Small–Angle Electron-Positron Scattering †

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Abstract

We consider small–angle electron–positron scattering in Quantum Electrodynamics. Leading logarithmic contributions to the cross–section are explicitly calculated to three loop. Next–to–leading terms are exactly computed to two loop. All the radiative corrections due to photons as well as pair production are taken into account. The impact of newly evaluated next-to-leading and higher order leading corrections is discussed and numerical results are explicitly given. The results obtained are generally valid for high and low energy e^+e^- colliders. At LEP and SLC these results can be used to reduce the uncertainty on the cross–section below the per mille level.

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An high accuracy, better than one *per mille,* measurement of the luminosity has been reached at LEP [1]. The small–angle electron–positron scattering (Bhabha) process is normally used as the reference cross–section to measure the luminosity.

A poorly known Bhabha cross–section has the consequence of producing a systematic error on the determination of relevant physical observables as, for example, the hadronic peak cross–section σ^h_{peak} or the leptonic widths $\Gamma_{e,\mu}$. This poor accuracy reflects itself on the extraction of the Standard Model parameters on the exclusion of new physics signals as, for example, those coming from a deviation from the Standard Model prediction for the number of light neutrinos [1]. The experimental uncertainty in the luminosity determination together with the theoretical one define the uncertainties on the above mentioned quantities. An adequate theoretical accuracy on the cross–section is therefore highly needed.

Various approaches have been used to obtain the Bhabha cross–section [3, 4]. Some of them (see ref. [3]) are based on Monte Carlo generators. Others use the structure function method to calculate the radiative corrections [4]. With both methods the contributions of next–to–leading as well as higher order radiative corrections have not been systematically evaluated and this yielding an important source of uncertainty above the level of $\delta\sigma/\sigma \simeq 0.001$ accuracy. Therefore an equally accurate theoretical determination of the Bhabha cross–section, even if it is approaching to, has not yet reached the experimental accuracy [2].

In this letter we describe the results obtained with a different approach based on the direct evaluation of Feynman diagrams. The results, given in analytical form, systematically take into account leading as well as next–to–leading contributions thus reducing the physical uncertainty to the 0.1% level.

The differential cross–section for the small-angle Bhabha process in the Standard Model is [5]:

$$\begin{aligned} \frac{\mathrm{d}\sigma_B}{\theta \mathrm{d}\theta} &= \frac{8\pi\alpha^2}{\theta^4}(1+\delta_\theta+\delta_{\mathrm{weak}}), \quad \theta \ll 1, \quad \delta_\theta = -\theta^2/2 + 9\theta^4/40, \qquad (1) \\ \delta_{\mathrm{weak}} &= 2g_v^2\xi - \frac{\theta^2}{4}(g_v^2+g_a^2)\Re\mathrm{e}\,\chi + \frac{\theta^4}{32}(g_v^4+g_a^4+6g_v^2g_a^2)|\chi|^2, \\ \chi &= s[(s-M_Z^2+iM_Z\Gamma_Z)\sin(2\theta_W)]^{-1}, \quad \xi = t[(t-M_Z^2)\sin(2\theta_W)]^{-1}, \end{aligned}$$

where θ is the scattering angle, $s=4E^2$, $t=-E^2\theta^2$, E is the beam (center-of-mass) energy, θ_W the Weinberg angle and $g_a=-1/2$, $g_v=-(1-4\sin^2\theta_W)/2$ the axial and vector couplings of the Z^0 boson. Experimental cross-sections are obtained by collecting events within particular angular portions of the detectors with given energy cuts. To compare with those observed distributions one has to take into account radiative corrections to higher orders in perturbation theory and it is convenient to introduce the dimensionless quantity:

