FRAGMENTATION FUNCTIONS FOR BARYONS IN A QUARK-DIQUARK MODEL*

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Abstract

A perturbative QCD calculation of heavy flavor quark fragmentation into heavy flavor baryons is developed along the lines of corresponding heavy meson models. The non-perturbative formation of the baryon is accomplished by implementing the quark-diquark model of the baryons. Diquark color form factors are used to enable the integration over the virtual heavy quark momentum. The resulting spin independent functions for charmed and bottom quarks to fragment into charmed and bottom baryons with spin 1/2 and 3/2 are compared with recent data. Predictions are made for the spin dependent fragmentation functions as well, particularly for the functions $\hat{g}_1$ and $\hat{h}_1$ in the case of spin 1/2 baryons.

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I. INTRODUCTION

Quarks produced in high energy processes materialize by evolving into jets of hadrons. The particular hadronic fragments and their kinematic dependences are of considerable interest. This fragmentation process reveals the features of the non-perturbative regime of QCD. The inclusive process of quark fragmentation into a single observed hadron along with any number of unobserved accompanying particles is described by a set of fragmentation functions. These fragmentation functions, defined in terms of appropriate light cone variables, kinematic variables and invariants, are probability distributions. These functions have received considerable attention in recent years. While experimental information beyond the pion distribution (presumably from light quarks) has been slow in accumulating, theoretical interest has been growing. The particular functional form for heavy flavored quarks to fragment into heavy flavored hadrons is of special interest.

Experimentally it is possible to identify heavy flavored jets by the production and characteristic flavor changing weak decays of the hadrons. Theoretically this situation has been studied using Operator Product Expansion techniques, light cone quantization, QCD perturbation theory, and Heavy Quark Effective Theory, among other methods. Some of these methods yield general properties that reflect the overall structure of QCD, as it is currently understood. Other approaches take particular models of the low energy behavior expected from QCD, but in regions that are not perturbatively calculable. The particulars of the various approaches are near the point of being tested against experiment. One general feature is known - the peak of the hadron distribution moves toward higher momenta as the quark mass increases. This feature is a result of the kinematics implicit in most models of the non-perturbative process, and is incorporated in the phenomenological Peterson function [1] that is used by experimenters to fit the sparse data on heavy quark fragmentation [2].

It is more difficult to test the spin dependences of the fragmentation processes experimentally. Yet these dependences are very important to know. They reflect the details of the primarily non-perturbative mechanism by which parton polarization is passed on to the hadrons. The spin-dependent fragmentation involves the reverse of the process by which the nucleon spin is shared by its partons (the"spin crisis"), and may reveal a similarly mysterious decoupling of valence quark spin and hadron spin for some regions of kinematics.

For the fragmentation of a heavy flavor quark into "doubly heavy" mesons, perturbative QCD may provide a starting point to a theoretical determination of the fragmentation functions. As examples, the fragmentation of a c-quark into the J/Ψ or the b-quark into the B_c, were calculated several years ago [3,4]. The reasoning follows the observation that a heavy quark or diquark pair must be produced to form the final hadron. At least one member of the pair will be nearly on-shell because the heavy flavor hadrons have small binding energies relative to their masses. So, the gluon producing the heavy pair must carry large squared time-like 4-momentum, k^2. This gluon will be shaken off by the virtual fragmenting quark and the relevent coupling will be of order $\alpha_s(k^2)$, which will be small. Hence perturbative QCD will be applicable. Non-perturbative effects will be incorporated into the binding of the initial quark with the pair produced heavy quark or diquark. If this is the case, the fragmentation functions are calculable, at an appropriate scale. The parton shower that accompanies the jet is a result of QCD radiative corrections, which can be obtained from the Renormalization Group or the Altarelli-Parisi equations.

Such calculations have been performed and scrutinized. It has been shown that in the heavy quark limit (i.e. the mass goes to infinity) the functions have the form expected from more general considerations [5]. This corresponds to the heavy meson taking all of the heavy quark's momentum; the distribution becomes a delta function at $z = 1$. The $1/m_Q$ corrections are calculated also. In any case, this approach can predict the spin-dependent fragmentation functions along with their momentum and mass dependences. In the heavy mass limit, of course, the spin of the heavy quark is conserved, so the spin dependence is simple. What is of phenomenological interest is the next order correction, at least, since that has non-trivial spin dependence.

The spin dependences of fragmentation are most readily studied experimentally by observing baryons rather than mesons. This is true for the production of hyperons or heavy hyperons (Λ_c, Λ_b, etc.), wherein the weak, parity violating decays provide polarization analyses [6]. To consider fragmentation into baryons in this perturbative scheme, the three quark system has to be confronted. A simple alternative is to consider the baryons as quark-diquark bound states [7], and to use the same perturbative method as for the mesons. In order for the perturbative calculation to be useful the creation of a heavy pair of quarks or diquarks must be an intermediate step. Ideally then, doubly heavy baryon fragmentation would be an appropriate testing ground for this scheme. Such data is sparse, however.

To begin to see the structure it will be worthwhile to stretch the region of applicability to the "singly" heavy baryons. We have been carrying out this program to see the expected spin and kinematic dependences, with the hope of providing an experimentally testable model [8]. A similar approach has been developed independently by Martynenko and Saleev [9], but with significantly different assumptions about how to represent the diquark structure. Of immediate interest is the question of whether the baryon fragmentation functions have the same kinematic dependence as the meson case. In general the answer is no in this model. Secondly, does this approach give the right magnitude for fragmentation into heavy flavor baryons? With our careful specification of the diquark chromodynamic form factors (unlike Martynenko and Saleev) the answer is yes. The spin dependent fragmentation is interestingly distinct from the naive heavy quark limit in detail. The calculations and results will be presented below, along with a comparison with some recent data.

II. PERTURBATIVE CALCULATION

The first calculations of the fragmentation functions in the perturbative scheme were applied to some of the inclusive heavy flavor meson decays of the Z^0, as produced at LEP [3,4]. The partial width for the inclusive decay process $Z^0 \to H + X$ can be written in general for any hadron H as

$$d\Gamma(Z^0 \to H(E) + X) = \sum_i \int_0^1 dz\, d\hat{\Gamma}(Z^0 \to i(E/z) + X, \mu)\, D_{i\to H}(z, \mu), \tag{1}$$

where H is the hadron of energy E and longitudinal momentum fraction z relative to the parton i, while μ is the arbitrary scale whose value will be chosen to avoid large logarithms. The quark and the hadron can carry spin labels as well, and appropriate spin-dependent fragmentation functions will be included, as we will show later. The fragmentation function $D_{i\to H}(z, \mu)$ enters here in a factorized form (that can be maintained through the evolution equations). Upon obtaining the $D_{i\to H}(z,\mu)$ in the model to be described, its evolution to observable scales is developed through summing leading logarithms via evolution equations.

Now, consider the final state with one heavy flavor meson, say the B_c for definiteness. We will soon replace this process by one involving a heavy flavor baryon. To leading order the B_c meson arises from the production of a pair of b-quarks, in which one of the quarks fragments into the meson. As Fig. 1 illustrates, with $Q=b, Q'=c, \bar{Q}'=\bar{c}$, the perturbative contribution involves the virtual b-quark radiating a hard gluon (We work in axial gauge, so that there is no contribution from the opposite quark). The hard gluon produces a heavy flavor pair of c-quarks. The gluon must have energy at least twice the charm mass, since the c and $\bar{c}$ are both on or near their mass shell. So the coupling $\alpha_s(k^2)$ is small, justifying the perturbative approach.

For nearly matching 4-velocities ($v=p/m$) the b and $\bar{c}$ form the B_c meson bound state at roughly the same 4-velocity, with amplitude given by various projection operators multiplying the Bethe-Salpeter wave function $\chi(p,q_r)$ (q_r is the $p(b)-p(\bar{c})$ relative momentum while p is their sum). Since the doubly heavy mesons are weakly bound objects (the sum of constituents' masses are near the bound state mass), the wave function is expected to dampen non-zero relative 3-momenta $\mathbf{q_r}$ in the hadron rest frame [10], so that the b and $\bar{c}$ 4-velocities are fixed at p/M. Then the integration over the relative momentum of the two heavy quarks in the full decay probability can be replaced by the squared amplitude evaluated at equal 4-velocities (for the two constituents and the hadron). The remaining integration over $\mathbf{q_r}$ applies to $|\chi(p,q_r)|^2$ which yields the square of the wave function at the origin in the hadron rest frame. That wave function is known from non-relativistic quark models for the heavy-heavy meson system, or, more directly, from the meson-to-vacuum decay constant.

