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Four-quark Operators Relevant to B Meson Lifetimes from QCD Sum Rules

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Abstract

At the order of $1/m_b^3$, the B meson lifetimes are controlled by the hadronic matrix elements of some four-quark operators. The nonfactorizable magnitudes of these four-quark operator matrix elements are analyzed by QCD sum rules in the framework of heavy quark effective theory. The vacuum saturation is found to be quite accurate at hadronic scale. The predicted lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ still contradicts experiments.

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Heavy hadron lifetimes provide us with testing ground to the Standard Model, especially to QCD in some aspects [1,2], because they can be systematically calculated within the framework of heavy quark expansion. Recent experimental results on the lifetime ratio of Λ_b baryon and B meson [3] showed some deviation from the theoretical expectation. This has drawn a lot theoretical attentions [1, 2, 4-10]. The current experimental values for the lifetime ratios which we are interested in are [3]

$$\begin{aligned} \frac{\tau(B^-)}{\tau(B^0)} &= 1.06 \pm 0.04 \ , \\ \frac{\tau(\Lambda_b)}{\tau(B^0)} &= 0.79 \pm 0.06 \ . \end{aligned} \tag{1}$$

Theoretically, if we do not assume the failure of the local duality assumption, the heavy hadron lifetime differences appear, at most, at the orders of $1/m_Q^2$ [4]. However, it has been found that the $O(\frac{1}{m_b^2})$ contribution is not enough for the explanation of above heavy baryon and heavy meson lifetime difference. Recently, in a model-independent analysis, it is pointed out that $1/m_b^3$ corrections are potentially important [6]. With the parameterization of the hadronic matrix elements of four-quark operators which appear in the hadron lifetimes at the order of $1/m_b^3$,

$$\begin{aligned} \langle \bar{B}|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle &\equiv B_1 f_B^2 m_B^2 \ , \\ \langle \bar{B}|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|\bar{B}\rangle &\equiv B_2 f_B^2 m_B^2 \ , \\ \langle \bar{B}|\bar{b}\gamma_\mu(1-\gamma_5)t_a q\bar{q}\gamma^\mu(1-\gamma_5)t_a b|\bar{B}\rangle &\equiv \epsilon_1 f_B^2 m_B^2 \ , \\ \langle \bar{B}|\bar{b}(1-\gamma_5)t_a q\bar{q}(1+\gamma_5)t_a b|\bar{B}\rangle &\equiv \epsilon_2 f_B^2 m_B^2 \ , \end{aligned} \tag{2}$$

and

$$\begin{aligned} \frac{1}{2m_{\Lambda_b}}\langle \Lambda_b|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)b|\Lambda_b\rangle &\equiv -\frac{f_B^2 m_B}{12} r \ , \\ \frac{1}{2m_{\Lambda_b}}\langle \Lambda_b|\bar{b}(1-\gamma_5)q\bar{q}(1+\gamma_5)b|\Lambda_b\rangle &\equiv -\tilde{B}\frac{f_B^2 m_B}{24} r \ , \end{aligned} \tag{3}$$

the hadron lifetime ratios are predicted to be

$$\begin{aligned} \frac{\tau(B^-)}{\tau(B^0)} &= 1 + 0.03 B_1 - 0.71\epsilon_1 + 0.20\epsilon_2 + O(\tfrac{1}{m_b^4}) \ , \\ \frac{\tau(\Lambda_b)}{\tau(B^0)} &= 0.98 - 0.17\epsilon_1 + 0.20\epsilon_2 - (0.013 + 0.022\tilde{B})r + O(\tfrac{1}{m_b^4}) \ . \end{aligned} \tag{4}$$

The parameters B_i, ϵ_i $(i = 1, 2)$, r and $\tilde{B}$ should be calculated by some nonperturbative QCD method. In above equation, the energy scale for them (including f_B) is at m_b.