$$\Sigma = \frac{Q_1^2}{4\pi\alpha^2}\int\limits_0^1 \mathrm{d}x_1 \int\limits_0^1 \mathrm{d}x_2 \Theta(x_1x_2-x_c)\int \mathrm{d}^2\boldsymbol{q}_1^\perp\Theta_1^c \int \mathrm{d}^2\boldsymbol{q}_2^\perp\Theta_2^c \frac{\mathrm{d}\sigma^{e^+e^-\to e^+(\boldsymbol{q}_2^\perp,x_2)e^-(\boldsymbol{q}_1^\perp,x_1)+X}}{\mathrm{d}x_1 d^2\boldsymbol{q}_1^\perp \mathrm{d}x_2\mathrm{d}^2\boldsymbol{q}_2^\perp}, \qquad (2)$$

where x_c is the energy fraction threshold for the detection of the final electron and positron $x_c \leq x_1x_2$, where $x_{1,2}$ are the energy fractions of the scattered leptons and $\boldsymbol{q}_1^\perp$, $\boldsymbol{q}_2^\perp$ are the components of their momenta transverse with respect to the beam directions. $\Theta^c_{1,2}$ are the step functions which account for the registration of the scattered leptons by the circular detectors situated close to the beams with an aperture bounded by the angles $\theta_{1,2,3,4}$:

$$\Theta_1^c = \Theta(\theta_3 - \frac{|\boldsymbol{q}_1^\perp|}{x_1 E})\Theta(\frac{|\boldsymbol{q}_1^\perp|}{x_1 E} - \theta_1), \qquad \Theta_2^c = \Theta(\theta_4 - \frac{|\boldsymbol{q}_2^\perp|}{x_2 E})\Theta(\frac{|\boldsymbol{q}_2^\perp|}{x_2 E} - \theta_2). \tag{3}$$

If θ_1 is the minimal aperture angle we define the minimal momentum transferred from the electron to the positron as $Q_1^2 = E^2\theta_1^2$. At LEP/SLC $Q_1 \simeq 1\ GeV/c$. For symmetric detectors: $\theta_1 = \theta_2$, $\theta_3 = \theta_4$, $\rho = \theta_3/\theta_1$.

$$\begin{aligned}\Sigma &= \Sigma_0 + \Sigma^\gamma + \Sigma^{\gamma\gamma} + \Sigma_\gamma^\gamma + \Sigma^{e^+e^-} + \Sigma^{3\gamma} + \Sigma^{e^+e^-\gamma} \\ &= \Sigma_0^0(1+\delta_{\mathrm{tot}}) = \Sigma_0^0(1+\delta_0+\delta^\gamma+\delta^{2\gamma}+\delta^{e^+e^-}+\delta^{3\gamma}+\delta^{e^+e^-\gamma}) \\ &= \Sigma_0^0(1+\sum_i \delta^i), \qquad \delta^{2\gamma} \equiv \delta^{\gamma\gamma} + \delta_\gamma^\gamma, \qquad \Sigma_0^0 = 1 - \frac{1}{\rho^2}\,,\end{aligned} \tag{4}$$

Σ_0 has the form:

$$\Sigma_0 = \int_1^{\rho^2} \frac{\mathrm{d}z}{z^2} U(-zQ_1^2)(1+\delta_{\mathrm{weak}}(\theta^2)+\delta_\theta(\theta^2))\Big|_{\theta^2 = z\theta_1^2} \tag{5}$$

with $U(t) = (1-\Pi(t))^{-2}$ where $\Pi(t)$ is the vacuum polarization operator of the exchanged photon [6]. The experimentally observable cross–section is given by the expression:

$$\sigma = \frac{4\pi\alpha^2}{Q_1^2}\Sigma_0^0(1+\delta_0+\delta^\gamma+\delta^{2\gamma}+\delta^{e^+e^-}+\delta^{3\gamma}+\delta^{e^+e^-\gamma}). \tag{6}$$

The quantity Σ^γ collects the radiative corrections to the Born amplitude related to the emission of single virtual, real soft or hard photon. Virtual and soft photon contributions to the differential cross–section are proportional to the Born amplitude: $\mathrm{d}\sigma = \mathrm{d}\sigma_B(1+\frac{\alpha}{\pi}(2(L-1)\ln\Delta + 3L/2 - 2))$. This quantity contains the *large logarithm* L, $L = \ln(zQ_1^2/m^2)$ (m is the electron mass), and $\Delta = \delta E/E$, where δE is the maximal energy carried by a soft photon, $\Delta \ll 1$. The Δ–dependence disappears in the total sum when the emission of a hard photon with energy fraction larger than Δ is also taken into account.

