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Dijet hadroproduction with rapidity gaps and QCD double logarithmic effects

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Abstract

We show that the hadroproduction of a pair of jets with large transverse energy in the central region bounded by rapidity gaps is an ideal process to see important double logarithmic QCD suppression effects. We compute the cross sections for both exclusive and inclusive double-diffractive dijet production at Tevatron and LHC energies for a range of rapidity configurations.

1. Introduction

Processes with large rapidity gaps in high energy pp (or $p\overline{p}$) collisions are being intensively studied both theoretically and experimentally, see for example refs.[1, 2, 3, 4]. One reason is that the requirement of a rapidity gap is a way to select events induced by QCD Pomeron exchange. Another reason is that events with a rapidity gap offer the opportunity to search for new heavy particles, such as Higgs bosons, in an environment in which the large QCD background is suppressed.

A particularly illuminating 'test' process with which to probe the underlying dynamics is the hadroproduction of a dijet system separated from the beam remnants by rapidity gaps[1]

$$p\overline{p} \to X \ + \ jj \ + \ \overline{X}, \tag{1}$$

where the two centrally produced jets each have large transverse energy (E_T). The 'plus' signs in (1) indicate the existence of rapidity gaps. This process has been observed at the Tevatron [2]. Moreover, it can be studied in more detail and, in particular, at larger E_T at the LHC. Exclusive dijet production, $p\overline{p} \to p + jj + \overline{p}$, was originally discussed at the Born level by Pumplin [3], and by Berera and Collins [4]. In this paper we are concerned with QCD effects which give significant modifications to the Born prediction. Indeed a novel and interesting effect is the strong suppression of the cross section by double logarithmic QCD form factors which reflect the fact that the emission of relatively soft gluons in the rapidity gap intervals is forbidden by the experimental cuts.

The first study [5] of such a suppression was in connection with the rapidity gap Higgs signal at the LHC. It was found that the cross section for the exclusive process $pp \to p + H + p$ was suppressed by Sudakov form factors by about a factor of 1000 in comparison with the Born cross section. Here we study the analogous effect in dijet production. This is an important process for two reasons. First, the prediction can be directly checked by experiment and, second, $b\overline{b}$ dijet production with rapidity gaps is the main source of QCD background for an intermediate mass Higgs boson + rapidity gaps signal. One way in which the dijet process differs from Higgs production [5] is that the Sudakov form factor suppression is partially alleviated by the special kinematics of the process. Just as the QED radiative corrections[2] have to be calculated for each particular choice of experimental cuts, so the QCD double logarithmic suppression needs to be evaluated for each specification of the rapidity gaps.

For pedagogical reasons we first study the exclusive dijet production process $p\overline{p} \to p + jj + \overline{p}$ in which the proton and antiproton remain intact. However we find, as expected, that the cross

[1]Since we shall assume that the process is mediated by gluon t channel exchanges our calculation applies equally well to $p\overline{p}$ and pp collisions.

[2]One of the best places to see experimental evidence of QED double logarithmic effects is in the J/ψ line shape in e^+e^- annihilation. The asymmetric widening of the line shape, arising from the radiative tail, is mainly due to these effects, see for example [6]. To obtain a sharp resonance peak it would be necessary to experimentally forbid QED radiation from the incoming e^+ and e^-, which would lead to a Sudakov-suppressed cross section. Clearly the analogous QCD effects which we will discuss here are much larger.

section is extremely small and so we then turn to the more realistic inclusive dijet production process given in (1). Our study will concentrate on the kinematic configuration where the "dijet" rapidity interval between the two rapidity gaps is not large. Here the predictions are particularly clear. If the interval is large then, as we shall see, the production of additional minijets will considerably complicate the theoretical framework without giving any additional insight. Of course we will also have to take into account the suppression of rapidity gap events due to parton-parton rescattering.

We will work in the double logarithmic approximation and use the leading power of all logarithms that occur. Provided that E_T is sufficiently large, this approach is rather well justified.

2. Dijet kinematics

Consider dijet hadroproduction at the quark level, $qq \to q + jj + q$, which is shown in Fig. 1. The experimental configuration of interest is where particle production is forbidden in the rapidity gaps

$$\Delta\eta(\text{veto}) \;=\; \pm\,(\eta_{\min}, \eta_{\max})$$

and a pair of larger E_T jets are produced with rapidities η_1, η_2 which both lie in a central rapidity interval denoted by $\Delta\eta(\text{dijet})$. The rapidity gap configuration is sketched on the right hand side of Fig. 1. It is necessary to choose the interval $\Delta\eta(\text{dijet})$ smaller than the interval $(-\eta_{\min}, \eta_{\min})$ between the gaps so as to ensure that the fragments of the large E_T jets can be collected and their momenta determined.

Let us denote the transverse momenta of the two jets by $\boldsymbol{P}_{1T}$ and $\boldsymbol{P}_{2T}$. Then the jet transverse energies are $E_{iT} = P_{iT}$. It is convenient to write the differential cross section for dijet production in the form $d\sigma/(d^2\boldsymbol{P}_T\, d^2\Delta\boldsymbol{P}_T\, d\eta\, d\Delta\eta)$ where

$$\boldsymbol{P}_T \;=\; \tfrac{1}{2}\,(\boldsymbol{P}_{1T} - \boldsymbol{P}_{2T}), \quad \eta \;=\; \tfrac{1}{2}(\eta_1 + \eta_2) \tag{2}$$

$$\Delta\boldsymbol{P}_T \;=\; \boldsymbol{P}_{1T} + \boldsymbol{P}_{2T}, \quad \Delta\eta = \eta_1 - \eta_2. \tag{3}$$

On the other hand the $gg \to jj$ hard subprocess is most naturally described in terms of the Mandelstam variables $\hat{s} = M^2$ and $\hat{t}$, where M is the invariant mass of the dijet system. For exclusive dijet production, $p\overline{p} \to p + jj + \overline{p}$, the proton form factors limit the momentum transfer from the proton (and from the antiproton). We thus have $|t_i| \ll P_T^2$ and consequently $(\Delta P_T)^2 \ll P_T^2$. The kinematics are much simpler when ΔP_T is small. It means that the rapidity axis defined with respect to the direction of the incoming p and $\overline{p}$ essentially coincides with the axis defined by the incoming hard gluons. In this limit

$$\hat{s} \;\equiv\; M^2 \;\simeq\; 4P_T^2 \cosh^2(\Delta\eta/2) \tag{4}$$

$$\hat{t} \;\simeq\; -P_T^2(1 + e^{-\Delta\eta}) \tag{5}$$

where $\Delta\eta$ can be defined by either the $p\overline{p}$ or the gg incoming 'beams'.