The QCD sum rule [11], which is regarded as a nonperturbative method rooted in QCD itself, has been used successfully to calculate the properties of various hadrons. In Ref. [7], in the framework of heavy quark effective theory (HQET), the baryonic parameters r and $\tilde{B}$ have been calculated by QCD sum rule, $r \sim 0.1 - 0.3$, $\tilde{B} \simeq 1$. For a complete analysis, the mesonic parameters B_i and ϵ_i should be also calculated from QCD sum rule. If they are taken as $B_i = 1$ and $\epsilon_i = 0$, which are obtained from vacuum saturation approximation, the measured lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ cannot be explained [7]. There are some argument, however, that the vacuum saturation maybe a poor approximation [8]. Especially from a naive large N_c analysis, ϵ_i's are about $1/N_c \sim 0.3$ [6]. We will explore the validity of the vacuum saturation approximation in detail from QCD sum rules in the framework of HQET.

Let us first consider the parameters B_i. We construct the following three-point Green's function,

$$\Gamma^O(\omega, \omega') = i^2 \int dx dy e^{ik'\cdot x - ik\cdot y} \langle 0|\mathcal{T}[\bar{q}(x)\gamma^\mu\gamma_5 h_v^{(b)}(x)] O(0) [\bar{q}(y)\gamma_\mu\gamma_5 h_v^{(b)}(y)]^\dagger|0\rangle \ , \quad (5)$$

where $\omega = 2v \cdot k$, $\omega' = 2v \cdot k'$; $h_v^{(b)}$ is the b-quark field in the HQET with velocity v. And O denotes the color-singlet operators given in Eq. (2),

$$O = \bar{b}\Gamma_1 q \bar{q}\Gamma_2 b \ , \quad (6)$$

with $\Gamma_1 = \Gamma_2 = \gamma^\mu(1 - \gamma_5)$ for B_1 and $\Gamma_1 = 1 - \gamma_5$, $\Gamma_2 = 1 + \gamma_5$ for B_2. In terms of the hadronic expression, the parameter B_i appears in the ground state contribution of $\Gamma^O(\omega, \omega')$,

$$\Gamma^O(\omega, \omega') = B_i \frac{f_B^4 m_B^2}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \text{resonances} \ , \quad (7)$$

where $\bar{\Lambda}$ is the heavy meson mass defined in the HQET. The resonance contribution will be simulated by the perturbative QCD contribution above some threshold energy due to the local duality assumption. On the other hand, this Green's function $\Gamma^O(\omega,\omega')$ will be calculated in terms of quarks and gluons, that is to say, by the operator product expansion method of QCD. The essential feature of the QCD sum rule is that in the QCD calculation, the vacuum condensates of quarks and gluons have to be included. Practically the calculation is performed at 1 GeV scale or so, only a few terms of the condensates with lowest dimensions are important. Note that this calculation will be reinforced by the Borel transformation, and its consistency should be checked through the finding of the so-called sum rule window.

The calculation of $\Gamma^O(\omega,\omega')$ in HQET is straightforward. The fixed point gauge [12] is used. The dominant nonvanishing Feynman diagrams are shown in Fig. 1 where the double lines denote the heavy quark. However, all these diagrams are factorizable, namely they do not contribute any deviation from vacuum saturation. And this is true even when the α_s radiative corrections to these diagrams are included. Note that there is no nonfactorizable gluon condensation contribution simply because O is a color-singlet operator. Generally, the nonfactorizable diagrams for a color-singlet four-quark operator are listed in Fig. 2. However, all of them are in fact zero for the operator O. Figs. 2(a-e) seem the leading contribution diagrams. But it is easy to see that they are zero due to the special structure of Γ_1 and Γ_2 in O (Eq. (6)). The four-gluon condensation Fig. 2(f) also gives vanishing contribution. This fact can be easily observed in the fixed point gauge. Note in this gauge, the heavy quark in Figs. 1 and 2 are free from interaction. Therefore, we conclude that the vacuum saturation is a very good approximation at hadronic scale for color-singlet operators,

$$B_i = 1 \qquad (i = 1\ ,\ 2)\ . \tag{8}$$

The parameters ϵ_i (i=1, 2), as we will see in the following analysis, also do not significantly deviate the expectation of the vacuum saturation at the hadronic scale. The procedure is similar to that for parameters B_i. The three-point Green's function in this case is constructed to be

$$\Gamma^T(\omega,\omega') = i^2 \int dxdy e^{ik'\cdot x - ik\cdot y}\langle 0|\mathcal{T}[\bar{q}(x)\gamma^\mu\gamma_5 h_v^{(b)}(x)]T(0)[\bar{q}(y)\gamma_\mu\gamma_5 h_v^{(b)}(y)]^\dagger|0\rangle \ , \tag{9}$$