A simple analytical expression for the small–angle Bhabha differential cross–section for one hard photon emission can be written within the the infinite-momentum frame formalism. One obtains:

$$\begin{aligned}&\frac{\mathrm{d}\sigma^H}{\mathrm{d}x\mathrm{d}^2\boldsymbol{q}_1^\perp\mathrm{d}^2\boldsymbol{q}_2^\perp} = \frac{2\alpha^2}{\pi^2}\frac{1+x^2}{((\boldsymbol{q}_2^\perp)^2)^2(1-x)}\Big[\frac{(\boldsymbol{q}_2^\perp)^2(1-x)^2}{d_1 d_2} - \frac{2m^2x(1-x)^2(d_1-d_2)^2}{(1+x^2)d_1^2d_2^2}\Big], \\ &d_1 = m^2(1-x)^2 + (\boldsymbol{q}_1^\perp - \boldsymbol{q}_2^\perp)^2 = (1-x)2p_1k, \\ &d_2 = m^2(1-x)^2 + (\boldsymbol{q}_1^\perp - x\boldsymbol{q}_2^\perp)^2 = x(1-x)2q_1k, \quad \boldsymbol{q}_2^\perp = \boldsymbol{q}_1^\perp + \boldsymbol{k}^\perp, \\ &x = \frac{q_1^0}{p_1^0}, \quad q_1 = xp_1 + \boldsymbol{q}_1^\perp, \quad p_1 + p_2 = q_1 + k + q_2\,,\end{aligned} \tag{7}$$

where p_1, p_2 and k are the 4-momenta of the initial electron, positron and emitted photon respectively, x and $1-x$ are the energy fractions of the scattered electron, $\boldsymbol{k}^\perp$ is the transverse component of the photon momentum with respect to the beam direction. The photon is supposed to be emitted from the electron line. The same result is obtained for the emission from the positron line. The above expression takes into account only scattering–type diagrams. According to the definition of eq.(2) we have that

$$\Sigma^{\gamma} = \frac{\alpha}{\pi}\int\limits_{1}^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1}\mathrm{d}x U(-zQ_1^2)\Big\{(L-1)P(x)[1+\Theta(x^2\rho^2-z)] \tag{8}$$

$$+ \frac{1+x^2}{1-x}k(x,z) - \delta(1-x)\Big\},$$

$$k(x,z) = \frac{(1-x)^2}{1+x^2}[1+\Theta(x^2\rho^2-z)] + L_1 + \Theta(x^2\rho^2-z)\,L_2 + \Theta(z-x^2\rho^2)L_3\,,$$

$$L_1 = \ln\left|\frac{x^2(z-1)(\rho^2-z)}{(x-z)(x\rho^2-z)}\right|, \quad L_2 = \ln\left|\frac{(z-x^2)(x^2\rho^2-z)}{x^2(x-z)(x\rho^2-z)}\right|, \quad L_3 = \ln\left|\frac{(z-x^2)(x\rho^2-z)}{(x-z)(x^2\rho^2-z)}\right|,$$

and

$$P(x) = \left(\frac{1+x^2}{1-x}\right)_+ = \lim_{\Delta\to 0}\left\{\frac{1+x^2}{1-x}\,\Theta(1-x-\Delta) + (\frac{3}{2}+2\ln\Delta)\,\delta(1-x)\right\} \tag{9}$$

is the kernel of the non–singlet (NS) evolution equation [8].