Even for inclusive dijet production, $p\overline{p} \to X + jj + \overline{X}$, the condition $(\Delta P_T)^2 \ll P_T^2$ is usually satisfied, unless the experimental criteria insist otherwise. Thus we shall assume the above kinematics in this paper.

Suppose for the moment we continue to work at the quark level, $qq \to q + jj + q$. We denote the amplitude for the process by T and write the differential cross section

$$\frac{d\sigma}{dP_T^2\, d\eta\, d\Delta\eta\, dt_1\, dt_2} \;=\; \frac{|T|^2}{(16\pi^2)^2}. \qquad (6)$$

Here we have used the identity

$$d^2\Delta P_T\, d^2q'_{1T}\, d^2q'_{2T}\, \delta^{(2)}\left(\Delta \boldsymbol{P}_T + \boldsymbol{q}_{1T} + \boldsymbol{q}_{2T}\right) \;=\; \pi^2\, dt_1 dt_2 \qquad (7)$$

where q'_1 and q'_2 are the momenta of the outgoing quarks. The amplitude T is proportional to the amplitude $\mathcal{M}$ describing the on-shell $gg \to jj$ subprocess. We normalise $\mathcal{M}$ by

$$\frac{d\hat{\sigma}}{d\hat{t}} \;=\; |\mathcal{M}|^2. \qquad (8)$$

To relate $d\hat{t}$ to dP_T^2 we note that at fixed M^2 we have

$$\frac{d\hat{t}}{dP_T^2} \;\simeq\; \frac{1 + e^{-\Delta\eta}}{1 - e^{-\Delta\eta}} \;\simeq\; M^2\, \frac{d(\Delta\eta)}{dM^2}. \qquad (9)$$

This identity means that the form of the differential cross section of the subprocess which emerges naturally from multi-Regge kinematics can be written as

$$\frac{d\hat{\sigma}}{dP_T^2}\, \frac{dM^2}{M^2} \;=\; \frac{d\hat{\sigma}}{d\hat{t}}\, d(\Delta\eta). \qquad (10)$$

3. Exclusive dijet production

We begin with the calculation of the double diffractive dijet production where the proton (antiproton) remains intact. The Born amplitude for quark-initiated production is described by the Feynman diagram shown by the solid lines in Fig. 1. We must explain the origin of the second t channel gluon. Without the rapidity gap restriction the dijet system could simply be produced by gluon-gluon fusion. However, the colour flow induced by such a single gluon exchange process would then produce many secondary particles which would fill up the rapidity gap. To screen the colour flow it is necessary to exchange a second t channel gluon. At lowest order in α_S this gluon couples only to the incoming quark lines. (The case in which the screening gluons also couple to the high E_T jets is of higher order in α_S). Thus the Born amplitude is given by

$$T(qq \to q + jj + q) \;=\; \frac{2}{9}\, 2 \int \frac{d^2 Q_T}{Q^2 k_1^2\, k_2^2}\, 8\alpha_S^2(Q_T^2) \hat{\mathcal{M}} \qquad (11)$$

where $\frac{2}{9}$ is the colour factor for the two-gluon colour-singlet exchange process and the factor 2 takes into account that both t channel gluons can radiate the dijet system, see, for example, [3, 5]. The amplitude $\hat{\mathcal{M}}$ represents the sum of the Feynman diagrams for the subprocess $gg \to jj$. In addition to the usual Mandelstam variables $\hat{s}$ and $\hat{t}$, the amplitude $\hat{\mathcal{M}}$ depends on the transverse momenta $\boldsymbol{k}_{iT}$ of the incoming gluons. We work in the limit where $k_{iT}^2 \ll E_T^2$. In this limit the off-shell amplitude is of the form

$$\hat{\mathcal{M}} \;=\; k_{1T}\, k_{2T}\, \mathcal{M}(\hat{s}, \hat{t}), \tag{12}$$

which reflects the gauge invariance of the amplitude. The remaining factor $\mathcal{M}$ is the amplitude which describes the on-shell $gg \to jj$ subprocess, which was introduced in (8) and which fixes the normalisation of (11) and (12). In the leading log approximation the origin of the k_{iT} factors in (12) is clear from the well-known Weizsäcker-Williams formula. The QCD analogue can be found in ref. [7]. The k_{iT} factors occur since the forward emission of massless vector particles without spin flip is forbidden [8].

If the momentum transfers are small ($k_1^2 \approx k_2^2 \approx Q^2$) then the integral in (11) behaves as $\int dQ_T^2/Q_T^4$. Thus small values of Q_T of the screening gluon are favoured. Fortunately, as we shall see, the existence of the rapidity gaps and the consequent Sudakov form factor suppression make the integral infrared convergent.

