IF-UERJ 007/97

BRS-BFT Quantization of the Chiral Schwinger Model on the Light-Front

Prem P. Srivastava[1]

Instituto de Física, Universidade do Estado de Rio de Janeiro, Rua São Francisco Xavier 524, Rio de Janeiro, RJ, 20550-013, Brasil.

ABSTRACT

Any theory quantized on the light-front (LF) gives rise in general to second class constraints. On the other hand Batalyin, Fradkin, and Tuytin (BFT) have proposed a systematic procedure for converting a second class system into (an abelian) first class system where possibly, according to Faddeev and Shatashvili, the genuinely anomalous theory can be distinguished from the apparently anomalous theory, e.g., the one which can be quantized selfconsistently in spite of the anomaly. An illustration was found in the study made by Jackiw and Rajaraman on the gauge non-invariant description of the Chiral Schwinger model (CSM) in its bosonized formulation.

We quantize the bosonized CSM in the case of the regularization ambiguity $a > 1$ by using BFT procedure on the LF, where we have two second class and one first class constraints. A first class theory is constructed in an extended phase space. The unitarizing involutive Hamiltonian which naturally generates all the secondary constraints and the BRS charge are obtained. Starting from the phase space partition function which employs the corresponding BRS Hamiltonian we obtain after performing the momentum integrations the corresponding first class Lagrangian in the configuration space.

The LF vacuum in the CSM is constructed and compared with the well known *condensate* or $\theta-$vacua in the Schwinger model. The discussion on the LF becomes quite transparent where, according to the proposal made earlier in the context of the description of the spontaneous symmetry breaking on the LF, the separation in the scalar field of the dynamical bosonic condensate variable has to be made first, before applying the Dirac procedure for the constrained dynamical systems. The condensate variable is seen to be neatly removed in the CSM by a field redefinition giving rise to a different vacuum structure in contrast to the case of the Schwinger model.

Keywords: Constrained Systems, Batalin-Fradkin-Tyutin Hamiltonian method, Batalin-Fradkin-Vilkovisky Formalism, chiral Schwinger Model

PACS numbers: 11.10.Ef, 11.15Tk

[1]E-mail: prem@lafexsu1.lafex.cbpf.br prem@vax.fis.uerj.br

1. Introduction

Dirac [1] in 1949 pointed out the advantages of studying the relativistic quantum dynamics of physical system on the hyperplanes of the light-front: $x^0 + x^3 = const.$, the *front form*. The LF coordinates $(x^+, x^-, x^\perp)$ where $x^\pm = (x^0 \pm x^3)/\sqrt{2} = x_\mp$ and $x^\perp = (x^1, x^2)$ are then convenient to use. We make the *convention* to regard [2] $x^+ \equiv \tau$ as the LF *time* coordinate while $x^- \equiv x$ is the longitudinal spatial coordinate and we study the evolution in τ. The transformation from the conventional to LF components is, however, *not a Lorentz transformation* and the structure of the *LF phase space* is notably different when compared with that of the conventional one and as a consequence, for example, the *LF vacuum* may become simpler than the one in the conventional (equal-time) theory. The SSB on the LF is also described [2] in a different fashion even though the physical outcome, as expected, is the same. An important advantge pointed out by Dirac of *front form* formulation is that here *seven* out of the ten Poincaré generators are *kinematical* against only six ones in the conventional theory constructed on the hyperplanes $x^0 = const.$, the *instant form*. Another distinguishing feature of theory in the *front form* is that it gives rise to a constrained dynamical system [3] which leads in general to a reduction in the number of independent field operators on the phase space. The vacuum structure may then become more tractable and the computation of physical quantities simpler. We recall that the LF field theory was rediscovered in 1966 by Weinberg [4] in his Feynman rules adapted for infinite momentum frame. It was demonstrated [5] latter that these rules correspond to the quantization of the theory on the LF. The recent revival [6-7] of the interest in LF quantization owes to the difficulties encountered in the computation of nonperturbative effects in the *instant form* QCD or in the study of the relativistic bound states of light fermions [7-6].

