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Prediction on CP Violation in Long Baseline Neutrino Oscillation Experiments

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ABSTRACT

We predict CP violation in the long baseline accelerator experiments taking into consideration the recent LSND data and the atmospheric neutrino data. The estimated upper bound of CP violation is 0.006, which may be observable in the long baseline accelerator experiments. It is found that the upper bound increases to 0.01 if the LSND data is excluded. The matter effect, which is not CP invariant, is found to be very small in the case we consider.

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1 Introduction

The origin of CP violation is still an unsolved problem in particle physics. In the quark sector, CP violation has been intensively studied in the KM standard model. [1] For the lepton sector, CP violation is also expected, unless neutrinos are massless. If neutrinos are Majorana particles, one finds three CP violating phases. [2] However, the effect of extra Majorana phases is suppressed by the factor $(m_\nu/E)^2$. [3] Therefore, CP violation in the neutrino flavor oscillations relates directly to the CP violating phase in the mixing matrix for massive neutrinos. [4]

Unfortunately, this CP violating effect is suppressed in the short baseline neutrino oscillation experiments if the neurinos have a hierarchical mass spectrum. However, suppression is avoidable [5, 6] in the long baseline accelerator experiments which are expected to operate in the near future. [7, 8] Observation of the CP violating effect may be possible in those experiments.

The recent indications of a deficit in the ν_μ flux of the atmospheric neutrinos [9] [11] has renewed interest in using accelerator neutrinos to perform long baseline neutrino oscillation experiments. Many possibilities of experiments have been discussed. [7, 8] The first experiment will begin in KEK-SuperKamiokande. [8] A few authors have expressed interest in CP violation in those experiments. [5, 6] We also discussed CP violation and estimated its upper bound. However, we did not take into account all available data, for example, the recent LSND data [12] in our previous estimate. By using the LSND data, we can obtain unambiguous results for CP violation. Thus, the purpose of this paper is to predict CP violation in those accelerator experiments taking into consideration both the recent LSND data [12] and atmospheric neutrino data. [11]

On the other hand, the background matter effect, which is not CP invariant, is

a significant obstacle in the attempt to observe CP violation originating from the phase of the neutrino mixing matrix. We present careful analyses for CP violation in the background matter. Therefore, we give in this paper more definite predictions compared with the previous ones. [5]

In §2, we discuss the neutrino mass pattern, in which the recent LSND data [12] and atmospheric neutrino data[11] are consistent with the predicted transition probabilities. In §3, the general framework of CP violation and the matter effect are discussed. We predict the magnitude of CP violation including the matter effect in §4. Section 5 is devoted to a conclusion.

2 Neutrino mass hierarchy and LSND data

One expects to extract the neutrino masses and flavor mixing matrix from the data of neutrino flavor oscillations. Although recent short baseline experiments have not yet confirmed the evidence of neutrino oscillation, they suggest some patterns of neutrino masses and mixings. For example, the data given by the LSND experiment [12] indicate at least one large mass difference, such as $\Delta m^2 \sim O(1\ \mathrm{eV}^2)$ with $\sin^2 2\theta_{\mathrm{LSND}} \simeq 10^{-3}$. In the near future, we can expect the data from the KARMEN experiment [13] which is also searching for the $\nu_\mu \to \nu_e (\overline{\nu}_\mu \to \overline{\nu}_e)$ oscillation as well as LSND, and the data from the CHORUS and NOMAD experiments, [14] which are looking for the $\nu_\mu \to \nu_\tau$ oscillation. The most powerful reactor experiments searching for the neutrino oscillation are those of Bugey [15] and Krasnoyarsk [16] at present. They provide excluded regions in the $(\sin^2 2\theta, \Delta m^2)$ parameter space by non-observation of the neutrino oscillation. The first long baseline reactor experiment CHOOZ [17] will also give us severe constraints. These experiments may determine patterns of neutrino masses and mixings. For the present, constraints are also given by the disappearance experiments in CDHS [18] and

CCFR, [19] and appearance experiments E531, [20] E776 [21] and new CCFR. [22] By use of those experimental bounds, some patterns are studied for neutrino masses and mixings. [23] [27]

In this paper, we take account of the LSND data seriously. Okada and Yasuda presented clearly [28] that the Bugey reactor experiment [15] and the CDHS [18] accelerator experiment exclude the mass hierarchical pattern $m_3 \gg m_2 \gg m_1 (\text{or} \simeq m_1)$ if the LSND data are used. Therefore, the mass pattern $m_3 \simeq m_2 \gg m_1$ is favored.

