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Dynamically generated inertia

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Abstract

A (Higgs vacuum)-(spacetime geometry) reciprocity principle is proposed and its consequences are explored. While it has been established that configurations of the spacetime metric tensor field, associated with acceleration with respect to local inertial frames, cause the vacuum to become thermalized, it is asserted that the converse is also possible. An appropriate thermal vacuum, through dynamical mass generation, can cause particles to propagate in a spacetime with a Minkowski metric, as if they were in a spacetime with a non-Minkowski metric. The two points of view are equivalent and interchangeable. Invoking the reciprocity principle in the case of the Unruh effect in an accelerated frame, a mechanism is analyzed whereby in the context of the minimal standard model a gradient in the vacuum expectation value of the Higgs field in the direction of acceleration produces an effective inertial force on an accelerated material body.

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I. INTRODUCTION

It is now well established, at least theoretically, that quantum fields on a spacetime manifold are significantly affected by the spacetime geometry. In every case where the existence of an event horizon limits the observation of particle modes, the vacuum becomes thermalized. Hawking, in a far-reaching paper [1], showed that this effect leads to pair production and particle radiation near the event horizon of a Schwarzschild black hole, with an energy spectrum corresponding to a temperature inversely proportional to the mass of the black hole. Israel [2], using the thermo-field dynamics of Umezawa and Takahashi [3], reproduced Hawking's result by identifying the hidden modes with the "fictitious" modes of TFD. An event horizon also exists in the Rindler wedge [4], and consequently an accelerating observer in flat spacetime also sees a thermal vacuum at a temperature proportional to the acceleration. This effect is usually referred to as the Davies-Unruh effect or sometimes just as the Unruh effect [6,7].

In general terms, the vacuum appears as a thermal state to an observer who is accelerating with respect to local inertial frames. The observer's rest frame may be an accelerated local frame in flat spacetime, or by the strong equivalence principle a stationary local frame in a curved spacetime (gravity field). A specific example has been given by Moreau, Ross, and Neutze [8] of the local equivalence and global similarity of the French geometry corresponding to an accelerated frame in flat spacetime and the Schwarzschild geometry. Also Rindler [4], through the use of Kruskal-Szekeres coordinates [5], has emphasized the similarity between Schwarzschild spacetime and a constant acceleration field in flat spacetime. Thus it is acceleration with respect to local inertial frames that causes an observer to see a thermal vacuum and not curvature per se.

In the absence of a complete theory of quantum gravity, it is interesting to note that in all of the work relating vacuum thermalization to spacetime geometry the classical metric tensor field is considered to be fundamental, or primary, and the quantum fields in spacetime secondary. In every case a classical configuration of the metric tensor field causes a thermalization of the vacuum quantum state of the fields in spacetime. Davies [6] has justified this approach by pointing out an analogy with the semiclassical treatment of atomic transitions where the electromagnetic field is considered classical and only the atomic system is treated quantum mechanically. The analogy is not quite complete, however, because, unlike the electromagnetic field interacting with an atom, the metric tensor field not only interacts with the fundamental quantum fields in spacetime, but it also determines the spatial distribution of the degrees of freedom of these fields. In the atomic case it would be like a configuration of the electromagnetic field affecting the radial coordinate of the atomic wavefunction. In reality, if a complete quantum theory exists, the metric tensor field well may be too intimately associated with the total quantum system to be treated separately in a classical context.

In addition to the necessity dictated by the lack of a complete quantum theory, there is also an historical reason why physicists attribute a primary (and classical) role to the spacetime metric tensor field. It defines the local inertial frames. The history of the concept of an inertial frame of reference goes back to Newton whose idea of an absolute space with respect to accelerations was criticized by Bishop Berkeley and later by Ernst Mach. While volumes have been written on this issue [9], I have no intention of giving a complete review here. Instead, let us begin with Einstein. In general relativity theory the metric tensor field determines the local inertial frames through Einstein's interpretation of Mach's Principle. The Einstein field equations for the universe as a whole can be solved in principle from initial value data on some spacelike hypersurface for a given configuration of the energy-momentum tensor field [10]. From the solution, $ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$, the local inertial frames are defined at any given point on the spacetime manifold by the set of local coordinate transformations, $\bar{x} = \bar{x}(x)$, that produce a Minkowski metric, $\eta_{\bar{\mu}\bar{\nu}} = g_{\mu\nu}\left(\partial x^\mu / \partial x^{\bar{\mu}}\right)\left(\partial x^\nu / \partial x^{\bar{\nu}}\right)$, at the point. As all of physics stems from the local inertial frames, one naturally attributes a fundamental role to the metric tensor field.

But the determination of the local inertial frames through general relativity is a strictly classical exercise, and once the inertial frames are established, inertia and gravitation, except for usually small tidal effects in the latter, seem to be primarily kinematic (acceleration with respect to the local inertial frames). While this classical account of inertia and gravitation is undoubtedly successful, at least phenomenologically and over distances characteristic of the solar system, is it fundamental? One is uneasy, for example, that contemporary quantum field theoretic concepts such as dynamical mass generation through spontaneous symmetry breaking apparently play no part, even though mass is at the core of gravitation and inertia. It was such unease that recently led Haisch, Rueda, and Puthoff [11] to seek a more tangible inertial force in terms of the *zitterbewegung* of a charged particle in a magnetic field of the vacuum predicted by stochastic electrodynamics.

In the present paper I continue the quest of Haisch et. al., but along a different route. My basic principle is one of reciprocity between the quantum vacuum and spacetime geometry. If a non-Minkowski configuration of the metric tensor field in an accelerated frame can cause an observer to perceive a thermal vacuum, then I propose that the converse is possible: a suitable thermal vacuum in an accelerated frame with a Minkowski metric can cause particles to propagate *as though they were* on geodesics corresponding to a non-Minkowski metric field.