For small scattering angles ($-t/s \sim \theta^2 \ll 1$) the elastic amplitude $A(s,t)$ has a generalized eikonal form: $A(s,t) = A_{\mathrm{Born}}(t)\ \Gamma^2(t)\ \sqrt{U(t)}\ \exp\{i\phi(t)\}$, where $\Gamma(t)$ is the Dirac electron form factor and $\phi(t)$ is the Coulomb phase [9]. The Coulomb phase factor collects the multiple exchange photon contributions in the scattering channel. This holds for elastic as well as for inelastic amplitudes. This form permits us to neglect diagrams with several exchanged photons since the phase factor $\exp\{i\phi(t)\}$ disappears in the physical cross-sections. To higher orders in the perturbative expansion, only Feynman diagrams with one exchanged photon between electron and positron line do contribute. For the second order photonic contributions we use the known results of the electron Dirac form factor[10]. Pauli form factor gives a negligible contributions of $\mathcal{O}(\theta^2(\alpha/\pi)^2)$. They include the cross-section for the emission of two soft photons as well as the emission a single real (soft or hard) photon with one–loop virtual radiative corrections [11] and the cross–section for the emission of two hard photons in the same or in opposite directions. For this last kinematical configuration we may use a generalization of the single bremsstrahlung cross-section in eq.(7) [12]. We obtain the following expression:

$$\Sigma_{\gamma}^{\gamma} = \frac{1}{4}(\frac{\alpha}{\pi})^2\int\frac{\mathrm{d}z}{z^2}U(-Q_1^2z)\Big\{L^2\int\limits_{x_c}^{1}\mathrm{d}x_1\int\limits_{x_c/x_1}^{1}\mathrm{d}x_2P(x_1)P(x_2) \tag{10}$$

$$\times\ [\Theta(z-1)\Theta(\rho^2-z)+\Theta(z-x_1^2)\Theta(x_1^2\rho^2-z)]$$

$$\times\ [\Theta(z-1)\Theta(\rho^2-z)+\Theta(z-x_2^2)\Theta(x_2^2\rho^2-z)]\ +\ L\Phi_{\gamma}^{\gamma}\Big\},$$

with Φ_{γ}^{γ} an analytical function (see [7]). To calculate within the single logarithmic accuracy the contribution of the emission of two hard photons from a single lepton line (the electon in our case) we separate, by introducing a new auxiliary parameter θ_0 ($1 \gg \theta_0 \gg m/\varepsilon$), in collinear and semi-collinear kinematical regions [13]. Namely, in the collinear region both emitted photons move in the narrow cones defined by the polar angle θ_0 with respect to the initial or final (as well as simultaneously) electron 3-momenta. This region gives leading and next–to–leading contributions. The semi–collinear region produces only next–to–leading terms and corresponds to the kinematical configurations when only one of the photons moves inside one of the defined narrow cones when the second is radiated outside. It can be shown that, as it should, the dependence on θ_0 disappears in the total sum:

$$\begin{aligned}\Sigma^{\gamma\gamma} &= \frac{1}{2}(\frac{\alpha}{\pi})^2 \int\limits_1^{\rho^2} \frac{\mathrm{d}z}{z^2} U(-Q_1^2 z)\Big\{L^2 \int\limits_{x_c}^{1} \mathrm{d}x \,\Big\{\frac{1}{2}P^{(2)}(x)\,[\,\Theta(x^2\rho^2 - z) + 1] \qquad (11)\\ &+ \int\limits_x^1 \frac{\mathrm{d}t}{t} P(t)\, P(\frac{x}{t})\, \Theta(t^2\rho^2 - z)\Big\} \,+\, L\Phi^{\gamma\gamma}\Big\},\\ P^{(2)}(x) &= \int\limits_x^1 \frac{\mathrm{d}t}{t} P(t)\, P(\frac{x}{t}) = \lim_{\Delta\to 0}\Big\{\,\Big[\Big(2\ln\Delta + \frac{3}{2}\Big)^2 - 4\zeta_2\Big]\,\delta(1-x) \qquad (12)\\ &+ 2\Big[\frac{1+x^2}{1-x}\Big(2\ln(1-x) - \ln x + \frac{3}{2}\Big) + \frac{1}{2}(1+x)\ln x - 1 + x\Big]\,\Theta(1-x-\Delta)\Big\}.\end{aligned}$$