The same procedure can be applied directly to the baryons, if the quark-diquark model of the baryons is used. The hard gluon in the process must produce a diquark–anti-diquark pair, $D-\bar{D}$, and the diquark (color anti-triplet) combines with the heavy flavor quark Q to form the baryon B_Q. The relevent wave function will be calculated from a fairly successful quark-diquark model [7].

Note that an alternative scenario has been proposed by Falk, *et al.*, in which the heavy quark fragments into a heavy diquark first, and then the diquark dresses itself to form the baryon [11] with probability of one for the latter. This leads to very different results, as pointed out in Ref. [9]. This scenario will not be used here, since the processes we are studying involve diquarks that do not necessarily carry the heavy flavor of the quark. The latter authors [9,12] have performed calculations that are similar in spirit to part of the procedure we follow below, although not emphasizing the spin dependent structure functions that we calculate below.

The tree level amplitude for Fig. 1, A_1, can be evaluated explicitly from perturbation theory. The decay rate for unpolarized $Z^0\to B_c+\bar{c}+b$ or $B_Q+\bar{Q}+D$, each an exclusive channel, can be written generically as

$$\Gamma_1 \;=\; \frac{1}{2M_Z}\int[d\bar{q}][dp][dp']\,(2\pi)^4\delta^4(Z-\bar{q}-p-p')\frac{1}{3}\sum|A_1|^2, \tag{2}$$

where $\bar{q}$, p, and p' are the 4-momenta of the $\bar{b}$, B_c and c (or the $\bar{Q}$, B_Q and $\bar{D}$), respectively, and $|A_1|^2$ is summed and averaged over unobserved spins and colors. We use the notation

$[dp] = d^3p/(16\pi^3 p_0)$ for the invariant phase space element. In spin dependent fragmentation the sum will only cover the unobserved outgoing parton spin labels (c or $\bar{D}$ in this explicit case). To isolate the fragmentation function, the production of the fragmenting quark ($d\hat{\Gamma}$ of Eqn. 1) must be factored out. The fictitious decay width for the $Z^0 \to b + \bar{b}$ or $Q + \bar{Q}$, with the b or Q-quark on shell is

$$\Gamma_0 = \frac{1}{2M_Z}\int [d\bar{q}][dq]\,(2\pi)^4\delta^4(Z - \bar{q} - q)\frac{1}{3}\sum |A_0|^2. \tag{3}$$

with q the b or Q-quark 4-momentum.

To obtain the full inclusive width the unobserved quark degrees of freedom must be integrated over. By introducing the variables q, the off-shell quark's 4-momentum, and $s = q^2$, the square of the virtual mass, the two body phase space for p and p' can be written as an integration over $z = (p_0 + p_L)/(q_0 + q_L)$ and s. Note that the transverse momentum of the hadron, $\mathbf{p_T}$, and the unobserved quark, $\mathbf{p'_T} = -\mathbf{p_T}$ (relative to the fragmenting quark momentum), are fixed for each pair of z and s values via the relation

$$s = q^2 = (q_0 + q_3)(q_0 - q_3) = \frac{M^2 + \mathbf{p_T}^2}{z} + \frac{m'^2 + \mathbf{p_T}^2}{1 - z}, \tag{4}$$

where M and m' are the masses of the hadron and the unobserved quark or anti-diquark, respectively. Then the phase space integration in Eq. 2 can be written for the on-shell hadron and unobserved quark and anti-diquark production as follows:

$$\begin{aligned}
&\int [d\bar{q}][dp][dp']\,(2\pi)^4\delta^4(Z - \bar{q} - p - p') \\
&= \int \frac{ds}{2\pi}\int [d\bar{q}][dq](2\pi)^4\delta^4(Z - q - \bar{q})\int [dp][dp']\,(2\pi)^4\delta^4(q - p - p') \\
&= \frac{1}{16\pi^2}\int ds \int [d\bar{q}][dq](2\pi)^4\delta^4(Z - q - \bar{q})\int_0^1 dz\,\frac{p_0}{zq_0}.
\end{aligned} \tag{5}$$

The variables p_0 and p_3 have been replaced by s and z, and the integration over $\mathbf{p}_T$ has been performed via the delta function that requires

$$\mathbf{p_T}^2 = z(1 - z)[s - s_{th}], \tag{6}$$

where $s_{th} = \frac{M^2}{z} + \frac{m'^2}{1-z}$ is the minimum value that s can assume. Note that the azimuthal integration in $\mathbf{p}_T$ has been performed assuming there is no such dependence in the amplitude. This will be true for spin averaged probabilities and for products of helicity amplitudes, but not for other orientations of quark or hadron spin. If spin projection operators are used in trace expressions, care must be taken to integrate out the azimuthal dependence first. The integrations over q and $\bar{q}$ will be common to the direct production of an on-shell quark in Γ_0 and the off-shell quark that fragments in Γ_1. Providing the production dependence ($|A_0|^2$) can be factored out of the full probability ($|A_1|^2$), this will allow the fragmentation process to be defined irrespective of the production mechanism, obviously an essential feature of any model. The factorization will be possible in the appropriate large momentum limit.

To match the integrand to the fragmentation function of Eq 1 the integration will be performed over the variable s, keeping z fixed and letting M_Z and $q_0 \to \infty$. The s integration ranges from s_{th} to $(M_Z - m_Q)^2$. Since the gluon propagator in the amplitude emphasizes low values of k^2, and the heavy quark propagator favors s not far from on-shell; the major contribution to the s integration appears at low s. Thence, the upper limit of the integration over s can be taken to ∞ to facilitate the evaluation of the definite integral. In the large M_Z or $q_0 \to \infty$ approximation the transverse momentum of the hadron is small relative to p_0 and p_3, since Eq. 6 shows the transverse momentum is independent of q_0 at fixed s and z. Thus it is sensible to ignore the transverse momentum in the relation $p = zq$ (after carefully evaluating s dependent terms in the integrand). Once the square of the amplitude A_1 is summed over spins and simplified by dropping non-leading contributions, the width for $Z^0 \to \bar{Q}Q$ can be factored out of the expression Eq. 1 via

$$D_{Q\to H}(z) = \frac{1}{16\pi^2} \lim_{q_0\to\infty} \int_{s_{th}}^{\infty} ds \frac{|A_1|^2}{|A_0|^2} \tag{7}$$

leaving an integral over the fragmentation function, since the production probability for the relevent quark has been factored out. Then

$$D_{Q\to H}(z) = \frac{8\alpha_s^2 |R(0)|^2}{27\pi m_Q} \int_{s_{th}}^{\infty} ds F(z,s), \tag{8}$$

where $R(0)$ is the Bethe-Salpeter wavefunction at the origin, and $F(z,s)$ is the remaining integrand, which depends on $s = q^2$, z and the quark masses, with the $q_0 \to \infty$ having been implemented. So the partial width for $Z^0 \to H + X$ is given by an integral over the virtuality of the heavy quark and the phase space of the unobserved degrees of freedom.

We now proceed with the calculation of fragmentation functions for (singly) heavy flavor baryons. The basic covariant coupling of diquarks to gluons was written long ago [7]. There is one coupling constant for the scalar diquark color octet vector current coupling to the gluon field—a color charge strength, along with a possible form factor F_s. The momentum space color octet current (which couples to the gluon field vector) is

$$J_\mu^{A(S)} = g_s F_s(k^2)(p+p')_\mu S^{\alpha\dagger} \lambda^A_{\alpha\beta} S^\beta, \tag{9}$$

where p and p' are the scalar diquark 4-momenta and $k = p' - p$. For the vector diquark there are three constants - color charge, anomalous chromomagnetic dipole moment κ, and chromoelectric quadrupole moment λ, along with the corresponding form factors, F_E, F_M, and F_Q.

$$J_\mu^{A(V)} = g_s(\lambda^A)_{\beta\alpha} \Big\{ F_E(k^2)[\epsilon^\alpha(p)\cdot\epsilon^{\beta\dagger}(p')](p+p')_\mu \tag{10}$$

$$+(1+\kappa)F_M(k^2)[\epsilon^\alpha_\mu(p) p\cdot\epsilon^{\beta\dagger}(p') + \epsilon^{\beta\dagger}_\mu(p') p'\cdot\epsilon^\alpha(p)] \tag{11}$$

$$+\frac{\lambda}{m_D^2} F_Q(k^2)[\epsilon^\alpha_\rho(p)\epsilon^{\beta\dagger}_\nu(p') + \frac{1}{2} g_{\rho\nu}\epsilon^\alpha(p)\cdot\epsilon^{\beta\dagger}(p')] k^\rho k^\nu (p+p')_\mu \Big\}, \tag{12}$$

where A is the color octet index, α, β, ..., are color anti-triplet indices, the ϵ's are polarization 4-vectors for the diquarks.