where T is the color-octet operators given in Eq. (2),

$$T = \bar{b}\Gamma_1 t_a q\bar{q}\Gamma_2 t_a b \ . \tag{10}$$

In the hadronic language, the parameter ϵ_i appears in the ground state contribution of $\Gamma^T(\omega,\omega')$,

$$\Gamma^T(\omega,\omega') = \epsilon_i \frac{f_B^4 m_B^2}{(2\bar{\Lambda}-\omega)(2\bar{\Lambda}-\omega')} + \text{resonances} \ . \tag{11}$$

In the calculation of $\Gamma^T(\omega,\omega')$, the dominant diagrams are found to be those given in Fig. 3. The four-quark condensate $\langle\bar{q}q\rangle^2$ diagram vanishes. We have neglected the perturbative diagrams which are three-loop diagrams of order α_s. That means the QCD radiative corrections to ϵ_i are not included. From general experience of QCD sum rule method, the condensate diagram plays more dominant role than the corresponding perturbative one. This neglect is expected viable.

The sum rules for the parameters ϵ_i are

$$\begin{aligned} \epsilon_1 &= \frac{1}{32\pi^2 f_B^4 m_B^2} m_0^2 \langle\bar{q}q\rangle \tilde{T} e^{4\bar{\Lambda}/\tilde{T}} \ , \\ \epsilon_2 &= \frac{-1}{16\pi^2 f_B^4 m_B^2}\left(\frac{\langle\alpha_s GG\rangle}{4\pi}\tilde{T} + m_0^2\langle\bar{q}q\rangle\right)\tilde{T} e^{4\bar{\Lambda}/\tilde{T}} \ , \end{aligned} \tag{12}$$

where $\tilde{T}$ is the Borel parameter. Note that there is no gluon condensate contribution to ϵ_1. Numerically we use the following values of the condensates,

$$\begin{aligned} \langle\bar{q}q\rangle &\simeq -(0.23\ \text{GeV})^3 \ , \\ \langle\alpha_s GG\rangle &\simeq 0.04\ \text{GeV}^4 \ , \\ \langle g\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle &\equiv m_0^2\langle\bar{q}q\rangle \ , \qquad m_0^2 \simeq 0.8\ \text{GeV}^2 \ . \end{aligned} \tag{13}$$

For consistence, the HQET sum rule of the decay constant f_B derived in Ref. [13] is used,

$$f_B^2 e^{-2\bar{\Lambda}/\tilde{T}} = \frac{3}{8\pi^2}\int_0^{\omega_c} d\nu \nu^2 e^{-\nu/\tilde{T}} - \langle\bar{q}q\rangle\left(1 - \frac{m_0^2}{4\tilde{T}}\right) \ , \tag{14}$$

where ω_c is the continuum threshold, which was determined as $\omega_c \simeq 2.0 \pm 0.3$ GeV. The Borel parameter $\tilde{T}$ in Eq. (12) should be the same as that of f_B, $0.6 \leq \tilde{T} \leq 1.0$ GeV. This point can be obviously seen from the sum rules for B_i if it is written down explicitly. In that sum rules, if all the non-factorizable contributions are neglected, then the resulting B_i is unity only if the Borel parameter is the same as that of f_B. There is no $\bar{\Lambda}$ dependences for ϵ_i's, because they are canceled actually. The numerical sum rule results for ϵ_1 and ϵ_2 are given in Fig. 4. From the figures, we see that ϵ_i's do not have perfect stability in the window of $\tilde{T}$. This is because we have not included the perturbative diagram. Finally the results for ϵ_1 and ϵ_2 are obtained as

$$
\begin{aligned}
\epsilon_1 &\simeq -(1.5 \pm 0.8) \times 10^{-3} \ , \\
\epsilon_2 &\simeq (2.3 \pm 1.4) \times 10^{-3} \ .
\end{aligned}
\tag{15}
$$

In spite of the large uncertainties in above equation, the vacuum saturation approximation for the color-octet matrix elements is nevertheless quite accurate, $\epsilon_i \sim O(10^{-3})$.

