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Gluonic Reggeons[1]

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Abstract: There are several reggeons with poles in the vicinity of angular momentum $j = 0$ contributing to the perturbative Regge asymptotics of QCD. One of them is a parity even colour octet like the leading reggeized gluon. In the odd parity colour octet channel there are 3 reggeons, two of them degenerating to one in the absence of fermions. Moreover there are reggeons in other gauge group representations. We obtain the terms in the high-energy effective action in the multi-Regge approximation describing these reggeons and their interaction with scattering quarks and gluons.

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1 Non-leading asymptotics

In the semi-hard region the behaviour of scattering processes is dominated by reggeized gluon exchange. The exchange of two reggeized and interacting gluons results in the BFKL pomeron [1].

In that region quark exchange is down by one power of $s^{\frac{1}{2}}$ for each exchange-fermion. This non-leading contribution is important if flavour quantum numbers in the t-channel exclude the pomeron exchange.

There are contributions from gluon exchange, which are down by one or more powers of s compared to the leading gluon exchange. Non-vacuum quantum numbers, odd P or C parity, can also be transferred by gluons. This is the situation where non-leading gluon exchange becomes relevant.

Our aim is to investigate the non-leading gluon exchange suppressed by one power of s and its physical effects. The Regge singularities describing these exchanges are located in the vicinity of angular momentum $j = 0$.

The high-energy effective action [2] is obtained from the QCD action by separating the modes of the gluon and quark fields relevant for scattering and exchange and by integrating over the remaining "heavy" modes [3]. The first approximation to the effective action involves the leading gluon exchange only and restricts the s-channel multi-particle intermediate states to the multi-Regge kinematics. The non-leading contributions from fermion exchange have been investigated [3], [4]. Now we are going to include the terms related to non-leading gluon exchange.

In this letter we show that there are several gluonic reggeons near $j = 0$. One of them transfers positive parity like the leading gluonic reggeon. Three reggeons transfer negative parity. Moreover, there are elementary exchanges with colour states different from the one of the gluon. The reggeization and the symmetry properties in the case of the non-leading exchanged gluons is still to be investigated. Here we point out a supersymmetry relation between the leading gluon and the non-leading gluon exchanges.

The high-energy effective action has been proposed originally as a tool for improving the leading $\ln s$ approximation. Some technical steps in our procedure are justified just for the perturbative Regge contributions. A treatment beyond the multi-Regge approximation is given in [5]. Referring to the application to the perturbative Regge asymptotics we shall give the inverse derivatives appearing in the calculations and in the final result a mathematical meaning. Technically they come in by adopting the light-cone gauge and by integrating out the "heavy" modes. Non-local interactions are an essential feature of the high energy effective action. With improvements the effective action concept will be useful also for a non-perturbative treatment.

2 Separation of modes

We follow the derivation of the effective action invented for the leading gluon exchange and quark exchange keeping now all terms of the non-leading (by one power of s) contributions. It is convenient to start from the Yang-Mills action in the light-cone axial gauge $A_- = 0$,

$$\begin{aligned} \mathcal{L} &= \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} \\ \mathcal{L}^{(2)} &= -2A^{a*}(\partial_+\partial_- - \partial\partial^*)A^a \\ \mathcal{L}^{(3)} &= -\frac{g}{2}J^a_-\mathcal{A}^a_+ - \frac{g}{2}j^a\mathcal{A}'^a \\ \mathcal{L}^{(4)} &= \frac{g^2}{8}J^a_-\partial_-^{-2}J^a_- - \frac{g^2}{8}j^aj^a \end{aligned} \tag{1}$$

We use light-cone components for the longitudinal part of vectors and complex numbers for the transverse part. The space-time derivatives are normalized such that $\partial_+ x_- = \partial_- x_+ = \partial x = \partial^* x^* = 1$. The gluon field is represented by the transverse gauge potential A^a, A^{a*}. It enters the interaction terms (1) in the combinations $\mathcal{A}^a_+ = \partial_-^{-1}(\partial A^a + \partial^* A^{a*})$, $\mathcal{A}'^a = i(\partial A^a - \partial^* A^{a*})$ and in the currents

$$J^a_- = i(A^* T^a \overleftrightarrow{\partial}_- A)\ , \quad j^a = (A^* T^a A) \tag{2}$$

In the following we encounter besides of J^a_- the transverse components J^a, J^{a*} of the vector current. We use the abbreviation $(AT^aB) = -if^{abc}A^bB^c$ with f^{abc} the structure constants of $SU(N)$.

