Production process dependence of neutrino flavour conversion

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Abstract

We perform a covariant wavepacket analysis of neutrino oscillations taking into account the lifetime of the neutrino production process . We find that flavour oscillations in space are washed out when the neutrinos are produced from long lived resonances - and what may be observed in appearance/disappearance experiments is a uniform conversion probability independent of distance. The lifetime of the resonance which produces the neutrinos acts as the the effective baseline of the experiment. For this reason the LSND experiment where neutrinos are produced from muon decay has two orders of magnitude more sensitivity to neutrino mass square difference than other experiments where the neutrinos are produced from pion or kaon decays. Using the covariant conversion probability formula we find that the region of Δm^2 and $sin^2 2\theta$ which is allowed by LSND muon at rest experiment $2.2 \times 10^{-3} eV^2 < \Delta m^2 < 6.3 eV^2$ and $0.002 < sin^2 2\theta < 0.15$. Recent LSND experiment with pion in flight neutrinos rules out a large part of the LSND μ DAR experiment.

There are two different formulas which describe flavour oscillations in spacetime; the one applied to neutrino oscillations [1] is derived in the high energy regime and the other applied to kaon oscillations [2] is valid in the non-relativistic limit. From the derivations of these formulas it is not clear what formula is applicable at intermediate energies (for example for kaon oscillations from stopped protons where $(m^2/E^2) \sim 25\%)$. A covariant derivation of a flavour oscillation formula which would be valid at both high and low energies would be of interest from the conceptual as well as the experimental point of view.

Kayser and Stodolsky [3] have advocated a covariant generalisation of the non-relativistic phase factor $exp(-imt)$ by replacing the absolute time t by the Lorenz invariant proper time $s = (t^2 - x^2)^{1/2}$. In the lab frame the covariant expression for the phase factor is $exp - im_i s_i = exp\ (-im_i^2 t/E_i)$. The phase difference between two mass eigenstates is $\Delta(m^2/E) \simeq (\Delta m^2/E)$ which is twice the the phase difference of the standard formula [1]. This result has prompted the claim [4] that kaon oscillations in Φ factory will have oscillation length which is half of what is given by the standard formula. This claim has been refuted by [5] who take the view that the interference phase difference should not be evaluated at different space-time points but should be evaluated at the average spacetime interval and taking the phase difference to be $(m_1 - m_2)(s_1 + s_2)/2$ instead of $(m_1 s_1 - m_2 s_2)$ they recover the standard formula. Other methods of showing that an extra factor of two does not appear in the kaon oscillations formula have been discussed in [6]. A covariant derivation of the neutrino oscillation formula has been given by Grimus and Stockinger [7] who treat the entire process of neutrino production ,propagation and detection as a single Feynman diagram. They show that on taking the large distance limit of the neutrino propagator the scattering cross section shows a space-time oscillatory behaviour and the oscillation length is identical to that given by the standard formula [1]. G-S assume the initial states to be plane waves therefore the concept of a coherence length [8] does not emerge in their formulation.

In this paper we derive the oscillation amplitude by evaluating the Feynman propagator of the neutrinos in the large time-like asymptotic limit. The asymptotic propagator of a position eigenstate has the form $K(x,t;m_i) \simeq (m_i/2\pi i s_i)^{3/2}\ exp{-im_i s_i}$ where s_i is the spacetime interval propagated by the m_i mass eigenstate. This is a field theoretic derivation of the Kayser-Stodolsky [3] phase factor. If the initial wavefunctions of the propagating particles were strictly delta functions in spacetime then no interference between different mass eigenstates can take place. We therefore generalise the delta function propagators to propagators of gaussian wavepackets. The interference term as a function of distance is obtained by taking the time-overlap of different mass eigenstate propagators . The expression for flavour conversion probability (for say two flavours with mixing angle θ) as a function of distance X turns out to be,

$$P(\alpha \to \beta; X) = \frac{1}{2} sin^2 2\theta (1 - cos(\frac{\Delta m^2}{2P} X)\ e^{-A}) \tag{1}$$

Therefore extra factor of two which came from naively subtracting the phases of plane wave propagators [4] goes away in the wavepacket averaging and the standard expression for the oscillations length is recovered. This is in agreement with [5]- [6]. The covariant wavepacket treatment introduces a new contribution to the exponential factor A when particles are produced from long lived resonances (for example for neutrinos from muon decay as opposed to neutrinos from Z decay). At distances smaller than the coherence length , the exponential suppression factor is,

