as I_n the integrals over k_1 from the subsequent terms with the different denominators in the common brackets and with the weight factor $(2\pi)^{1-D}\mu^{4-D}$, where μ is the renormalization point. Then we have

$$\int\frac{\kappa d\kappa d\sigma_{RRGG}}{dx d^{D-2}k_1}d^{D-2}k_1=\frac{2\pi g^4\mu^{2\epsilon}N^2}{x(1-x)}\left[\sum_{n=1}^{15}I_n+(x\leftrightarrow 1-x)\right]. \tag{3.8}$$

All integrals I_n are convergent at large k_1 for $D<5$ and have only logarithmic divergencies in the infrared region when $D\to 4$. Some of these integrals were calculated by us for arbitrary D, for others we could obtain only several terms of the expansion in powers of $\epsilon=(D-4)/2$. Since the BFKL equation (1.1) contains integration over q_2, or equivalently, over Δ, and the kernel is singular at $\Delta=0$, we should be careful in order to keep all terms which give nonvanishing contribution in the physical limit $\epsilon\to 0$ after integration over x and Δ. The results of calculation of the integrals I_n can be found in the Appendix B. Using this table of integrals one can obtain the x - distribution for the gluon pair production in the QMRK. We present here the results for both, for the totally differential distribution and for the x - distribution, because they may be applied for description of experiments on two-jet production in the QMRK.

4 The contribution to the BFKL kernel

According to Eq.(3.3), in order to obtain the two-gluon contribution to the BFKL kernel from (3.8) we have to integrate it over x using the expressions for I_n from Appendix B. But it is not the best way to do, because of the complexity of these expressions. Really it is more convenient to go one step back and to start with a totally differential distribution. The matter is that after integration over k_1, using the Feynman parametrization, additional integrations over the Feynman parameters arise. In the final expressions of Appendix B these integrations have been performed at fixed x. But calculating the contribution to the BFKL kernel we can change the orders of integrations over x and over these Feynman parameters in the most convenient way so that integrations could be performed much more easily.

One can get from Eq.(3.6) the following convenient expression for the integral from the RRGG cross section σ_{RRGG} over the squared invariant mass of the Reggeons κ

$$\int \frac{\kappa d\kappa d\sigma_{RRGG}}{4\pi g^4\mu^{2\epsilon}N^2} = \int_{\delta_R}^{1-\delta_R} \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \Big\{ (1+\epsilon)(1-x)^2 \frac{4(k_1q_1)^2+(1-4x)(q_1^2)^2}{t^2}$$
$$+\frac{x(1-x)q_1^2}{\kappa Z}\Big[2q_2^2+(1+\epsilon)\left(2(1-x)(k_1q_1)-x(1-x)q_1^2-k_2^2\right)-\epsilon x(1-x)(\Delta^2-q_2^2)\Big]$$
$$+\left(\frac{x(1-x)q_1^2}{Z}\right)^2\left[\frac{(1+\epsilon)}{2}-(3+2\epsilon)x(1-x)\right]+\left(\frac{-xq_1^2q_2^2}{2(1-x)\kappa k_1^2}+\frac{x^2q_1^2q_2^2}{2k_1^2Z}+\frac{xq_1^2q_2^2}{k_1^2k_2^2}\right)$$
$$+\frac{1}{\kappa t}\Big[-2(1-x)q_1^2q_2^2+(1+\epsilon)(1-x)(q_1^2)^2+\frac{(1+\epsilon)}{2}x(1-x)\left(2(1-x)q_1^2(\Delta^2-q_2^2)-x(q_1^2)^2\right.$$
$$\left.-x(\Delta^2-q_2^2)^2\right)-2(1+\epsilon)(2-x)(1-x)q_1^2(k_1q_1)+2(1+\epsilon)(1-x)\left((k_1q_1)^2+((k_1-x\Delta)q_1)^2\right)$$
$$+(1+\epsilon)q_1^2k_2^2\Big]-\frac{xq_1^2k_2^2\left((1+\epsilon)k_2^2-2q_2^2\right)}{\kappa Zt}-\frac{xq_1^2(q_2^2)^2}{2\kappa Zt}-\frac{xq_2^2(q_1^2)^2}{2\kappa tk_1^2}+\frac{(1-x)(2(k_1q_1)-q_1^2)q_1^2q_2^2}{2tk_1^2k_2^2}$$
$$+\frac{1}{2t\tilde{t}}\Big[-2q_1^2q_2^2+\frac{(1+\epsilon)}{2}x(1-x)\left((\Delta^2-q_2^2)^2+(q_1^2)^2\right)-4(1+\epsilon)x(1-x)(k_1q_1)^2\Big]+\frac{(q_1^2)^2(q_2^2)^2}{4t\tilde{t}k_1^2k_2^2}\Big\}. \tag{4.1}$$

The intermediate parameter $\delta_R \ll 1$ is introduced here in order to exclude the contribution of the multi-Regge kinematics, which is accounted by the leading term in the BFKL kernel. In the expression for the contribution to the BFKL kernel (3.3) the dependence on δ_R vanishes.

Let us again denote the contribution of the n-th term in the integrand in the RHS of (4.1) as J_n and calculate these integrals separately. Note, that contrary to the case of the differential distribution some of the integrals J_n contain ultraviolet divergencies at large k_1; but these divergencies can be regularized by the same parameter ϵ as infrared ones and, as it is clear from Eq.(3.6), they must cancel in the final expression after summing all J_n. Using the expressions for these integrals from Appendix C and noting that the subtraction of the multi-Regge contribution in Eq.(3.3) is equivalent to omitting the terms with $\ln(\delta_R)$ we get finally for the correction to the BFKL kernel the following result:

$$\mathcal{K}_{RRGG}(\vec{q}_1,\vec{q}_2) = \frac{4\bar{g}_\mu^4\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\Big\{\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^2}\left(\frac{\vec{\Delta}^2}{\mu^2}\right)^\epsilon\Big[\frac{1}{\epsilon^2}-\frac{11}{6}\frac{1}{\epsilon}-\frac{2\pi^2}{3}+\frac{67}{18}+\epsilon\Big(\frac{11\pi^2}{36}-\frac{202}{27}$$
$$+9\zeta(3)\Big)\Big]-\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3(\vec{q}_1\vec{q}_2)^2)}{8\vec{q}_1^{\,2}\vec{q}_2^{\,2}}-\left(\frac{11}{3}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}+\frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3(\vec{q}_1\vec{q}_2)^2)}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\right)$$
$$\times\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{2}{3}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\Big[\left(1-\frac{2(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)\left(\vec{q}_1^{\,4}-\vec{q}_2^{\,4}-2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)+(\vec{q}_1\vec{q}_2)\Big(2\vec{q}_1^{\,2}-$$
$$-2\vec{q}_2^{\,2}-(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\Big)\Big]+\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{\Delta}(\vec{q}_1+\vec{q}_2))}{\vec{\Delta}^2(\vec{q}_1+\vec{q}_2)^2}\Big[\frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{\Delta}^4}{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^4}\right)+$$

$$
+L\left(-\frac{\vec{q_1}^{\,2}}{\vec{q_2}^{\,2}}\right)-L\left(-\frac{\vec{q_2}^{\,2}}{\vec{q_1}^{\,2}}\right)\Bigg]-\left[4\vec{q_1}^{\,2}\vec{q_2}^{\,2}+\frac{(\vec{q_1}^{\,2}-\vec{q_2}^{\,2})^2}{4}+(2\vec{q_1}^{\,2}\vec{q_2}^{\,2}-3\vec{q_1}^{\,4}-3\vec{q_2}^{\,4})\frac{(2\vec{q_1}^{\,2}\vec{q_2}^{\,2}-(\vec{q_1}\vec{q_2})^2)}{16\vec{q_1}^{\,2}\vec{q_2}^{\,2}}\right]
$$

$$
\times\int_0^\infty \frac{dx}{(\vec{q_1}^{\,2}+x^2\vec{q_2}^{\,2})}\ln\left|\frac{1+x}{1-x}\right|-\vec{q_1}^{\,2}\vec{q_2}^{\,2}\left(1-\frac{(\vec{\Delta}(\vec{q_1}+\vec{q_2}))^2}{\vec{\Delta}^2(\vec{q_1}+\vec{q_2})^2}\right)\left(\int_0^1-\int_1^\infty\right)dz\frac{\ln\left((z\vec{q_1})^2/\vec{q_2}^{\,2}\right)}{(\vec{q_2}-z\vec{q_1})^2}\Bigg\}. \tag{4.2}
$$

Here we passed to the $(D-2)$- dimensional Euclidian vectors and

$$
\bar{g}_\mu^2=\frac{g_\mu^2 N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}},\;\; g_\mu=g\mu^\epsilon,\;\; \psi(z)=\frac{\Gamma'(z)}{\Gamma(z)};\qquad L(z)=\int_0^z\frac{dt}{t}\ln(1-t). \tag{4.3}
$$

Eq.(4.2) takes into account all terms, giving in the physical case $\epsilon\to 0$ non zero contributions at subsequent integration over $\vec{\Delta}$. The terms in this correction, which are singular at $\epsilon\to 0$, cancel with the corresponding terms in the corrections to the BFKL kernel from the RRG vertex and the gluon Regge trajectory, as it was shown in Ref. [9].

Appendix A

For finding the convolutions (3.4) it is convenient to present the tensor $c^{\mu\nu}(k_1,k_2)$ (2.9) in the following form:

$$c^{\mu\nu}(k_1,k_2)=l_1^{\mu\nu}+l_2^{\mu\nu}+l_3^{\mu\nu}, \tag{A.1}$$

where

$$l_1^{\mu\nu}=-\frac{xq_1^2}{\kappa}\Big[\frac{k_2^\mu k_2^\nu}{Z}+\frac{k_1^\mu((1-x)k_1-k_2)^\nu}{(1-x)k_1^2}-\frac{g^{\mu\nu}}{2}\frac{(1-x)}{Z}\Big(\frac{Z}{(1-x)}-\frac{k_1^2}{x}\Big)\Big], \tag{A.2}$$

$$l_2^{\mu\nu}=\frac{1}{\kappa}\Big[-\frac{q_1^\mu\Lambda^\nu}{(1-x)}+\Big(\Delta-\frac{x\Delta^2}{k_1^2}k_1\Big)^\mu q_1^\nu-\frac{2x}{k_1^2}k_1^\mu\Big(\frac{(q_1k_1)}{(1-x)}k_2-(q_1\Delta)k_1\Big)^\nu+(q_1\Lambda)g^{\mu\nu}\Big], \tag{A.3}$$

$$\Lambda=k_1-x\Delta, \tag{A.4}$$

$$l_3^{\mu\nu}=\frac{xk_1^\mu(q_1-k_1)^\nu}{k_1^2}+\frac{(q_1-k_1)^\mu(q_1-k_1)^\nu}{t}-\frac{g^{\mu\nu}}{2}\frac{(1-x)}{t}(q_1^2-2k_1q_1). \tag{A.5}$$

The calculation of each product $l_i^{\mu\nu}l_{j\,\mu\nu}$ is significantly simpler than calculation of the whole convolution $c^{\mu\nu}(k_1,k_2)c_{\mu\nu}(k_1,k_2)$, though still rather tedious. The results are:

$$l_1^{\mu\nu}l_{1\,\mu\nu}=\frac{x^2(q_1^2)^2}{\kappa^2}\Big\{\kappa\Big(\frac{(k_1k_2)}{k_1^2Z}+\frac{(1-x)}{xZ}-\frac{1}{x(1-x)k_1^2}\Big)+$$
$$+\Big(\frac{1+\epsilon}{2}\Big)\Big[\frac{(1-x)}{Z}\kappa\Big(1-2x-\frac{(2\Lambda\Delta)}{\kappa}\Big)\Big]^2\Big\}, \tag{A.6}$$

$$l_2^{\mu\nu}l_{2\,\mu\nu}=\frac{1}{\kappa^2}\Big\{x(1-x)\kappa\Big(-\frac{q_1^2}{(1-x)^2}-\frac{q_1^2\Delta^2}{k_1^2}+\frac{2(q_1k_1)(q_1\Lambda)}{(1-x)k_1^2}\Big)+2(1+\epsilon)(q_1\Lambda)^2\Big\}, \tag{A.7}$$