The Sudakov form factor F_S is the probability not to emit bremsstrahlung gluons (one of which is shown by p_T in Fig. 1). We have

$$F_S \;=\; \exp(-S(Q_T^2, E_T^2)) \tag{13}$$

where S is the mean multiplicity of bremsstrahlung gluons

$$S(Q_T^2, E_T^2) \;=\; \int_{Q_T^2}^{E_T^2} \frac{dp_T^2}{p_T^2} \int_{p_T}^{\frac{1}{2}M} \frac{d\omega}{\omega}\, \frac{3\alpha_S(p_T^2)}{\pi} \;=\; \frac{3\alpha_S}{4\pi}\, \ln^2\left(\frac{E_T^2}{Q_T^2}\right). \tag{14}$$

Here ω and p_T are the energy and transverse momentum of an emitted gluon in the dijet rest system. Note that E_T is the transverse energy of the jet adjacent to the 'hard' gluon from which the bremsstrahlung takes place. The last equality in (14) assumes a fixed coupling α_S and $E_T \simeq M/2$, and is shown only for illustration. The lower limit of itegration in (14) reflects the destructive interference of amplitudes in which the bremsstrahlung gluon is emitted from a 'hard' gluon k_i and from the soft screening gluon Q. That is there is no emission when the wavelength of the bremsstrahlung gluon ($\simeq 1/p_T$) is larger than the separation, $\Delta\rho \sim 1/Q_T$, of the two t channel gluons in the transverse plane, since then they act as a single coherent colour-singlet system. However, the situation is a little more complicated. As it stands (13) and (14) represent to double logarithmic accuracy, the probability to have no bremsstrahlung at all. But, in a realistic experiment we do not exclude bremsstrahlung in the rapidity interval embracing the two larger E_T jets. That is in practice it is difficult to distinguish the bremsstrahlung gluons from gluons which belong to the jets. Only emission in some fixed rapidity interval $\Delta\eta(\text{veto})$ is vetoed in an experiment. For example, the D0 collaboration [2] at the Tevatron choose a rapidity gap interval of $\Delta\eta(\text{veto}) \;=\; (\eta_{\min} = 2, \eta_{\max} = 4.1)$. The suppression of (14) should therefore only act in the rapidity interval $\Delta\eta(\mathrm{veto})$. Now the rapidity of the bremsstrahlung gluon is $\eta_b = \ln(\omega/2p_T)$. The relevant integration in (14) becomes

$$\int \frac{d\omega}{\omega} \rightarrow \int d\eta_b \tag{15}$$

where we must restrict the η_b integration to the rapidity interval $\Delta\eta(\mathrm{veto})$.

In addition to the form factor suppression we must also include the ladder evolution gluons (shown by the dashed lines in Fig. 1) and to consider the process at the proton-antiproton level rather than the quark level. Both changes are achieved by making the replacements [9]

$$\frac{4\alpha_S(Q^2)}{3\pi} \rightarrow f(x, Q^2) = \frac{\partial(xg(x,Q^2))}{\partial \ln Q^2} \tag{16}$$

in (11), where $x = x_1$ or x_2 for the upper or lower ladders in Fig. 1 respectively, and where $f(x, Q^2)$ is the unintegrated gluon density of the proton. The identification (16) is valid for small momentum transfer from the proton, which is the dominant region for the exclusive process. We may therefore set $k_{1T}^2 \approx k_{2T}^2 \approx Q_T^2 \approx Q^2$. Strictly speaking even at zero transverse momentum transfer, $q_{1T} - q_{1T}' = 0$, we do not obtain the exact gluon structure function, as a non-zero component of longitudinal momentum is transferred through the two-gluon ladder. However, in the region of interest, $x \sim 0.01$, the value of $|t_{\mathrm{min}}| = m_p^2 x^2$ is so small that we may safely put $t = 0$ and identify the ladder coupling to the proton with $f(x, Q^2)$ [9].

When we take the modifications (15) and (16) into account the Born amplitude (11) becomes

$$T(p\overline{p} \rightarrow p + jj + \overline{p}) = 2\pi^3 \int \frac{dQ^2}{Q^4}\, e^{-S(Q_T^2, E_T^2)}\, f(x_1, Q^2) f(x_2, Q^2) \mathcal{M}, \tag{17}$$

where $E_T = P_T$. In the limit $Q_T^2 \ll E_T^2$ the amplitude $\mathcal{M}$ essentially becomes the on-shell amplitude introduced in (8) and (12), which is simply a function of $\hat{s}$ and $\hat{t}$, and so may be taken outside the loop integral. Equation (17) can then be expressed in the symbolic form

$$T \equiv 16\pi^2 \mathcal{L}^{\frac{1}{2}} \mathcal{M} \tag{18}$$

where $\mathcal{L}$ may be regarded as the pomeron-pomeron luminosity factor. Clearly $\mathcal{L}$ depends on the choice of the rapidity gap configuration. The factor $16\pi^2$ arises due to the choice of normalisation in (8).

The differential cross section for $p\overline{p} \rightarrow p + jj + \overline{p}$ is given in terms of $|T|^2$ by

$$\frac{d\sigma}{dP_T^2\, d\eta\, d\Delta\eta} = \frac{|T|^2}{(16\pi^2)^2 b^2} = \frac{1}{b^2}\, \mathcal{L}\, \frac{d\hat{\sigma}}{d\hat{t}}, \tag{19}$$

where the last equality follows from (18) and (8). To obtain (19) we have integrated over $d(\Delta P_T)^2$ and the proton momentum transfer, see (7). These integrals are governed by the proton form factors. We obtain a factor of $1/b$ from both the proton and antiproton where exp $(-b|t_i|)$ is taken to be the approximate form of the proton form factor.

In the standard calculation of the cross section of a hard scattering process, such as $gg \to jj$, we would average over the colours and the polarisations of the incoming gluons. Here we have to be more careful. First, the cross section $d\hat{\sigma}/d\hat{t}$ describes dijet production in a colour singlet configuration. Second, in exclusive dijet production the polarisation vectors of the incoming gluons are directed along $\boldsymbol{k}_{iT}$, and hence are strongly correlated[3] since $\boldsymbol{k}_{1T} \simeq \boldsymbol{k}_{2T} \simeq \boldsymbol{Q}_T$. To determine $d\hat{\sigma}/d\hat{t}$ we perform the appropriate colour and polarisation averaging and obtain

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{9}{4} \frac{\pi\, \alpha_S^2(P_T^2)}{P_T^4},$$

which is in agreement with the cross section obtained in ref. [4].