For $\Phi^{\gamma\gamma}$ see [7]. To the same order of perturbation theory we have to take into account the e^+e^- pair production processes. For hard pair production we again consider four collinear kinematical regions, when the created pair moves close to the directions of the projectiles or of the scattered particles, and six semi–collinear regions which having $2 \to 3$ like kinematics i.e. when one of the final particles moves close to the beam direction or when one component of the created pair is close the other. The cancellation of the auxiliary parameter θ_0 can be explicitly verified [14].

The relevant two–loop contribution from the electron form factor [10] contains L^3 terms which disappear in the sum with the contribution due to soft pair production [14]:

$$\begin{aligned}\Sigma^{e^+e^-} &= \frac{1}{4}(\frac{\alpha}{\pi})^2 \int\limits_1^{\rho^2} \frac{\mathrm{d}z}{z^2} U(-zQ_1^2)\Big\{L^2 \int\limits_{x_c}^1 \mathrm{d}x[1 + \Theta(z-1)\Theta(x^2\rho^2 - z)]\; R(x) \,+\, L\Phi^{e^+e^-}\Big\} \qquad (13)\\ R(x) &= \frac{2}{3}P(x) + 2(1+x)\ln\, x + \frac{1-x}{3x}(4 + 7x + 4x^2),\end{aligned}$$

where $\Phi^{e^+e^-}$ collects nonleading corrections which can be found in [7].

Within the third order of perturbation theory it is sufficient, to the accuracy of the $\mathcal{O}(10^{-3})$, to consider only leading contributions related to the initial particle radiation. Leading contributions can be systematically included by using QED evolution equations [8].

We consider the channels of $\gamma\gamma\gamma$ and γe^+e^- production real or virtual in all possible combinations. We obtain

$$\begin{aligned}\Sigma^{3\gamma} &= \frac{1}{4}\,(\frac{\alpha}{\pi}\mathcal{L})^3 \int \frac{\mathrm{d}z}{z^2} \int\limits_{x_c}^1 \mathrm{d}x_1 \int\limits_{x_c/x_1}^1 \mathrm{d}x_2\, \Big[\frac{1}{6}\delta(1-x_2)\; P^{(3)}(x_1) \qquad (14)\\ &\times\; \Theta(x_1^2\rho^2 - z)\Theta(z-1) + \frac{1}{2}P^{(2)}(x_1)P(x_2)\Theta_1\Theta_2\Big],\\ \Theta_{1,2} &\equiv \Theta(z - x_{1,2}^2)\Theta(\rho^2 x_{1,2}^2 - z),\end{aligned}$$

$$P^{(3)}(x) = \int\limits_x^1 \frac{\mathrm{d}y}{y} P^{(2)}(y) P(\frac{x}{y}), \qquad \mathcal{L} = \ln\frac{Q_1^2}{m^2}, \tag{15}$$

$$\begin{aligned} \Sigma^{e^+e^-\gamma} &= \frac{1}{4}(\frac{\alpha}{\pi}\mathcal{L})^3 \int \frac{\mathrm{d}z}{z^2} \int\limits_{x_c/x_1}^{1} \mathrm{d}x_1 \int\limits_{x_c}^{1} \mathrm{d}x_2 \Big\{ \frac{1}{3}[R^P(x_1) - \frac{1}{3}R^s(x_1)] \\ &\times \delta(1-x_2)\Theta(x_1^2\rho^2 - z)\Theta(z-1) + \frac{1}{2}\, P(x_2) R^r(x_1)\, \Theta_1\Theta_2 \Big\}, \end{aligned} \tag{16}$$

where

$$R^r(x) = R^s(x) + \frac{2}{3}P(x), \qquad R^s(x) = \frac{1-x}{3x}(4+7x+4x^2) + 2(1+x)\ln x, \tag{17}$$