$$l_3^{\mu\nu}l_{3\,\mu\nu}=\frac{x^2q_1^2}{k_1^2}+\frac{x(1-x)(q_1k_1)(q_1^2-2k_1q_1)}{k_1^2t}+\Big(\frac{1+\epsilon}{2}\Big)\Big(\frac{(1-x)(q_1^2-2k_1q_1)}{t}\Big)^2, \tag{A.8}$$

$$2l_1^{\mu\nu}l_{2\,\mu\nu}=-\frac{xq_1^2}{\kappa^2}\Big\{-2(1+\epsilon)(1-x)\Big(1-2x-\frac{(2\Lambda\Delta)}{\kappa}\Big)\frac{(q_1\Lambda)\kappa}{Z}+\kappa(q_1k_1)$$
$$\times\Big(\frac{x\kappa}{k_1^2Z}+\frac{(1-x)}{Z}+\frac{(1-3x+x^2)}{(1-x)k_1^2}\Big)+\kappa(q_1\Delta)\Big(\frac{(1-2x)}{Z}-\frac{2}{k_1^2}\Big)\Big\}, \tag{A.9}$$

$$2l_1^{\mu\nu}l_{3\,\mu\nu}=-\frac{xq_1^2}{\kappa}\Big\{(1+\epsilon)(1-x)^2\frac{\kappa}{Zt}(q_1^2-2k_1q_1)\Big(1-2x-\frac{(2\Lambda\Delta)}{\kappa}\Big)+\frac{1}{2(1-x)k_1^2t}$$
$$\times\Big(q_1^2(q_2^2+(1-x)(q_1^2-\Delta^2))+2(1-x)\kappa(q_1^2-k_1q_1)\Big)+\frac{q_2^2}{2Zt}\Big(q_2^2-4(1-x)(q_1^2-2k_1q_1)\Big)+$$
$$+\frac{4(1-x)q_1^2-q_2^2}{2x(1-x)t}+\frac{x\Delta^2(2q_1k_2-\Delta^2)}{2k_1^2Z}+\frac{1}{Z}\Big((1-x)(q_1^2-k_1q_1)+\frac{(1-2x)}{2}(q_2^2-\Delta^2)\Big)+$$
$$+\frac{1}{2(1-x)k_1^2}\Big(-2(1-3x+x^2)(q_1k_1)+(1-x)\Delta^2+xq_2^2-2q_1^2\Big)\Big\}, \tag{A.10}$$

$$2l_2^{\mu\nu}l_{3\,\mu\nu}=\frac{1}{\kappa}\Big\{-2(1+\epsilon)(1-x)(q_1^2-2k_1q_1)\frac{(q_1\Lambda)}{t}-x(1-x)(q_1^2-2k_1q_1)\frac{(q_1k_1)\kappa}{k_1^2t}-$$

$$-\frac{x^2q_1^2\kappa}{k_1^2}-\frac{2x(q_1k_1)(q_1\Lambda)}{k_1^2}+\frac{xq_1^2}{(1-x)}+\frac{xq_1^2}{2(1-x)k_1^2t}\times$$

$$\times\Big(2(1-x)(q_1^2-q_1k_1)\kappa+q_1^2(q_2^2+2(1-x)q_2\Delta)\Big)+\frac{q_1^2}{2(1-x)t}\Big(4(1-x)^2q_1^2-(1-2x)^2q_2^2\Big)+$$

$$+\frac{xq_1^2}{2(1-x)k_1^2}\Big(-2q_1k_1-q_1^2+2xq_1\Delta+(1-4x+2x^2)\Delta^2\Big)\Big\}. \qquad \text{(A.11)}$$

After the summation of similar terms we obtain

$$(c^{\mu\nu}(k_1,k_2))^2=\frac{1}{\kappa t}\Big[2xq_1^2q_2^2-\Big(\frac{1+\epsilon}{2}\Big)(1-x)\Big((q_1^2-2xq_1\Delta)^2+(q_1^2)^2\Big)+(1+\epsilon)(1-x)^2q_1^2(q_1^2-2k_1q_1)\Big]+$$

$$+\frac{x(1-x)q_1^2}{\kappa Z}\Big[(1+\epsilon)(1-x)^2q_1^2-\epsilon x(1-x)(\Delta^2-q_2^2)-2(1+\epsilon)x(q_1\Lambda)\Big]+\Big(\frac{x(1-x)q_1^2}{Z}\Big)^2$$

$$\times\Big[\Big(\frac{1+\epsilon}{2}\Big)-(3+2\epsilon)x(1-x)\Big]+\Big(\frac{1+\epsilon}{2}\Big)\frac{(1-x)^2q_1^2}{t^2}\Big[(1-4x)q_1^2-4(q_1(k_1-xq_1))\Big]-$$

$$-\frac{xq_1^2q_2^2}{2(1-x)\kappa k_1^2}+\frac{x^2q_1^2q_2^2}{2k_1^2Z}-\frac{x(q_1^2)^2q_2^2}{2\kappa tk_1^2}-\frac{2xq_1^2}{\kappa Zt}\Big[\Big(\frac{q_2^2}{2}-(1-x)(q_1^2-2k_1q_1)\Big)^2-\Big(\frac{1-\epsilon}{2}\Big)(1-x)^2$$

$$\times(q_1^2-2k_1q_1)^2\Big]+2(1+\epsilon)\Big(\frac{q_1^2-x\Delta^2-2k_1q_2}{\kappa t}\Big)\Big(\frac{(1-x)(k_1q_1)^2}{t}+\frac{(q_1\Lambda)^2}{\kappa}\Big)-2(1+\epsilon)(1-2x)$$

$$\times(x(1-x)q_1^2)^2\frac{(\Lambda\Delta)}{\kappa Z^2}+\Big(\Big(\frac{x(1-x)q_1^2}{\kappa Z}\Big)^2\Delta^\mu\Delta^\nu-\frac{2x(1-x)q_1^2}{\kappa^2Z}\Delta^\mu q_1^\nu\Big)\Big(2(1+\epsilon)\Lambda_\mu\Lambda_\nu-\Lambda^2g_{\mu\nu}\Big). \qquad \text{(A.12)}$$