It is easy to show that the integral in (17) has a saddle point given by

$$\ln(E_T^2/Q^2) = (2\pi/3\alpha_S(Q^2))(1-2\gamma) \tag{20}$$

where γ is the anomalous dimension of the gluon, $f(x,Q^2) \propto (Q^2)^\gamma$.

4. Inclusive dijet production

As is usual, the cross section for inclusive production is expected to be larger than for the exclusive process. Here the initial protons may be destroyed and the transverse momentum ΔP_T of the dijet system, (3), is no longer limited by the proton form factor, but it is still smaller than P_T in the leading log approximation. The process is shown in Fig. 2 in the form of the amplitude multiplied by its complex conjugate. The partonic quasielastic subprocess is $ab \to a' + jj + b'$. If the partons a, b are quarks then the Born amplitude for the subprocess is given by (11). However, the form factor suppressions are more complicated than for the inclusive process. As the momenta transferred, $t_i = (Q - k_i)^2$, are large we can no longer express the upper and lower 'blocks' in terms of the gluon structure function, but instead they are given by BFKL non-forward amplitudes.

We begin with the expression for the Born cross section for the subprocess $gg \to g + jj + g$

$$\frac{d\sigma}{dP_T^2 d\eta\, d\Delta\eta} = \alpha_S^4 \frac{81}{64\pi^2} \mathcal{I} \frac{d\hat{\sigma}}{d\hat{t}} \tag{21}$$

with

$$\mathcal{I} = \int \frac{dQ^2}{Q^2} \frac{dQ'^2}{Q'^2} \frac{dk_{1T}^2}{k_1^2\, k_1'^2} \frac{dk_{2T}^2}{k_2^2\, k_2'^2}\, k_{1T}\, k'_{1T}\, k_{2T}\, k'_{2T}, \tag{22}$$

where the six propagators of Fig. 2 are evident.

Again care is needed in the computation of the subprocess cross section $d\hat{\sigma}/d\hat{t}$. As we noted from (12) the off-shell $gg \to jj$ amplitude is proportional to the k_{iT} of the incoming gluons, and the remaining on-shell cross section $d\hat{\sigma}/d\hat{t}$ should be therefore computed averaging over gluon polarisations. The polarisations are described by vectors $\boldsymbol{\epsilon}_i$ which are proportional to the

[3] The correlation is absent for inclusive dijet production.

gluon $\boldsymbol{k}_{iT}$. Now the leading log contribution comes from the strongly-ordered region $k_{iT} \ll k_{jT}$ with $i \neq j$. As before comparatively small values of the momentum Q of the screening gluon are favoured. However now, without the presence of proton form factors, the total momentum transfer $Q - k_i$ may be large. In the limit $Q^2 \ll k_i^2$ this means that k_i has to be balanced by k_i' for both $i = 1, 2$. That is we have

$$t_i \;=\; (Q - k_i)^2 \;\simeq\; -k_{iT}^2 \;\simeq\; -k_{iT}'^2. \tag{23}$$

The consequence for the polarisation averaging is that we require $\boldsymbol{\epsilon}_i \;\simeq\; \boldsymbol{\epsilon}_i'$, but that $\boldsymbol{\epsilon}_i$ is no longer correlated to $\boldsymbol{\epsilon}_j$ (as it was for exclusive production). After averaging, the on-shell $gg \to gg$ cross section is found to be

$$\frac{d\hat{\sigma}}{d\hat{t}} \;=\; \frac{\pi \alpha_S^2(P_T^2)}{P_T^4}\, \frac{9}{2} \left(1 - \frac{P_T^2}{M^2}\right)^2, \tag{24}$$

while that for $gg \to q\bar{q}$ is

$$\frac{d\hat{\sigma}}{d\hat{t}} \;=\; \frac{\pi \alpha_S^2(P_T^2)}{P_T^2\, M^2}\, \frac{1}{6} \left(1 - \frac{2P_T^2}{M^2}\right) \tag{25}$$

for each flavour of quark.

We have chosen the scale of the coupling α_S to be P_T^2, that is the value which for single inclusive jet production gave small higher-order corrections and which led to predictions in agreement with the data.

Again we must estimate the suppression due to gluon bremsstrahlung filling up the rapidity gaps. Now the mean number of gluons emitted, with transverse momenta $Q_T < p_T < k_{iT}$, in the rapidity interval $\Delta\eta_i \;=\; \Delta\eta(\text{veto})$ is

$$n_i \;=\; \frac{3\alpha_S}{\pi}\, \Delta\eta_i\, \ln\left(\frac{k_{iT}^2}{Q_T^2}\right). \tag{26}$$

The amplitude for no emission in the gap $\Delta\eta_i$ is therefore $\exp(-n_i/2)$. In this way we see that the Born integral (22) is modified to

$$\mathcal{I} \;=\; \int \frac{dQ^2}{Q^2}\, \frac{dQ'^2}{Q'^2}\, \frac{dt_1}{t_1}\, \frac{dt_2}{t_2} \exp\Big(-(n_1 + n_1' + n_2 + n_2' + S_1 + S_1' + S_2 + S_2')/2\Big) \tag{27}$$

where the exponential factor represents the total form factor suppression in order to maintain the rapidity gaps $\Delta\eta(\text{veto})$. The Sudakov form factors, $\exp(-S(k_{iT}^2, E_T^2)/2) \;\equiv\; \exp(-S_i/2)$, arise from the insistence that there is no gluon emission in the interval $k_{iT} < p_T < E_T$, see (13) and (14). Again note that the Sudakov form factors are multilated due to the imposition of a specific rapidity gap interval $\Delta\eta(\text{veto})$, see the replacement given in (15).