We separate the field into modes $A = A_t + A_s + A_1$. A_t are the momentum modes typical for exchanged gluons, $k_+k_- \ll |\kappa|^2$, where k_+, k_- represent the longitudinal and the complex number κ the transverse part of the four-momentum k. A_s are the modes of a scattering gluon, $|k_+k_- - |\kappa|^2| \ll |\kappa|^2$ and A_1 contains the heavy modes $k_+k_- \gg |\kappa|^2$.

Introducing this mode separation into the action (1) the kinetic term decomposes into three. We write the contribution from $\mathcal{L}^{(3)}$, where two fields are in the modes $A_s + A_1$ and one in the mode A_t.

$$
\begin{aligned}
\mathcal{L}_1^{(3)} &= -\frac{g}{2}[J_-^a - \frac{1}{2}\cdot\frac{\partial_-}{\partial\partial^*}(\partial J^a + \partial^* J^{a*}) \\
&- i\frac{\partial_-^2}{\partial^*}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A) - i\frac{\partial_-^2}{\partial}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A^*)]\mathcal{A}_+^a \\
&- g[j^a + \frac{i}{4}\frac{1}{\partial\partial^*}(\partial J^a - \partial^* J^{a*}) - \frac{1}{2}\frac{\partial_-}{\partial^*}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A) \\
&+ \frac{1}{2}\frac{\partial_-}{\partial}(\frac{1}{\partial_-}(\partial A + \partial^* A^*)T^a A^*)]\mathcal{A}'^a
\end{aligned}
\tag{3}
$$

It is understood that the second factor in each term ($\mathcal{A}_+^a$ or $\mathcal{A}'^a$) is in the A_t modes.
In the case of scattering with a large momentum component k_- the term with J_-^a contributes to order $\mathcal{O}(k_-)$, the terms with J^a, J^{a*} and j^a to $\mathcal{O}(k_-^0)$ and the other terms to $\mathcal{O}(k_-^{-1})$. In describing scattering with large k_+ the ordering goes in the reverse direction.

3 Quasi-elastic scattering

Consider the simplest contribution from the quartic term $\mathcal{L}^{(4)}$ arrising in the mode separation, where all four gluon fields are in the scattering modes A_s. We write the contribution to high-energy scattering with two fields having momenta with large k_- and two with large k_+

$$
\mathcal{L}_{scatt}^{(4)} = \frac{g^2}{4} J_-^a \, \frac{1}{\partial_-^2} \, J_-^a - \frac{g^2}{2} j^a \, j^a - \frac{g^2}{4} j_D^r \, j_D^r \quad . \tag{4}
$$

It is implied that the first current in each term carries the modes with large k_-. This expression applies also in the more general case if the second current in each term is built from fields with all modes $A_t + A_s + A_1$ with large k_+. The currents j_D^r are not in the adjoint representation (indices a, b, c, d, e) but in the reducible representation (index r) appearing as the symmetric part in the decomposition of the product of two adjoint representations,

$$
\begin{aligned}
& j_D^r = (A^* D^r A) \ , \\
& (T^c)_{ab}(T^c)_{cd} + (T^c)_{ac}(T^c)_{bd} = (D^r)_{ad}(D^r)_{cb}
\end{aligned}
\tag{5}
$$

We write down all quartic terms contributing to the scattering of a gluon with large k_- with the two other fields in all modes

$$\mathcal{L}^{(4)}_{tot} = \mathcal{L}^{(4)}_{scatt} + < \mathcal{L}^{(3)}_1 \, \mathcal{L}^{(3)}_1 >_{A_t} + < \mathcal{L}^{(3)}_1 \, \mathcal{L}^{(3)}_1 >_{A_1} \quad . \tag{6}$$

The second term is obtained by substituting the product of the two fields in the exchange modes A_t by their propagator. The analogous contraction is done in the last term with A_1 modes, where $\mathcal{L}^{(3)}_1$ is restricted to the first order in A_1. This is the simplest but essential contribution from the integration over "heavy" modes.

Disregarding terms $\mathcal{O}(k_-^{-1})$ $\mathcal{L}^{(4)}_{tot}$ can be written as a sum of terms in each of which the two fields carrying large k_- appear in the currents J^a_-, J^a, J^{a*}, j^a, j^r_D.