If the excess of the electron events at LSND are due to the neutrino oscillation, the $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillation transition probability is equal to $0.31^{+0.11}_{-0.10} \pm 0.05$ % [12]. The plot of the LSND favored region of Δm^2 vs $\sin^2 2\theta$ allows the mass region to be larger than $\Delta m^2 \simeq 0.04$ eV^2. However, the constraint by Bugey [15] has excluded a mass region lower than $\Delta m^2 = 0.27$ eV^2. On the other hand, the constraint by E776 [21] has excluded a region larger than $\Delta m^2 = 2.3$ eV^2. Thus, the mass squared difference Δm^2_{31} is obtained in the range

$$\Delta m^2_{31} \simeq 0.27 \text{ eV}^2 \sim 2.3 \text{ eV}^2 \ . \tag{1}$$

There are only two hierarchical mass difference scales Δm^2 in the three-flavor mixing scheme. If the highest neutrino mass scale is taken to be $O(1\text{eV})$, which is appropriate for cosmological hot dark matter (HDM), [29] the other mass scale is either the atmospheric neutrino mass scale $\Delta m^2 \simeq 10^{-2}$ eV^2 [9] [11] or the solar neutrino mass scale $\Delta m^2 \simeq 10^{-5} \sim 10^{-6}$ eV^2. [30] Since the long baseline experiments correspond to the atmospheric neutrino mass scale, we take $\Delta m^2 \simeq 10^{-2}$ eV^2 as the lower mass scale. Then, one should introduce the sterile neutrino [31] if one wishs to solve the solar neutrino problem from the standpoint of a neutrino oscillation.

Since we fixed the neutrino mass scales, we can now discuss the pattern of the 3×3

neutrino mixing matrix. In general, there are three allowed regions of the mixings, which are derived from the reactor and accelerator disappearance experiments.

The Bugey reactor experiment [15] and CDHS [18] and CCFR [19] accelerator experiments give bounds for the neutrino mixing parameters at the fixed value of Δm^2_{31} for three cases, as seen in Appendix A:

$$(A)\ |U_{e1}| \simeq 1,\ \ |U_{\mu 1}| \ll 1, \quad (B)\ |U_{e1}| \ll 1,\ \ |U_{\mu 1}| \ll 1, \quad (C)\ |U_{e1}| \ll 1,\ \ |U_{\mu 1}| \simeq 1. \tag{2}$$

The cases (A) and (C) are consistent with the LSND data because $|U_{e1}U_{\mu 1}|$ could be $O(10^{-3})$, as seen in Eq. (A· 6). [5, 24, 27] On the other hand, the cases (A) and (B) are consistent with the atmospheric neutrino data. The following transition probability formulae for the atmospheric neutrino may be helpful to understand the above situation:

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &= -4U_{\mu 2}U_{\tau 2}U_{\mu 3}U_{\tau 3}\sin^2(\frac{\Delta m^2_{32}L}{4E}) + 2U^2_{\mu 1}U^2_{\tau 1}\ , \\ P(\nu_\mu \to \nu_e) &= -4U_{e2}U_{\mu 2}U_{e3}U_{\mu 3}\sin^2(\frac{\Delta m^2_{32}L}{4E}) + 2U^2_{e1}U^2_{\mu 1}\ . \end{aligned} \tag{3}$$

We calculate the CP violation effect in the following pattern of the mixing matrix (case (A)), which is only consistent with both LSND and atmospheric data:

$$|U| \simeq \begin{pmatrix} 1 & \epsilon_3 & \epsilon_4 \\ \epsilon_1 & U_{\mu 2} & U_{\mu 3} \\ \epsilon_2 & U_{\tau 2} & U_{\tau 3} \end{pmatrix}\ , \tag{4}$$

where ϵ_i $(i = 1 \sim 4)$ are small numbers.

3 CP violation and matter effect

The direct measurements of CP violation originating from the phase of the neutrino mixing matrix are the differences of the transition probabilities between CP-conjugate

channels (see Appendix B):

$$\begin{aligned}\Delta P \equiv P(\overline{\nu}_\mu \to \overline{\nu}_e) - P(\nu_\mu \to \nu_e) &= P(\nu_\mu \to \nu_\tau) - P(\overline{\nu}_\mu \to \overline{\nu}_\tau) \\ = P(\overline{\nu}_e \to \overline{\nu}_\tau) - P(\nu_e \to \nu_\tau) &= 4J^\nu_{CP}(\sin D_{12} + \sin D_{23} + \sin D_{31}) \ , \end{aligned} \tag{5}$$

where

$$D_{ij} = \Delta m^2_{ij} \frac{L}{2E} \ , \tag{6}$$

and [32]

$$J^\nu_{CP} = \mathrm{Im}(U_{\mu 3} U^*_{\tau 3} U^*_{\mu 2} U_{\tau 2}) = s_{12} s_{23} s_{13} c_{12} c_{23} c^2_{13} \sin\phi \ . \tag{7}$$