For the calculation of $\tilde c^{\mu\nu}\tilde c'_{\nu\mu}$, where

$$\tilde c^{\mu\nu}=\Big(g^{\mu\rho}-2\frac{k_1^\mu k_1^\rho}{k_1^2}\Big)c_\rho{}^\nu(k_1,k_2) \qquad \text{(A.13)}$$

and $'$ means the substitution $x\leftrightarrow 1-x,\ k_1\leftrightarrow k_2$, it is convenient to use the following relations:

$$2\tilde c^{\mu\nu}\tilde c'_{\nu\mu}=(\tilde c^{\mu\nu}+\tilde c'^{\nu\mu})\left(\tilde c_{\mu\nu}+\tilde c'_{\nu\mu}\right)-\tilde c^{\mu\nu}\tilde c_{\mu\nu}-(\tilde c^{\mu\nu}\tilde c_{\mu\nu})' \qquad \text{(A.14)}$$

and

$$\tilde c^{\mu\nu}\tilde c_{\mu\nu}=c^{\mu\nu}c_{\mu\nu}. \qquad \text{(A.15)}$$

Therefore, we have

$$2\tilde c^{\mu\nu}\tilde c'_{\nu\mu}=(\tilde c^{\mu\nu}+\tilde c'^{\nu\mu})\left(\tilde c_{\mu\nu}+\tilde c'_{\nu\mu}\right)-c^{\mu\nu}c_{\mu\nu}-(c^{\mu\nu}c_{\mu\nu})'. \qquad \text{(A.16)}$$

Using

$$\tilde c^{\mu\nu}+\tilde c'^{\nu\mu}=b^{\mu\nu}+b'^{\nu\mu},$$

$$b^{\mu\nu}=\frac{q_1^2}{2k_1^2k_2^2}k_1^\mu k_2^\nu-\frac{(1-x)}{xt}k_1^\mu q_1^\nu+\frac{q_1^\mu(q_1-k_1)^\nu}{t}-\frac{q_1^2}{k_1^2t}k_1^\mu(q_1-k_1)^\nu-\frac{g^{\mu\nu}}{2}\frac{(1-x)}{t}(q_1^2-2k_1q_1), \qquad \text{(A.17)}$$

we obtain comparatively easily

$$(\tilde c^{\mu\nu}+\tilde c'^{\nu\mu})\left(\tilde c_{\mu\nu}+\tilde c'_{\nu\mu}\right)=\frac{(q_1^2)^2(q_2^2)^2}{2t\tilde tk_1^2k_2^2}+\frac{q_1^2q_2^2}{2k_1^2k_2^2}-\frac{2q_1^2q_2^2}{t\tilde t}+\frac{xq_1^2q_2^2}{2(1-x)k_1^2\tilde t}+\frac{(1-x)q_1^2q_2^2}{2xk_2^2t}+$$

$$+\left(\frac{1+\epsilon}{2}\right)\left(\frac{(1-x)}{t}(q_1^2-2k_1q_1)+\frac{x}{\tilde{t}}(q_1^2-2k_2q_1)\right)^2. \tag{A.18}$$

Substituting (A.18) in (A.16) and simplifying similar terms we come to the result:

$$2\tilde{c}^{\mu\nu}\tilde{c}'_{\nu\mu}=\Big\{-c^{\mu\nu}c_{\mu\nu}+\frac{q_1^2q_2^2}{2k_1^2k_2^2}+\frac{1}{2t\tilde{t}}\Big[-2q_1^2q_2^2+\left(\frac{1+\epsilon}{2}\right)x(1-x)\Big((\Delta^2-q_2^2)^2+(q_1^2)^2\Big)\Big]+$$

$$+\left(\frac{1+\epsilon}{2}\right)\frac{(1-x)^2q_1^2}{t^2}\Big[(1-4x)q_1^2-4(q_1(k_1-xq_1))\Big]+2(1+\epsilon)(1-x)(q_1^2-x\Delta^2-2k_1q_2)$$

$$\times\frac{(k_1q_1)^2}{\kappa t^2}+\frac{(1-x)(2k_1q_1-q_1^2)q_1^2q_2^2}{2tk_1^2k_2^2}-2(1+\epsilon)(1-x)(q_2^2+x\Delta^2+2k_1q_2)\frac{(k_1q_1)^2}{\kappa t\tilde{t}}+$$

$$+\frac{(q_1^2)^2(q_2^2)^2}{4t\tilde{t}k_1^2k_2^2}\Big\}+\Big\{x\leftrightarrow 1-x,\quad k_1\leftrightarrow k_2\Big\}. \tag{A.19}$$

Now, using Eqs.(A.12) and (A.19), one can easily obtain Eq.(3.6) for the differential distribution.

Appendix B

Here we calculate the transverse momentum integrals appearing in Eq.(3.8). Because the transverse momenta are spacelike we pass to $(D-2)$- dimensional Euclidian vectors in the final expressions. Let us remind, that the expressions for the invariants $\kappa, Z, t, \tilde{t}$ in terms of the integration variables are given by Eqs.(2.10), (3.7) and $\epsilon = (D-4)/2$.

$$I_1 = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{1}{\kappa t} \Big[2xq_1^2 q_2^2 - \left(\frac{1+\epsilon}{2}\right)(1-x)\left((q_1^2 - 2xq_1\Delta)^2 + (q_1^2)^2\right) + (1+\epsilon)(1-x)^2$$

$$\times q_1^2(q_1^2 - 2k_1q_1)\Big] = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}(-x^2(1-x))\left(\frac{x(1-x)\vec{q}_1^{\,2}}{\mu^2}\right)^\epsilon \Big\{ \frac{1}{((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})} \Big[2\vec{q}_1^{\,2}\vec{q}_2^{\,2} +$$

$$+ \frac{(1+\epsilon)}{2}(1-x)\left((x-2)\vec{q}_1^{\,4} - x(\vec{\Delta}^2 - \vec{q}_2^{\,2})^2\right)\Big]\left(\frac{1}{\epsilon} + 2\ln\left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right)\right) -$$

$$- \frac{(1-x)^2}{x}\frac{(2\vec{q}_1\vec{q}_2)\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \ln\left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right)\Big\}. \quad \text{(B.1)}$$

$$I_2 = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} 4(1+\epsilon)(1-x)(q_1^2 - x\Delta^2 - 2k_1q_2)\frac{(k_1q_1)^2}{\kappa t^2} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}(-4x^2(1-x)^2)$$