$$A \quad = \quad (\frac{\Delta m^2}{\sqrt{2} E}\ \tau)^2 \tag{2}$$

where τ is the lifetime of the resonance which produces the particle which undergo flavour oscillations. If the uncertainty in position of the initial particle $v\tau$ is larger than the detector distance X , the exponential term washes out the oscillations in (1). In neutrino experiments where the source is pions, kaons, muons or nuclear fission, the spatial oscillations of the conversion probability cannot be observed. What can be observed in these experiments is a constant (distance independent) conversion probability. The conversion probability is sensitive to values of $\Delta m^2 \simeq (1/\sqrt{2} v\tau)$, which means that the experimental bound on Δm^2 is lower with longer -lifetime sources. For this reason the LSND experiment [9] which uses neutrinos from muon decay ($\tau_\mu = 2.19 \times 10^{-6} s$) is sensitive to two orders of magnitude lower neutrino mass square difference compared to other accelerator experiments like BNL-E776 [11], Karmen [12] and the recent LSND experiment [10] which use neutrinos from pion and kaon decays ($\tau \sim 10^{-8} sec$). We fit the experimental data from LSND muon decay experiment along with BNL-E776, Karmen, LSND pion decay and Bugey [14] experiments with the conversion probability formula (1) and plot the allowed range for the mass difference and mixing angle. We find that there is a large region of LSND allowed band which is not ruled out by the other experiments. The region of Δm^2 and $sin^2 2\theta$ which is allowed by LSND and all other experiments is $2.2 \times 10^{-3} eV^2 < \Delta m^2 < 6.3 eV^2$ and $0.002 < sin^2 2\theta < 0.15$. The lower bound on allowed Δm^2 is two orders lower than what is obtained by fitting the LSND data with the standard oscillation formula (equation (1) with A set to zero). Recent LSND experiment [10] with pion in flight neutrinos rules out a large part of the LSND μ DAR experiment.

Field theoretic derivation of phase factor: The phenomenon of flavour oscillations takes place because particles are produced and detected as weak interaction eigenstates (the neutrino states ν_e, ν_μ, ν_τ or the Kaon states K^0, $\bar{K}^0$ etc) but the propagators are diagonal in the mass eigenstates (the neutrino mass eigenstates $\nu_i, i = 1-3$ or the Kaon mass eigenstates K_L, K_S). The probability amplitude for oscillations of a gauge eigenstate ($|\alpha>$) to another ($|\beta>$) is a linear superposition of the propagation amplitude of the mass eigenstates ($|i>$).

$$\mathcal{A}(\alpha \to \beta; t) = \sum_i \quad <\beta|i> \quad <i|e^{-iHt}|i><i|\alpha> \tag{3}$$

In relativistic field theory the propagation amplitude of the mass eigenstates $<i|e^{-iHt}|i>$ can be identified with the Feynman propagator $S_F(x_f - x_i, m_i) =< T\ \nu_i(x_i)\bar{\nu}_i(x_f)>$. The Feynman propagator in position space is ,

$$K(x, m_i) = -i \int \frac{d^4p}{(2\pi)^4} \frac{(i\ \not{\partial} + m_i)}{p^2 - m_i^2 - i\epsilon} e^{-ip\cdot x}. \tag{4}$$

This integration can be done by expressing the denominator as an exponential,

$$\frac{-i}{p^2 - m_i^2 - i\epsilon} = \int_0^\infty d\alpha \quad exp\ i\alpha(p^2 - m_i^2 - i\epsilon) \tag{5}$$

and integrating over the resulting Gaussian in p. The remaining integral over α,

$$\int_0^\infty d\alpha\ \alpha^{-2}\ exp\ i\left(\alpha m_i^2+\frac{(t^2-|\mathbf{x}|^2)}{4\alpha}\right) \tag{6}$$

can be performed by substituting $s=(t^2-|\mathbf{x}|^2)^{1/2}$, $z=ims$ and $\eta=2(\alpha m/s)$ and making use of the integral formula [15] for the Bessel function

$$\frac{1}{2}\int_0^\infty d\eta\ \eta^{-(\nu+1)}\ exp\ -\frac{z}{2}(\eta+\frac{1}{\eta})=K_\nu(z). \tag{7}$$

The resulting expression for the Feynman propagator (4) is

$$K(x,m_i)=(\frac{i}{4\pi^2})(i\ \partial\!\!\!/+m_i)\ (\frac{m_i}{s_i})\ K_1(im_is_i) \tag{8}$$

where $s_i=(t^2-\mathbf{x}_i^2)^{1/2}$ is the invariant spacetime interval propagated by the ν_i mass eigenstate. If this interval is large ($s>>m_i^{-1}$) and time-like ($t\geq|\mathbf{x}|$) then we can use the asymptotic expansion of the Bessel function

$$K_1(ims)\simeq\left(\frac{2}{\pi ims}\right)^{1/2}\ e^{-ims} \tag{9}$$

to obtain from (8) the expression for the propagation amplitude at large time-like separation

$$K(x,t;m_i)=\left(\frac{m}{2\pi i\sqrt{(t_f-t)^2-(\mathbf{x_f}-\mathbf{x_i})^2)}}\right)^{3/2}\ exp-im\sqrt{(t_f-t)^2-(\mathbf{x_f}-\mathbf{x_i}^2)} \tag{10}$$

The phase factor of the amplitude (10) is Lorentz-invariant and can be written in terms of the neutrino energy E and the time of flight t as measured from the lab frame as $-im_is_i=-im_i\ (1-v_i^2)^{1/2}\ t=-i(m_i^2\ /E_i)\ t$.

An extra factor of two appears on subtracting the phases at different spacetime points. A flavour eigenstates neutrino or kaon is observed at a single spacetime point and one should therefore compute the phase difference at the overlap of the two mass eigenstate wavepackets. This averaging over spacetime is done formally by considering the propagators of gaussian wavefunctions as opposed to plane waves , and the standard expression for the oscillation length is recovered.