I take as fundamental the fields of the Minimal Standard Model of particle physics [12], and in particular the scalar Higgs field. I define an inertial frame as one in which the Higgs vacuum is at a uniform temperature. This definition is entirely local and invariant under Lorentz transformation, because all points in an inertial frame are physically equivalent. It avoids use of the property of inertia itself (free test particles do not accelerate), or any nonlocal concept such as reference to the "fixed stars". From an operational point of view, this definition is implementable everywhere since, unlike radiation, the vacuum cannot be screened out of any region.

Then, as we shall see from an examination of the Davies-Unruh effect and the principle of reciprocity, an accelerated frame is one in which there exists a vacuum temperature gradient in the direction opposed to that of acceleration. The temperature gradient causes the spontaneous symmetry breaking of the Higgs field to be nonuniform in the accelerated frame and the vacuum expectation value of the Higgs field acquires a gradient in the direction of acceleration. Consequently, for example, a quark, propagating inside an accelerating nucleon, with a mass that is dynamically generated through a Yukawa interaction with the nonuniformly broken Higgs field, does so with a mass that increases in the direction of acceleration. Such propagation results in what can be interpreted as an inertial force on the nucleon and ultimately on the nucleus and the macroscopic body of which the nucleon is a constituent.

In any local frame with coordinates $x^\lambda \equiv \{ct, x, y, z\}$, the classical, relativistic equation of motion for a "free" particle can be written as

$$m\frac{d^2x^\lambda}{d\tau^2} = F^\lambda - m\Gamma^\lambda{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}, \tag{1.1}$$

where the $d^2x^\lambda/d\tau^2$, $\lambda = 0, 1, 2, 3$, are the components of the particle's proper acceleration, the term $-m\Gamma^\lambda{}_{\mu\nu}\,(dx^\mu/d\tau)\,(dx^\nu/\tau)$ represents a so-called fictitious inertial force [13], and F^λ is any "applied" force which I will take to be an inertial force produced by a temperature gradient of the vacuum. If the frame is inertial, then the two right-hand terms in Eq. (1.1) are zero as the Christoffel symbols $\Gamma^\lambda{}_{\mu\nu}$ are all zero and the vacuum has a uniform temperature. If the frame is noninertial, then we can choose one of two alternative descriptions. Either (1) we ignore the vacuum, in which case the metric must be non-Minkowskian, some of the Γ's are nonzero and thus the fictitious inertial force is nonzero and the real inertial force F^λ is zero, or (2) we consider a thermal vacuum with a temperature gradient, in which case the metric must be Minkowskian, the Γ's are all zero and the F^λ are nonzero. Description (1) is the traditional way of doing physics and the inertial force, $-m\Gamma^\lambda{}_{\mu\nu}\,(dx^\mu/d\tau)\,(dx^\nu/d\tau)$, is called fictitious. I have called it kinematic as it is simply the result of viewing inertial motion from a noninertial frame. In description (2), on the other hand, the inertial force F^λ is real and is the result of interaction of the particle with an altered vacuum. While both descriptions may be equally valid for predicting world lines of massive particles, (2) must be considered more fundamental as it involves the quantum fields of the Standard Model while (1) does not.

We can construct a complete and consistent kinematic picture involving real inertial forces in which the traditional way of doing physics is a special case. Suppose we choose a uniformly accelerating, noninertial frame S in which to do physics and assign a Minkowski metric $\eta_{\mu\nu}$ to it so that there are no fictitious inertial forces. As S is noninertial, however, there will be a vacuum temperature gradient producing a real inertial force F^{λ} causing a free particle to accelerate. Now we wish to describe the motion with respect to an inertial frame $\bar{S}$ which is accelerating with respect to the noninertial frame S. The coordinate trasformation, $x^{\mu} = x^{\mu}(\bar{x})$, corresponding to the accelerated motion of $\bar{S}$ with respect to S, leads to a non-Minkowski metric in the inertial frame $\bar{S}$ given by

$$g_{\bar{\mu}\bar{\nu}}(\bar{x}) = \frac{\partial x^{\mu}}{\partial x^{\bar{\mu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}\eta_{\mu\nu}(x) \ , \tag{1.2}$$

and a transformed inertial force given by

$$F^{\bar{\lambda}} = \frac{\partial x^{\bar{\lambda}}}{\partial x^{\lambda}}F^{\lambda} \, . \tag{1.3}$$

In the inertial frame $\bar{S}$ the fictitious force, $-m\Gamma^{\bar{\lambda}}{}_{\bar{\mu}\bar{\nu}}\,(dx^{\bar{\mu}}/d\tau)\,(dx^{\bar{\nu}}/d\tau)$, associated with the non-Minkowski metric, $g_{\mu\nu}(\bar{x})$, cancels with the transformed inertial force $F^{\bar{\lambda}}$ so that the free particle does not accelerate.

Alternatetively we could choose the inertial frame $\bar{S}$ to do physics and assign a Minkowski metric to it so that there are no fictitious inertial forces. The vacuum temperature is uniform in $\bar{S}$, so there is no real inertial force as well, and a free particle does not accelerate. Now we wish to describe the motion with respect to the noninertial frame S which is accelerating with respect to $\bar{S}$. The coordinate transformation, $x^{\bar{\mu}} = x^{\bar{\mu}}(x)$, corresponding to the accelerated motion of S with respect to $\bar{S}$, leads to a non-Minkowski metric in the noninertial frame S given by

$$g_{\mu\nu}(x) = \frac{\partial x^{\bar{\mu}}}{\partial x^{\mu}}\frac{\partial x^{\bar{\nu}}}{\partial x^{\nu}}\eta_{\bar{\mu}\bar{\nu}}(\bar{x}) \ , \tag{1.4}$$

and a transformed inertial force given by

$$F^{\lambda} = \frac{\partial x^{\lambda}}{\partial x^{\bar{\lambda}}}F^{\bar{\lambda}} = 0 \, , \tag{1.5}$$

since $F^{\bar{\lambda}} = 0$. Any frame can be chosen, inertial or noninertial, and a real inertial force determined with respect to that frame. Thereafter, a transformation can be made to any other frame, and the real inertial force transforms in the usual way. The traditional way of doing physics is to always choose an inertial frame with zero real inertial force, a special case of the more general kinematic picture envisaged here.