The justification of the non-Sudakov form factors, $\exp(-n_i/2)$ is a little subtle. First we notice from (27) that due to the asymmetric configuration of the t-channel gluons, $Q_T \ll k_{iT}$, we have, besides $\Delta\eta_i$, a second logarithm, $\ln(k_{iT}^2/Q_T^2)$, in the BFKL evolution. These double logs are resummed[4] to give the BFKL non-forward amplitude $\exp(-n_i/2)\Phi(Y_i)$, where the remaining factor $\Phi(Y_i)$ accounts for the usual longitudinal BFKL logarithms[5],

$$Y_i \;\equiv\; (3\alpha_S/2\pi)\Delta\eta_i. \tag{28}$$

For rapidity gaps with $\Delta\eta_i \lesssim 4$ we have $Y_i \lesssim 0.5$, and it is sufficient to include only the $\mathcal{O}(Y_i)$ term, which gives $\Phi \;\simeq\; 1 + Y_i Q_T^2/k_{iT}^2 \;\simeq\; 1.1 \pm 0.1$ [10]. At our level of accuracy we may neglect the enhancement due to Φ, and hence we obtain (27), which is valid in the double log approximation.

To evaluate $\mathcal{I}$ of (27) we first perform the Q^2 and Q'^2 integrations and obtain $(Y_1+Y_2)^{-2}$. Then we integrate over $\ln(t_1/t_2)$ which gives $\frac{1}{2}(1/Y_1 + 1/Y_2)$ where, at large $\Delta\eta_i$, we neglect the t_i dependence of S_i. Thus (27) becomes

$$\mathcal{I} \;=\; \frac{1}{2Y_1Y_2(Y_1+Y_2)} \int^{E_T^2} \frac{dt}{t}\, \exp(-2S(t, E_T^2)). \tag{29}$$

For fixed α_S the final (dt) integration gives $\pi(6\alpha_S)^{-\frac{1}{2}}$ in the double log approximation. However, to predict the cross section for inclusive production we must convolute the parton-parton cross sections with the parton densities $a(x_a, t)$ of the proton, with $a = g$ or q, and evaluate the dt integral numerically. There is a subtlety when we come to include these parton luminosity factors

$$\int_{x_{\rm min}}^1 dx_a\, a(x_a, k_{1T}^2)\, \ldots,$$

where a summation over $a = g, q$ is implied. At first sight we might expect $x_{\rm min} = M/\sqrt{s}$ for central dijet production. However, at large k_{iT} the rapidities of the a', b' jets are small in the dijet rest frame; $\eta_{a'} = \ln(x_{a'}\sqrt{s}/k_{1T})$. Thus in order to maintain the rapidity gaps $(\eta_{a'} > \eta_{\rm max})$, we must take

$$(x_1)_{\rm min} \;=\; [Me^{\eta} \,+\, k_{1T}\, e^{\eta_{\rm max}}]/\sqrt{s}, \tag{30}$$

and similarly for $(x_2)_{\rm min}$.

So far we have considered the simplest $gg \to jj$ hard subprocess. However, if the virtuality k_i^2 of the incoming gluons is much smaller than P_T^2 of the outgoing jets, then we must discuss the possibility of DGLAP evolution in the (k_{iT}, P_T) interval. The evolution means that one (or more) extra jets may be emitted with transverse momentum q_T in this interval. At first sight it appears that the order α_S correction will be enhanced by a factor $\ln(P_T^2/k_{iT}^2)$. Indeed such a contribution would arise if $\eta_{\rm min}$ is sufficiently large so that $\alpha_S\, \eta_{\rm min} \;\sim\; 1$. The situation can be described with reference to Fig. 2. The incoming proton 'fragments' into a system of partons with rapidities $\eta > \eta_{\rm max}$. This process is described by DGLAP evolution which effectively sums up the collinear logs. The leading log contribution comes from the configuration

[4]The resummation corresponds to the Reggeization of the t-channel gluons.

[5]Here $\Delta\eta_i$ (or Y_i) plays the role of $\ln(1/x)$ in the BFKL evolution.

where the angles of the secondary partons are strongly ordered. The effect is described by the parton distribution $a(x_a, k_{1T}^2)$. The emission of partons with larger opening angles in the range $\theta_{\rm min} < \theta < \theta_{\rm max}$ (corresponding to the rapidity gap $\eta_{\rm max} < \eta < \eta_{\rm min}$) is experimentally vetoed. The resulting suppression of the cross section is taken into account by the $\exp(-S_i/2)$ and $\exp(-n_i/2)$ factors in (27). Nevertheless, starting from $\theta = \theta_{\rm max}$, the DGLAP evolution may be continued to larger angles. Thus, as well as the Born process $gg \to jj$, we should include more complicated inclusive subprocesses such as $gg \to jjg$ shown in Fig. 3. Fortunately if $\theta_{\rm max}$ is sufficiently large, or equivalently if $\eta_{\rm min}$ is sufficiently small, the probability of extra jet emission (which is proportional to $\alpha_S \eta_{\rm min}$) may be neglected. In other words, the presence of the rapidity gap removes the main part of the DGLAP enhancement, that is it kills the $\ln(P_T^2/k_{iT}^2)$ factor which would have occurred due to emission in the rapidity gap interval in the absence of the experimental veto.

5. Predictions for the double-diffractive dijet cross section

Our main objective is to estimate the dependence of the cross section for central dijet production in pp (or $p\overline{p}$) collisions on the imposition of rapidity gaps. An understanding of this double diffractive process is important. On the one hand the $b\overline{b}$ dijet channel is the background to Higgs production from either WW or pomeron-pomeron fusion. On the other hand, dijet production provides an observable test of novel QCD double logarithmic effects. That is it is possible to study how the event rate varies according to the experimental choice of the rapidity gaps in which QCD double logarithmic gluon emission is forbidden.