It should be noted that the vacuum saturation we have proved above is valid at some hadronic scale, other than the scale of m_b. Because we have been working in the framework of HQET, in which the natural scale is $\mu \ll m_b$. Information on parameters B_i and ϵ_i at scale m_b is necessary in the analysis for the hadron lifetimes by using Eq. (4). In Ref. [6], the renormalization group evolution of the four-quark operators was given. There is a notation difference between this paper and Ref. [6]. The decay constant f_B in this paper has been taken value at hadronic scale in the analysis, whereas that in Ref. [6] at scale of m_b. Taking this difference into consideration, we have the following evolution relations,

$$
\begin{aligned}
B_i(m_b) &\simeq B_i(\mu) - 0.24\epsilon_i(\mu) \ , \\
\epsilon_i(m_b) &\simeq -0.05 B_i(\mu) + 0.72\epsilon_i(\mu) \ .
\end{aligned}
\tag{16}
$$

That is

$$
\begin{aligned}
B_1(m_b) &\simeq B_2(m_b) \simeq 1 + O(10^{-4}) \ , \\
\epsilon_1(m_b) &\simeq \epsilon_2(m_b) \simeq -0.05 + O(10^{-3}) \ .
\end{aligned}
\tag{17}
$$

We see that the vacuum saturation approximation is also good at scale of m_b.

The b-hadron lifetime ratios are therefore predicted by

$$\frac{\tau(B^-)}{\tau(B^0)} \simeq 1.06 \qquad \text{and} \qquad \frac{\tau(\Lambda_b)}{\tau(B^0)} \simeq 0.96 \ . \tag{18}$$

In obtaining above numbers, the QCD sum rule results for the baryonic parameters r and $\tilde{B}$ [7] have been also taken values at scale m_b. From Eq. (18) we see that, while $\tau(B^-)/\tau(B^0)$ is agree with measurement, the QCD sum rule result for $\tau(\Lambda_b)/\tau(B^0)$, after including $1/m_b^3$ corrections, still contradicts the experimental measurement.

In summary, the nonfactorizable contributions of the hadronic matrix elements of four-quark operators relevant to the B meson lifetimes have been calculated by QCD sum rules in the framework of HQET. The vacuum saturation is justified at a quite accurate level at hadronic scale. The discrepancy between theory and experiment on $\tau(\Lambda_b)/\tau(B^0)$ has not been solved. It is unlikely that higher $1/m_b$ corrections will give the solution. If the experimental data of $\tau(\Lambda_b)/\tau(B^0)$ is further confirmed in the future, that may imply either the failure of local duality assumption or some new physics.

Acknowledgments

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References

[1] M. Neubert, Int. J. Mod. Phys. A11 (1996) 4173.

[2] I.I. Bigi, UND-HEP-95-BIG02, hep-ph/9508408; UND-HEP-96-BIG06, hep-ph/9612293.

[3] J. Alcaraz et al. (B Lifetime Group), http://wwwcn.cern.ch/~ claires/lepblife.html.

[4] I.I. Bigi et al., in B Decays, ed. S. Stone, 2nd ed. (World Scientific, Singapore, 1994), and references therein.

[5] I.I. Bigi and N.G. Uraltsev, Z. Phys. C62 (1994) 623; Nucl. Phys. B423 (1994) 33.

[6] M. Neubert and C.T. Sachrajda, Nucl. Phys. B483 (1997) 339.

[7] P. Colangelo and F. De Fazio, Phys. Lett. B387 (1996) 371.

[8] V. Chernyak, Nucl. Phys. B457 (1995) 96.

[9] N.G. Uraltsev, Phys. Lett. B376 (1996) 303;
J.L. Rosner, Phys. Lett. B379 (1996) 267;
G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, Phys. Lett. B382 (1996) 409;
H.Y. Cheng, IP-ASTP-02-97, hep-ph/9704260;
B. Guberina and B. Melić, IRB-TH 1/97, hep-ph/9704445;
H.J. Lipkin and P.J. O'Donnell, TAUP 2424-97, WIS-97/12/MAY-PH, UTPT-97-11, hep-ph/9705363.