$$\times \Big\{ \frac{x(\vec{q}_1\vec{\Delta})^2}{((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})}\left(\frac{1}{\epsilon} + 1 + \ln\left(\frac{x((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})^2}{(1-x)\vec{q}_1^{\,2}\mu^2}\right)\right) + \left((\vec{q}_1\vec{q}_2)^2 - \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2}\right)$$

$$\times \left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{x\vec{q}_2^{\,4}}\right)\ln\left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right) + \frac{2(\vec{q}_1\vec{\Delta})(\vec{q}_1\vec{q}_2)}{\vec{q}_2^{\,2}}\ln\left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right) -$$

$$- \frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_2^{\,2}} + \frac{(1-2x)\vec{q}_1^{\,2}}{(1-x)}\Big\}. \quad \text{(B.2)}$$

$$I_3 = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} 2(1+\epsilon)(q_1^2 - x\Delta^2 - 2k_1q_2)\frac{(q_1(k_1 - x\Delta))^2}{\kappa^2 t} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}(2x^2(1-x)^2)$$

$$\times \Big\{ \vec{q}_1^{\,2}\left(\frac{1}{2\epsilon} - \frac{1}{2} + \frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} + \frac{1}{2}\ln\left(\frac{x(1-x)\vec{q}_1^{\,2}}{\mu^2}\right)\right) + \frac{\vec{q}_1^{\,2}((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})}{x\vec{q}_2^{\,2}}\left(\frac{1}{2} - \frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)$$

$$\times \ln\left(\frac{(1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right)\Big\}. \quad \text{(B.3)}$$

$$I_4 = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} (-2(1+\epsilon))(1-x)(q_2^2 + x\Delta^2 + 2k_1q_2)\frac{(k_1q_1)^2}{\kappa t\tilde{t}} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}(x^2(1-x)^2)$$

$$\times \Big\{ \frac{2x(\vec{q}_1\vec{\Delta})^2}{((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})}\left(\frac{1}{\epsilon} + 1 + \ln\left(\frac{x(1-x)\vec{q}_1^{\,2}}{\mu^2}\right)\right) + \left(\frac{4x(\vec{q}_1\vec{\Delta})^2}{((1-x)\vec{q}_1^{\,2} + x\vec{q}_2^{\,2})} - \vec{q}_1^{\,2} +\right.$$

$$+\frac{(2\vec{q}_1\vec{q}_2)}{\vec{q}_2^{\,2}}\left(2\vec{q}_1^{\,2}-(\vec{q}_1\vec{q}_2)\right)+\frac{(1-x)\vec{q}_1^{\,4}}{x\vec{q}_2^{\,2}}\left(\frac{2(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}-1\right)\Bigg)\ln\left(\frac{(1-x)\vec{q}_1^{\,2}+x\vec{q}_2^{\,2}}{(1-x)\vec{q}_1^{\,2}}\right)+\Big(\vec{q}_1^{\,2}\vec{q}_2^{\,2}+$$

$$+4x(1-2x)\vec{q}_1^{\,4}-2(\vec{q}_1\vec{q}_2)^2\left(1+\frac{2x(1-x)\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+4x\vec{q}_1^{\,2}(\vec{q}_1\vec{q}_2)\Big)\frac{1}{\sqrt{\vec{q}_2^{\,2}(\vec{q}_2^{\,2}+4x(1-x)\vec{q}_1^{\,2})}}$$

$$\times\ln\left(\frac{\sqrt{\vec{q}_2^{\,2}+4x(1-x)\vec{q}_1^{\,2}}+\sqrt{\vec{q}_2^{\,2}}}{\sqrt{\vec{q}_2^{\,2}+4x(1-x)\vec{q}_1^{\,2}}-\sqrt{\vec{q}_2^{\,2}}}\right)\Bigg\}. \qquad \text{(B.4)}$$

$$I_5=\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(-2)xq_1^2}{\kappa Zt}\left[\left(\frac{q_2^2}{2}-(1-x)(q_1^2-2k_1q_1)\right)^2-\left(\frac{1-\epsilon}{2}\right)(1-x)^2(q_1^2-2k_1q_1)^2\right]=$$

$$=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{2x\vec{q}_1^{\,2}}{\vec{\Delta}^2}\Bigg\{\frac{1}{((1-x)\vec{q}_1^{\,2}+x\vec{q}_2^{\,2})}\frac{1}{\epsilon}\left(\frac{x(1-x)\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\Bigg[\frac{\vec{q}_2^{\,4}}{4}-(1-x)\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))+$$

$$\frac{(1+\epsilon)}{2}(1-x)^2(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))^2\Bigg]+\Bigg[-\frac{\vec{q}_2^{\,4}}{4}+(1-x)\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))-\frac{(1-x)^2}{2}(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))^2$$

$$-2x(1-x)^3\frac{\vec{\Delta}^2}{\vec{q}_2^{\,2}}\left((\vec{q}_1\vec{q}_2)^2-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\right)-\frac{(1-x)}{\vec{q}_2^{\,2}}\left(x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}-\vec{\Delta}^2)\right)\Big\{(\vec{q}_1\vec{q}_2)\Big(\vec{q}_2^{\,2}-(1-x)$$

$$\times(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))\Big)+\frac{(1-x)}{2\vec{q}_2^{\,2}}\left(x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}-\vec{\Delta}^2)\right)\left(2(\vec{q}_1\vec{q}_2)^2-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\right)\Big\}\Bigg]$$

$$\times\frac{1}{\sqrt{\left(x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}+\vec{\Delta}^2)\right)^2-4(1-x)^2\vec{q}_1^{\,2}\vec{\Delta}^2}}$$

$$\times\ln\left(\frac{x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}+\vec{\Delta}^2)+\sqrt{\left(x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}+\vec{\Delta}^2)\right)^2-4(1-x)^2\vec{q}_1^{\,2}\vec{\Delta}^2}}{x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}+\vec{\Delta}^2)-\sqrt{\left(x\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}+\vec{\Delta}^2)\right)^2-4(1-x)^2\vec{q}_1^{\,2}\vec{\Delta}^2}}\right)+$$

$$+\Bigg[\frac{1}{((1-x)\vec{q}_1^{\,2}+x\vec{q}_2^{\,2})}\Big\{\frac{\vec{q}_2^{\,4}}{4}-(1-x)\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))+\frac{(1-x)^2}{2}(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))^2\Big\}+\frac{(1-x)}{\vec{q}_2^{\,2}}$$