5.1 The inclusive cross section

Recall that the calculation is done in the leading log approximation and that we anticipate that the *inclusive* configuration, $pp \to X + jj + X$, plays the dominant role. We present the cross section in the differential form of (21) for central dijet production with rapidity $\eta \equiv \frac{1}{2}(\eta_1 + \eta_2) = 0$. The results are shown in Table 1 for both FNAL and LHC energies of $\sqrt{s} = 1.8$TeV and 14TeV respectively.

For the FNAL predictions the rapidity gaps were chosen to be those used by the D0 collaboration [2], that is $\eta_{\rm min} = 2$ and $\eta_{\rm max} = 4.1$. The jets were taken to have $P_T = 15$GeV. At the LHC energy we took $P_T = 50$GeV, and besides the above choice of rapidity gap, we also present results for a larger gap with $\eta_{\rm min} = 2$, $\eta_{\rm max} = 6$ so as to explore the sensitivity to the gap size $\Delta\eta$(veto). We calculated the cross section using various recent sets of partons. The values in Table 1 were obtained using the MRS(R2) set of partons [11].

¿From the Table we see that, for $\eta = 0$ and $\Delta\eta \sim 1$, the inclusive dijet cross sections at LHC and FNAL are

$$\frac{d\sigma}{dP_T^2\, d\eta\, d\Delta\eta} \approx \begin{cases} 2\,\text{pb/GeV}^2 & \text{at } \sqrt{s} = 14\text{TeV} \quad (\text{with } P_T = 50\text{GeV}) \\ 100\text{pb/GeV}^2 & \text{at } \sqrt{s} = 1.8\text{TeV} \quad (\text{with } P_T = 15\text{GeV}). \end{cases} \tag{31}$$

The larger value at the Tevatron energy simply reflects the $1/P_T^4$ behaviour. Note that the cross sections are rather large. For example if we integrate over P_T^2 using the above values of P_T as the lower bounds then the cross sections are approximately given by multiplying the quoted values by P_T^2 in GeV^2. That is an integrated cross section of about 5nb at LHC with $P_T > 50\text{GeV}$, and 20nb at the Tevatron with $P_T > 15\text{GeV}$.

The above large cross section values do not take into account the possibility of multiple parton-parton scattering (see, for example, ref. [5]). The secondary hadrons produced in such a rescattering will tend to fill up the original rapidity gaps. We thus have to multiply the cross sections in the table by a factor W which is the probability not to have an inelastic rescattering. To estimate W we may use [12]

$$W \;=\; \left(1 - \frac{2(\sigma_{\rm el} + \sigma_{\rm SD} + \sigma_{\rm DD})}{\sigma_{\rm tot}}\right)^2 \;=\; 0.06 \quad \text{at } \sqrt{s} \;=\; 1.8\text{TeV}, \tag{32}$$

where $\sigma_{\rm el}$, $\sigma_{\rm SD}$, $\sigma_{\rm DD}$ and $\sigma_{\rm tot}$ are the elastic, single and double diffractive and total $p\bar{p}$ cross sections respectively. The numerical estimate in (32) is obtained using the CDF measurements [13] of $\sigma_{\rm tot}$, $\sigma_{\rm el}$ and $\sigma_{\rm SD}$. For the double diffractive cross section we use the factorization relation $\sigma_{\rm DD} \;=\; (\sigma_{\rm SD})^2/\sigma_{\rm el}$. We note that earlier $p\bar{p}$ cross section measurements [14] would give $W = 0.025$. The smaller value is mainly due to the smaller measured $\sigma_{\rm tot}$.

Alternative ways to estimate W can be found in refs.[15]. W is a common overall suppression factor which affects the cross section for any process with one or more rapidity gaps[6]. Thus, although it gives an added uncertainty to the overall normalisation of the cross section, it should not modify the form of the η, $\Delta\eta$ and P_T dependence.

¿From the Table we also see that the $pp \to X + b\bar{b} + X$ cross section is about 1% of the whole cross section for inclusive double-diffractive dijet production, $pp \to X + jj + X$. That is the dijets are dominantly gluon-gluon jets. As mentioned above, the estimate of $b\bar{b}$ production with rapidity gaps is relevant in assessing the background to the production of a Higgs boson of intermediate mass.

The predictions for *inclusive* dijet production are stable to the use of different recent sets of parton distributions and to different treatments of the infrared region. The reason is that the saddle point of the integration of (27) lies in the perturbative region. For the LHC energy it occurs at $k_T^2 \simeq 20\text{GeV}^2$ and $Q^2 \simeq 4\text{GeV}^2$, while for the FNAL energy it is at $k_T^2 \simeq 2.5\text{GeV}^2$ and $Q^2 > 1\text{GeV}^2$. Even in the latter case the uncertainty due to partons and the infrared contribution is only $\pm 10\%$.

We find that there is about 50% suppression due to the Sudakov form factor. The suppression arising from the presence of the BFKL non-forward amplitudes is strongly dependent on the size of the rapidity gap. For example at LHC energies the cross section for dijet production with the larger gap, $\Delta\eta(\text{veto}) \;=\; (2,\, 6)$, is a factor of 100 smaller than that for $\Delta\eta(\text{veto}) \;=\; (2,\, 4.1)$.

[6] If for very high energy pp collisions the suppression factor W becomes extremely small, then the subprocess $\gamma\gamma \to jj$ could become competitive. The reason is that this subprocess arises from large impact parameters where rescattering is essentially absent.

The rapid decrease with increasing $\Delta\eta$(veto) is a characteristic feature of perturbative Pomeron effects on this process. It comes mainly from the presence of non-forward BFKL amplitudes which in turn arise from the asymmetric gluon exchange configurations where Reggeization is important. It should be readily observable.