The paper is organized as follows. In Sec. II a negative vacuum temperature gradient in a rigid, accelerating frame of reference with a Fermi metric is determined. Invoking reciprocity, I postulate a corresponding fundamental vacuum temperature gradient in an accelerating frame. In Sec. III I consider nonuniform spontaneous symmetry breaking of the Higgs field in such a temperature gradient and show that the Higgs vacuum expectation value has a gradient in the direction of acceleration. Since all mass is dynamically generated by Yukawa interactions with the spontaneously broken Higgs field, any massive particle will propagate with respect to an accelerated frame with a mass that increases in the direction of acceleration. In Sec. IV I show an example by calculating the propagator for the variable-mass Higgs particle itself. The resulting expression can also be interpreted as the propagator for a particle of fixed mass moving against a potential which rises in the direction of acceleration. In Sec. V I put all these ideas together and present a simple model of an accelerated nucleon. Finally in Sec. VI I give a summary and draw some conclusions.

II. THE VACUUM TEMPERATURE IN AN ACCELERATING FRAME

Let $\bar{S}$ be an inertial frame in flat spacetime, and consider a local frame S, with axes parallel to those of $\bar{S}$, whose origin O is uniformly accelerating at a constant rate a with respect to $\bar{S}$ in the direction of the positive x axis. Such a frame can be constructed by the extension of the axes of a Fermi-Walker transported tetrad [14], and the coordinate transformation between the two frames is given by

$$\frac{a\bar{t}}{c} = \left(1 + \frac{ax}{c^2}\right) \sinh \frac{at}{c} \,, \tag{2.1}$$

$$\frac{a\bar{x}}{c^2} = \left(1 + \frac{ax}{c^2}\right)\cosh\frac{at}{c} - 1\,, \tag{2.2}$$

$\bar{y} = y$, and $\bar{z} = z$. For time-like intervals the line element in S is given by

$$-c^2 d\tau^2 = -\left(1 + \frac{ax}{c^2}\right)^2 c^2 dt^2 + dx^2 + dy^2 + dz^2\,, \tag{2.3}$$

indicating that the spatial coordinates measure proper distance in S.

Let us further consider a set of stations located at fixed positions along the x axis of the accelerating frame. According to Eqs. (2.2) and (2.3), the proper accelerations of these points are not uniform, but vary with their x position according to

$$\left(\frac{d^2\bar{x}}{d\tau^2}\right)_x = a\left(1 + \frac{ax}{c^2}\right)^{-1}\cosh\frac{at}{c}\,. \tag{2.4}$$

We are interested in the distribution of proper acceleration along the x axis of the accelerated frame with respect to an instantaneously comoving inertial frame so we choose $t = \bar{t} = 0$. Also we are interested exclusively in magnitudes of acceleration and points in the vicinity of the origin O for which $|ax/c^2| \ll 1$. With the value of x limited to the extent of a nucleon located at the origin ($\sim 10^{-15}$ m), this inequality gives as an upper limit for acceleration $|a| \ll 9 \times 10^{30}$ g, a limit far beyond the range of any experimental test of Newton's second law. Thus for $|ax/c^2| \ll 1$ we can rewrite Eq. (2.4) as

$$a\left(x\right) \equiv \left(\frac{d^2\bar{x}}{d\tau^2}\right)_x = a\left(1 - \frac{ax}{c^2}\right). \tag{2.5}$$

Equation (2.5) indicates that the stations along the x axis have proper accelerations that vary linearly with x with magnitudes that decrease in the direction of acceleration. According to the Unruh effect [15], [16], the vacuum at each of these points appears to have a Planckian frequency distribution with an associated temperature related to proper acceleration by

$$T\left(x\right) = \frac{\hbar a\left(x\right)}{2\pi ck}\,, \tag{2.6}$$

where $a\left(x\right)$ is given by Eq. (2.5) and k is Boltzmann's constant. Thus the vacuum temperature near the origin in the accelerated frame S is given as a function of x by

$$T(x) = T_0 \left(1 - \frac{ax}{c^2}\right) , \tag{2.7}$$

where $T_0 \equiv T(0)$ is the nominal temperature of the vacuum at the origin.

The vacuum temperature distribution given by Eq. (2.7) is a consequence of the Fermi metric in the accelerated frame given by Eq. (2.3). The Fermi metric also produces some nonzero Christoffel symbols which give fictitious inertial forces and inertia is described kinematically. Instead, I opt for description (2), as discussed in the introduction, where I assume the vacuum temperature in the accelerated frame given by Eq. (2.7) as a postulate along with a Minkowski metric.

One might argue that if a vacuum temperature gradient produces a real inertial force, then such a force should be produced in the accelerated frame with the Fermi metric, and the description (1) of inertia is redundant. But if you choose description (1) you must ignore all effects of the vacuum on inertia, and the description is purely kinematic. If you choose description (2) the metric must be Minkowskian so that there is no fictitious inertial force, and the real inertial force is entirely due to an altered vacuum. We can choose description (1) or (2), but not both. Thus Eq. (2.7) only suggests the postulate for description (2) by reciprocity.