In the perturbative diagrams involved here, the virtual heavy quark emits a time-like off-shell gluon, that,in turn, produces a diquark-antidiquark pair while attaining nearly on-shell 4-momentum. The diquark combines with the heavy quark to form a heavy flavor baryon, whose amplitude for formation is related to the Bethe-Salpeter wavefunction for the diquark-quark system. As in the meson production calculations, it is assumed that the constituents are heavy enough so that the binding is relatively weak, i.e. the quark and diquark are both on-shell and the binding energy is negligibly small. This is expected to be true for constituents with masses well above Λ_{QCD}, and even the light flavor diquarks almost satisfy this constraint. The basic perturbative amplitude is shown in Fig. 1 with the Q$'$-quark line replaced by an (anti-)diquark D line.

It should be realized that the integration (over s, the square of the virtual heavy quark mass) involved in the calculation would diverge for point-like vector diquarks, since the gluon coupling to a pair, Eq. 12 carries momentum factors. The virtual mass in the integration, $\sqrt{s}$, is passed on to the gluon and, subsequently, to the gluon-diquark vertex. Hence it is essential to regulate the integrand by some means. This is best accomplished via the chromoelectromagnetic form factors for the gluon coupling to the diquark. The form factor approach makes physical sense - it is a result of the compositeness of the diquarks. And for consistency, once the vector has form factors, the scalar diquark must have one also.

There is no direct information about the chromoelectromagnetic form factors. We may expect that the ordinary electromagnetic form factors will have the same functional form as their QCD counterparts—the source of both sets of form factors is the matrix element of a conserved vector current operator. In the relevent case here, though, the vector operator is the gluon field — a color octet. Also, what is of concern here is the time-like region of the form factor. For diquarks, of course, there is not any direct empirical evidence about their electromagnetic form factors, but diquark-quark models of the nucleon have constrained the parameterization of the form factors. For one thing, the dimensional counting rules lead to $1/|q|^4$ asymptotic behavior of nucleon form factors (at asymptotic momentum transfer the baryon is a three quark system). A quark-diquark nucleon must approach this asymptotic behavior also, for consisitency. For a *point-like* scalar diquark bound to a quark, the asymptotic behavior will be $1/|q|^2$ from dimensional counting for the exclusive pair production. Hence the composite scalar diquark must have an effective coupling to the gluon, i.e. a form factor, that approaches asymptotia as $1/|q|^2$. Since the vector particle has a polarization 4-vector associated with it, an extra power of momentum arises in the asymptotic amplitude. It becomes necessary for the charge and magnetic form factors to have $1/|q|^4$ asymptotic dependence, and the quadrupole $1/|q|^6$ behavior.

Using a quark-diquark model of the nucleon, and the well measured electromagnetic form factors of the nucleon, Kroll, *et al.* [13], have obtained *electromagnetic* form factors for the diquarks. Two vector form factors are given $1/|q|^4$ asymptotic behavior (the third is set to zero) and the scalar behaves as $1/|q|^2$. For the nucleon form factor study [13] as well as a recent study of higher twist contributions to the nucleon structure functions [14], the scalar diquark form factor and vector diquark form factor are assumed to have simple pole and dipole forms, respectively, with pole positions M_S and M_V above 1 GeV,

$$F_S(k^2) = 1/(1 - k^2/M_S^2),\ \ F_E(k^2) = 1/(1 - k^2/M_V^2)^2,$$
$$F_M(k^2) = (1 + \kappa)F_E(k^2),\ \ F_Q(k^2) = 0. \tag{13}$$

The pole position values are somewhat higher than the dipole position for the overall nucleon form factors - near 800 MeV If we make the assumption that the color form factors have the same functional form as the electromagnetic form factors, we can proceed. However,the region of most relevence for the fragmentation functions is time-like k^2, below the $4m_N^2$ threshold. In the s integration that will be performed here, the time-like k^2 region begins at $4m_{Diquark}^2$ for the value $z = 1/(1 + m_D/m_B)$, and at higher values for other choices of z. This implies that the integration region either overlaps or comes near to overlapping the pole positions. The pole singularities have to be tamed, and the final integration may be very dependent on the method used to moderate the singularities. Treating the poles as real resonance positions, including a small imaginary part, on the order of the nearby vector meson width, would be sensible physically. However, since the color octet form factor would be dominated by color octet vector mesons, and the latter are not expected to be strongly bound or narrow resonances, pole positions with large widths may be preferred. This would hide our ignorance and provide an interpolation between the space-like and time-like asymptotic regions. That is the ansatz we adopt.

The neccessity for diquark form factors has an important consequence theoretically. In the light cone expansion, the baryon production via three quarks would contribute to the leading twist fragmentation functions. Dimensional counting requires the $1/|q|^4$ behavior to which we alluded above. But the diquarks depart from this behavior except at asymptotia. Hence the diquark form factors produce non-leading twist behavior for the fragmentation functions. Gluon contributions are buried in those form factors. In terms of the full set of such functions [15], we have more than just $\hat{f}_1$, $\hat{g}_1$ and $\hat{h}_1$. There are non-leading twist functions like $\hat{e}_1$ and $\hat{h}_2$ that will receive contributions at next-to-leading twist. When we determine what we call $\hat{f}_1$, $\hat{g}_1$ and $\hat{h}_1$, we actually have some non-leading twist contributions that have not been disentangled.

The amplitudes for the baryon production can now be calculated. The spin 1/2 ground state baryons are composed of a scalar diquark and a heavy quark in an s-state. There is only one coupling, and it involves the F_S. The amplitude is

$$A_{S\,1/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_S(k^2) \bar{U}_B g_s [k_\lambda - 2m_d v_\lambda] P^\lambda, \tag{14}$$

where

$$P^\lambda = \triangle^{\lambda\nu} g_s \gamma_\nu \frac{m_Q(1+\mathbf{v}) + \mathbf{k}}{(\mathbf{s} - \mathbf{m_Q^2})} \Gamma. \tag{15}$$

For the vector diquark baryons, there are two form factors (we take the quadrupole to be zero – it falls as $1/|q|^6$ asymptotically). The chromomagnetic coupling involves a parameter κ, the “anomalous chromomagnetic moment”. This is taken to be -1.10, as will be explained below in Section IV on comparing with data. The s-state baryons are spin 3/2 and 1/2, which we will refer to as $1/2'$. The $1/2'$ lies between the 3/2 and the ground state 1/2 baryon. The amplitude for vector diquarks to be produced, along with the heavy quark, contributes to both 3/2 and $1/2'$ states. The amplitude is conveniently divided into a chromoelectric and chromomagnetic part, involving the two distinct form factors. The chromoelectric part contributing to the spin $1/2'$ baryon is

$$A_{E\,1/2} = -\frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2) \bar{U}_B \gamma_5 \gamma^\mu \frac{1+\mathbf{v}}{\mathbf{2}} g_s \epsilon_\mu^\dagger [k_\lambda - 2m_d v_\lambda] P^\lambda. \tag{16}$$

The chromomagnetic contribution to the spin $1/2'$ baryon is

$$A_{M\,1/2} = \frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2)(1+\kappa)\bar{U}_B\gamma_5\gamma^\mu \frac{1+\mathbf{v}}{\mathbf{2}} g_s[g_{\mu\lambda}(\epsilon^\dagger v)m_d - \epsilon^\dagger_\lambda k_\mu]P^\lambda. \tag{17}$$

For the spin 3/2 baryon the corresponding amplitudes are

$$A_{E\,3/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2)\bar{\Psi}^\mu_B g_s \epsilon^\dagger_\mu [k_\lambda - 2m_d v_\lambda]P^\lambda, \tag{18}$$

and

$$A_{M\,3/2} = \frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2)(1+\kappa)\bar{\Psi}^\mu_B g_s[g_{\mu\lambda}(\epsilon^\dagger v)m_d - \epsilon^\dagger_\lambda k_\mu]P^\lambda. \tag{19}$$