5.2 The exclusive cross section

The calculation of the cross section for the *exclusive* process, $pp \to p + jj + p$, is much more dependent on the infrared region. The problem is that the main contribution to the integral in (17) comes from rather small values of Q, even when the Sudakov form factor is included. The predictions are therefore sensitive to the gluon density in the region $Q^2 \approx 1\text{GeV}^2$, or less, where it is not well defined. Of course double-diffractive dijet hadroproduction is dominated by the inclusive process and so the computation of the exclusive cross section is not so important. Nevertheless, for completeness, we give an estimate of the cross section.

The procedure that we follow is similar to that used in refs.[3, 4]. The idea is to use an unintegrated gluon density $f(x, l_T^2)$ which is truncated at low transverse momentum l_T in such a way as to reproduce the observed value of the total (inelastic) pp cross section. We start from the Low-Nussinov two-gluon exchange model for the cross section. The Born amplitude gives

$$\sigma_{pp} \;=\; 4\pi\alpha_S^2 \int \frac{dl_T^2}{l_T^4}\,\frac{2}{9}\,(3)(3), \tag{33}$$

where we integrate over the transverse momentum l_T of the gluons exchanged between 3 (valence) quarks of the protons. As usual, $\frac{2}{9}$ is the colour factor. We may improve this estimate by rewriting (33) in terms of the unintegrated gluon density, see the quark level formula (16),

$$\sigma_{pp} \;=\; \frac{\pi^3}{2} \int \frac{dl_T^2}{l_T^4}\, f(x, l_T^2)\, f(x, l_T^2) \tag{34}$$

where $x = 2l_T/\sqrt{s}$. In the perturbative region $l_T^2 > l_0^2$ the right hand side is known. We can therefore insert the value of the inelastic cross section, $\sigma_{pp} \approx 45\text{mb}$, measured at FNAL to determine the infrared contribution to the integral. We may either use a "sharp cut-off", putting $f = 0$ for $l_T^2 < l_0^2$, or employ a "soft cut" by extrapolating into the region $l_T^2 < l_0^2$ using the linear form

$$f(x, l_T^2) \;=\; (l_T^2/l_0^2)\, f(x, l_0^2). \tag{35}$$

To reproduce the observed cross section we find that we need to take the sharp cut-off at $l_0 = 1.1\text{GeV}$, or alternatively to choose the soft cut starting at $l_0 = 1.5\text{GeV}$.

We assume that this procedure can be taken over to evaluate the infrared contribution to the exclusive dijet cross section of (17). The predicted values of the cross section are shown in Table 1 for both treatments of the infrared region. We see that the exclusive double diffractive dijet cross section is a factor of about 20 or 150 smaller than the inclusive one at the Tevatron or LHC energies respectively. The suppression due to the double logarithmic Sudakov factor is

$\frac{1}{2}$ for $P_T = 15$GeV jets at the Tevatron, whereas it is $\frac{1}{10}$ for $P_T = 50$GeV jets at the LHC.

6. Discussion

The observation of processes with rapidity gaps is of great interest for understanding the structure of the perturbative Pomeron. In this respect the central production of dijets with a rapidity gap on either side is an ideal "perturbative laboratory". The process has a large cross section and may be studied in detail at the Tevatron and at the LHC. Here we have obtained a formalism which allows an estimate of the cross section and which systematically takes into account the main effects, some of which have not been considered before. The dijet system is produced by the fusion of two gluons (of momenta k_1 and k_2). A second t channel gluon exchange (of momentum Q) is needed to neutralise the colour flow. The main contribution to the cross section comes when the screening gluon is comparatively soft ($Q^2 \ll k_i^2$), yet Q^2 is large enough to allow the inclusive cross section to be reliably estimated by perturbative QCD. The cross section is found to be suppressed by Sudakov form factors and by non-forward BFKL amplitudes, the latter arising from the asymmetric two-gluon exchange configuration.

We presented sample results for the cross section which demonstrate the scale of the effects. We chose the rapidity gaps to correspond to those used by the D0 collaboration [2] at FNAL. However, at the LHC energy we also presented results for larger rapidity gaps. The main uncertainty is from rescattering effects, which will populate the gaps. The cross sections presented in Table 1 do not include the suppression factor arising from the requirement to have no rescattering. We gave an estimate of this factor in (32).

It is interesting to note that the calculation of dijet production with a single rapidity gap is much more complicated, and less informative theoretically, than that of the process with a rapidity gap on either side of the dijet. The problem is the gluon that screens the colour flow many now couple, not only to the initial parton, but to any gluon or secondary parton on the side where we have no rapidity gap. Typically the main contribution comes from the graphs where it interacts with the parton which has q_{Ti} of the order of the Q of the screening gluon. Thus one has to follow the history and evolution from some cut-off μ up to the transverse momentum P_T of the dijet system, and to sum up the resulting "interacting" diagrams. As there is no second rapidity gap there is no strong suppression of the cross section. Moreover, the dominant region of Q, in the integral analagous to (27), will be closer to the infrared region than it was for dijet production with two rapidity gaps.

Finally we emphasize that we have worked at the partonic level. That is the rapidity intervals are defined with respect to the emitted partons. In a realistic experimental situation our results therefore correspond to smaller rapidity gaps for the hadrons since a gluon produced just outside the $\Delta\eta$(veto) interval may produce a secondary inside the gap. To obtain an indication of the size of the effect we recomputed the cross section at the Tevatron energy (for $\eta = 0,\ \Delta\eta = 0,\ P_T = 15$GeV) with $\eta_{\min} \to \eta_{\min} - 0.5$ and $\eta_{\max} \to \eta_{\max} + 0.5$ and found that it was suppressed by a further factor of 12.