Conservation laws for long distance propagators: A flavour eigenstate propagates as a linear combination of different mass eigenstates which are on-shell. The phase difference is obtained by some authors by assuming that the different mass eigenstates have common energy, and by some by assuming that they have a common momentum [16]. In this section we show that the linear combination of different mass eigenstates have neither the same energy nor the same momentum. We show that particles propagating over large distances ($X >> P/m^2$) are constrained by the conservation laws at the vertex to be on shell. The on-shell condition is all that is needed to fix the phase difference and the oscillation length.

Consider a diagram with a propagator between vertices at spacetime points (x_1, x_2) with a number of external legs at the vertices. The amplitude is proportional to

$$\int d^4x_1 \;\; d^4x_2 \;\; exp(-i\sum_i q_i \cdot x_1) \;\; K(x_1, x_2) \;\; exp(i\sum_f q_f \cdot x_2) \tag{11}$$

Where q_i are the incoming four momenta from the external legs at x_1 and q_f are the incoming four mommenta from the external legs at x_2. Substituting the propagator

$$K(x_1, x_2) = \int d^4p \frac{e^{-ip\cdot(x_1-x_2)}}{p^2 - m^2 + i\epsilon} \tag{12}$$

in (11) and integrating over x_1 and x_2 gives the usual energy mommentum conservation $\delta^4(\sum_i q_i - p)$ and $\delta^4(\sum_f q_f - p)$ at the vertices. When the proper distance between the two vertices s is larger than m^{-1}, then the propagator used in (11) is the asymptotic form (10), which in mommentum space can be written as [17],

$$\begin{aligned} K(x_1, x_2; s >> m^{-1}) &= \frac{1}{(2ms)^{3/2}} e^{-ims} \\ &= \int \frac{d^4p}{\sqrt{\mathbf{P^2} + m^2}} \;\; exp \;\; -i((\sqrt{\mathbf{P^2} + m^2})(t_f - t_i) - \mathbf{P} \cdot (\mathbf{x_f} - \mathbf{x_i})) \end{aligned} \tag{13}$$

Substituting (13) in (11) and integrating over x_1 we obtain

$$\begin{aligned} &\int d^4x_1 \; exp \;\; -i\{(\sum_i E_i - \sqrt{\mathbf{P^2} + m^2} \;\;)t_1 + (\sum_i \mathbf{q_i} - \mathbf{P})\} \cdot \mathbf{x_1} \\ &\qquad = \;\; \delta(\sum_i E_i - \sqrt{\mathbf{P^2} + m^2}) \;\; \delta^3(\sum_i \mathbf{q_i} - \mathbf{P}) \end{aligned} \tag{14}$$

This implies that at a vertex energy and mommentum are conserved and in addition those particles which propagate without freely over distances larger than ($|\mathbf{P}|/m^2$) obey the mass shell constraint , $E = \sqrt{\mathbf{P}^2 + m^2}$.

Wavepacket analysis: The propagator $K(x_f, x_i)$ given in (10) is the amplitude for the propagation of a particle localised at x_i (a delta function initial wave function) to be detected at x_f. The propagator of some general initial wavepacket $\Psi_{in}(x-x_i)$ is obtained by using the expressions for the delta function propagator given in (10) and the superposition principle,

$$\tilde{K}(x_f - x_i) = \int d^4x \; K(x_f, x) \; \Psi_{in}(x - x_i) \tag{15}$$

If the initial wave function is a gaussian ,

$$\Psi_{in}(x - x_i) = N \; exp{-i}(E_a(t - t_i) + i\mathbf{P_a} \cdot (\mathbf{x} - \mathbf{x_i})) \;\; exp \;\; -\frac{(\mathbf{x} - \mathbf{x_i})^2}{4\sigma_x^2} - \frac{(t - t_i)^2}{4\sigma_t^2} \tag{16}$$

where σ_x and σ_t are the uncertainties is the initial position and time of production of the particle respectively and N is the normalisation constant. In the earlier wave-packet analyses of oscillation problem [8], the uncertainty in the time of production was neglected. In a covariant treatment both should be included. In the next section we show that the time uncertainty gives rise to a novel phenomenon of conversion without oscillations in many experimentally relevant situations. Substituting the expression for $K(x_f, x)$ given in (10) and the gaussian initial wavepacket (16) in (15) and evaluating the integral by the stationary phase method [18] we obtain the expression for the propagation amplitude of a gaussian wavepacket ,

$$\tilde{K}(\mathbf{X}, T; m_a) = \left(\frac{1}{4\pi|\mathbf{X}|^2}\right)^{1/2} \left(\frac{\pi}{\sigma_x^2 + v_a^2\sigma_t^2}\right)^{-1/4} \;\; exp(-i\frac{m_a^2}{E_a}T \;\; + i\mathbf{P_a} \cdot (\mathbf{X} - \mathbf{v_a}\, T))$$
$$\times \;\; exp - \frac{(\mathbf{X} - \mathbf{v_a}\, T)^2}{4(\sigma_x^2 + v_a^2\sigma_t^2)} \tag{17}$$