III. SPONTANEOUS SYMMETRY BREAKING IN AN ACCELERATING FRAME

I assume that the rules of quantum mechanics are universally valid without change in frames of reference that are accelerating with respect to inertial frames, with the proviso that in place of fictitious forces associated with a non-Minkowski metric one must take into account an altered vacuum. My position is that, with this one proviso, a local accelerated frame is the physical equivalent of an inertial frame. A quantum field theory exists with respect to an accelerated frame. Energy-momentum is defined in the standard way and is conserved at all vertices.

I now postulate a negative vacuum temperature gradient given by Eq. (2.7) in an accelerated frame with a Minkowski metric. Since in the Standard Model all mass is dynamically generated by Yukawa interactions with a spontaneously broken Higgs field, I am led to consider the finite-temperature quantum theory of a scalar Higgs field. For simplicity I consider only a real, isoscalar field with an action given in natural units by

$$S[\phi] = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \right), \tag{3.1}$$

where $m^2 > 0$ and $g = -1$ because there is a Minkowski metric. Because of the + sign the term quadratic in the field in Eq. (3.1) is not a mass term, and the field has spontaneous broken symmetry. The vacuum expectation value of the Higgs field at a finite temperature is determined by the minimum of an effective potential [17,18],

$$\begin{aligned} \hat{V}_{\hat{\beta}}\left(\hat{\phi}\right) \equiv \frac{V_\beta(\phi)}{\lambda \eta^4} &= -\frac{1}{2}\hat{\phi}^2 + \frac{1}{4}\hat{\phi}^4 + \frac{\lambda}{64\pi^2}\hat{M}^4 \ln \hat{M}^2 \\ &+ \frac{\lambda}{2\pi^2 \hat{\beta}^4} \int_0^\infty dx\, x^2 \ln\left[1 - e^{-\sqrt{x^2 + \hat{\beta}^2 \hat{M}^2}}\right], \end{aligned} \tag{3.2}$$

where $\hat{M}^2 \equiv -1 + 3\hat{\phi}^2$, $\hat{\phi} \equiv \phi/\eta$ is a reduced Higgs field, $\eta \equiv \sqrt{m^2/\lambda}$ is the field point, λ is the Higgs field self-coupling constant, and $\hat{\beta}^2 \equiv \lambda \eta^2 \beta^2$ where $\beta = 1/T$. Equation (3.2) is the one-loop approximation, and the range of ϕ is restricted to $\hat{\phi}^2 > 1/3$. While the higher-loop terms make significant contributions, the one-loop approximation is good enough for our exploratory purpose. In Fig. 1 I show the effective potential, $\hat{V}_{\hat{\beta}}\left(\hat{\phi}\right)$, plotted against $\hat{\phi}$ for several values of constant temperature in the range $0 \leq \hat{T} \leq \hat{T}_c$, where $\hat{T}_c$ is the critical temperature above which the vacuum is no longer spontaneously broken. In Fig. 2 the value of $\hat{\phi}^2$ at minimum effective potential, $\left\langle \hat{\phi}^2 \right\rangle_{\min \hat{V}_{\hat{\beta}}}$, is plotted against temperature, $\hat{T}$. From the figure we note that $d\left\langle \hat{\phi}^2 \right\rangle_{\min \hat{V}_{\hat{\beta}}} / d\hat{T}$ is always negative for $\hat{T} > 0$ and goes to zero as $\hat{T} \to 0$. Accordingly, in an accelerated frame with a negative temperature gradient, the expectation value of the Higgs field will increase with x in the direction of acceleration.

IV. PROPAGATION OF THE HIGGS PARTICLE IN THE ACCELERATED FRAME

In the minimal Standard Model the mass of the Higgs particle is given by

$$m_H^2 = \lambda \left\langle \phi^2 \right\rangle = \lambda \eta^2 \left\langle \hat{\phi}^2 \right\rangle = m^2 \left\langle \hat{\phi}^2 \right\rangle , \tag{4.1}$$

where $m^2 > 0$. According to Eq. (4.1) and Fig. 2, when the Higgs particle propagates in the accelerated frame its mass varies because the vacuum expectation value of the square of the Higgs field varies from point to point in x. From Eq. (2.7) we see that $(T - T_0)/T_0 \ll 1$ for $|ax/c^2| \ll 1$, and we can expand $m^2 \left\langle \hat{\phi}^2 \right\rangle$ in a Taylor series about $\hat{T}_0$, the temperature at $x = 0$, and retain only up to the linear term.

$$m^2 \left\langle \hat{\phi}^2 \right\rangle = m^2 \left\langle \hat{\phi}^2 \right\rangle_{T_0} + m^2 \left(\frac{d \left\langle \hat{\phi}^2 \right\rangle}{d\hat{T}} \right)_{T_0} \left(\hat{T} - \hat{T}_0 \right) . \tag{4.2}$$

Then substituting Eq. (2.7) divided through by $\eta\sqrt{\lambda}$ into Eq. (4.2) for $\hat{T}$ and the result into Eq. (4.1), we have for the Higgs mass

$$m_H^2 = m_0^2 \left(1 + \mu a x \right) , \tag{4.3}$$

where $m_0^2 \equiv m^2 \left\langle \hat{\phi}^2 \right\rangle_0$ is the mass at $x = 0$, and

$$\mu \equiv -\hat{T}_0 \left(\frac{d \ln \left\langle \hat{\phi}^2 \right\rangle}{d\hat{T}} \right)_{\hat{T}_0} \tag{4.4}$$

is a positive dimensionless parameter. I have set $c = 1$ in Eq. (2.7) before substituting in order to be compatible with natural units.