In the short baseline experiments, we obtain $D_{31} \simeq -D_{12} = O(1)$ if $\Delta m^2_{31} = O(1\ \mathrm{eV}^2)$. The factor $(\sin D_{12} + \sin D_{23} + \sin D_{31})$ is suppressed because the two largest terms almost cancel due to their opposite signs. Another term is still small. Therefore, the CP violating quantity in Eq. (5) is significantly reduced. So one has no chance to observe CP violation for the present in the short baseline neutrino oscillation experiments. However, the situation is different in the long baseline accelerator experiments. The oscillatory terms $\sin D_{12}$ and $\sin D_{31}$ can be replaced by the average value 0 since the magnitude of $D_{31}(\simeq -D_{12})$ is $10^3 \sim 10^4$. Then CP violation is dominated by the $\sin D_{23}$ term, which is $O(1)$. Thus the CP violating quantity ΔP is not suppressed unless J^ν_{CP} is very small.

In order to predict the magnitude of CP violation, one should estimate J^ν_{CP}. In case (A), which is consistent with the LSND data and the atmospheric neutrino data, we should take $c_{12},\ c_{13} \simeq 1$ and $c_{23} \simeq 1/\sqrt{2}$. The LSND data determine the value of

$$c_{12} c_{13} | c_{23} s_{12} + s_{23} s_{13} c_{12} e^{i\phi} | \ , \tag{8}$$

as seen in Eq. (A$\cdot$ 6), if Δm^2_{31} is fixed. In the future, the observation of the process $\nu_e \to \nu_\tau$ or $\nu_\mu \to \nu_\tau$ will give a constraint for

$$c_{12} c_{13} | s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\phi} | \ . \tag{9}$$

On the other hand, the disappearance experiment in the work by Bugey[15] gives a bound for $c_{12}c_{13}$ which is not so confining. For example, we obtain: [24]

$$|c_{12}c_{13}|^2 \geq 0.984 \quad \text{at} \quad \Delta m^2_{31} = 2 \text{ eV}^2 \ . \tag{10}$$

Using these constraints, we find the allowed parameter regions of s_{12}, s_{13} and ϕ.

We show the allowed curves in the $s_{12} - s_{13}$ plane for the fixed values of ϕ and $s_{23} = 1/\sqrt{2}$ in Fig. 1, in which we have taken the typical LSND data: $\Delta m^2_{31} = 2\text{eV}^2$ and $\sin^2 2\theta_{\text{LSND}} = 2.6 \times 10^{-3}$. It is easily understood that larger s_{12} and s_{13} are allowed as ϕ increases to 180°. As seen in Eq. (9), the larger s_{12} and s_{13} near 180° are expected to lead to a larger transition probability of $\nu_e \to \nu_\tau$ or $\nu_\mu \to \nu_\tau$. The predicted maximal probabilities at CHORUS and NOMAD are $P(\nu_e \to \nu_\tau) \simeq 3.3 \times 10^{-4}$ and $P(\nu_\mu \to \nu_\tau) \simeq 2.6 \times 10^{-8}$, which are still below the sensitivity of those experiments.

It may be useful to comment that the atmospheric neutrino anomaly is attributed to the $\nu_\mu \to \nu_\tau$ oscillation due to $s_{23} = 1/\sqrt{2}$ in this case.

Now, we discuss the matter effect in the case of $m_3 \simeq m_2 \gg m_1$. The general discussion of the matter effect in the long baseline experiments was given by Kuo and Pantaleone. [33] The T violation effects in the earth were also studied numerically by Krastev and Petcov. [34] Their results indicate that there exist relatively large regions of masses and mixings for which the T violation effect can be considerably enhanced due to the presence of matter when neutrinos cross the earth along its diameter.

Although the distance travelled by neutrinos is less than 1000 Km in the long baseline experiments, those data include the background matter effect which is not CP invariant. The matter effect should be carefully analyzed since the effect depends strongly on the mass hierarchy, mixings, and the incident energy of the neutrino, as shown in previous works. [33, 34]

The analytic formulation of the matter effect is given for the three generation case in

Appendix C. [35] The matter effect in the long baseline accelerater experiments is rather easily calculated by assuming a constant electron density, which is $n_e = 2.4$ mol/cm^3. However, the mixing angles and the CP phase, in matter depend in a quite complicated way on the vacuum mass-squared differences, vacuum mixing angles and vacuum CP phase as shown in Appendix C.