Any theory quantized on the light-front (LF) gives rise in general to second class constraints along with some first class ones. Recently, Batalyin, Fradkin, and Tuytin (BFT) [8,9] have proposed a systematic procedure for converting a second class system into (an abelian) first class system where possibly, according to Faddeev and Shatashvili [10], the genuinely anomalous theory can be distinguished from the apparently anomalous theory, e.g., the one which can be quantized selfconsistently in spite of the anomaly. An illustration was found in the study made by Jackiw and Rajaraman [11] on the gauge non-invariant description of the Chiral Schwinger model (CSM) in its bosonized formulation. They showed that a consistent and unitary quantum field theory is even possible in the gauge non-invariant formulation. Alternatively, a gauge-invariant version [12], can be obtained by adding a Wess-Zumino term to the gauge non-invariant original theory as was proposed in ref. [10]. The CSM is considered as an archetype of anomalous gauge theories and has received attention of many authors [13].

Recently, it was shown [14] that the well known $\theta - vacua$ in the Schwinger model could be obtained by a straightforward quantization of the theory on the light-front

[2] We can of course make the convention with the role of x^+ and x^- interchanged.

(LF). The procedure adopted was the one proposed earlier in connection with the *front form* description of the SSB (and the tree level Higgs mechanism) [2,14]. The scalar field (of the equivalent bosonized Schwinger model) is separated, based on physical considerations, into the *dynamical bosonic condensate* and the quantum fluctuation fields. The Dirac procedure [3] is then followed in order to construct the Hamiltonian formulation and the quantized theory. The $\theta-$vacua come out naturally along with their continuum normalization [14]. It is then rather important to understand as to how and why the vacuum structure in the LF quantized CSM should come out to be quite different; as is known already from the studies on CSM in the conventional framework. We could work with the standard Dirac method but the recently proposed BFT procedure which is elegant and avoids the computation of Dirac brackets. It would thus get tested on the LF as well and it also allows for constructing (new) effective Lagrangian theories.

The organization of the paper is as follows: In section 2, we convert the two second class constraints of the bosonized CSM with $a > 1$ into first class constraints according to the BFT formalism. In section 3, we obtain the first class Hamiltonian from the canonical Hamiltonian and recover the DB using Poisson brackets in the extended phase space. In section 4, we find the corresponding first class Lagrangian after performing the momentum integrations, and section 5 is describe the Conclusion.

2. Conversion on the LF of the CSM Model to First Class Constrained Dynamical System

The bosonized CSM model (for $a > 1$) is described by the action

$$S_{CSM} \;=\; \int d^2x \;\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\nu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi + \frac{1}{2}ae^2A_\mu A^\mu \;\right], \quad (1)$$

where a is a regularization ambiguity [14], which is defined for calculating the fermionic determinant of the fermionic CSM. The LF coordinates are $(x^+ \equiv \tau, x^- \equiv x)$ where $x^{\pm} = (x^0 \pm x^1)/\sqrt{2} = x_{\mp}$ and the $\pm$ components of any tensorial index are defined in analogous fashion. The nonvanishing elemets of the metric tensor $\eta_{\mu\nu}$ are $\eta_{+-} = \eta_{-+} = 1$ while $\epsilon^{-+} = \epsilon^{01} = 1$. The Eq. (1) in the LF coordinates takes the form

$$S_{CSM} \;=\; \int d^2x^- \;\left[\frac{1}{2}(\partial_+A_- - \partial_-A_+)^2 + \partial_-\phi\,\partial_+\phi + 2eA_-\partial_+\phi + ae^2A_+A_-\right], \qquad (2)$$