A particle whose mass varies linearly in a given spatial direction as in Eq. (4.3) propagates as though it had a fixed mass m_0 in a region with a linear potential. I establish this correspondence by determining the propagator for a scalar Higgs particle whose mass varies according to Eq. (4.3). I continue with natural units for this calculation. For acceleration in the x^k direction, $k = 1, 2$ or 3, the free Euclidean action is obtained by substituting Eq. (4.3) into Eq. (3.1) for m^2 and setting $\lambda = 0$. It is given by

$$S=\frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\phi\partial^\mu\phi+m_0^2\left(1+\mu a x^k\right)\phi^2\right], \tag{4.5}$$

where in natural units a $(= a/c^2)$ has a unit of inverse length, and $g=-1$ since we still have a Minkowski metric. I have omitted the self-coupling term from the action because I am interested in the effect of acceleration on the otherwise free propagator. For $\mu a x^k \ll 1$, I treat the term $\mu a x^k$ as a small perturbation and follow the perturbative procedure given by Ramond [19].

The shape to shape amplitude in the presence of an arbitrary current $J(x)$ is given by the functional integral

$$\langle\phi''(\infty)|\,\phi'(\infty)\rangle = N\int[d\phi]\,e^{-\left\langle\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{1}{2}m_0^2\phi^2+V(\phi)-J\phi\right\rangle_x}, \tag{4.6}$$

where

$$V(\phi)\equiv\frac{1}{2}m_0^2\mu a x^k\phi^2. \tag{4.7}$$

Here and in the following equations $\langle\cdots\rangle_x\equiv\int d^4x\cdots$. The vacuum persistence amplitude is given by

$$\begin{aligned}e^{-Z[J]} &= \langle 0_+|\,0_-\rangle_J\\ &= \int[d\phi''][d\phi']\langle 0_+|\,\phi''\rangle\langle\phi''|\,\phi'\rangle_J\langle\phi'|\,0_-\rangle\\ &= e^{-\left\langle V\left(\frac{\delta}{\delta J}\right)\right\rangle_x}e^{-Z_0[J]},\end{aligned} \tag{4.8}$$

where $Z[J]$ and $Z_0[J]$ are the generating functionals for connected Green's functions with the perturbation V turned on and off, respectively. Ramond's procedure, with the perturbation given by Eq. (4.7), gives the following expression.

$$\begin{aligned}Z[J] &= \frac{1}{2}\langle J_x\Delta_{xy}J_y\rangle_{xy}\\ &+\frac{1}{2}m_0^2\mu a\left[\left\langle x^k\Delta_{xx}\right\rangle_x+\left\langle x^k\Delta_{xy}\Delta_{xz}J_yJ_z\right\rangle_{xyz}\right]\\ &-\frac{1}{2}m_0^4\mu^2a^2\left[\left\langle J_w\Delta_{wx}x^k\Delta_{xy}y^k\Delta_{yz}J_z\right\rangle_{wxyz}+\frac{1}{2}\left\langle x^ky^k\Delta_{xy}^2\right\rangle_{xy}\right]+\cdots,\end{aligned} \tag{4.9}$$

where the $\Delta_{xy}\equiv\Delta(x-y)$ are Feynman propagators and $J_x\equiv J(x)$, etc.

The propagator for the variable-mass Higgs particle in the accelerating frame is given by the two-point Green's function,

$$\begin{aligned} G^{(2)}\left(x_1,x_2\right) &= -\left.\frac{\delta^2 Z\left[J\right]}{\delta J\left(x_1\right)\delta J\left(x_2\right)}\right|_{J=0} \\ &= \Delta\left(x_1-x_2\right) - m_0^2\mu a\int d^4x\Delta\left(x_1-x\right)x^k\Delta\left(x-x_2\right) \\ &+ m_0^4\mu^2a^2\int d^4x\int d^4y\Delta\left(x_1-x\right)x^k\Delta\left(x-y\right)y^k\Delta\left(y-x_2\right)+\cdots. \end{aligned} \tag{4.10}$$

With m_0 and a each having units of inverse length, μ dimensionless, and each Feynman propagator having two inverse powers of length, the terms of Eq. (4.10) are dimensionally consistent.

Equation (4.10) gives the propagator for a variable mass (but otherwise free) Higgs boson between spacetime points x_1 and x_2 in a frame accelerating in the x^k direction. The complete propagator, including self interaction, would consist of diagrams containing four-legged vertices connected by variable-mass propagators of the form given by Eq. (4.10). The variable-mass propagator of Eq. (4.10) can also be interpreted as the propagator for a particle of fixed mass m_0 moving in a potential, $V\left(x^k\right) = m_0^2\mu a x^k$, which rises in the direction of acceleration x^k. Engineering units are restored by substituting the inverse Compton wavelength, $m_0c/\hbar$, for m_0 and letting $a \to a/c^2$. Then with the x axis the axis of acceleration $(k=1)$, the potential is

$$V\left(x\right) = \frac{m_0^2c^2}{\hbar^2}\frac{\mu a x}{c^2}\,. \tag{4.11}$$

V. A MODEL OF AN ACCELERATED NUCLEON

We now have a picture of a virtual Higgs boson propagating inside a nucleon that is subject to acceleration. Propagating in the direction of acceleration, against the potential gradient, it loses energy-momentum and vice versa. The energy-momentum that the Higgs boson had at the beginning of its propagation was borrowed from an up or down quark at a Higgs-quark vertex. At the end of its propagation at a second Higgs-quark vertex the quark gets back less energy-momentum than it lent if the motion is in the direction of acceleration, and vice versa. At the boundary of the nucleon, where the gluon flux tubes are fully stretched, the momentum deficit of the quarks in the direction of acceleration and surplus in the opposite direction are transferred to the nucleon as a whole. This model of dynamical inertia generation may be oversimplified, but it also may contain a grain of physical reality. In the Standard Model, the Yukawa coupling terms in $\mathcal{L}_{\mathrm{Higgs}}$,

$$-\frac{h}{2}\eta\bar{q}q - \frac{h}{2}\frac{1}{\sqrt{2}}\bar{q}\sigma q\,, \tag{5.1}$$

are usually interpreted as a quark mass term and a quark-Higgs vertex term. But they can also be interpreted as simply a quark-Higgs vertex term, $-(h/2)\,\bar{q}\phi q$, with a Higgs field, $\phi = \eta + \sigma/\sqrt{2}$, measured from the origin instead of the field point.