$$\times\Big\{(\vec{q}_1\vec{q}_2)\left(\vec{q}_2^{\,2}-(1-x)(\vec{q}_1^{\,2}-2x(\vec{q}_1\vec{\Delta}))\right)+\frac{(1-x)}{2\vec{q}_2^{\,2}}\left(x\vec{q}_2^{\,2}+(1-x)\vec{q}_1^{\,2}\right)\left(2(\vec{q}_1\vec{q}_2)^2-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\right)\Big\}\Bigg]$$

$$\times\ln\left(\frac{((1-x)\vec{q}_1^{\,2}+x\vec{q}_2^{\,2})^2}{(1-x)^2\vec{q}_1^{\,2}\vec{\Delta}^2}\right)-\frac{(1-x)^3\vec{\Delta}^2}{2\vec{q}_2^{\,4}}\left(2(\vec{q}_1\vec{q}_2)^2-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{\Delta}^2}\right)\Bigg\}. \qquad \text{(B.5)}$$

$$I_6=\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{1}{2t\tilde{t}}\left[-2q_1^2q_2^2+\left(\frac{1+\epsilon}{2}\right)x(1-x)\left((q_1^2)^2+(\Delta^2-q_2^2)^2\right)\right]=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}$$

$$\times\frac{x(1-x)}{\sqrt{\vec{q}_2^{\,2}(\vec{q}_2^{\,2}+4x(1-x)\vec{q}_1^{\,2})}}\left[-2\vec{q}_1^{\,2}\vec{q}_2^{\,2}+\frac{x(1-x)}{2}\left(\vec{q}_1^{\,4}+(\vec{\Delta}^2-\vec{q}_2^{\,2})^2\right)\right]$$

$$\times \ln\left(\frac{\sqrt{\vec{q}_2{}^2+4x(1-x)\vec{q}_1{}^2}+\sqrt{\vec{q}_2{}^2}}{\sqrt{\vec{q}_2{}^2+4x(1-x)\vec{q}_1{}^2}-\sqrt{\vec{q}_2{}^2}}\right). \tag{B.6}$$

$$I_7=\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{x(1-x)q_1^2}{2\kappa Z}\Big[(1+\epsilon)(1-2x(1-x))q_1^2-2\epsilon x(1-x)(\Delta^2-q_2^2)\Big]=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}$$

$$\times\frac{x(1-x)\vec{q}_1{}^2}{2\vec{\Delta}^2}\frac{1}{\epsilon}\left(\frac{x(1-x)\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\Big[-(1+\epsilon)(1-2x(1-x))\vec{q}_1{}^2+2\epsilon x(1-x)(\vec{\Delta}^2-\vec{q}_2{}^2)\Big]. \tag{B.7}$$

$$I_8=\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\left(\frac{x(1-x)q_1^2}{Z}\right)^2\Big[\Big(\frac{1+\epsilon}{2}\Big)-(3+2\epsilon)x(1-x)\Big]=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{x(1-x)\vec{q}_1{}^4}{\vec{\Delta}^2}$$

$$\times\left(\frac{x(1-x)\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\left(\frac{(1+\epsilon)}{2}-(3+2\epsilon)x(1-x)\right). \tag{B.8}$$

$$I_9=\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}(1+\epsilon)(1-4x)\left(\frac{(1-x)q_1^2}{t}\right)^2=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}x(1-x)(1-4x)\vec{q}_1{}^2$$

$$\times(1+\epsilon)\left(\frac{x(1-x)\vec{q}_1{}^2}{\mu^2}\right)^{\epsilon}. \tag{B.9}$$

The integrals I_7, I_8 and I_9 are calculated for arbitrary D.

$$I_{10}=\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(1-x)q_1^2q_2^2(2k_1q_1-q_1^2)}{2tk_1^2k_2^2}=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\mu^{-2\epsilon}\frac{(1-x)\vec{q}_1{}^2\vec{q}_2{}^2}{2}$$

$$\times\Big[-\frac{1}{\epsilon}\frac{(x(1-x)\vec{q}_1{}^2)^{\epsilon}}{(x\vec{q}_2{}^2+(1-x)\vec{\Delta}^2)}+\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\left(\frac{1}{(x\vec{q}_2{}^2+(1-x)\vec{\Delta}^2)^{1-\epsilon}}-\frac{1}{(\vec{\Delta}^2)^{1-\epsilon}}\right)-$$

$$-\frac{2\epsilon(\vec{\Delta}^2)^{\epsilon}}{(x\vec{q}_2{}^2+\vec{\Delta}^2)}\int_{\vec{\Delta}^2/(x\vec{q}_2{}^2+\vec{\Delta}^2)}^{1}\frac{dz}{z}\ln(1-z)\Big]. \tag{B.10}$$

$$I_{11}=\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(-x)(q_1^2)^2q_2^2}{2\kappa tk_1^2}=\frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{x(1-x)\vec{q}_1{}^2\vec{q}_2{}^2}{2}\Big[\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(x^2\vec{\Delta}^2/\mu^2)^{\epsilon}}{x\vec{\Delta}^2}+$$

$$+\frac{(2(\vec{q}_1\vec{\Delta})-\vec{\Delta}^2)}{\vec{\Delta}^2(x\vec{q}_2{}^2+(1-x)\vec{q}_1{}^2)}\left(\frac{1}{\epsilon}+\ln\left(\frac{x((1-x)\vec{q}_1{}^2+x\vec{q}_2{}^2)^2}{(1-x)\vec{q}_1{}^2\mu^2}\right)\right)-\frac{2(\vec{\Delta}(\vec{q}_1-x\vec{\Delta}))}{\vec{\Delta}^2(\vec{q}_1-x\vec{\Delta})^2}$$

$$\times\ln\left(\frac{(1-x)\vec{q}_1{}^2+x\vec{q}_2{}^2}{x(1-x)\vec{\Delta}^2}\right)\Big]. \tag{B.11}$$

$$I_{12} = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(q_1^2)^2 (q_2^2)^2}{4t\tilde{t} k_1^2 k_2^2} = \frac{1}{2}\Bigg[-I_{10} + \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} (1-x) \vec{q}_1^{\,2} \vec{q}_2^{\,2}$$