Each amplitude should be multiplied by the color factor $4/3\sqrt{3}$. In these amplitudes, $\psi(0)$ is the Bethe-Salpeter wavefunction at the origin (for the s-state Q-diquark system), m_d is the appropriate diquark mass, U_B is a spin 1/2 Dirac spinor for the baryon, $\epsilon^\dagger(p')$ is the polarization 4-vector for the unobserved anti-diquark, Ψ^μ_B is the Rarita-Schwinger spinor for the spin 3/2 baryon, $v = p/M$ is the 4-velocity for the heavy baryon of mass M, Γ is the production vertex for the heavy quark–antiquark pair, k is the 4-momentum of the gluon and, with $n = (1,0,0,-1)$, the corresponding propagator in axial gauge is

$$\triangle^{\mu\nu} = \frac{1}{k^2}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{(nk)}). \tag{20}$$

Considerable simplification of these amplitudes follows. Recall that the formation of the baryon requires that the quark and diquark carry the same 4-velocity as the baryon, so that $k = rp + p'$, with $r = m_D/M$ and $m_Q + m_D = M$ in the weak coupling approximation. This leads to many simplifying relations among the kinematic variables. Of particular importance is the relation $k^2 = r(s - m_Q^2)$, which ties the gluon propagator to the heavy quark propagator as the s integration is performed. The resulting simplified forms for each of the amplitudes become

$$A_{S\,1/2} = \frac{\psi(0)}{\sqrt{2m_d}} F_S(k^2)\bar{U}_B 2g_s^2 \left(\frac{2M^2(1-r) + M\,\not{k}}{(s-m_Q^2)^2} - \frac{(np)}{(nk)(s-m_Q^2)} \right) \Gamma, \tag{21}$$

$$A_{E\,1/2} = -\frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2)\bar{U}_B \gamma_5 \frac{2g_s^2}{M(s-m_Q^2)^2} [(\epsilon^\dagger p) + M\,\not{\epsilon}^\dagger]$$

$$*[2M^2(1-r) - 2\frac{(np)}{(nk)}(kp) + M\,\not{k}]\Gamma, \tag{22}$$

$$A_{M\,1/2} = \frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2)(1+\kappa)\bar{U}_B \gamma_5 \frac{g_s^2}{rM(s-m_Q^2)^2} \{-2(kp)(p\epsilon^\dagger)(1-r)$$

$$+2\frac{(n\epsilon^\dagger)}{(nk)}(kp)^2 - 2r(p\epsilon^\dagger)\frac{(np)}{(nk)}(kp) + (3r-2)M(p\epsilon^\dagger)\,\not{k} + 2M\frac{(n\epsilon^\dagger)}{(nk)}(kp)\,\not{k}$$

$$-2rM(p\epsilon^\dagger)(kp)\frac{\not{n}}{(nk)} + \ 2r(kp)M\,\not{\epsilon}^\dagger - (kp)\,\not{\epsilon}^\dagger\,\not{k}\}\Gamma, \tag{23}$$

$$A_{E\,3/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2)\bar{\Psi}_B^\mu g_s \epsilon_\mu^* 2g_s^2 \left(\frac{2M^2(1-r) + M\,\not{k}}{(s-m_Q^2)^2} \right.$$

$$\left. - \frac{(np)}{(nk)(s-m_Q^2)} \right) \Gamma, \tag{24}$$

$$A_{M\,3/2} = \frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2)(1+\kappa)\bar{\Psi}_B^\mu g_s^2 \frac{1}{r(s-m_Q^2)^2} \{-2r(p\epsilon^\dagger)(kp)\frac{n_\mu}{(nk)}$$

$$-2(1-r)k_\mu(p\epsilon^\dagger) + 2(kp)\frac{(n\epsilon^\dagger)}{(nk)}k_\mu - k_\mu\,\not{\epsilon}^\dagger\,\not{k}\}\Gamma. \tag{25}$$

There are 3 cases to consider for each heavy quark flavor—3 final state baryons. For the two states resulting from the vector diquark, the electric and magnetic amplitudes must be added together. Then for each baryon, the amplitude is squared and a trace is taken to sum over spins (including spin projection operators for the spin dependent cases). The analog of Eq. 2 is obtained for each baryon. By carefully organizing the terms in the integrand, the width for the inclusive production of the virtual heavy quark can be divided out to yield the analog of Eq. 8 for each baryon. Finally the integration over $s = q^2$ can be performed numerically—the form factors make it difficult to write an analytic expression for each case. The resulting z dependent fragmentation functions are the "boundary" functions, obtained at a scale μ^2 at the threshold $(2m_D + m_Q)^2$, with the strong coupling constant $\alpha(\mu = 2m_D)$. To consider higher momentum scales, the Altarelli-Parisi evolution equations are used. This strategy essentially sums the leading log contribution of the parton shower that is generated by the off-shell heavy quark.

We have taken some particular cases to illustrate the results. For the c(su) or c(sd) baryons, the Ξ_c states, the diquark is given a mass of 0.9 GeV/c^2 and the ratio of diquark to hadron mass is $r = 0.33$. Form factor parameterizations are discussed in Section 4. The resulting function, $\hat{f}_1(z, Q^2)$ is shown in Fig. 2 for the boundary value at $\mu = 2m_D + m_Q$ and for $Q_0 = 5.5$ GeV, the jet energy obtained at CESR. The three s-states lead to different behavior and overall probability. Note that the 1/2 ground state is produced roughly as frequently as the $1/2'$ while the latter is produced about twice as often as the 3/2 state. The observed 1/2 ground states are produced from the decays of these vector diquark states approximately as often as they are directly produced.

In Fig. 3 the corresponding fragmentation functions for charmed states with non-strange diquarks are shown. These functions are evolved to 45 GeV, the jet energy attained at LEP. Fig. 4 shows the b-quark states with non-strange diquarks, also evolved to 45 GeV. It is clear that these spin-averaged fragmentation functions peak at high z, even after evolution. The vector diquark baryon states have noticable secondary peaks at the low scale, which get diluted at higher scales.The secondary peaks arise from the polynomials in s in the chromoelectric and chromomagnetic contributions and depend on the amount of the latter, as fixed by the parameter $1+\kappa$. The shapes of the curves are significantly different from the Peterson function shape [1] and eventually should be distinguishable experimentally. The peak position moves towards higher z as the heavy quark mass increases, as expected from heavy quark QCD.

III. SPIN DEPENDENT FRAGMENTATION FUNCTIONS

In the preceding, the spin orientations of the virtual quark and heavy baryon have been summed. In general, however, there is a spin dependence to the fragmentation process. For a spin 1/2 baryon there are two fragmentation functions that characterize the leading twist spin dependence, $\hat{g}_1(z)$ and $\hat{h}_1(z)$. They correspond, at the parton model level, to the transfer from the quark to the baryon of longitudinal polarization (or helicity) and transversity [6]. As a consequence of the development leading to Eq. 7, the spin dependent fragmentation function $\hat{g}_1$ can be written in the form

$$\hat{g}_1(z) = D_{Q(+)\to H(+)}(z) - D_{Q(+)\to H(-)}(z) = \frac{1}{16\pi^2}\lim_{q_0\to\infty}\int_{s_{th}}^{\infty} ds \frac{|A_{++}|^2 - |A_{+-}|^2}{|A_{0+}|^2}, \tag{26}$$

where $|A_{+\lambda}|^2$ is the probability for the helicity $+1/2$ quark to produce a helicity λ baryon, with a sum over the unobserved diquark degrees of freedom implied. The A_{0+} represents the corresponding production of an on-shell helicity $+1/2$ heavy quark, at large $q_0 + q_3$. The reader may ask, how can a heavy quark flip its helicity in hadronization? For the scalar diquark combining with the heavy quark, this has to be an effect that would vanish in the heavy mass limit, relative to the spin independent fragmentation, i.e. $\hat{g}_1(z)$ would coincide with $\hat{f}_1(z)$. For the corresponding vector diquark case this need not be true, since the diquark can carry negative helicity leading to the opposite helicity for the baryon.

The analogous transversity function requires the superposed helicity states $(|+\rangle \pm i|-\rangle)/\sqrt{2} = |y_\pm\rangle$.

$$\hat{h}_1(z) = D_{Q(y_+)\to H(y_+)}(z) - D_{Q(y_+)\to H(y_-)}(z) = \frac{1}{16\pi^2}\lim_{q_0\to\infty}\int_{s_{th}}^{\infty} ds \frac{\left|A_{y_+,y_+}\right|^2 - \left|A_{y_+,y_-}\right|^2}{\left|A_{0y_+}\right|^2}, \tag{27}$$

Now in the model we are considering the transversity can not flip, since there is no chirality change in the matrix elements. We have the transitions $\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^+$ or $\frac{1}{2}^+ + 1^+$, analogous to a virtual quark decay into a baryon and a positive parity boson. To get a non-trivial result there needs to be an opposite parity bosonic state as well.