We give qualitative discussion about the matter effect by using the approximate formula of the effective mixing angle θ_{m12}, which is given in terms of vacuum mixings as [33]

$$\sin 2\theta_{m12} = \frac{\Delta m^2_{21} \sin 2\theta_{12}}{\sqrt{(A\cos^2\theta_{13} - \Delta m^2_{21}\cos 2\theta_{12})^2 + \Delta m^2_{21}\sin^2 2\theta_{12}}} , \tag{11}$$

to zeroth order in $A \sin 2\theta_{13}$. In the present case of $m_3 \simeq m_2 = O(1 \text{ eV}) \gg m_1$, we have the relation $\Delta m^2_{21} \gg A \simeq 5 \times 10^{-4}$ eV^2($A \equiv 2\sqrt{2} G_F n_e E$). Then, one can easily find that the the matter effect A contributes slightly to $\sin\theta_{m12}$ in Eq. (11) as long as $\theta_{12} \ll 1$. The matter effect on the angles θ_{m23} and θ_{m13} are also expected to be very small. Thus the matter effect is expected to be small in the case we consider. Numerical studies are discussed in the next section.

4 CP violation in long baseline accelerator experiments

Long baseline accelerator experiments are planed to operate in the near future. [7, 8] The most likely possiblities are in KEK-SuperKamiokande (250 Km), CERN-Gran Sasso (730 Km) and Fermilab-Soudan 2 (730 Km) experiments (MINOS). The average energy of the ν_μ beams are approximately 1 GeV, 6 GeV and 10 GeV at KEK-PS (12 GeV), CERN-SPS (80 GeV) and the Fermilab proton accelerator (120 GeV), respectively.

We can estimate the factor $\sin D_{12} + \sin D_{23} + \sin D_{31}$ in those experiments. [5]

Although estimated values depend on Δm^2_{31} and E, these could be close to 1 for $\Delta m^2_{31} = 1 \sim 10$ eV^2. In order to predict the magnitude of CP violation at the KEK-SuperKamiokande (250 Km) experiment, we fix $L = 250$ Km and take $E = 1.3$ GeV with $\Delta m^2_{32} = 0.01$ eV^2. This experiment will be a disappearance experiment, since such neutrinos will not produce tau leptons. But there is a manifestation of CP violation even in the case of disappearance if final states are summed over.

Now we can calculate $\Delta P(\nu_\mu \to \nu_\tau) \equiv P(\nu_\mu \to \nu_\tau) - P(\overline{\nu}_\mu \to \overline{\nu}_\tau)$ and $\Delta P(\nu_\mu \to \nu_e) \equiv P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e)$, which are the direct measurement of CP violation in the long baseline experiments. In the present case, $P(\nu_\mu \to \nu_\tau)$ is large ($\simeq 0.4$) but $P(\nu_\mu \to \nu_e)$ is very small ($\simeq 0.01$). By use of the constraints in Fig. 1, we predict $\Delta P(\nu_\mu \to \nu_\tau)$ and $\Delta P(\nu_\mu \to \nu_e)$ in the long baseline experiment with $L = 250$ Km and $E = 1.3$ GeV. We show ΔP in the matter versus the vacuum mixing s_{12} with $\phi = 159^\circ$ in Fig. 2, in which the predicted $\Delta P(\nu_\mu \to \nu_\tau)$ in the vacuum is also presented by the dotted curve. The predicted $\Delta P(\nu_\mu \to \nu_e)$ in the vacuum cannot be distiguished from that in matter. It is found that the matter effect is very small. As ϕ changes from 90° to 180°, larger s_{12} and s_{13} are allowed, but $\sin\phi$ decreses. So the maximal J^ν_{CP} is realized between 90° to 180°. Actually, the largest value of CP violation is given at $\phi = 159^\circ$. The predicted largest value is 0.006, which may be obserbable in the KEK-SuperKamiokande experiment in the future.

It is noticed that the CP non-invariant quantity $|\Delta P(\nu_\mu \to \nu_\tau)|$ is different from $|\Delta P(\nu_\mu \to \nu_e)|$ in matter. On the other hand, these have the same magnitudes for CP violation originating from the phase in the neutrino mixing matrix, as seen in Eq. (5).