$$\times \Bigg\{ \frac{\left(\vec{q}_2^{\,2} (\vec{\Delta}(\vec{q}_1 + \vec{q}_2)) - 2x(\vec{q}_1\vec{q}_2)(\vec{q}_2\vec{\Delta}) \right)}{\mathcal{D}(x)\vec{\Delta}^2} \ln\left(\frac{(1-x)\vec{q}_2^{\,2} + x\vec{\Delta}^2}{x\vec{q}_1^{\,2}} \right)$$

$$- \frac{\vec{q}_2^{\,4} \left(\vec{q}_2^{\,2} (\vec{\Delta}(\vec{q}_1 + \vec{q}_2)) - 4x(1-x)(2\vec{q}_1^{\,2}\vec{q}_2^{\,2} - (\vec{q}_1\vec{q}_2)^2) \right)}{\mathcal{D}(x)\mathcal{D}(1-x)\sqrt{\vec{q}_2^{\,2}(\vec{q}_2^{\,2} + 4x(1-x)\vec{q}_1^{\,2})}} \ln\left(\frac{\sqrt{\vec{q}_2^{\,2} + 4x(1-x)\vec{q}_1^{\,2}} + \sqrt{\vec{q}_2^{\,2}}}{\sqrt{\vec{q}_2^{\,2} + 4x(1-x)\vec{q}_1^{\,2}} - \sqrt{\vec{q}_2^{\,2}}} \right) \Bigg\} \Bigg]$$

$$+ \frac{1}{2}\Big[x \leftrightarrow (1-x) \Big], \qquad \mathcal{D}(z) = \frac{\left((\vec{q}_2^{\,2} - 2z\vec{q}_1\vec{q}_2)\vec{q}_1 + \vec{q}_1^{\,2}\vec{q}_2 \right)^2}{\vec{q}_1^{\,2}}. \qquad \text{(B.12)}$$

$$I_{13} = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(-x) q_1^2 q_2^2}{2(1-x)\kappa k_1^2} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{\Delta}^2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left(\frac{x^2\vec{\Delta}^2}{\mu^2} \right)^{\epsilon}. \qquad \text{(B.13)}$$

The last expression is valid for arbitrary D.

$$I_{14} = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{x^2 q_1^2 q_2^2}{2k_1^2 Z} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{\Delta}^2} \left(\frac{\vec{\Delta}^2}{\mu^2} \right)^{\epsilon} x^2 \int_0^1 \frac{dz}{\left(xz(1-xz) \right)^{1-\epsilon}} \approx \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}$$

$$\times \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{\Delta}^2} \left(\frac{\vec{\Delta}^2}{\mu^2} \right)^{\epsilon} x \left[\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} - \frac{(1-x)^{\epsilon}}{\epsilon} + \ln x + \frac{\epsilon}{2} \ln x \ln(x(1-x)^2) - 2\epsilon \int_0^{1-x} \frac{dz}{z} \ln(1-z) \right]. \qquad \text{(B.14)}$$

The first equality here is exact and in the second we performed the expansion in ϵ with the required accuracy.

$$I_{15} = \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{q_1^2 q_2^2}{2k_1^2 k_2^2} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{\Delta}^2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left(\frac{\vec{\Delta}^2}{\mu^2} \right)^{\epsilon}. \qquad \text{(B.15)}$$

This relation is exact.

Appendix C

Here we calculate the integrals appearing in expression (4.1). The first six integrals are calculated exactly without any problems and the result is

$$J_1+...+J_6=\int_{\delta_R}^{1-\delta_R}\frac{dx}{x(1-x)}\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\Big\{(1+\epsilon)(1-x)^2\frac{4(k_1q_1)^2+(1-4x)(q_1^2)^2}{t^2}$$

$$+\frac{x(1-x)q_1^2}{\kappa Z}\Big[2q_2^2+(1+\epsilon)\left(2(1-x)(k_1q_1)-x(1-x)q_1^2-k_2^2\right)-\epsilon x(1-x)(\Delta^2-q_2^2)\Big]$$

$$+\left(\frac{x(1-x)q_1^2}{Z}\right)^2\left[\frac{(1+\epsilon)}{2}-(3+2\epsilon)x(1-x)\right]+\left(\frac{-xq_1^2q_2^2}{2(1-x)\kappa k_1^2}+\frac{x^2q_1^2q_2^2}{2k_1^2Z}+\frac{xq_1^2q_2^2}{k_1^2k_2^2}\right)\Big\}$$

$$=\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{2\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)}\Big[\Big(4\ln(1/\delta_R)+\psi(1)+\psi(1+\epsilon)-2\psi(1+2\epsilon)-\frac{(11+7\epsilon)}{2(1+2\epsilon)(3+2\epsilon)}\Big)$$

$$\times\left(\frac{\vec{\Delta}^2}{\mu^2}\right)^\epsilon\frac{\vec{q}_1{}^2\vec{q}_2{}^2}{\vec{\Delta}^2}-\frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)}\left(\frac{\vec{q}_1{}^2}{\mu^2}\right)^\epsilon\vec{q}_1{}^2\Big]. \tag{C.1}$$

All other integrals don't contain the dependence on δ_R at $\delta_R\to 0$, therefore we will put $\delta_R=0$ below. The next two integrals are not singular at $|\vec{\Delta}|\to 0$. Besides that, using the following substitution of integration variables, corresponding to left-right symmetry [8],

$$\vec{k}_1\leftrightarrow\vec{k}_2,\qquad x\leftrightarrow\frac{x\vec{k}_2^2}{((1-x)\vec{k}_1^2+x\vec{k}_2^2)}, \tag{C.2}$$

we obtain

$$J_8=\int_0^1\frac{dx}{x(1-x)}\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(-x)q_1^2k_2^2\left((1+\epsilon)k_2^2-2q_2^2\right)}{\kappa Zt}=$$

$$\Big[\int_0^1\frac{dx}{x(1-x)}\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(-x)q_2^2\left((1+\epsilon)k_1^2-2q_1^2\right)}{\kappa t}\Big](q_1\leftrightarrow -q_2), \tag{C.3}$$

and therefore J_8 has the same structure as J_7 and it is convenient to consider these two integrals together. After the rather complicated calculation we get

$$J_7+J_8=\int_0^1\frac{dx}{x(1-x)}\int\mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\Big\{\frac{1}{\kappa t}\Big[-2(1-x)q_1^2q_2^2+(1+\epsilon)(1-x)(q_1^2)^2+\frac{(1+\epsilon)}{2}$$