where $\mathbf{X} = \mathbf{x_f} - \mathbf{x_i}$ and $T = t_f - t_i$ are the space and time intervals propagated by the center of the gaussian wavepacket and $\mathbf{v_a} \equiv \mathbf{P_a}/E_a$. The normalisation constant has been fixed such that $\int \; dT \; d^3\mathbf{X} \;\; \tilde{K}(\mathbf{X}, T; m_a) \;\;\; \tilde{K}^\dagger(\mathbf{X}, T; m_b) = 1$. The factor of $(1/4\pi|\mathbf{X}|^2)^{1/2}$ in (17) shows that the particle flux decreases inversely with the sqaure of distance. Neutrino disappearance experiments look for evidence of depletion of a certain neutrino species over and above the expected inverse square decrease in flux. In the following we will not display this factor $(1/4\pi|\mathbf{X}|^2)$ in the probability expressions. The amplitude for the oscillation of one type of neutrino flavour to another is obtained by substituting the propagation amplitudes for mass eigenstates (17) in ,

$$\mathcal{A}(\alpha \to \beta; X, T) = \sum_{a}^{3} U_{\alpha a} \; \tilde{K}(m_a; X, T) \quad U^*_{a\beta} \tag{18}$$

where α, β denote the flavor eigenstates (ν_e, ν_μ, ν_τ or $K^0, \bar{K}^0$) and the summation index a denotes a mass eigenstates (ν_1, ν_2, ν_3 or K_L, K_S) . $U_{\alpha a} =< \alpha|a >$ and $U^*_{\beta a} =< a|\beta >$ are the elements of the mixing matrix which relate the flavor eigenstates with the mass eigenstates. The probabilty of flavour oscilation as a function of space-time is the modulus squared of the amplitude (29)

$$\begin{aligned} P(\alpha \to \beta; X, T) &= |\sum_{a}^{3} U_{\alpha a} \; \tilde{K}(m_a; X, T) \quad U^*_{a\beta}|^2 \\ &= \sum_a |U_{\beta a}|^2 \; |U_{\alpha a}|^2 \quad + \quad \sum_{a \neq b} U_{\beta a} \; U^*_{\alpha a} U^*_{\beta b} \; U_{\alpha b} \quad \tilde{K}(\mathbf{X}, T; m_a) \quad \tilde{K}^\dagger(\mathbf{X}, T; m_b) \end{aligned} \tag{19}$$

In interference experiments the time of flight of the particle is not measured and only the distance between the source and the detector is accurately known [16] . The probability for the flavour conversion as a function of distance is given by the time integral of (19) ,

$$\begin{aligned} P(\alpha \to \beta; X) &= \int \; dT \; |\sum_{a}^{3} U_{\alpha a} \; \tilde{K}(m_a; X, T) \quad U^*_{a\beta}|^2 \\ &= \sum_a |U_{\beta a}|^2 \; |U_{\alpha a}|^2 \quad + \quad \sum_{a \neq b} U_{\beta a} \; U^*_{\alpha a} U^*_{\beta b} \; U_{\alpha b} \quad \int \; dT \tilde{K}(\mathbf{X}, T; m_a) \quad \tilde{K}^\dagger(\mathbf{X}, T; m_b) \end{aligned} \tag{20}$$

The interference term given by the time-overlap of the propagation amplitudes (wave-functions) of different mass eigenstates can be evaluated for the gaussian propagator (17) and is given by,

$$\begin{aligned} Re \int dT \quad \tilde{K}(\mathbf{X}, T; m_a) \quad \tilde{K}^\dagger(\mathbf{X}, T; m_b) = \quad & \frac{2}{v} \, cos \left((\mathbf{P_a} - \mathbf{P_b}) \cdot \mathbf{X} - (E_a - E_b) \frac{(\mathbf{v_a} + \mathbf{v_b})}{2v^2}) \cdot \mathbf{X} \right) \\ & \times \, exp - \left((E_a - E_b)^2 \frac{(\sigma_x^2 + v^2 \sigma_t^2)}{2v^2} + \frac{(\mathbf{v_a} - \mathbf{v_b})^2 \; X^2}{8(\sigma_x^2 + v^2 \sigma_t^2) v^2} \right) \end{aligned} \tag{21}$$

where $v \equiv \sqrt{(v_a^2 + v_b^2)/2}$. In terms of the average momentum $\mathbf{P}$ and energy E and their respective differences $\Delta\mathbf{P}$and ΔE the interference term (21) ,to the leading order in $(\Delta P/P)$ and $(\Delta E/E)$, reduces to the form,

$$\frac{2}{v}\ cos\left(\frac{X}{P}(E\Delta E - \mathbf{P}\cdot\Delta\mathbf{P})\right)\ exp - (A) \tag{22}$$

where the exponetial damping factor,

$$A = (\Delta E)^2\ 2\bar{\sigma}^2\ (\frac{E^2}{P^2}) + (\frac{\Delta P}{P})^2\ \frac{X^2}{4\bar{\sigma}^2} \tag{23}$$

and $\bar{\sigma}^2 \equiv (\sigma_x^2 + (P/E)^2\sigma_t^2)$.