$$\begin{aligned} R^P(x) &= R^s(x)\Big(\frac{3}{2} + 2\ln(1-x)\Big) + (1+x)\Big(-\ln^2 x - 4\int\limits_0^{1-x} \mathrm{d}y \frac{\ln(1-y)}{y}\Big) \\ &+ \frac{1}{3}(-9-3x+8x^2)\ln x + \frac{2}{3}\Big(-\frac{3}{x} - 8 + 8x + 3x^2\Big) + \frac{2}{3}P^{(2)}(x). \end{aligned}$$

By combining the partial results in eqs.(5),(8),(10–14), and (16) one obtains the final result for the observable cross–section eq.(6).

In Table 1 we give our results for different values of the threshold energy fraction x_c for the defined angular acceptance of $\theta_1 = 1.6^\circ$ and $\theta_2 = 2.8^\circ$. Nonleading second order photonic corrections are negative and turn out to be larger in magnitude than both third order and second order ones due to pair production. This is shown in Fig. 1. Photonic corrections (leading and nonleading) (see Fig. 2) due to double photon emission from both fermions dominate the ones from a single lepton line. This fact is a consequence of the sign–changing $P^{(2)}(x)$ function (see eq.(12)), which describes double bremsstrahlung from a single lepton line. The importance of radiative corrections to Bhabha cross–section is illustrated in Fig. 3. The decreasing of the cross–section in the region $x_c \to 1$ can be understood as a reduction by increasing x_c of the positive contribution of real photon emission, while first order virtual corrections, being negative, remain unchanged.

The accuracy of this result is implicitely defined by the terms omitted in the perturbative series. Typically they are of the type:

$$\frac{\alpha}{\pi}\theta^2,\ (\frac{\alpha}{\pi})^2,\ (\frac{\alpha}{\pi})^3 L^2,\ (\frac{\alpha}{\pi}L)^4. \tag{18}$$

An estimate of their magnitudes permits us to state that the uncertainty of our result to be at the level 10^{-4}. A detailed analysis of the attained accuracy is given in [7]. From the analysis given above clearly emerges the importance of the next–to–leading contributions to reach an accuracy adequate to the one attained in present LEP and SLC experiments. The above formulae can be applied to future high–energy e^+e^- colliders as well.

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Figure Captions

Fig.1: The ratios a) $\delta^{2\gamma}_{nonleading}/\delta_{tot}$, b) $\delta^{e^+e^-}/\delta_{tot}$ and c) $(\delta^{3\gamma}+\delta^{e^+e^-\gamma})/\delta_{tot}$ as functions of x_c for $\theta_1 = 1.6°$, $\theta_2 = 2.8°$ and $\sqrt{s} = 91.161$.

Fig.2: Behaviour of a) $\delta^{\gamma\gamma}_{leading}$ (solid line) and $\delta^{\gamma\gamma}_{leading} + \delta^{\gamma\gamma}_{nonleading}$ (dashed line), b) $\delta^{\gamma}_{\gamma\, leading}$ (solid line) and $\delta^{\gamma}_{\gamma\, leading} + \delta^{\gamma}_{\gamma\, nonleading}$ (dashed line) as functions of x_c (parameters are as in Fig.1).

Fig.3: Corrected cross-section of Bhabha scattering according to eq.(6) as a function of x_c: a) $\sigma_0 + \mathcal{O}(\alpha)$ corrections, b) with $\mathcal{O}(\alpha^2 L^2)$ photonic corrections added, c) with all other corrections (see eq.(4)) added. σ_0 is the Born cross–section. For the parameters of as in Fig.1 one has $\sigma_0 = 106.33$ nb.