[10] For earlier discussions, see B. Guberina, S. Nussinov, R. Peccei and R. Rückl, Phys. Lett. B89 (1979) 111;
N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B248 (1984) 261;
B. Guberina, R. Rückl and J. Trampetic, Z. Phys. C33 (1986) 297;
M.A. Shifman and M. Voloshin, Yad. Fiz. 41 (1985) 463; JETP 64 (1986) 698.

[11] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 488.

[12] For a review, see V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Fortschr. Phys. 32 (1984) 585.

[13] M. Neubert, Phys. Rev. D45 (1992) 2451; Phys. Rep. 245 (1994) 259.

Figure captions

Fig. 1. Dominant nonvanishing Feynman diagrams for $\Gamma^O(\omega, \omega')$.

Fig. 2. Nonfactorizable diagrams for a general color-singlet four-quark operator.

Fig. 3. Condensate contribution to $\Gamma^T(\omega, \omega')$.

Fig. 4. Sum rules for ϵ_1 (a) and ϵ_2 (b). The sum rule window is $\tilde{T} = 0.6 - 1.0$ GeV.

Figures

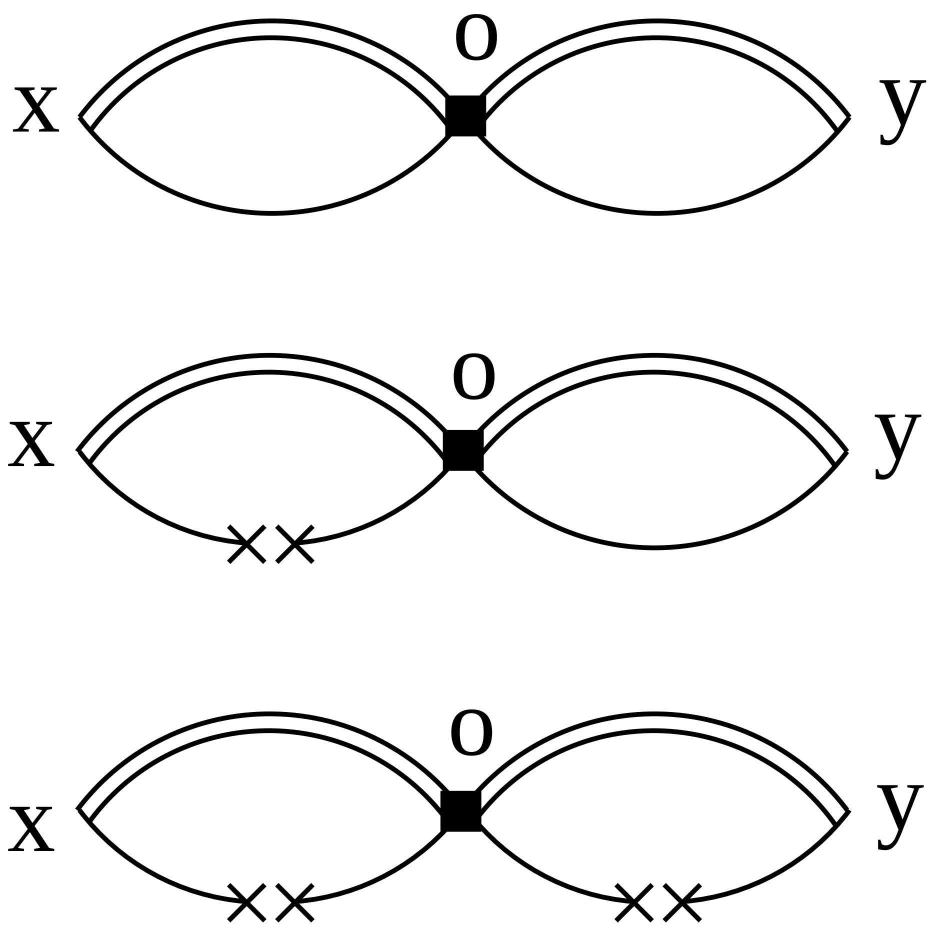


Fig. 1

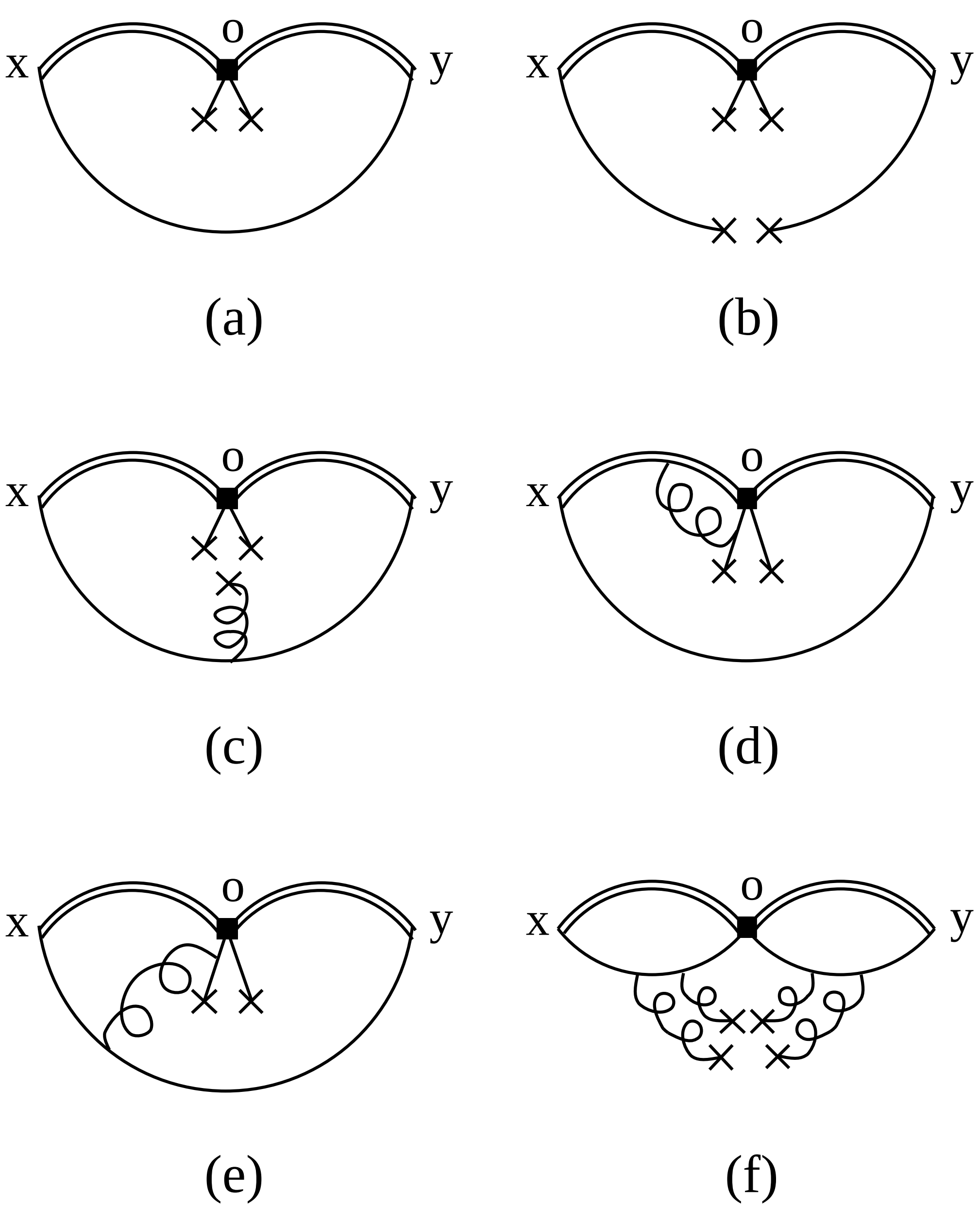


Fig. 2

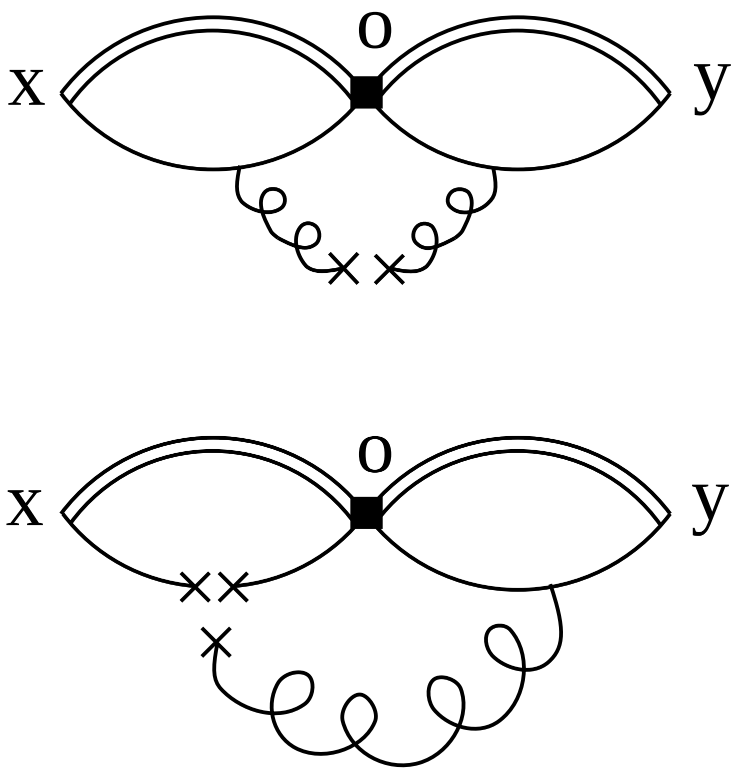
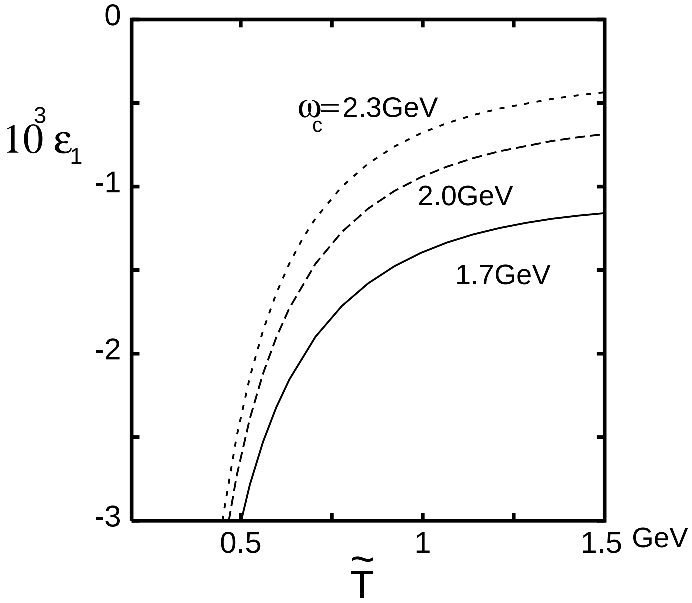


Fig. 3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-2
-3
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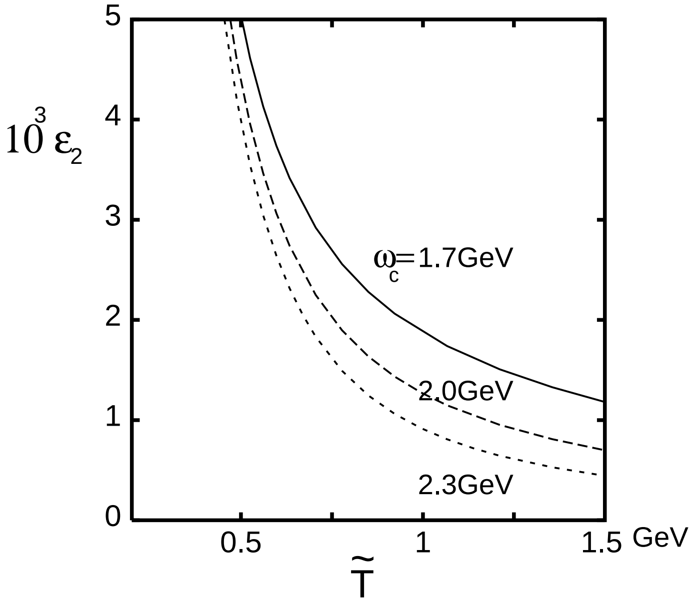
1.7GeV
0.5
1
1.5
GeV
$\tilde{T}$

Fig. 4(a)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GeV
$\tilde{T}$

Fig. 4(b)