In general neither ΔE nor ΔP is zero and they depend upon how the state is prepared. For example if the mommentum of the initial and the associated final state is measured then $\Delta P = 0$. In the last section we have shown that for long distance propagation ($s > m^{-1}$) $\mathbf{P}$ and E are not independent and are related by the mass shell constraint. The particular combination that appears in the phase difference turns out to be independent of the preparation and is fixed by the condition that each of the mass eigenstates be on shell,

$$E_i{}^2 - P_i{}^2 = m_i{}^2 \qquad i = a, b$$

$$\Rightarrow\quad 2E\Delta E - 2\mathbf{P}\cdot\mathbf{\Delta P} = \Delta m^2 \tag{24}$$

Substituting (24) in (22) we see that the interference term of (21) is given by ,

$$\frac{2}{v}\ cos(\frac{\Delta m^2}{2P}X)\ \ e^{-A} \qquad , \tag{25}$$

The probability for the flavour conversion as a function of distance (20) is therefore,

$$P(\alpha\to\beta; X) = \sum_a\ \frac{1}{v_a}\ |U_{\beta a}|^2\ |U_{\alpha a}|^2$$

$$+\quad \sum_{a\neq b}\ \frac{1}{v}\ \ |U_{\beta a}\ U^*_{\alpha a}U^*_{\beta b}\ U_{\alpha b}| cos(\frac{(m_a^2 - m_b^2)}{2P}X\quad +\quad \delta)\ \ e^{-A} \tag{26}$$

where $\delta = arg(U_{\beta a}\ U^*_{\alpha a}U^*_{\beta b}\ U_{\alpha b})$.

We see that the standard oscillation formula which was obtained for relativistic particles [1] is actually valid at all energies. In other words although the formula (26) is usually derived by taking the leading term in a (m^2/P^2) series, our covariant calculation shows that there are actually no $O(m^2/P^2)$ corrections to (26).

In the non-relativistic regime where $P = (m_a + m_b)v/2$, the standard kaon oscillation formula $cos(m_a - m_b)T$ is recovered.

There is no extra factor of two in the relativistic kaon oscillation formula as would have appeared without the wavepacket averaging . We see that although the two mass eigenstates have different proper times the wave packet overlap results in the average proper time appearing in the interference term. Therefore the interfernce phase is actually $(m_a - m_b)\bar{s} = (m_a - m_b)(m_a + m_b)T/(E_a + E_b) = (m_a^2 - m_b^2)T/2E$. The average time prescription of [5] can therefore be justified using the covariant propagator method. Other methods of showing that an extra factor of two does not appear in the flavour oscillations formula have been discussed in [6].

Conversion without oscillation: The suppression factor A which goes with the oscillation term has some interesting new implications. If the parameters of the experiment are such that A becomes large then no oscillations is spacetime can be observed. What can be observed is a constant conversion probability.

For example, the survival probability of a ν_μ after propagating over a distance X in the large A limit is, (for two neutrino flavours (ν_e, ν_μ) with the mixing matrix elements $U_{e1} = U_{\mu 2} = cos\theta$ and $U_{e2} = -U_{\mu 1} = sin\theta$),

$$P(\nu_\mu \rightarrow \nu_\mu; X) = sin^4\theta + cos^4\theta \quad + 2sin^2\theta cos^2\theta \;\; cos(\frac{\Delta m^2}{2P}X)\; e^{-A} \simeq \; sin^4\theta + cos^4\theta \qquad (27)$$

and the expression for the conversion probability when A is large is,

$$P(\nu_\mu \rightarrow \nu_e; t) = 2sin^2\theta \; cos^2\theta \; (1 - cos(\frac{\Delta m^2}{2P}X)\; e^{-A}) \quad \simeq \quad 2sin^2\theta \; cos^2\theta \qquad (28)$$

It has been noted ealier [8] one condition A must be small which implies that X must be smaller than the coherence length $L_{coh} = (2\bar{\sigma}P/\Delta P)$. We shall show below that the constraint $L_{osc} < L_{coh}$ is not a sufficient condition to ensure the occurrence of flavour ocillations in space. In the analysis of refs [8] only the uncertainty in the initial position σ_x was considered, in our analysis we have included the contribution of σ_t the uncertainty in time at which the neutrino was produced , which as it turns out, makes the larger contribution to the suppression term A. This happens when the neutrinos are produced from long lived resonances. The neutrino wavepacket which is produced has a spread in time with width σ_t which cannot be smaller than the lifetime of the resonance whose decay produces it. The dominant contribution in that case arises from the first term of A,

$$A \simeq 2(\Delta E)^2 \; \bar{\sigma}^2 \simeq (\frac{\Delta m^2 \; \tau}{\sqrt{2} E})^2 \tag{29}$$

where we have equated the initial time uncertainity σ_t with τ - the lifetime of the resonance that produces the neutrino, and we have ignored the spatial spread of the wave-packet σ_x which in most experiments is many orders of magnitude smaller than σ_t. In terms of the oscillation length $L_{osc} = 4\pi E/(\Delta m^2)$ the suppression factor $A = (2\pi L_{osc})^2 2\tau^2$. In order to observe oscillations the source-detector distance L must be larger than L_{osc}. Which that the minimum source-detector distance in order that neutrino oscillations be observed is given by condition, $L_{min} = (2\pi)\sqrt{2}\tau$. We shall show below that most neutrino experiments do not satisfy this criterion for observability of space oscillations. On the other hand spatial oscillations of flavour can be observed in the Φ and B factories owing to the large width of ϕ and Υ resonances.