In our predictions, we have used the LSND data. However, it may asserted that the LSND result has not yet been confirmed. One has to wait for the data from the KARMEN experiment which is also searching for the $\nu_\mu \to \nu_e(\overline{\nu}_\mu \to \overline{\nu}_e)$ oscillation

as well as LSND. In order to clarify the role of the constraint from the LSND data, we discuss the case excluding the LSND data. As long as the highest neutrino mass scale is taken to be $O(1\text{ eV})$, which is appropriate for the cosmological HDM, and the atmospheric neutrino anomaly is due to the large $\nu_\mu \to \nu_\tau$ oscillation, the mass pattern $m_3 \simeq m_2 \gg m_1$ is also favored, even if the LSND data is excluded (here, the hierarchy of masses is assumed). Then, the most significant constraint is given by the E776 data. The mass squared difference Δm^2_{31} is allowed in the region larger than 2.3 eV^2, which is the upper bound given by LSND. In the range $\Delta m^2_{31} = 2 \sim 10$ eV^2, we find the largest bound of $\sin^2 2\theta = 3 \times 10^{-3}$ at $\Delta m^2_{31} = 8$ eV^2, which is not allowed by the LSND data. By use of this bound, we show the curves of the upper bounds in the $s_{12} - s_{13}$ plane, fixing $s_{23} = 1/\sqrt{2}$ in Fig. 3, where the larger of s_{12} and s_{13} are allowed. The largest value of the CP violation is given at $\phi = 163^\circ$. Predicted curves of the upper bounds in the matter are shown in Fig. 4. The predicted largest value is 0.01 if the LSND data is excluded. Thus, the LSND data provides a somewhat severe constraint.

5 Conclusions

We have studied the direct measurements of CP violation originating from the phase of the neutrino mixing matrix in the long baseline neutrino oscillation experiments. We can predict CP violation in these accelerator experiments taking into consideration both recent LSND data [12] and atmospheric neutrino data. [11] These data favor the mass hierarchy $m_3 \simeq m_2 \gg m_1$ with $m_3 = O(1\text{ eV})$ and the flavor mixings $|U_{e1}| \simeq 1$ and $|U_{\mu 1}| \ll 1$ with $|U_{\mu 2}| \simeq 1/\sqrt{2}$. The estimated upper bound of CP violation is 0.006, which may be observable in the long baseline accelerator experiments in the future. The matter effect, which is not CP invariant, is found to be very small.

In order to clarify the role of the constraint from the LSND data, the case excluding

the LSND data has been discussed. Then, the predicted upper bound is 0.01. It is noted that the LSND data provides the somewhat tight constraint.

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A Experimental Constraints

We show constraints for the neutrino mixings from the reactor and accelerator disappearance experiments. Since no indications in favor of neutrino oscillations were found in these experiments, we only obtain the allowed regions in $(U_{\alpha 1}^2, \Delta m_{31}^2)$ parameter space, where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ is defined. The Bugey reactor experiment [15] and CDHS [18] and CCFR [19] accelerator experiments give bounds for the neutrino mixing parameters at the fixed value of Δm_{31}^2. We follow the analyses given by Bilenky et al.[24]

Since the CP violating effect can be neglected in those short baseline experiments, we use the following formula without CP violation for the probability in the disappearance experiments:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 1}|^2(1 - |U_{\alpha 1}|^2)\sin^2(\frac{\Delta m_{31}^2 L}{4E}) \ , \tag{12}$$

where the mass hierarchy $\Delta m_{31}^2 \simeq \Delta m_{21}^2 \gg \Delta m_{32}^2$ is assumed. The mixing parameters can be expressed in terms of the oscillation probabilities as [24]

$$|U_{\alpha 1}|^2 = \frac{1}{2}(1 \pm \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) \ , \tag{13}$$

with

$$B_{\nu_\alpha\nu_\alpha} = \{1 - P(\nu_\alpha \to \nu_\alpha)\} \sin^{-2}(\frac{\Delta m^2_{31} L}{4E}) \ , \tag{14}$$

where $\alpha = e$ or μ. Therefore the parameters $U^2_{\alpha 1}$ at the fixed value of Δm^2_{31} should satisfy one of the following inequalities:

$$|U_{\alpha 1}|^2 \geq \frac{1}{2}(1 + \sqrt{1 - B_{\nu_\alpha\nu_\alpha}}) \equiv a^{(+)}_\alpha \ , \quad \text{or} \quad |U_{\alpha 1}|^2 \leq \frac{1}{2}(1 - \sqrt{1 - B_{\nu_\alpha\nu_\alpha}}) \equiv a^{(-)}_\alpha \ . \tag{15}$$

The negative results of Bugey, CDHS and CCFR experiments have given the values of $a^{(\pm)}_e$ and $a^{(\pm)}_\mu$. [24] For example, one obtains $a^{(+)}_e = 0.984$, $a^{(-)}_e = 0.016$, $a^{(+)}_\mu = 0.985$ and $a^{(-)}_\mu = 0.015$ for $\Delta m^2_{31} = 2$ eV^2.