In Figs. 5 and 6 the longitudinal fragmentation function $\hat{g}_1(z, Q^2)$ is plotted for the two spin 1/2 states with the same physical parameters as in the preceding figures. Note that

$\hat{g}_1$ is very similar in shape to the spin averaged case for the Λ states, as expected when the diquark is a scalar. This shows that the helicity flip contribution is relatively small for the values of $r = m_D/M$ relevent for the bottom and charm baryons. Hence the longitudinal polarization of the heavy quark is passed on to the baryon. But we will see that this spin preservation is less than 100%. Furthermore, the production of excited baryons will dilute that spin preservation. The spin 1/2 Σ states have very different behavior, showing the importance of non-zero helicity for the vector diquarks.

The spin dependent fragmentation functions for the spin 3/2 baryons are even richer in complexity. There are seven such functions at leading twist, many of which will be accessible from the decay distributions of these states into the 1/2 state plus a pion. While these fragmentation functions have not been classified in the light-cone expansion formalism, it is clear that the number of independent leading twist functions coincides with the number of forward amplitudes for parity conserving elastic scattering of spin 1/2 on spin 3/2. In general there will be 2(2S+1)-1 such leading twist fragmentation functions for a quark to fragment into a spin S hadron. In the heavy quark limit the helicity of the quark will be preserved in the hadron, so we expect the analog of $\hat{g}_1$ to be near the spin averaged function.

IV. COMPARISON WITH DATA

The analysis of charmed baryon production data has been extensive at CESR and, more recently, at LEP. Fragmentation data now exist from the CLEO collaboration [2] for some of the Ξ_c states. The Λ_c is studied at both LEP and CESR. Bottom fragmentation into baryons is now being studied at LEP. CLEO, in particular, has determined spin independent fragmentation functions for the lowest mass spin 1/2 and 3/2 states Ξ_c. Polarization asymmetry has been measured for Λ and Λ_b at LEP. Production rates for Λ_c from c-quarks have been determined. All of this data provide a testing ground for the model being proposed here.

The parameters that enter our calculations of fragmentation functions (and their integrals over z) are the masses of the constituents, the poles in the chromodynamic form factors, the widths of those poles, and the anomalous chromomagnetic moment of the vector diquark. For the diquark masses we take $m(ud) = 0.6$ GeV and $m(us) = 0.9$ GeV. The baryon masses are taken from the data, so the ratios, $r = m_D/M$ of diquark to baryon masses are determined thereby for each baryon. It is assumed that the difference between a baryon mass and the constituent heavy quark mass plus the diquark mass is negligible, in order that the Bethe-Salpeter wave functions need be evaluated only at the origin. The poles that enter the form factors of Eq 13 for the (ud) diquarks are taken from the electromagnetic form factors [13], $M_S(ud) = 1.8$ GeV and $M_V(ud) = 1.2$ GeV. The full width at half maximum for the scalar form factor is set at 0.88 GeV and, for simplicity, the vector form factor is assumed to have the same width. Recall that the width is introduced so that there are no singularities in the physical region of s, the virtuality of the fragmenting quark, or, correspondingly, k^2, the square of the gluon 4-momentum. It will transpire that the fragmentation probabilities will depend critically on the pole positions and width, since the integration region is dominated by the lowest values of s, where the heavy quark is nearly on mass shell. In that region the poles are nearby. The pole and width parameters are expected to be different for the (su) diquarks. We take a cue from the electromagnetic form factors of the charged π's and K's, where the charge radii are roughly in a ratio of 1.3:1, corresponding to a pole postion that increases by 1.3 for the strange meson. This is qualitatively understandable by analogy. The ρ vector meson contributes to the π charge form factor, while the ϕ vector meson contributes to the kaon electric charge form factor. The latter has a mass 1.3 times that of the ρ. So, to fix the (us) diquark chromodynamic form factors we choose an overall scale factor of 1.4 for the pole positions and the width, slightly bigger than the meson case.

The anomalous magnetic parameter κ_{EM} was determined [13] for the (ud) vector diquarks to be a positive number - a result of fitting the composite nucleon form factors. For the chromomagnetic case, however, a negative value is preferred for κ from calculations of the mass spectrum of excited baryons [7]. Furthermore, using a diquark-quark model of the nucleon to calculate the electromagnetic charge radii and polarizabilities preferred a negative value for κ_{EM}. It is unclear what value to take for this parameter, given the divergence of different methods.

We will determine a value for κ by optimizing our model predictions compared to data for the ratio of production probabilities, $R(\Sigma_b) = \Sigma_b/(\Sigma_b + \Sigma_b^*)$. In the model, Σ_b and Σ_b^* are b+vector$\{u, d\}$ diquark states of spin 1/2 and 3/2. The difference in production probabilities or $\hat{f}_1(z)$ for these two states depends sensitively on κ. Using the measured value from DELPHI [16] of 0.24 ± 0.12 for the ratio, we choose $\kappa = -1.10$. This makes the overall chromomagnetic coupling small and negative $(1 + \kappa = -0.10)$. The reason for this small value is that the ratio $R(\Sigma_b)$ would be exactly 1/3 from spin counting if there were no chromomagnetic term at all; the 1/3 is compatable with the data.

The simplest states to study, from our point of view, are the $\frac{1}{2}^+$ ground states, since they involve the scalar diquark. For these states the integral over z of $\hat{f}_1(z)$ should correspond to the total production probability for producing the state from the corresponding heavy quark. However, there are contributions to the same probabilities from the excited states that decay into these ground states. Consider the Λ_c fragmented from a c-quark, for which OPAL [17] measures $5.6 \pm 2.6\%$ and CLEO [2] finds $9.5 \pm 1.3\%$. These measurements include directly fragmenting Λ_c's along with any state that decays into this ground state. The Σ_c's (both spin 1/2 and 3/2 states) decay strongly into $\Lambda_c + \pi$, so contribute to the rate. With the parameters chosen, we find 0.5% for the directly fragmented Λ_c and 3.3% when the Σ_c's are included. This is consistent with the LEP data. For the analogous b-quark system we have fixed the $\Sigma_b + \Sigma_b^*$ rate to be 4.8%, consistent with experiment [16], $4.8 \pm 1.6\%$. The total Λ_b is then 5.8%, comparing nicely with the measurement [19] of $7.6 \pm 4.2\%$. These and the following results are summarized in the Table below.

It is significant to note that we have obtained these sizeable baryon fragmentation rates, in contrast with the similar model of Martynenko and Saleev [9]. Saleev [12] obtains only 0.2% for the Λ_b. This indicates the importance of our form factors in getting the correct normalizations.

Having confidence in the overall normalization, we have reason to trust the full fragmentation functions. These are shown in Fig. 3 and 4 for the c and b states. The input, unevolved "boundary data" show a large peak at high z and a secondary peak at medium z. That fairly severe behavior is moderated considerably after evolving to the scale of CESR or LEP. But even at the LEP scale, the functions are distinguishable from the Peterson function, being peaked at higher z and more skewed. As sufficient data is gathered, it will be possible to see such a difference.

For the singly strange diquark, the lowest charmed baryon $1/2^+$ states are the $c+[u,s]$ and $c+[d,s]$ states, Ξ_c^+ and Ξ_c^0, involving the antisymmetric, spin 0 diquarks. These, along with the spin $3/2^+$ states ($c+\{u,s\}$ and $c+\{d,s\}$ baryons), Ξ_c^{*+} and Ξ_c^{*0}, involving the symmetric, spin 1 diquarks, have been seen and measured in sufficient quantities for CLEO to sketch their fragmentation functions [2]. The spin $1/2^+$ partners, $\Xi_c^{'}$, of $3/2^+$ states have not been seen yet. They are presumed to have a mass below the $\Xi_c+\pi$ threshold, so must be seen in radiative decay channels. Note that these latter $\Xi_c^{'}$ $1/2^+$ states have the same isospin as the lower lying ground states Ξ_c $1/2^+$ and could mix with them, in principle. In any case, the measured fragmentation functions provide a crude test of the model. The data are fit by the experimenters with a common parameterization of the Peterson function [1]. It is easy to see in Fig. 2 that the data fall nicely on $\hat{f}_1$ of our model, evolved to $Q=5.5$ GeV, with the possible exception of the highest z data point. These data are not sufficiently accurate to be a crucial test of the model, but do exhibit the trends we expect. Note that the experimental variable x_p [2] does not correspond exactly to our z, the light cone variable.