For relativistic particles produced from long lived resonances the formula for the conversion probability which should be fitted with the experiments is ,

$$P(\nu_\alpha \rightarrow \nu_\beta) = \frac{1}{2} \; sin^2 2\theta \quad (1 - \; cos(\frac{2.53 \Delta m^2 L}{E}) \;\; Exp - (\frac{3.58 \Delta m^2 \tau}{E})^2 \;\;) \tag{30}$$

where Δm^2 is the mass square difference in eV^2 ,L is the detector distance in m (km), τ is the lifetime of the parent particle in the lab frame in m (km) and E is the energy in MeV (GeV).

We shall now fit the oscillation formula (30) with the results of some terrestrial neutrino oscillation experiments , classified according to the type of the source of neutrinos.

FIGURES

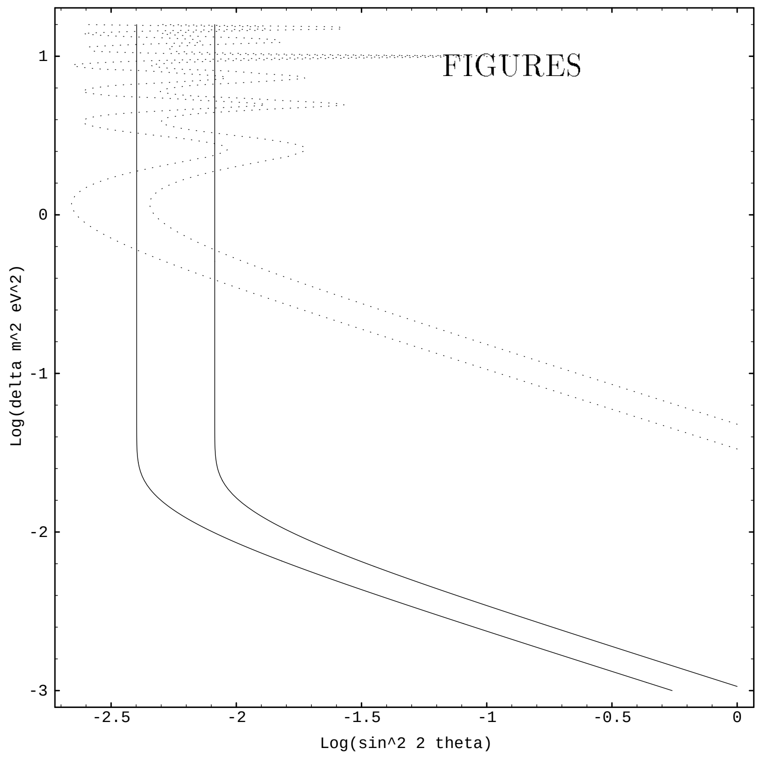

FIG. 1. Lsnd allowed regions with the standard formula (between the dotted curves)and the covariant oscillation formula (continuous curves).

Stopped Muons : The LSND experiment [9] has searched for $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ in a beam of anti-muon neutrinos produced from stopped muons. The minimum uncertainty in the time of $\bar{\nu}_{\mu}$ production is the muon lifetime $\tau_{\mu} = 658.65m$. The average neutrino energy $E_{\nu} \simeq 30MeV$ and the detector is at a distance $L = 30m$ from the source. In order that neutrino oscillations from muon decay neutrinos be seen the minimum source detector distance must be $L_{min} = \sqrt{2}2\pi\tau_{\mu} = 5852.6m$. This means that the conversion probability does not oscillate as a function of distance. Fitting the reported [9] range of conversion probability $P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}) = (0.31+0.11-0.1\pm0.05)\times10^{-2}$ with the formula (30) , the regions of allowed Δm^2 and $sin^2 2\theta$ which gives rise to this probability band are shown as dotted band in Fig. 1. The same range of conversion probability when fitted with the standard oscillation formula gives the shaded region shown in Fig.1. We have averaged the conversion probability by assuming a gaussian distribution of neutrino energy, with average energy $30MeV$ and a half width of $10MeV$. The asymptotic bounds are as follows. If $\Delta m^2 > (\sqrt{2}E/\tau_\mu) = 1.27 \times 10^{-2} eV^2$ then mixing angle constrained in the range $0.3 \times 10^{-2} \leq sin^2 2\theta \leq 0.28 \times 10^{-1}$. For large mixing angles $\sin^2 2\theta \simeq 1$ the mass square difference lies in the range, $10^{-3} eV^2 \leq \Delta m^2 \leq 1.5 \times 10^{-3} eV^2$. This bound is two orders lower than what is obtained by the use the standard oscillation formula [9].