Table 1: Per cent values of δ^i as defined in eq.(4) for $\sqrt{s} = 91.161$ GeV, $\theta_1 = 1.61°$, $\theta_2 = 2.8°$, $\sin^2\theta_W = 0.2283$, $\Gamma_Z = 2.4857$ GeV. Quantity $\delta^{2\gamma}$ is equal to the sum $\delta^{\gamma\gamma} + \delta^{\gamma}_{\gamma}$ (see eq.(4)).

x_c	δ_0	δ^{γ}	$\delta^{2\gamma}_{\text{leading}}$	$\delta^{2\gamma}_{\text{nonleading}}$	$\delta^{e^+e^-}$	$\delta^{e^+e^-\gamma}$	$\delta^{3\gamma}$	$\sum \delta^i$
0.1	4.120	−8.918	0.657	0.162	−0.016	−0.017	−0.019	−4.031±0.006
0.2	4.120	−9.226	0.636	0.156	−0.027	−0.011	−0.016	−4.368±0.006
0.3	4.120	−9.626	0.615	0.148	−0.033	−0.008	−0.013	−4.797±0.006
0.4	4.120	−10.147	0.586	0.139	−0.039	−0.005	−0.010	−5.356±0.006
0.5	4.120	−10.850	0.539	0.129	−0.044	−0.003	−0.006	−6.115±0.006
0.6	4.120	−11.866	0.437	0.132	−0.049	−0.002	−0.001	−7.229±0.006
0.7	4.120	−13.770	0.379	0.130	−0.057	−0.001	0.005	−9.194±0.006
0.8	4.120	−17.423	0.608	0.089	−0.069	0.001	0.013	−12.661±0.006
0.9	4.120	−25.269	1.952	−0.085	−0.085	0.005	0.017	−19.379±0.006

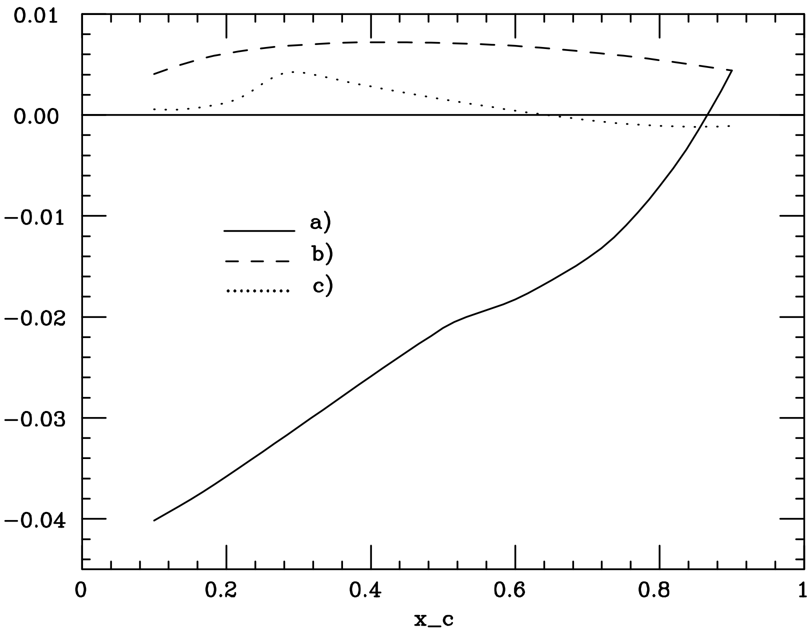

Figure 1: The ratios a) $\delta^{2\gamma}_{nonleading}/\delta_{tot}$, b) $\delta^{e^+e^-}/\delta_{tot}$ and c) $(\delta^{3\gamma}+\delta^{e^+e^-\gamma})/\delta_{tot}$ as functions of x_c for $s^{1/2} = 91.161$ GeV and $\theta_1 = 1.6°$, $\theta_2 = 2.8°$.