The ratio of the 3/2 to 1/2 production can be extracted from the data with some uncertainty [18]. The percentage of all Ξ_c^+ states that arose from decays $\Xi_c^{*0}\to\Xi_c^++\pi^-$ is given as $(27\pm 8)\%$ and the percentage of all Ξ_c^0 states that arose from decays $\Xi_c^{*+}\to\Xi_c^0+\pi^+$ is given as $(17\pm 6)\%$. (Note that we have combined the statisitical and systematic errors here.)

The experimenters do not see the π^0 channels, $\Xi_c^{*+}\to\Xi_c^++\pi^0$ and $\Xi_c^{*0}\to\Xi_c^0+\pi^0$. From isospin conservation these channels account for 1/3 of the decays into $\Xi_c+\pi$, while the reported charged π channels constitute 2/3. Suppose $N\,\Xi_c^*$ states of both charges are produced. Then 2/3 N will be seen in the charged π decay mode. The total number of $\Xi_c^{+,0}$'s seen will be $N_{+,0}=\frac{2}{3}N/(0.27,0.17)$ (supressing errors until the end). The number of $\Xi_c^{+,0}$'s not coming from the decays of the 3/2 states will be $N_{+,0}-N$. Assume that $n_{+,0}$ of the $\Xi_c^{+,0}$'s come from other fragmented states' decays. Then $N_{+,0}-N-n_{+,0}$ is the number of direct fragmentation products of the charmed quark. The ratio $R(+\,\mathrm{or}\,0)$ of directly fragmented $\Xi_c^{+,0}$ to $\Xi_c^{*+,0}$ is given thereby as $R(+)=1.5\pm 0.7-n_+/N:1$ and $R(0)=2.9\pm 1.4-n_0/N:1$.

The numbers $n_{+,0}$ will come from the radiative decays of the heavier 1/2 states, as well as higher Ξ_c states (radial and orbital excitations of the $c+(su)$ and $c+(sd)$ systems). We have calculated the fragmentation functions for the spin 1/2 quark–vector-diquark states and hence the number of $\Xi_c^{'}$ spin $1/2'$ states vs. Ξ_c^* spin 3/2 states. That is 0.5:1 for the parameterization used in Fig. 2. Assuming $n_{+,0}$ is due entirely to these $1/2'$ states decaying 100% into the ground state $\Xi_c^{+,0}$, we have for the different charge states $R(+)=0.9\pm 0.7$ and $R(0)=2.3\pm 1.4$, both of which are consistent with the ratio of 1.4:1 predicted by the same model calculation. Hence, if the model is taken seriously, and the experimental uncertainties are firm, the data do not require large contributions from fragmentation of the c-quark into higher excitations of the Ξ_c states.

There are two reasons to be cautious about these experimental numbers, however. First, the errors are quite large, leaving considerable variation possible within two standard deviations. Secondly, the CLEO results are obtained at e^++e^- energy near 10 GeV. For the c-quark jet at roughly 5 GeV, the extraction of asymptotically meaningful fragmentation functions is somewhat dubious.

In a previous version of our model [8] our parameterization gave a much larger vector diquark to scalar diquark production probability. With the more reasonable values now adopted these diquark states and the corresponding baryons are produced with roughly the same probabilities, as the calculations for heavy-heavy baryons by Martynenko and Saleev [9] favored.

Finally we consider the spin dependent fragmentation. At this time there is not enough data to determine z dependence for polarization in heavy quark fragmentation. However, there is a determination of the net longitudinal polarization of Λ_b produced at LEP. That number is -0.23 ± 0.25 as determined by ALEPH [19], using a technique suggested by Bonvicini and Randall [20]. Given that the b-quark produced at the Z^0 pole is expected to have longitudinal polarization of -0.94, this measurement gives 0.24 ± 0.27 for the net transfer of helicity from the b-quark to the Λ_b. This is rather low if one anticipates that the heavy baryon carries most of the helicity of the heavy quark - the expectation of heavy quark field theory [21].

Now the longitudinal polarization of the Λ_b fragmenting from a positive helicity b-quark is a function of z - the ratio $\hat{g}_1(z)/\hat{f}_1(z)$ for the directly produced Λ_b (calculated to leading twist). The integral over z of that ratio would give a net polarization. Experimentally, though, the net polarization is obtained by taking each event, regardless of its z (and p_T), and calculating a quantity related to its polarization. The result of this process is to give a net polarization that will be the integral of $\hat{g}_1(z)$ over z divided by the corresponding integral of $\hat{f}_1(z)$. Both of these integrals are scale independent. Using our fragmentation functions we obtained 0.90, only marginally lower than the heavy quark limiting value, but still not the small result extracted from the data. However, from our spin independent calculation above and the LEP data, we know that the Σ_b and Σ_b^* are produced in relative abundance, and will decay into $\Lambda_b + \pi$, so that the polarization will be diluted by these other channels. A heavy quark limit calculation by Falk and Peskin [22] anticipated this circumstance. They summed the contributions of all of these states in determining the net Λ_b polarization. Using simplifying assumptions, they obtained about 0.72 for the fractional helicity transfer from the b-quark to the hadron. That estimate assumed the ratio (called A in their paper) of Σ_b and Σ_b^* (vector diquark states) to Λ_b (scalar diquark states) of 0.45. Taking that ratio to be 4.8 instead, which is our result and consistent with experiment, the resulting fractional helicity transfer becomes 0.26. Falk and Peskin also define a parameter w_1 which measures the amount of helicity ± 1 diquark that combines with the heavy quark. They take that parameter to be zero, whereas we can calculate w_1, which averages over all z to be $\simeq 0.6$. With our values for both A and w_1 we obtain the fractional helicity transfer of 0.46. Both of these results are near the central value of the measurement. In obtaining these results we have assumed the heavy quark limiting values of $\pm 1/3$ for the polarization of the secondary Λ_b's resulting from the Σ_b and Σ_b^* decays. These results will obtain when the chromomagnetic contribution is small, as it is for the κ we have chosen.

The application of a similar model by Saleev [12] to Λ_b production and polarization yields very different results. As we noted above, his production probability is too small. The polarization for direct Λ_b is less than ours (0.6 to 0.7). However, it seems that it is the chirality asymmetry (Left - Right) that Saleev has calculated, rather than helicity. It is the latter that is measured experimentally. We have a much weaker dependence on the diquark mass in our calculation as a result.

V. SUMMARY

A model for fragmentation into heavy flavored baryons has been developed using perturbative QCD and a Bethe-Salpeter wave function for a quark-diquark system. This provides the starting point for QCD evolved fragmentation functions. The parameterization of the diquark structure through the chromodynamic form factors for scalar and vector diquarks is accomplished by using the pole form applied to the electrodynamic form factors. The kinematic region near these poles is quite important because the integral over the fragmenting quark's virtuality, s (or indirectly, the baryon's transverse momentum at a fixed z), emphasizes the nearly on-shell region where the corresponding gluon k^2 is near the poles. It is this pole parameterization that is crucial for determining the overall magnitudes of the various production probabilities.

The production probabilities were all close to experimental values, which supports our reasoning about the form factors. The z and Q^2 dependences predicted have the common feature of being very sharply peaked at high z for the input scale μ_0, and more broadly peaked at high Q^2. For the $1/2^+$ ground states, the z dependence of the spin dependent $\hat{g}_1$ is close to the form for $\hat{f}_1$, but their ratio (which will determine the baryon polarization as a function of z) is striking. For the higher mass $1/2^+$ state containing the vector diquark the $\hat{g}_1$ has a very different z dependence. It will be particularly interesting to see if the peak and dip structures for both spin 1/2 states are reproduced by the data.

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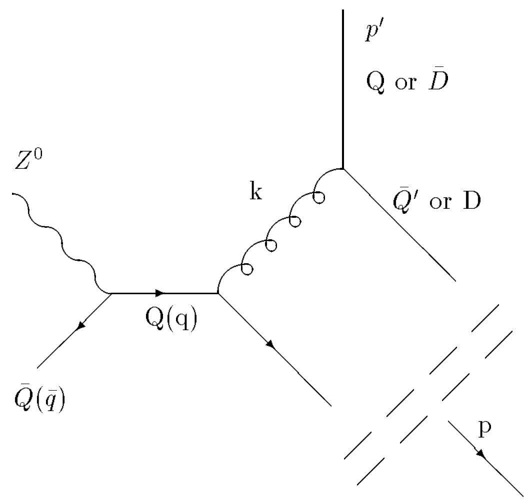

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FIG. 1. The amplitude for $Z^0 \to \mathrm{Meson}(Q\bar{Q}') + X$ or $\mathrm{Baryon}(QD) + X$.