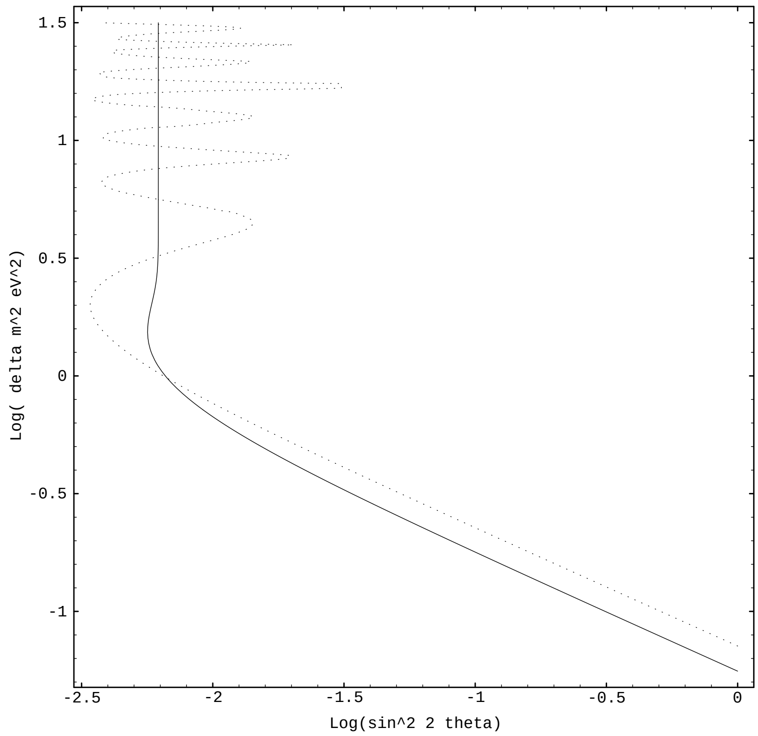

FIG. 2. Karmen allowed regions with the standard formula (dotted line) and the covariant oscillation formula (continuous line).

Stopped Pions : The Karmen experiment [12] searches for $\nu_\mu \rightarrow \nu_e$ conversion in a beam of ν_μ produced by the decay of stopped pions . The time at which the ν_μ is prduced cannot be localised to less than the pion lifetime $\tau_{\pi^+} = 7.804m$. The ν_μ energy is $E_\nu = 29.8MeV$.

In order for oscillations to be observable the detector distance must be larger than $L = 2\sqrt{2}\pi\tau_{\pi^+} = 69.31m$. Since the detector distance is $17.5m$, it is not possible to observe spatial oscillations in the Karmen experiment. The experimental bound $P(\nu_\mu \to \nu_e) < 3\times10^{-3}$, fitted with the conversion probabilty formula (30) constrains $sin^2 2\theta < 0.6\times10^{-2}$ if $\Delta m^2 > (\sqrt{2}E/\tau_{\pi^+}) = 1 \quad eV^2$. In the $sin^2 2\theta \sim 1$ limit $\Delta m^2 < 0.08 eV^2$. We have averaged the conversion probability by assuming a gaussian distribution of neutrino energy, with average energy $30MeV$ and a half width of $10MeV$. The regions of the parameter space allowed by the covariant wavepacket formula and by the standard formula are shown in Fig.2. Since the pion decay length is of the same order as the experimental baseline, the improvement in sensitivity to Δm^2 is marginal compared to LSND.

High energy Pions or Kaons: When high energy neutrino beams are obtained from the decay of a pion or kaon in flight the both τ and E are larger by the Lorentz factor γ and consequently A remains the same as that with stopped pions or kaons. In the BNL E776 experiment [11] most of the neutrinos are produced in $\pi^\pm$ decay and have an average energy $E \simeq 5Gev$ and the detector is at a distance $L = 1km$. In the lab frame the neutrino time uncertainty is $\tau = \tau_{\pi^+}(E_{\pi^+}/m_{\pi^+})$. Taking the mean $\pi^\pm$ energy to be $10GeV$, the lifetime of the pions in the lab frame is $\tau = 7.804(10/0.135)m = 0.578km$. Fitting the E776 experimental limit $P(\nu_\mu \to \nu_e) < 1.5\times10^{-3}$ with the conversion probability formula (30), we see from Fig.3 that $sin^2 2\theta < 0.3\times10^{-2}$ if $\Delta m^2 > (\sqrt{2}E/\tau_{\pi^+}) = 2.5 \quad eV^2$. In the $sin^2 2\theta sim 1$ limit , $\Delta m^2 < 0.1eV^2$.

Recently the LSND group has reported [10] measuring a conversion probability $P(\nu_\mu \to \nu_e) = (2.6\pm1.0\pm0.5)\times10^{-3}$ with muon neutrinos produced by pions in flight. The average neutrino energy is $130MeV$. The plot of the allowed region in Fig 4. shows that the LSND pion in flight result rules out the small Δm^2 region which was allowed by the LSND muon at rest experiment [9].