The bound-state problem of interacting quark, gluon, and Higgs fields inside a nucleon has not been solved quantum field theoretically, even in an inertial frame of reference. Instead, I neglect the quarks and gluons and consider a simple, relativistic, semiclassical model consisting of a standing wave for the Higgs field in a cubic potential well which is square in the y and z directions and has a sloping floor in x, the direction of acceleration.

The wave equation corresponding to the action of Eq. (4.5) is the Klein-Gordon equation with a potential term given by Eq. (4.11).

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m_0^2c^2}{\hbar^2}\left(1+\mu\frac{ax}{c^2}\right)\right]\Psi = 0. \tag{5.2}$$

For a standing wave solution I substitute $\Psi(t,\mathbf{r}) = \exp(-iEt/\hbar)\,\psi(\mathbf{r})$ into Eq. (5.2) and obtain the time-independent equation,

$$\left[-\nabla^2 + \frac{m_0^2c^2}{\hbar^2}\left(1+\mu\frac{ax}{c^2}\right)\right]\psi(\mathbf{r}) = \frac{E^2}{\hbar^2c^2}\psi(\mathbf{r})\,. \tag{5.3}$$

Writing $\psi(\mathbf{r}) = X(x)\,Y(y)\,Z(z)$ and $E^2 = E_x^2 + E_y^2 + E_z^2$, Eq. (5.3) separates in the usual way giving

$$-\frac{d^2X}{dx^2} + \frac{m_0^2c^2}{\hbar^2}\left(1+\mu\frac{ax}{c^2}\right)X = \frac{E_x^2}{\hbar^2c^2}X, \tag{5.4}$$

$$\frac{d^2Y}{dy^2}+\frac{E_y^2}{\hbar^2c^2}Y=0,\qquad \frac{d^2Z}{dz^2}+\frac{E_z^2}{\hbar^2c^2}Z=0. \tag{5.5}$$

Imposing the boundary conditions, $Y(0)=Y(s)=0$, on the solution, $Y=\sin(E_y y/\hbar c)$, gives $E_y/\hbar c=\pi/s$ for the ground state, and similarly for E_z. Then we can write Eq. (5.4) in terms of the total energy as

$$-\frac{d^2X}{dx^2}+\frac{m_0^2c^2}{\hbar^2}\left(1+\mu\frac{ax}{c^2}\right)X=\left(\frac{E^2}{\hbar^2c^2}-\frac{2\pi^2}{s^2}\right)X. \tag{5.6}$$

Making the change of variable,

$$\rho=\frac{x}{l}+\lambda\,, \tag{5.7}$$

where l is a length defined by

$$\frac{1}{l^3}\equiv\left(\frac{m_0^2c^2}{\hbar^2}\right)\left(\frac{\mu a}{c^2}\right), \tag{5.8}$$

and λ is a dimensionless energy parameter defined by

$$\lambda\equiv-\frac{c^2}{\mu a l}\left(\frac{E^2}{m_0^2c^4}-\frac{2\pi^2\hbar^2}{m_0^2c^2s^2}-1\right), \tag{5.9}$$

Eq.(5.6) becomes Airy's equation,

$$\frac{d^2X}{d\rho^2}-\rho X=0, \tag{5.10}$$

which has a general solution in terms of Airy functions [20]:

$$X(\rho)=C_1Ai(\rho)+C_2Bi(\rho)\,, \tag{5.11}$$

where C_1 and C_2 are constants to be determined by boundary conditions.

In applying the two boundary conditions at $x=0$ and $x=s$ it is useful to make the following definitions:

$$u\equiv s/l\ , \tag{5.12}$$

$$v\equiv\frac{c^2}{\mu a s}\left(\frac{E^2}{m_0^2c^4}-\frac{2\pi^2\hbar^2}{m_0^2c^2s^2}-1\right). \tag{5.13}$$

Then $\lambda = -uv$ and $x = 0, s$ corresponds to $\rho = -uv, u\,(1-v)$, respectively, and we can write the two boundary conditions as

$$C_1 Ai\,[-uv] + C_2 Bi\,[-uv] = 0, \tag{5.14}$$

$$C_1 Ai\,[u\,(1-v)] + C_2 Bi\,[u\,(1-v)] = 0. \tag{5.15}$$

Setting the determinant to zero, we have

$$Ai\,(-uv)\,Bi\,(u\,(1-v)) - Ai\,(u\,(1-v))\,Bi\,(-uv) = 0. \tag{5.16}$$

For a given value of u, determined by the magnitude of the acceleration a and the size of the nucleon s through Eqs. (5.8) and (5.12), we seek the least root of Eq. (5.16), $v \equiv v_0$, corresponding to the lowest positive energy $E = E_0$ through Eq. (5.13). The amplitude of the corresponding standing wave is given by

$$\psi\,(\mathbf{r}) = A\left\{Ai\,[u\,(x/s - v_0)] - \frac{Ai\,[-uv_0]}{Bi\,[-uv_0]} Bi\,[u\,(x/s - v_0)]\right\}\sin\left(\frac{\pi y}{s}\right)\sin\left(\frac{\pi z}{s}\right), \tag{5.17}$$

and is shown in Fig. 3 for $y = z = s/2$.