The rescattering and rapidity gap broadening effects give the main uncertainties in our cross section estimates. The first estimates indicate that the combined effect will diminish the perturbative predictions in Table 1 by more than two orders of magnitudes. The broadening of the gap can be simulated by Monte Carlo studies and can be readily accounted for when the data is analysed. However the rescattering estimate is model dependent and requires confirmation by studying multiplicity distributions and the energy behaviour of diffractive cross sections.

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References

[1] J.D. Bjorken, *Int. J. Mod. Phys.* **A7** (1992) 4189; *Phys. Rev.* **D45** (1992) 4077; *Phys. Rev.* **D47** (1993) 101.

[2] D0 collaboration: M. Abolins, talk presented at INFN Eloisatron project 33 workshop, Erice, Italy, October 1996; S. Abachi et al., Fermilab preprint - CONF-96-247-E (contribution to Int. Conf. on HE Physics, Warsaw, July 1996).

[3] J. Pumplin, *Phys. Rev.* **D52** (1995) 1477.

[4] A. Berera and J.C. Collins, *Nucl. Phys.* **B474** (1996) 183.

[5] V.A. Khoze, A.D. Martin and M.G. Ryskin, Durham preprint DTP/97/12, *Phys. Lett.* (in press).

[6] Ya. I. Azimov, V.A. Khoze, L.N. Lipatov and A.I. Vainstein, *JETP Lett.* **21** (1975) 172.

[7] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298.

[8] V.N. Baier, V.S. Fadin and V.A. Khoze, *Nucl. Phys.* **B65** (1973) 381.

[9] M.G. Ryskin, R.G. Roberts, A.D. Martin and E.M. Levin, Durham preprint DTP/95/96, *Z. Phys.* **C** (in press).

[10] J.R. Forshaw and M.G. Ryskin, *Z. Phys.* **C68** (1995) 137.

[11] A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Lett.* **B387** (1996) 419.

[12] A. Rostovtsev and M.G. Ryskin, *Phys. Lett.* **B390** (1997) 375.

[13] CDF collaboration, F. Abe et al., *Phys. Rev.* **D50** (1994) 5550.

[14] E710 collaboration, N.A. Amos et al., *Phys. Lett.* **B243** (1990) 158.

[15] Yu.L. Dokshitzer, V.A. Khoze and T. Sjöstrand, *Phys. Lett.* **B274** (1992) 116; E. Gotsman, E.M. Levin and U. Maor, *Phys. Lett.* **B309** (1993) 199 and **B353**(1995) 526; T.L. Lungov and C.O. Escobar, *Phys. Rev.* **D53** (1996) 4857.

$\Delta\eta$	$\sigma_{\rm inc}$ (pb/GeV^2)	$\sum_q \frac{\sigma_{\rm inc}^{q\overline{q}}}{\sigma_{\rm inc}}$	$\frac{\sigma_{\rm inc}^{b\overline{b}}}{\sigma_{\rm inc}}$	$(k_T^2)_{\rm sad.pt}$ (GeV^2)	$\sigma_{\rm exc}$ (pb/GeV^2)
LHC ($\sqrt{s} = 14$TeV) :					
0	2.4 (0.021)	4%	0.8%	20	0.026 (0.027)
1	2.4 (0.019)	3%	0.7%	22	0.020 (0.021)
2	2.0 (0.014)	2%	0.4%	28	0.010 (0.011)
FNAL ($\sqrt{s} = 1.8$TeV) :					
0	110 (8.7)	4%	0.8%	2.5	3.9 (5.5)
1	110 (8.2)	3%	0.6%	2.7	2.9 (4.2)
2	88 (5.8)	2%	0.3%	3.4	1.2 (1.9)

Table 1: The double-diffractive inclusive dijet cross section $\sigma_{\rm inc}$ as in (31) evaluated at $\eta = 0$ for three values of rapidity difference of the jets $\Delta\eta = \eta_1 - \eta_2$. The first number for $\sigma_{\rm inc}$ corresponds to the rapidity gap choice $\Delta\eta({\rm veto}) = (2, 4.1)$, whereas the number in brackets corresponds to $\Delta\eta({\rm veto}) = (2, 6)$ for LHC and (1.5, 4.6) for the Tevatron. Also shown are the percentage of events where the dijets are quark-antiquark pairs summed over all types of quarks, and the percentage of $b\overline{b}$ events. $(k_T^2)_{\rm sad.pt}$ is the saddle point of the integration of (27). The final column is the exclusive dijet cross section, first evaluated using a sharp cut-off in (17) and, second (in brackets), using the soft cut-off of (35). The transverse momenta of the jets are taken to be $P_T = 50(15)$GeV at LHC(Tevatron) energies.

Figure Captions

Fig. 1 The Born amplitude for exclusive double-diffractive dijet production shown at the quark level, together with the QCD radiative corrections arising from 'evolution' gluons (dashed lines) and the Sudakov-type form factor suppression associated with the $\Delta\eta$(veto) rapidity gaps. The comparatively soft screening gluon has four-momentum Q. The rapidity gaps are indicated by $\Delta\eta$(veto) and the two large P_T jets are required to lie in the rapidity interval $\Delta\eta$(dijet).

Fig. 2 The amplitude multiplied by its complex conjugate for inclusive double-diffractive dijet production with rapidity gaps $\Delta\eta_1$ and $\Delta\eta_2$ on either side, $\Delta\eta_i = \pm(\eta_{\min}, \eta_{\max})$. The suppression due to QCD radiative effects comes from the double log resummations $\exp(-n_i/2)$ in the BFKL non-forward amplitudes and from the Sudakov-like form factor suppressions $\exp(-S_i/2)$ associated with the rapidity gaps along the hard gluon lines.

Fig. 3 A schematic diagram of the subprocess $gg \to jjg$, where the additional jet with transverse momentum q_T is emitted with $\theta > \theta_{\max}$. Only the vetoed region in the forward direction is shown. There is also a rapidity gap in the backward hemisphere.