Reactor neutrinos: Reactor neutrinos are produced primarily from the beta decay of ^{235}U and ^{229}Pu which have a half life $\sim 10^2 sec$ [14]. The minimum detector distance for observing neutrino oscillations is $10^{11}m$ which means that it is impossible to detect spatial oscillations with reactor neutrinos. The upper bounds on conversion probability in the Bugey experiment is $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu \ or \ \bar{\nu}_\tau) < 0.075$. The largest detector distance $L = 95m$ and the average neutrino energy is $E \simeq 5MeV$. The conversion probability formula is independent of Δm^2 if $\Delta m^2 > 10^{-9} eV^2$. Fig 3. shows that in this limit the reactor experiments rule out $sin^2 2\theta \geq 0.15$.

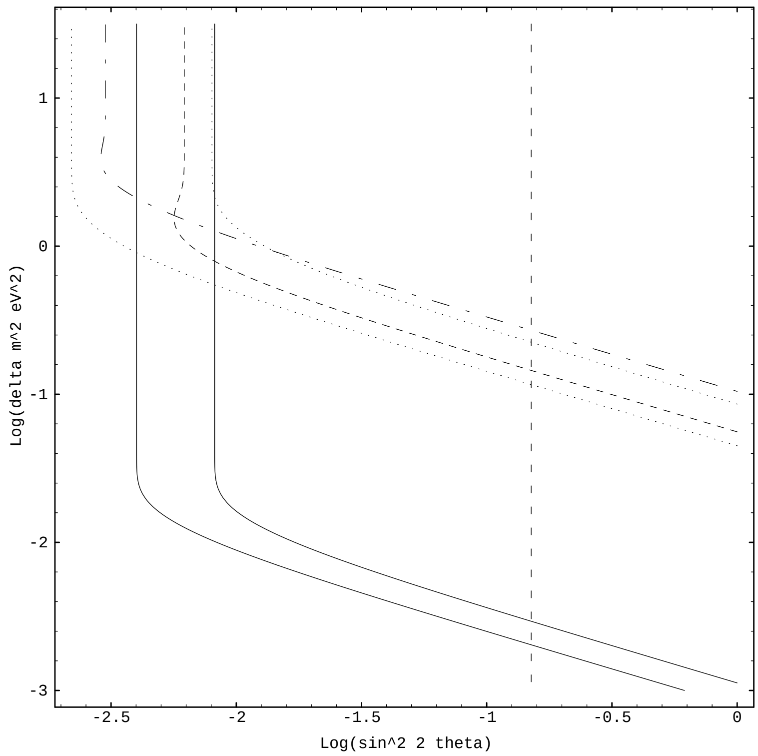

FIG. 3. The region between the continuous lines is allowed by the LSND μ decay in flight experiment .The region between the dotted lines is allowed by the LSND π decay in flight experiment . Region ruled out by E776 is above the dashed-dotted curve and by Karmen is above dashed curve. Region to the right of dashed verticle line is ruled out by Bugey.

In the standard oscillation formula the oscillatory term averages to zero when $\Delta m^2 > (4\pi E/L)$. Using the wavepacket formula (30) however we see that the oscillatory term is is exponentially damped at much lower values of Δm^2 (provided $\tau > (\sqrt{2}L/4\pi)$). If neutrino sources have a large decay time which is the case in most experiments , the conversion probabilty is sensitive to mass differences $\Delta m^2 > (\sqrt{2}E/\tau)$. For this reason the LSND experiment which uses neutrinos from muon decay has two orders of magnitude more sensitive than KARMEN, BNL-E776 etc where the neutrinos are from π and K decay. This is evident from the combined plot of these experimental results, shown in Fig.3 fitted with the covariant wavepacket formula (30). The reactor experiments where $\tau \sim 10^2 s$ are in fact sensitive to values of Δm^2 as low as $10^{-9} eV^2$. But the probability measurement in reactor experiments is poor compared to the accelerator experiments which is why they rule out only a small region of parameter space. From Fig 3. we see that the region of Δm^2 and $sin^2 2\theta$ which is allowed by LSND muon DAR experiment and the upper bounds from other experiments is $2.2 \times 10^{-3} eV^2 < \Delta m^2 < 6.3 eV^2$ and $0.002 < sin^2 2\theta < 0.15$. Including the LSND with pion in flight experiment result rules out a large part of the LSND μ DAR allowed region.

Meson oscillations at B and Φ factories: The mesons produced by ϕ or $\Upsilon(4S)$ decay are non-relativistic and the expression for the damping factor is $A = 2(\Delta m\ \tau)^2$. For the kaons produced from from ϕ decays, $\Delta m_K = 3.5 \times 10^{-12} MeV$ and the ϕ lifetime is $\tau_\phi = 0.225 MeV^{-1}$, and the damping factor $A = 2.4 \times 10^{-24}$ is negligible. For the case of $B\bar{B}$ oscillations produced from $\Upsilon(4S)$ decays, the $\Delta m_B = 3.53 \times 10^{-10}\ MeV$ and the lifetime of $\Upsilon(4S)$ is $\tau_\Upsilon = 0.042 MeV^{-1}$ and again the damping factor, $A = 8.6 \times 10^{-22}$ is negligible. This is important for tests of EPR [19] and Quantum Mechanics [20] where the measurement of the oscillatory term of the probability is essential.

Acknowledgments I thank Terry Goldman, Walter Grimus and Marek Nowakowski for useful correspondence.

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