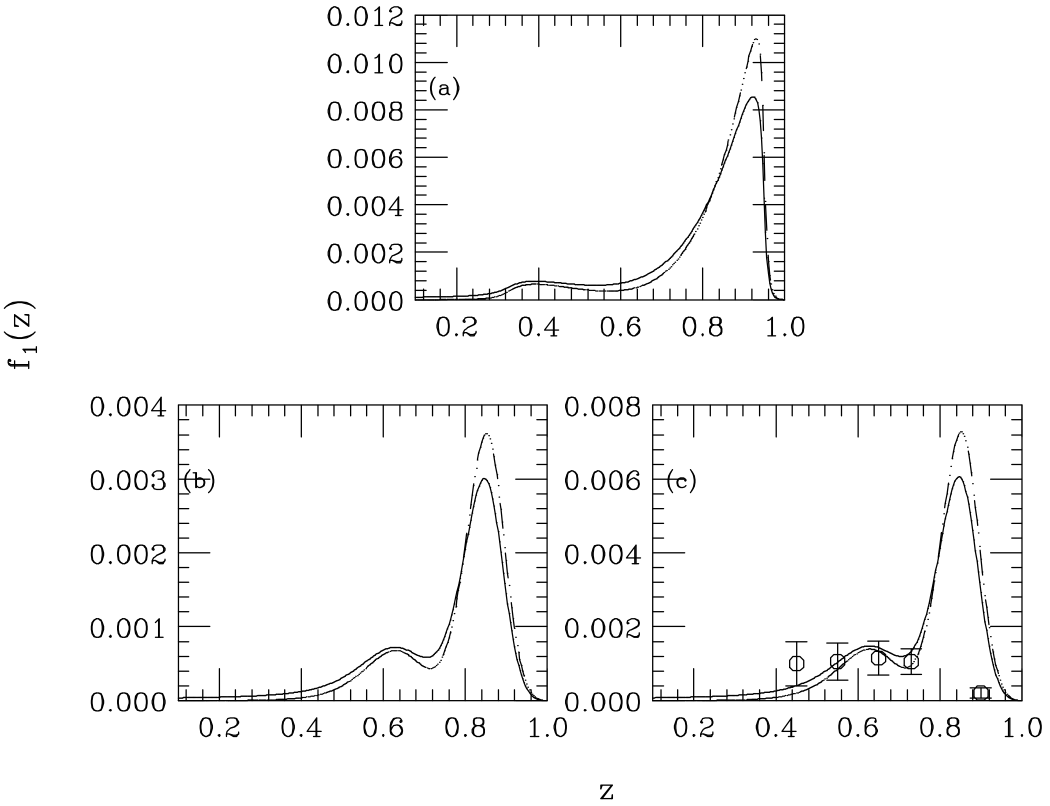

FIG. 2. Approximate $\hat{f}_1(z, Q^2)$ for a.$\Xi_c(1/2)$, b.$\Xi_c'(1/2)$, and c.$\Xi_c^*(3/2)$, each at $Q = \mu_0$ and 5.5 GeV.

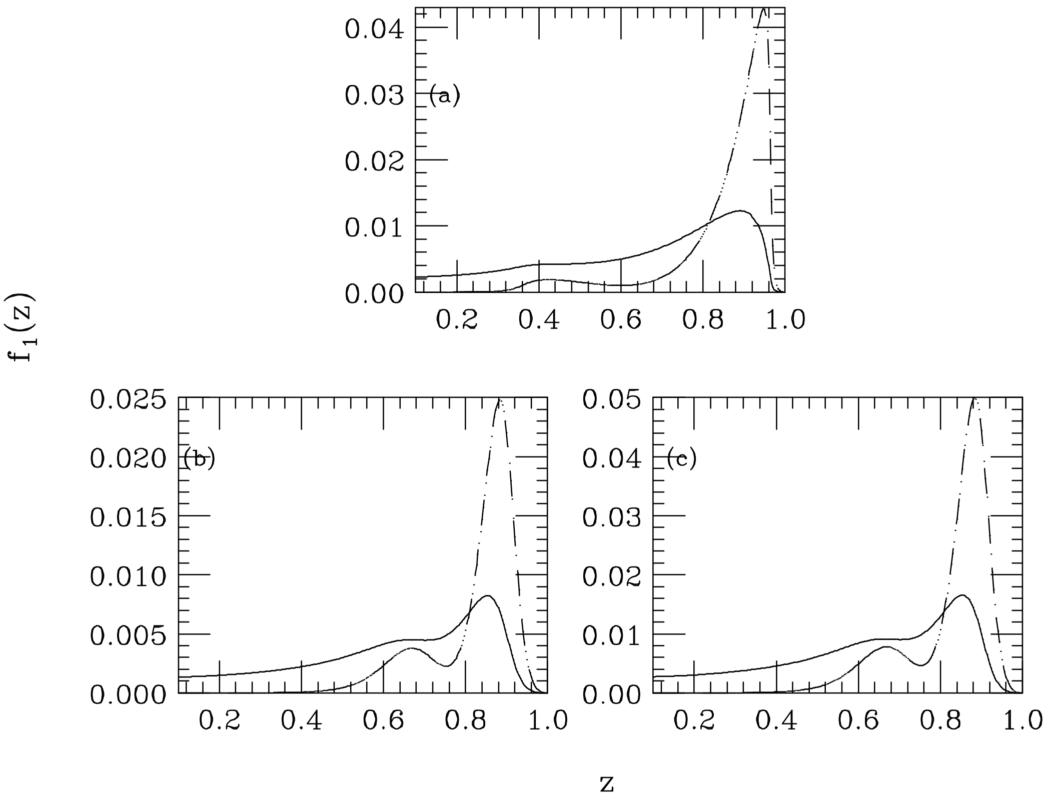

FIG. 3. Approximate $\hat{f}_1(z, Q^2)$ for a.Λ_c, b.Σ_c, and Σ_c^*, each at $Q = \mu_0$ and 45 GeV.

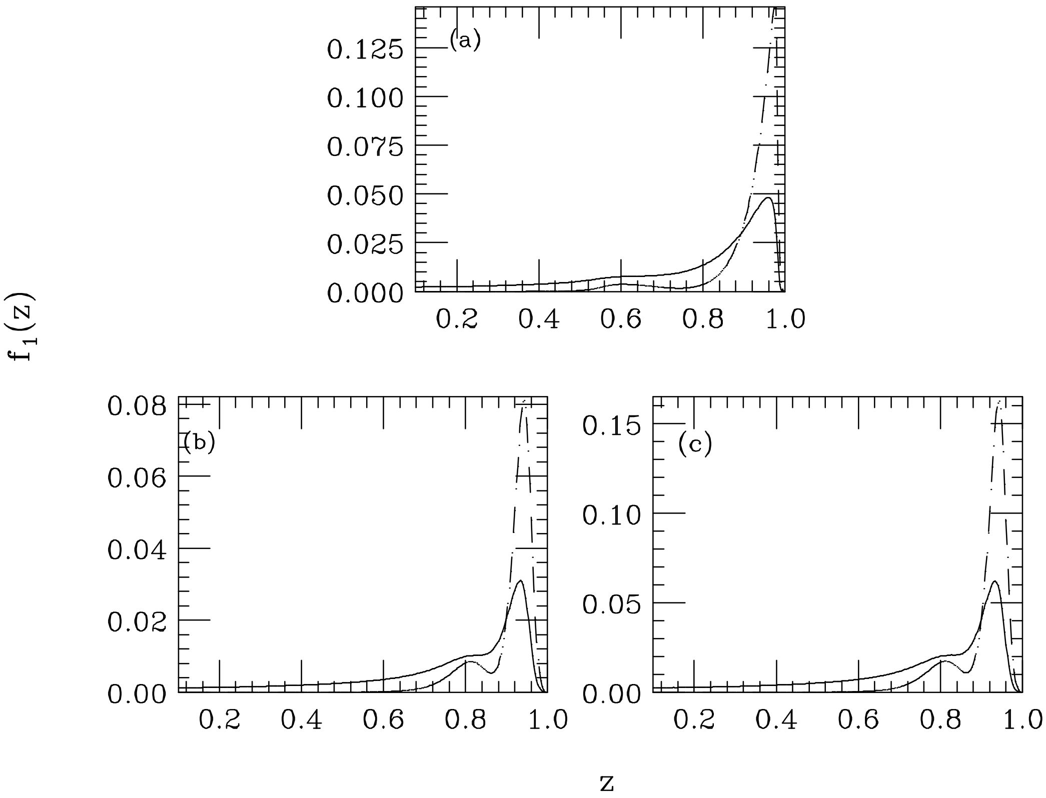

FIG. 4. Approximate $\hat{f}_1(z, Q^2)$ for a.Λ_b, b.Σ_b, and Σ_b^*, each at $Q = \mu_0$ and 45 GeV.