The net force on the nucleon is the (vector) sum of the forces on the boundaries at $x = x_1 = 0$ and $x = x_2 = s$. Each of the forces on a boundary is the negative rate of change of the energy E_0 when the boundary is moved, holding the other boundary fixed. Thus we have for the net force

$$F = -\left.\frac{\partial E_0}{\partial x_1}\right|_{x_1=0,\,x_2=s} - \left.\frac{\partial E_0}{\partial x_2}\right|_{x_1=0,\,x_2=s}, \tag{5.18}$$

where the forces are taken to be positive in the positive x direction, and E_0 corresponds to the least root v_0. In order to take the partial derivatives we must re-express the boundary conditions as $X\,(x = x_1) = X\,(x = x_2) = 0$ and set $x_1 = 0$ and $x_2 = s$ only after differentiation. The corresponding values of ρ are $\rho_1 = u\,(\xi_1 - v)$ and $\rho_2 = u\,(\xi_2 - v)$, where $\xi \equiv x/s$. Thus we have at the two boundaries

$$C_1 Ai\,[u\,(\xi_1 - v)] + C_2 Bi\,[u\,(\xi_1 - v)] = 0, \tag{5.19}$$

$$C_1 Ai\left[u\left(\xi_2 - v\right)\right] + C_2 Bi\left[u\left(\xi_2 - v\right)\right] = 0, \tag{5.20}$$

and setting the secular determinant to zero gives

$$Ai\left[u\left(\xi_1 - v\right)\right] Bi\left[u\left(\xi_2 - v\right)\right] - Ai\left[u\left(\xi_2 - v\right)\right] Bi\left[u\left(\xi_1 - v\right)\right] = 0. \tag{5.21}$$

I now partially differentiate Eq. (5.21) with respect to ξ_1 holding ξ_2 constant and vice versa, finally setting $\xi_1 = 0$, $\xi_2 = 1$, and $v = v_0$ in each instance. I solve the resulting equations respectively for $\partial v/\partial\xi_1|_{\xi_1=0,\,\xi_2=1,\,v=v_0}$ and $\partial v/\partial\xi_2|_{\xi_1=0,\,\xi_2=1,\,v=v_0}$ and obtain

$$\begin{aligned}
&\left.\frac{\partial v}{\partial \xi_1}\right|_{\xi_1=0,\xi_2=1,\,v=v_0} \\
&= \left\{Ai\left[u\left(1-v_0\right)\right] Bi'\left[-uv_0\right] - Ai'\left[-uv_0\right] Bi\left[u\left(1-v_0\right)\right]\right\} \\
&\times \left\{Ai\left[u\left(1-v_0\right)\right] Bi'\left[-uv_0\right] + Ai'\left[u\left(1-v_0\right)\right] Bi\left[-uv_0\right]\right. \\
&\quad \left. - Ai\left[-uv_0\right] Bi'\left[u\left(1-v_0\right)\right] - Ai'\left[-uv_0\right] Bi\left[u\left(1-v_0\right)\right]\right\}^{-1},
\end{aligned} \tag{5.22}$$

and a similar equation for $\partial v/\partial\xi_2|_{\xi_1=0,\,\xi_2=1,\,v=v_0}$ where in each term the prime (denoting a derivative with respect to the argument of the Airy function) is switched to the other factor.

The derivatives of v are related to derivatives of energy through Eq. (5.13):

$$\begin{aligned}
&\left.\frac{\partial v}{\partial \xi_1}\right|_{\xi_1=0,\,\xi_2=1,\,v=v_0} \\
&= \frac{dx_1}{d\xi_1}\frac{\partial}{\partial x_1}\left[\frac{c^2}{\mu a}\left(\frac{E^2}{m_0^2 c^4} - \frac{2\pi^2\hbar^2}{m_0^2 c^2 s^2} - 1\right)\right]\Bigg|_{x_1=0,\,x_2=s,\,E=E_0} \\
&= \frac{2E}{\mu m_0^2 c^2 a}\frac{\partial E}{\partial x_1}\Bigg|_{x_1=0,\,x_2=s,\,E=E_0},
\end{aligned} \tag{5.23}$$

and similarly for $\partial v/\partial\xi_2|_{\xi_1=0,\,\xi_2=1,\,v=v_0}$. Solving Eq. (5.23) for $\partial E/\partial x_1$ and $\partial E/\partial x_2$ and substituting into Eq. (5.18), we have for the net force

$$F = -\frac{\mu a}{c^2}\frac{m_0^2 c^4}{2E_0}\left[\frac{\partial v}{\partial \xi_1} + \frac{\partial v}{\partial \xi_2}\right]_{\xi_1=0,\,\xi_2=1,\,v=v_0}. \tag{5.24}$$

The quantity in the square bracket in Eq. (5.24) is identically unity (and dimensionless), as can be demonstrated by putting the sum of Eq. (5.22) and its counterpart over a common denominator. Furthermore, from Eq. (5.13), the energy E_0 is given in terms of the least root v_0 by

$$E_0 = m_0 c^2 \sqrt{1 + \frac{2\pi^2\hbar^2}{m_0^2 c^2 s^2} + \frac{v_0 \mu a s}{c^2}}\,. \tag{5.25}$$

Putting it all together, we have for the net force on the nucleon

$$F = -\frac{1}{2}\frac{\mu m_0 a}{\sqrt{1 + \frac{1}{2}\left(\frac{\lambda_0}{s}\right)^2 + \mu v_0 \left(\frac{as}{c^2}\right)}}\,, \tag{5.26}$$

where $\lambda_0 = h/m_0 c$ is the Compton wavelength of the Higgs boson. As of 1992 the estimated lower bound for the Higgs mass was 48 GeV [21], which corresponds to a Compton wavelength $\lambda_0 < 2.9 \times 10^{-34}$ m, giving a value of λ_0/s of the order of 10^{-19}. Also, since μ and v_0 are the order of unity or less, for values of acceleration such that $|as/c^2| \ll 1$ the third term in the square root is small compared to unity. Thus we are justified in expanding the square root in Eq. (5.26) to obtain

$$F = -\frac{1}{2}\mu m_0 a \left[1 - \frac{1}{4}\left(\frac{\lambda_0}{s}\right)^2 - \frac{1}{2}\mu v_0 \left(\frac{as}{c^2}\right) + \cdots\right]. \tag{5.27}$$