Consider now the contribution of $\mathcal{L}^{(4)}_{tot}$ to high-energy scattering where the two other fields are in the scattering modes A_s too, but with large k_+. We obtain

$$\begin{aligned}\mathcal{L}^{(4)}_{tot,scatt} = {} & \frac{g^2}{8} J^a_- \, \frac{1}{\partial\partial^*} \, J^a_{+R} - \frac{g^2}{16}(\partial_+ J^a_-) \, \frac{1}{\partial\partial^*} \, (\partial_- J^a_{+R}) \\ & + g^2 j^a_s \, j^a_{sR} - \frac{g^2}{2} j^a \, j^a_R - \frac{g^2}{4} j^r_D \, j^r_{DR} + \mathrm{O}(s^{-1}) \quad .\end{aligned} \tag{7}$$

The first current in each term carries the scattering modes with large k_- and the second current the scattering modes with large k_+. The current with subscript R is obtained from the expression of the corresponding current without this subscript by substituting $A^a \to A^a_R = -\frac{\partial^*}{\partial} A^{a*}$. This corresponds to the gauge transformation leading from the gauge $A^a_- = 0$ to the gauge $A^a_+ = 0$. The result (7) has the form expected from parity symmetry.

Each of the current times current terms in (7) can be viewed as the result of an elementary t-channel exchange (pre-reggeon). Here we can read off which reggeons arise. The interaction of the pre-reggeons and in particular their reggeization has to be derived from the further analysis (Sec. 5).

In pure gluodynamics there are two terms describing colour octet odd parity exchange, where

$$j^a_s = j^a + \frac{i}{4} \frac{1}{\partial\partial^*} (\partial J^a - \partial^* J^{a*}) \quad . \tag{8}$$

Technically it is useful to notice that the analogous result without the substitution $A^a \to A^a_R$ in the currents with label R is obtained from the first two terms of (6) only if $\mathcal{L}_1^{(3)}$ is truncated by omitting those terms which are $\mathcal{O}(k_-^{-1})$ in the large k_- scattering regime.

The quasi-elastic high-energy scattering can be described by the following effective action

$$\begin{aligned}
&\mathcal{L}_{eff,scatt} = \mathcal{L}_{kin.} + \mathcal{L}_{s-} + \mathcal{L}_{s+} \\
&\mathcal{L}_{kin.} = -2A_s^{a*}(\partial_-\partial_+ - \partial\partial^*)A_s^a - 2\mathcal{A}_+^a\partial\partial^*\mathcal{A}_- + \mathcal{A}_{(+)}^a\mathcal{A}_{(-)}^a \\
&-\mathcal{A}_s'^{a(+)}\mathcal{A}_s'^{a(-)} + 2\mathcal{A}_2'^{a(+)}\mathcal{A}_2'^{a(-)} + B^{r(+)}B^{r(-)} \\
&\mathcal{L}_{s-} = -\frac{g}{2}J_-^a\mathcal{A}_+^a - \frac{g}{4}(\frac{\partial_+}{\partial\partial^*}J_-^a)\mathcal{A}_{(+)}^a - gj_s^a\mathcal{A}_s'^{a(+)} - gj^a\mathcal{A}_2'^{a(+)} - \frac{g}{2}j_D^rB^{r(+)}
\end{aligned} \tag{9}$$

$\mathcal{L}_{s+}$ is obtained from $\mathcal{L}_{s-}$ by replacing the labels $+ \leftrightarrow -$ and the currents by their partners with label R. Each pre-reggeon is represented by a pair of fields. The distinction by labels +, - is related to the fact that the reggeon exchange is oriented in rapidity. The fields with label + couple to scattering particles with large k_- only and vice versa.

4 Including fermions

In the kinematics of high-energy scattering it is natural to decompose Dirac fields ψ describing quarks into light-cone parts, $\psi = \psi_- + \psi_+$, $\gamma_-\psi_+ = \gamma_+\psi_- = 0$. Adopting the gauge $A_- = 0$ we eliminate ψ_+ and decompose ψ_- with respect to a spinor basis u_{ij}, $i,j = \pm$,

$$\psi_- = fu_{--} + \bar{f}u_{-+}\ , \quad \gamma u_{-+} = \gamma^* u_{--} = 0 \tag{10}$$

with two complex component field f and $\bar{f}$.

To describe the essential features of the fermionic terms in the high-energy effective action it is enough to look at the case of supersymmetric Yang-Mills theory, i.e. restricting to one fermion chirality f and changing to the adjoint gauge group representation.