$$\times x(1-x)\left(2(1-x)q_1^2(\Delta^2-q_2^2)-x(q_1^2)^2-x(\Delta^2-q_2^2)^2\right)-2(1+\epsilon)(2-x)(1-x)q_1^2(k_1q_1)$$

$$+2(1+\epsilon)(1-x)\left((k_1q_1)^2+((k_1-x\Delta)q_1)^2\right)+(1+\epsilon)q_1^2k_2^2\Big]-\frac{xq_1^2k_2^2\left((1+\epsilon)k_2^2-2q_2^2\right)}{\kappa Zt}\Big\}=$$

$$\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{2\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)}\frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)}\left(\left(\frac{\vec{q}_1{}^2}{\mu^2}\right)^\epsilon-\left(1+\frac{\epsilon}{2}\right)\left(\frac{\vec{q}_2{}^2}{\mu^2}\right)^\epsilon\right)\vec{q}_1{}^2-\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\Big[\frac{2}{3}\Big(\vec{q}_1{}^2-2$$

$$\times\frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_2{}^2}\Big)-\frac{\vec{q}_1{}^2}{3}\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec{q}_1{}^2}{\mu^2}\right)-\frac{2}{3}\right)+(2\vec{q}_1\vec{\Delta})\left(\frac{2}{3}\ln\left(\frac{\vec{q}_1{}^2}{\vec{q}_2{}^2}\right)+1\right)-2\vec{\Delta}^2+\frac{2}{3}\frac{\vec{q}_1{}^2(6\vec{q}_2{}^2-\vec{\Delta}^2)}{(\vec{q}_1{}^2-\vec{q}_2{}^2)}$$

$$\times \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + \frac{\vec{\Delta}^2\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\left(\vec{q}_1^{\,2}(\vec{q}_1^{\,2}-3\vec{q}_2^{\,2})\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + 2\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\right) + \frac{2}{3(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\Big(\vec{q}_1^{\,2}$$

$$\times(3\vec{q}_2^{\,2}-\vec{q}_1^{\,2})-(\vec{q}_1\vec{q}_2)(3\vec{q}_1^{\,2}+\vec{q}_2^{\,2})+2\vec{q}_1^{\,2}\frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_2^{\,2}}\Big)\left(2\vec{q}_2^{\,4}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - 2\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2}) + (\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2\right)\Bigg]. \tag{C.4}$$

Using the change of integration variables (C.2) one can get

$$J_{10} = J_9(q_1 \leftrightarrow -q_2), \tag{C.5}$$

and it is enough to calculate only one of these integrals. For the sum of J_9 and J_{10} we obtain

$$J_9 + J_{10} = \int_0^1 \frac{dx}{x(1-x)}\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\Big\{-\frac{xq_1^2(q_2^2)^2}{2\kappa Zt} - \frac{xq_2^2(q_1^2)^2}{2\kappa t k_1^2}\Big\}$$
$$= \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\frac{2}{\epsilon^2}\left(\frac{\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^2}. \tag{C.6}$$

$$J_{11} = \int_0^1 \frac{dx}{x(1-x)}\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(1-x)(2(k_1q_1)-q_1^2)q_1^2q_2^2}{2tk_1^2k_2^2} = -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}$$
$$\times\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^4}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+4\epsilon\psi''(1)\right]\left(\frac{\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^2}. \tag{C.7}$$

$$J_{12} = \int_0^1 \frac{dx}{x(1-x)}\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{1}{2t\tilde{t}}\Big[-2q_1^2q_2^2 + \frac{(1+\epsilon)}{2}x(1-x)\left((\Delta^2-q_2^2)^2+(q_1^2)^2\right)$$
$$-4(1+\epsilon)x(1-x)(k_1q_1)^2\Big] = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\Big[\frac{\vec{q}_1^{\,2}}{3}\left(\frac{1}{\epsilon}+\ln\left(\frac{\vec{q}_1^{\,2}}{\mu^2}\right)-\frac{1}{6}\right)+\frac{\vec{q}_2^{\,2}}{2}\Big] - \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}$$
$$\times\Big\{\frac{3}{8}\frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-(\vec{q}_1\vec{q}_2)^2)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}) + \frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3(\vec{q}_1\vec{q}_2)^2)}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + \Big[4\vec{q}_1^{\,2}\vec{q}_2^{\,2}+$$
$$\frac{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}{4} + \left(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3\vec{q}_1^{\,4}-3\vec{q}_2^{\,4}\right)\frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-(\vec{q}_1\vec{q}_2)^2)}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\Big]\int_0^\infty \frac{dx}{(\vec{q}_1^{\,2}+x^2\vec{q}_2^{\,2})}\ln\left|\frac{1+x}{1-x}\right|\Big\}. \tag{C.8}$$

$$J_{13} = \int_0^1 \frac{dx}{x(1-x)}\int \mu^{-2\epsilon}\frac{d^{D-2}k_1}{(2\pi)^{D-1}}\frac{(q_1^2)^2(q_2^2)^2}{4t\tilde{t}k_1^2k_2^2} = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\Big\{\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^2}\left(\frac{\vec{\Delta}^2}{\mu^2}\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\Big[\frac{1}{\epsilon^2}$$
$$+\frac{1}{\epsilon}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{\Delta}^4}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+4\epsilon\psi''(1)\Big]+\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{\Delta}(\vec{q}_1+\vec{q}_2))}{\vec{\Delta}^2(\vec{q}_1+\vec{q}_2)^2}\Big[\frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{\Delta}^4}{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^4}\right)$$

$$+ L\left(-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - L\left(-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\Bigg] - \vec{q}_1^{\,2}\vec{q}_2^{\,2}\left(1 - \frac{(\vec{\Delta}(\vec{q}_1+\vec{q}_2))^2}{\vec{\Delta}^2(\vec{q}_1+\vec{q}_2)^2}\right)\left(\int_0^1 - \int_1^\infty\right) dz \frac{\ln\left((z\vec{q}_1)^2/\vec{q}_2^{\,2}\right)}{(\vec{q}_2 - z\vec{q}_1)^2}\Bigg\}, \tag{C.9}$$

where

$$L(x) = \int_0^x \frac{dt}{t}\ln(1-t). \tag{C.10}$$

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