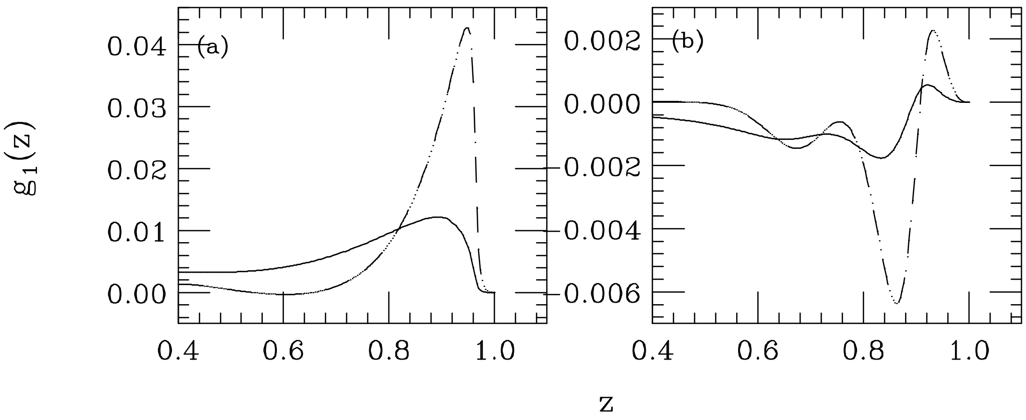

FIG. 5. Approximate $\hat{g}_1(z, Q^2)$ for a.Λ_c, b.Σ_c, each at $Q = \mu_0$ and 45 GeV.

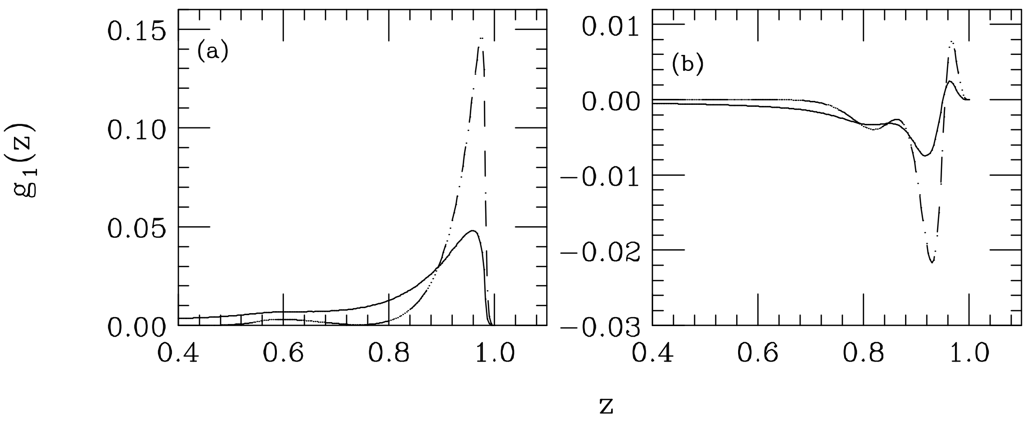

FIG. 6. Approximate $\hat{g}_1(z,Q^2)$ for a.Λ_b, b.Σ_b, each at $Q=\mu_0$ and 45 GeV.

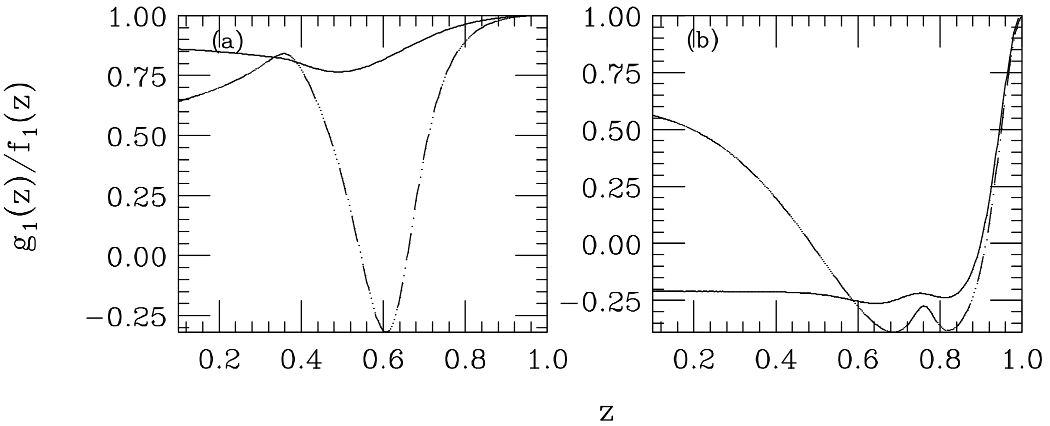

FIG. 7. Approximate ratio $\hat{g}_1(z, Q^2)/\hat{f}_1(z, Q^2)$ for a.Λ_c, b.Σ_c, each at $Q = \mu_0$ and 45 GeV.

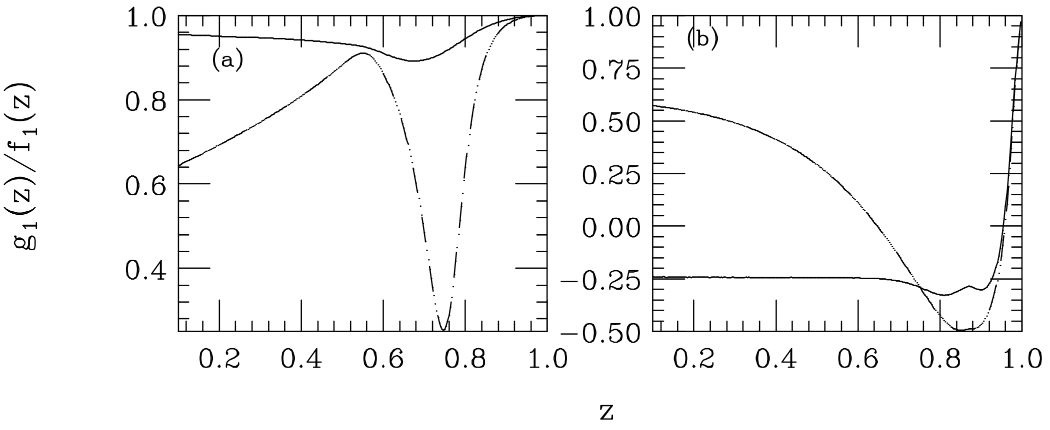

FIG. 8. Approximate ratio $\hat{g}_1(z,Q^2)/\hat{f}_1(z,Q^2)$ for a.Λ_b, b.Σ_b, each at $Q=\mu_0$ and 45 GeV.

Particle	Experiment	Prediction
$P(c \to \Lambda_c)$ (including decays of Σ_c and Σ_c^*)	5.6±2.6% [OPAL]	3.26%
$P(b \to \Lambda_b)$ (including decays of Σ_b and Σ_b^*)	7.6±4.2% [ALEPH]	5.8%
$P(b \to \Sigma_b + \Sigma_b^*)$	4.8±1.6% [DELPHI]	4.8% (fixed)
$\frac{P(b\to\Sigma_b)}{P(b\to\Sigma_b)+P(b\to\Sigma_b^*)}$	0.24±0.12 [DELPHI]	0.33
$\frac{P(c\to\Xi_c^*)}{P(c\to\Xi_c)}$	2.3±1.4 and 0.9±0.7 [CLEO]	1.4
$P(c \to \Xi_c)$ (including decays of Ξ_c' and Ξ_c^*)	-	0.53%
$P(c \to \Xi_c)$ (direct production)	-	0.17%
$P(c \to \Xi_c^{*+}$ or $\Xi_c^{*0})$ (direct production)	-	0.12%
$P(c \to \Lambda_c)$ (direct production)	-	0.52%
$P(c \to \Sigma_c^{*++}$, Σ_c^{*+} or $\Sigma_c^{*0})$ (direct production)	-	0.62%
$P(b \to \Lambda_b)$ (direct production)	-	1%
$P(b \to \Sigma_b^{*+}$, Σ_b^{*0} or $\Sigma_b^{*-})$ (direct production)	-	1.06%