There is a supersymmetry subgroup leaving A_- unchanged and acting on A and f as

$$\begin{aligned}
&\delta_i A = f, \quad \delta_i A^* = f^*, \quad \delta_i f = 2i\partial_- A, \quad \delta_i f^* = 2i\partial_- A^* \\
&\delta_r A = f, \quad \delta_r A^* = -f^*, \quad \delta_r f = -2i\partial_- A, \quad \delta_r f^* = 2i\partial_- A^* \ .
\end{aligned} \tag{11}$$

This subgroup can be used to reconstruct the fermionic terms in the light-cone action (1) or in the separated triple terms (3) from the gluonic ones.

Here we shall not discuss the terms in the effective action related to fermion exchange considered earlier [3] and rather concentrate on the fermionic contributions to the currents appearing in the scattering with gluon exchange.

The fermionic contributions to the currents ($J \to J + J_F$, where J is any of the above gluonic currents) are given by

$$J^a_{-F} = (f^* T^a f) \ , \quad J^{a*}_F = -\frac{1}{2}(\frac{1}{\partial_-} f^* T^a \overleftrightarrow{\partial} f) \ . \tag{12}$$

It turns out that with fermions included the current j^a splits up into two j^a and $\mathcal{K}^a$, both with the gluonic part $(A^* T^a A)$ and with the fermionic parts

$$j^a_F = 0 \ , \quad \mathcal{K}^a_F = -\frac{i}{4}(f^* T^a \overleftrightarrow{\partial_-^{-1}} f) \ . \tag{13}$$

The separated triple terms with gluonic exchange have the form (3) with the fermionic parts (12), (13) included and with the replacement

$$j^a \to \frac{1}{2}(j^a + \mathcal{K}^a) \ . \tag{14}$$

The high-energy scattering terms (7) change also by including the fermionic parts in the currents and by replacing

$$j^a \ j^a_R \to \frac{1}{2} j^a \ j^a_R + \frac{1}{2}\mathcal{K}^a \ \mathcal{K}^a_R \ . \tag{15}$$

The currents with label R are expressed analogously to the ones without this label where the fields are replaced by A^a_R, f^a_R,

$$A^a_R = -\frac{\partial^*}{\partial} A^{a*} \ , \quad f^a_R = \frac{\partial^*}{\partial_-} f^{a*} \ . \tag{16}$$

It is remarkable that the exchange fields $\mathcal{A}^a_+$ and $\mathcal{A}'^a$ together with two combinations of the exchange modes f^a_t form a quartet under (11). Correspondingly, the currents J^a_- and j^a are the even members of a supersymmetry quartet. J^{a*} is part of doublet and $\mathcal{K}^a$ is a member of a quartet again. The other even member of the latter multiplet is $k^a = (\frac{1}{\partial_-} f^* T^a \frac{1}{\partial_-} f)$ and it is related to the subasymptotics $\mathcal{O}(s^{-1})$.

Thus we have obtained that with fermions there are three terms contributing to the high-energy scattering in the colour octet odd parity channel and we find three reggeons near $j = 0$ in this channel related to the exchange of $\mathcal{A}'$.

A similar effect of removing degeneracy as in (14), (15) leading to one additional reggeon compared to gluodynamics arises in the case of the other gauge group representations j_D^r.

5 Reggeon interaction

Besides of the effective vertices of scattering $\mathcal{L}_{s\pm}$ (9) the high-energy effective action involves vertices $\mathcal{L}_p$ of emission of particles (described by the scattering modes A_s) from the pre-reggeons (related to the exchange modes $\mathcal{A}_t$). These imply reggeization and reggeon interactions.

$$\mathcal{L}_{eff} = \mathcal{L}_{kin} + \mathcal{L}_{s+} + \mathcal{L}_{s-} + \mathcal{L}_p \tag{17}$$

A way to obtain $\mathcal{L}_p$ in the multi-Regge approximation is to consider the inelastic $2 \to 3$ scattering where the final state particles are separated by large rapidity gaps. We construct the terms of order 5 in the fields which give an effective description of this inelastic scattering on the tree level

$$\begin{aligned} \mathcal{L}^{(5)} = &< \mathcal{L}_{tot}^{(4)} \, \mathcal{L}_1^{(3)} >_{A_t} + < \mathcal{L}_{tot}^{(4)} \, \mathcal{L}_1^{(3)} >_{A_1} \\ &+ < \mathcal{L}_1^{(3)} \, \mathcal{L}_{scatt}^{(4)} >_{A_t} + < \mathcal{L}_1^{(3)} \, \mathcal{L}_{scatt}^{(4)} >_{A_1} \quad . \end{aligned} \tag{18}$$

The technical remark above referring to (7) applies also here. The second term contributes mainly to replacing $A^a \to A_R^a$ in the currents of large k_+ particles. The structure of the result can be read off from the other 3 terms only.