It is noted from Eq. (15) there are three allowed regions of $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ as follows:

$$\begin{aligned}
&(A) \quad |U_{e1}|^2 \geq a^{(+)}_e \ , \qquad |U_{\mu 1}|^2 \leq a^{(-)}_\mu \ , \\
&(B) \quad |U_{e1}|^2 \leq a^{(-)}_e \ , \qquad |U_{\mu 1}|^2 \leq a^{(-)}_\mu \ , \\
&(C) \quad |U_{e1}|^2 \leq a^{(-)}_e \ , \qquad |U_{\mu 1}|^2 \geq a^{(+)}_\mu \ .
\end{aligned} \tag{16}$$

In addition to these constraints, we should take account of the constraints of E531 [20] and E776 [21] experimental data by using the transition probabilities

$$\begin{aligned}
P(\nu_\mu \to \nu_e) &\simeq 4|U_{e1}|^2|U_{\mu 1}|^2 \sin^2(\frac{\Delta m^2_{31} L}{4E}) \ , \\
P(\nu_\mu \to \nu_\tau) &\simeq 4|U_{\mu 1}|^2|U_{\tau 1}|^2 \sin^2(\frac{\Delta m^2_{31} L}{4E}) \ .
\end{aligned} \tag{17}$$

These constraints often become severer than the ones of the disappearance experiments.

B CP Violation

The amplitude of the $\nu_\alpha \to \nu_\beta$ transition with the neutrino energy E after traversing the distance L can be written as

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = e^{-iEL} \left\{ \delta_{\alpha\beta} + \sum_{k=2}^{3} U_{\alpha k} U^*_{\beta k} \left[\exp\left(-i \frac{\Delta m^2_{k1} L}{2E} \right) - 1 \right] \right\} \ , \tag{18}$$

where the $U_{\alpha i}$ denote the elements of the 3×3 neutrino flavor mixing matrix, in which α and i refer to the flavor eigenstate and the mass eigenstate, respectively. The amplitude $\mathcal{A}(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$ is given by replacing U with U^* on the right-hand side of Eq. (18). The direct measurements of CP violation originating from the phase of the neutrino mixing matrix are the differences of the transition probabilities between CP-conjugate channels:[4]

$$\begin{aligned} \Delta P \quad &\equiv P(\overline{\nu}_\mu \to \overline{\nu}_e) - P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\tau) - P(\overline{\nu}_\mu \to \overline{\nu}_\tau) \\ &= P(\overline{\nu}_e \to \overline{\nu}_\tau) - P(\nu_e \to \nu_\tau) = 4J^\nu_{CP}(\sin D_{12} + \sin D_{23} + \sin D_{31}) \ , \end{aligned} \tag{19}$$

where

$$D_{ij} = \Delta m^2_{ij} \frac{L}{2E} \ , \tag{20}$$

and J^ν_{CP} is defined for the rephasing invariant quantity of CP violation in the neutrino mixing matrix as well as that in the quark sector.[32] In terms of the standard parametrization of the mixing matrix,[36]

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13} \end{pmatrix} \ , \tag{21}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ are vacuum mixings, and ϕ is the CP violating phase, we have

$$J^\nu_{CP} = \mathrm{Im}(U_{\mu 3}U^*_{\tau 3}U^*_{\mu 2}U_{\tau 2}) = s_{12}s_{23}s_{13}c_{12}c_{23}c^2_{13}\sin\phi \ . \tag{22}$$

The oscillatory terms are periodic in L/E, and $D_{12} + D_{23} + D_{31} = 0$ is satisfied.

C Matter Effect

The effective mass squared in matter M^2_m for the neutrino energy E in the weak basis [35] is

$$M_m^2 = U_m \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_m^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{23}$$

where $A \equiv 2\sqrt{2}G_F n_e E$. For antineutrinos, the effective mass squared is given by replacing $A \to -A$ and $U_m \to U_m^*$. The effective mixing matrix U_m is

$$\begin{pmatrix} c_{m13}c_{m12} & c_{m13}s_{m12} & s_{m13}e^{-i\phi_m} \\ -c_{m23}s_{m12} - s_{m23}s_{m13}c_{m12}e^{i\phi_m} & c_{m23}c_{m12} - s_{m23}s_{m13}s_{m12}e^{i\phi_m} & s_{m23}c_{m13} \\ s_{m23}s_{m12} - c_{m23}s_{m13}c_{m12}e^{i\phi_m} & -s_{m23}c_{m12} - c_{m23}s_{m13}s_{m12}e^{i\phi_m} & c_{m23}c_{m13} \end{pmatrix} \tag{24}$$

where $s_{mij} \equiv \sin\theta_{mij}$ and $c_{mij} \equiv \cos\theta_{mij}$ are effective mixings, and ϕ_m is the effective phase in matter.