This result differs from Newton's second law by a leading term of order as/c^2. Since $s \sim 10^{-15}$ m, this term is extremely small compared to unity even for very large values of acceleration. For example, if the nucleon were part of an atom vibrating in a crystal lattice with a maximum acceleration of 10^{17} ms^{-2}, the leading non Newtonian term is of the order of 10^{-15}. The other term in Eq. (5.27) is even less at $O(10^{-38})$. So to a very high degree of accuracy we can write

$$F = -\frac{1}{2}\mu m_0 a \equiv -m_n a\,, \tag{5.28}$$

where the $\equiv$ defines the inertial mass of the nucleon m_n by Newton's second law.

The key parameter in Eq. (5.28) is the dimensionless vacuum temperature parameter μ. It is determined by the variation of spontaneous symmetry breaking of the Higgs vacuum with temperature at the temperature of the vacuum inside the nucleon as shown in Fig. 2. Although the model of an acclerating nucleon is not complete (the quarks and the gluons have been neglected), nevertheless it is my conjecture that the nucleons and other baryons have different inertial masses because the temperature of the Higgs vacuum inside them is different. For example, the mass of a neutron is slightly greater than that of a proton because the presence of a down quark in the place of an up quark causes the Higgs vacuum to be at a slightly higher temperature and the vacuum temperature parameter a little greater.

Assuming the nucleon is a proton with a mass $m_p = 0.938$ GeV and using the lower bound of 48 GeV for the Higgs mass, we have from the second equality in Eq. (5.28) that $\mu < 0.039$. Then, using Eq. (4.4) and the numbers from Fig. 2, we get an estimate of a lower bound on the temperature of the Higgs vacuum inside the proton of 3.2×10^{14} K.

VI. SUMMARY AND CONCLUSIONS

A mechanism for the dynamical generation of inertia has been proposed. It consists of the following ideas: (1) in an inertial frame of reference the vacuum has a uniform temperature; (2) in an accelerated frame the temperature of the vacuum varies linearly and decreases in the direction of acceleration; (3) consequently the spontaneous symmetry breaking of the Higgs vacuum is nonuniform in the accelerating frame and the resulting mass of the Higgs boson varies linearly as it propagates over short distances; (4) the resulting propagator corresponds to a particle of fixed mass moving in a linearly rising potential in the direction of acceleration; (5) propagating Higgs bosons inside nucleons lose energy-momentum in the direction of acceleration and gain it in the opposite direction; (6) the energy-momentum deficits and surpluses are transferred to quarks propagating in the respective directions at Higgs-quark vertices; the momentum deficits in the direction of acceleration and surpluses in the opposite direction are transferred from the quarks to the nucleon as a whole at the nucleon boundary.

Although the model presented of an accelerating nucleon certainly needs refining, the mechanism for dynamic generation of inertia has yielded a result for an inertial force that is consistent with Newton's prescription through the Second Law. But while the Newtonian inertial force is interpreted as fictitious, or kinematic, the dynamically generated inertial force is real, a reality that is perceived in everyday experience. The dynamically generated inertial force is not a resistance to velocity, but only to acceleration because the vacuum temperature is uniform in any inertial frame.

There is an important issue which has not been addressed. The inertial mass of a macroscopic body is not just the sum of the inertial masses of its constituent partons. There is a mass deficit associated with binding energy. In dynamically generated inertia a mass deficit must be caused by a change in the Higgs vacuum when binding takes place. A possibility in the proposed mechanism is that binding causes the Higgs vacuum locally to be at a lower temperature T_0 than otherwise, moving $\left\langle \hat{\phi} \right\rangle^2$ to a point of less slope in Fig. 2 (μ decreases) and resulting in a lower inertial mass.

Spacetime alone does not exist without the fundamental quantum fields in it. The vacuum state of these fields is all pervasive. From a field theorist's perspective, spacetime merely serves as a label for the degrees of freedom of the fields. From this point of view it is questionable whether it is logically sound to base physics on inertial frames determined by classical configurations of the spacetime metric tensor field, completely ignoring the fundamental quantum fields. Such an approach to physics confers on spacetime a stand-alone reality which is somewhat doubtful. In this paper I have given alternative determinations of inertial and noninertial frames involving the all-pervasive vacuum state of the Higgs field.

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FIGURES

FIG. 1. The finite-temperature effective potential $\hat{V}_{\hat{\beta}}\left(\hat{\phi}\right)$ versus the magnitude of the Higgs field $\hat{\phi}$ for several temperatures in the range $0 \leq \hat{T} \leq \hat{T}_c$ where $\hat{T}_c$ is the critical temperature above which the vacuum is no longer spontaneously broken. Axes: vbetahat $\equiv \hat{V}_{\hat{\beta}}\left(\hat{\phi}\right)$; phihat $\equiv \hat{\phi}$.

FIG. 2. The expectation value of the square of the Higgs field corresponding to minimum effective potential $\left\langle\hat{\phi}^2\right\rangle_{\min \hat{V}_{\hat{\beta}}}$ as a function of temperature $\hat{T}$. Axes: minphihatsq $\equiv \left\langle\hat{\phi}^2\right\rangle_{\min \hat{V}_{\hat{\beta}}}$; That $\equiv \hat{T}$.

FIG. 3. The unsymmetric factor of the Higgs field standing wave, $X\left(x\right)$, versus distance x in the cubic model of an accelerated nucleon. Axes: psi $\equiv \psi$; x/s $\equiv x/s$.