The production vertices $\mathcal{L}_p$ are read off from the result by representing it as

$$\mathcal{L}^{(5)} = < \mathcal{L}_{s-} \, \mathcal{L}_p \, \mathcal{L}_{s+} >_{A_t} \tag{19}$$

and factorizing the scattering vertices $\mathcal{L}_{s-}$, $\mathcal{L}_{s+}$ in view of the kinetic terms $\mathcal{L}_{kin}$ (9).

There is a number of terms in $\mathcal{L}_p$ differing by the kind of pre-reggeons in the two t-channels. Here we present as an example the part referring to the odd parity reggeons ($\mathcal{A}'_s$, $\mathcal{A}'$) in both channels (without fermions)

$$
\begin{aligned}
\mathcal{L}_p \mid_{\mathcal{A}'_s\mathcal{A}'_2} = -\frac{ig}{4}\{ \quad & 2(\frac{1}{\partial}\mathcal{A}'^{(-)}_s T^a \partial \mathcal{A}'^{(+)}_s) + 2(\partial^* \mathcal{A}'^{(-)}_s T^a \frac{1}{\partial^*}\mathcal{A}'^{(+)}_s) \\
+ \quad & (\frac{\partial^*}{\partial}\mathcal{A}'^{(-)}_s T^a \frac{\partial}{\partial^*}\mathcal{A}'^{(+)}_s) + (\mathcal{A}'^{(-)}_s T^a \mathcal{A}'^{(+)}_s) \\
+ \quad & 4(\mathcal{A}'^{(-)}_2 T^a \mathcal{A}'^{(+)}_2) \qquad (20) \\
+ \quad & 2(\frac{1}{\partial}\mathcal{A}'^{(-)}_s T^a \partial \mathcal{A}'^{(+)}_2) + 2(\mathcal{A}'^{(-)}_s T^a \mathcal{A}'^{(+)}_2) \\
+ \quad & 2(\partial^* \mathcal{A}'^{(-)}_2 T^a \frac{1}{\partial^*}\mathcal{A}'^{(+)}_s) + 2(\mathcal{A}'^{(-)}_2 T^a \mathcal{A}'^{(+)}_s)\}\frac{1}{\partial^*}A^{a*} + c.c.
\end{aligned}
$$

The details about the effective production vertices related to the non-leading $\mathcal{O}(s^0)$ gluonic reggeons will be given elsewhere.

6 Discussion

At high energy, in the perturbative Regge region, gluon interaction gives rise not only to the leading asymptotics (singularity near $j = 1$, resulting in the BFKL pomeron) but also to subleading terms in the asymptotic expansion. There are quantities of phenomenological interest where these terms are not just small corrections. The flavour singlet spin structure function $g_1(x)$ is determined in the region of small x by gluon exchange, where one of the exchanged gluons is of the non-leading type discussed here. The spin structure function of the photon F_3^γ related to the helicity flip amplitude is dominated at small x by the exchange of two subleading gluonic reggeons.

Extending the high-energy effective action in the multi-Regge approximation to the sub-leading terms $\mathcal{O}(s^0)$ we have identified the reggeons arising near angular momentum $j = 0$. There is a partner of the leading gluon exchange with the same parity property. In the odd parity channel we find two reggeons in pure Yang-Mills theory in the adjoint colour representation but with fermions they split off into three.

Presently we are missing a good understanding of the role of the different reggeons in the odd parity channel. Also it would be desirable to look for physical effects of the reggeons in the other colour representations. Among them there is a colour singlet reggeon.

With this analysis we did a step towards a field theoretic understanding of non-leading Regge singularities. The methods and the validity of the results are restricted to the perturbative region.

The high-energy effective action can be considered as a new approach to the classical question of reggeization in field theory. It provides an alternative to the approach based on amplitude analysis proposed by Gell-Mann et al [6] and developed in [7]. Comparing we see that the t-channel factorizability of the amplitudes is a crucial point in both approaches.

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