If we use the constant electron density $n_e = 2.4\ \mathrm{mol/cm}^3$, the effective mixing angles and phase are given in terms of vacuum mixings and the effective neutrino masses M_1, M_2 and M_3, which are eigenvalues in Eq. (23), as follows: [35]

$$\begin{aligned} s_{m12}^2 &= \frac{-(M_2^4 - \alpha M_2^2 + \beta)\Delta M_{31}^2}{\Delta M_{32}^2(M_1^4 - \alpha M_1^2 + \beta) - \Delta M_{31}^2(M_2^4 - \alpha M_2^2 + \beta)} \ , \\ s_{m13}^2 &= \frac{M_3^4 - \alpha M_3^2 + \beta}{\Delta M_{31}^2 \Delta M_{32}^2} \ , \\ s_{m23}^2 &= \frac{E^2 s_{23}^2 + F^2 c_{23}^2 + 2EFc_{23}s_{23}\cos\phi}{E^2 + F^2} \ , \\ e^{-i\phi_m} &= \frac{(E^2 e^{-i\phi} - F^2 e^{i\phi})s_{23}c_{23} + EF(c_{23}^2 - s_{23}^2)}{\sqrt{(E^2 s_{23}^2 + F^2 c_{23}^2 + 2EFc_{23}s_{23}\cos\phi)(E^2 s_{23}^2 + F^2 c_{23}^2 - 2EFc_{23}s_{23}\cos\phi)}} \ , \end{aligned} \tag{25}$$

where

$$\begin{aligned} \alpha &= m_3^2 c_{13}^2 + m_2^2(c_{12}^2 c_{13}^2 + s_{13}^2) + m_1^2(s_{12}^2 c_{13}^2 + s_{13}^2) \ , \\ \beta &= m_3^2 c_{13}^2(m_2^2 c_{12}^2 + m_1^2 s_{12}^2) + m_2^2 m_1^2 s_{13}^2 \ , \\ E &= [\Delta m_{31}^2(M_3^2 - m_2^2) - \Delta m_{21}^2(M_3^2 - m_3^2)s_{12}^2]c_{13}s_{13} \ , \\ F &= (M_3^2 - m_3^2)\Delta m_{21}^2 c_{12}s_{12}c_{13} \ , \end{aligned} \tag{26}$$

and

$$M_1^2 \simeq \frac{1}{2}[m_1^2 + m_2^2 + Ac_{13}^2 - \sqrt{(Ac_{13}^2 - \Delta m_{21}^2\cos 2\theta_{12})^2 + (\Delta m_{21}^2 \sin 2\theta_{12})^2}] \ ,$$

$$M_2^2 \simeq \frac{1}{2}[m_1^2 + m_2^2 + Ac_{13}^2 + \sqrt{(Ac_{13}^2 - \Delta m_{21}^2 \cos 2\theta_{12})^2 + (\Delta m_{21}^2 \sin 2\theta_{12})^2}] ,$$
$$M_3^2 \simeq m_3^2 + As_{13}^2 . \quad (27)$$

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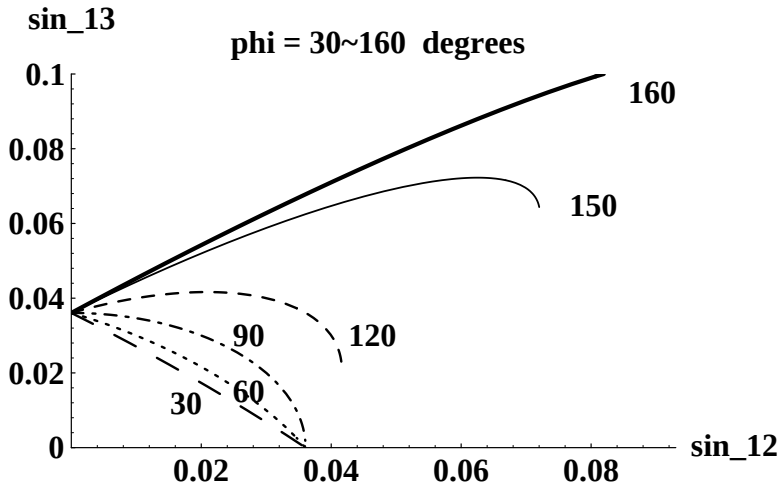

Figure 1:

Allowed curves in the $s_{12} - s_{13}$ plane, fixing $s_{23} = 1/\sqrt{2}$, in the case $\Delta m^2_{31} = 2$ eV^2 and $\sin^2 2\theta_{\rm LSND} = 2.6 \times 10^{-3}$. The thick-solid curve, thin-solid curve, short-dashed curve, dash-dotted curve, dotted curve and long-dashed curve corrspond to ϕ=160°, 150°, 120°, 90°, 60° and 30°, respectively.