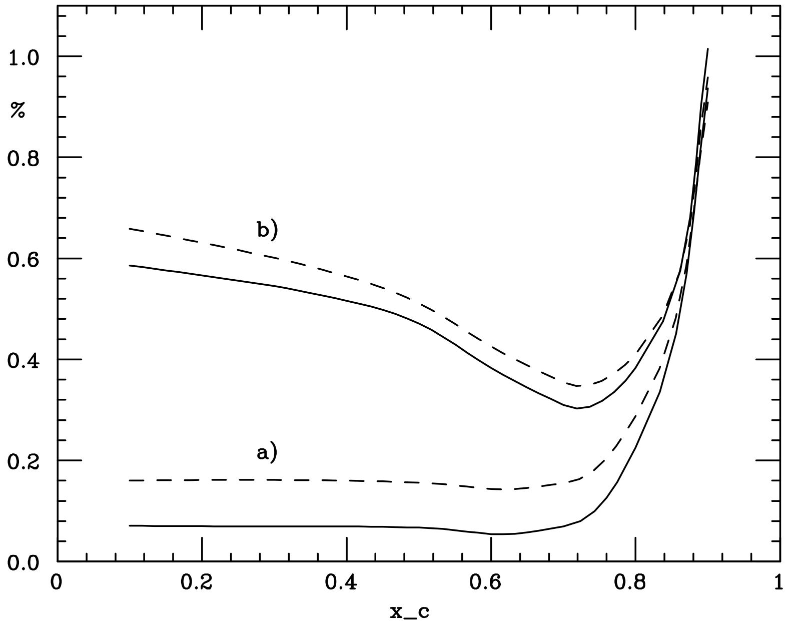

Figure 2: Behaviour of a) $\delta^{\gamma\gamma}_{leading}$ (solid line) and $\delta^{\gamma\gamma}_{leading} + \delta^{\gamma\gamma}_{nonleading}$ (dashed line), b) $\delta^{\gamma}_{\gamma_{leading}}$ (solid line) and $\delta^{\gamma}_{\gamma_{leading}} + \delta^{\gamma}_{\gamma_{nonleading}}$ (dashed line) as functions of x_c (parameters are as in Fig.1).

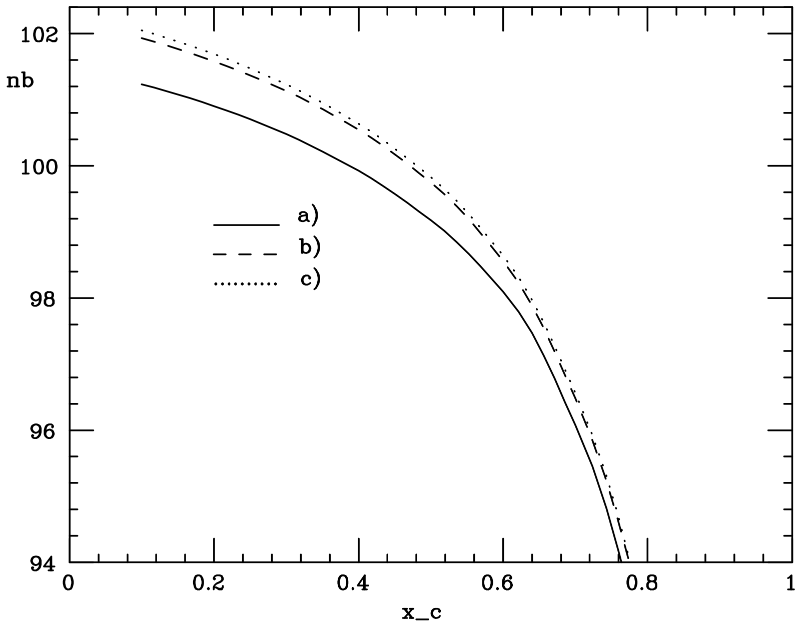

Figure 3: Corrected cross–section of Bhabha scattering according to eq.(6) as a function of x_c: a) $\sigma_0 + \mathcal{O}(\alpha)$ corrections, b) with $\mathcal{O}(\alpha^2 L^2)$ photonic corrections added, c) with all other corrections added, where σ_0 is the Born cross–section. For the parameters of as in Fig.1 one has $\sigma_0 = 106.33$ nb.