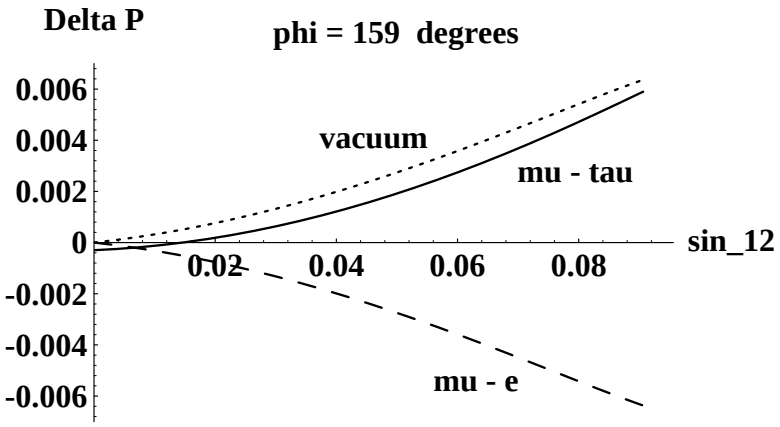

Figure 2:

Predicted $\Delta P(\nu_\mu \to \nu_\tau)$(solid curve) and $\Delta P(\nu_\mu \to \nu_e)$(dashed curve) in the matter versus s_{12} with $\phi = 159^\circ$. $\Delta P(\nu_\mu \to \nu_\tau)$ in the vacuum are also presented by the dotted curve. Here, $E = 1.3$ GeV and $L = 250$ Km with $\Delta m^2_{32} = 10^{-2}$ eV^2 are used.

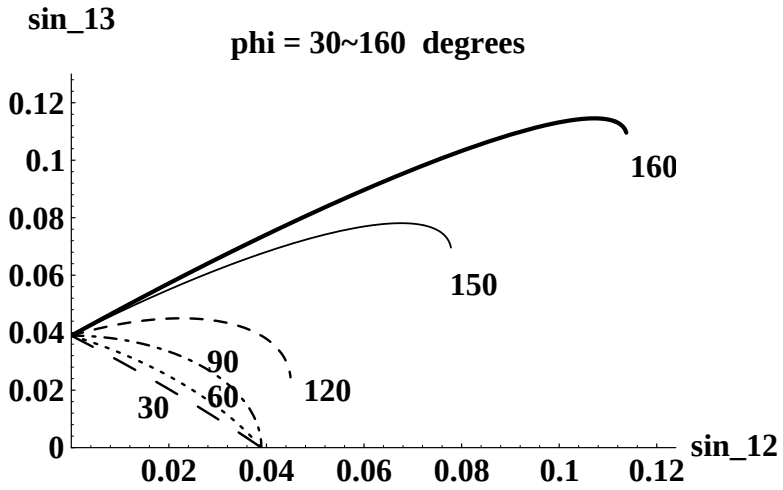

Figure 3:

Upper bounds in the $s_{12} - s_{13}$ plane, fixing $s_{23} = 1/\sqrt{2}$, in the case $\Delta m^2_{31} = 8$ eV^2 and $\sin^2 2\theta_{\rm E776} = 3.0 \times 10^{-3}$. The notation is the same as that in Fig. 1.

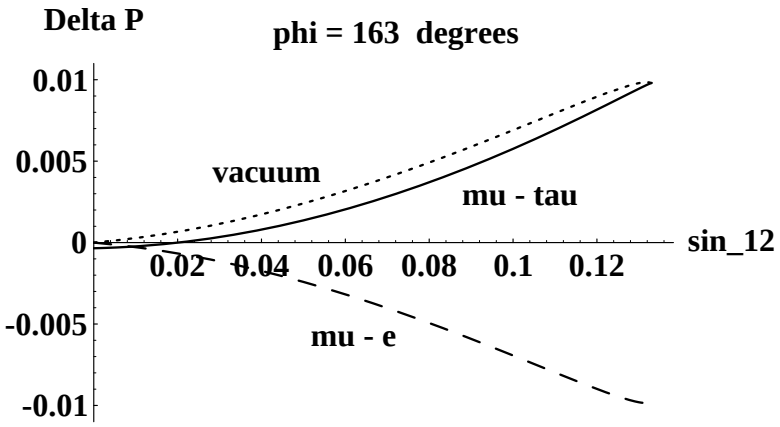

Figure 4:

Predicted upper bound of CP violation in matter versus s_{12} with $\phi = 163^\circ$. The notation is the same as that in Fig. 2.