We now make the *separation*, in the scalar field (a generalized function) : $\phi(\tau, x^-) = \omega(\tau) + \varphi(\tau, x^-)$, where $\omega(\tau)$ is the (dynamical) bosonic *condensate* variable and the field φ represents the quantum fluctuations. This procedure enabled us to give [2] earlier the description of the SSB and (tree level) Higgs mechanism on the LF and there the variable ω was shown to be a c-number or a background field. In the QED_2, e.g., the Schwinger model, on the contrary, it was shown to be a q-number operator and whose eigenvalues were shown there to label the vacuum states. Such a separation as noted earlier also in ref. [14,15] is rather required so that the problem under consideration may be described by means of a *well posed* mathematical problem. We set $\int dx^- \varphi(\tau, x^-) = 0$ so that the entire zero-momentum mode of ϕ is represented by the condensate variable. The *chiral transformations* is defined [15,14] by: $\omega \rightarrow \omega + const.$, $\varphi \rightarrow \varphi$, and $A_\mu \rightarrow A_\mu$ so that the *boundary conditions* at infinity on the quantum fluctuation field φ are kept unaltered under these transformations. Unlike in the Schwinger model the $U(1)$ gauge symmetry is now broken due to the presence of the mass term for the gauge field. The Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2}(\partial_+ A_- - \partial_- A_+)^2 + \partial_- \varphi \, \partial_+ \varphi + ae^2 [A_+ + \frac{2}{ae}(\partial_+ \varphi + \partial_+ \omega)] A_-, \tag{3}$$

We note that the dynamical fields are A_- and φ while A_+ has no kinetic term. On making a redefinition of the (auxiliary) field A_+ we can recast the action on the LF in the following form

$$S_{CSM} \;=\; \int dx^- \left[\dot{\varphi}\varphi' + \frac{1}{2}(\dot{A}_- - {A_+}')^2 - 2e\dot{A}_- \varphi + ae^2 A_+ A_- \right], \tag{4}$$

Here, an overdot (a prime) indicates the partial derivative with respect to τ (x^-), and φ is assumed, in order to connect the mathematical structure above with the physical interpretations, to satisfy the conditions for the existence of the Fourier transform in the spatial variable x^-. We note that unlike in the case of the Schwinger model the condensate variable is totally removed in the present case. This is an indication of that the vacuum structure of the CSM would be quite different from that of the Schwinger model.

The canonical momenta are given by

$$\begin{aligned} \pi^+ &= 0, \\ \pi^- &= \dot{A}_- - A'_+ - 2e\varphi, \\ \pi_\varphi &= \varphi'. \end{aligned} \tag{5}$$

Since the Hamiltonian framework must be self-consistent with the Lagrangian equations we give a look at them before applying the Dirac procedure. We have

$$\begin{aligned} \partial_+ \partial_- \phi &= -e\partial_+ A_-, \\ \partial_+ \partial_+ A_- - \partial_+ \partial_- A_+ &= ae^2 A_+ + 2e\partial_+ \phi, \\ \partial_- \partial_- A_+ - \partial_+ \partial_- A_- &= ae^2 A_-, \end{aligned} \tag{6}$$

We derive easily $[(a-1)\partial_\mu \partial^\mu + a^2 e^2] A_- = 0$, $(2-a)\partial_+ A_- = a\partial_- A_+$, and $\partial_\mu \partial^\mu \phi = -2e\partial_+ A_-$. It follows that for $a > 1$

$$\partial_\mu \partial^\mu \phi \;=\; -\frac{ea}{(a-1)} E, \tag{7}$$

$$\left[\partial_\mu \partial^\mu + \frac{e^2 a^2}{(a-1)} \right] E \;=\; 0, \tag{8}$$

where $E = F_{+-}$, and consequently both the massive and massless scalar excitations are present in the model under considerations. The massless one is not a Goldstone boson. The vacuum structures of the Schwinger model, which requires $\theta - vacua$, and that of the CSM are quite different.

We follow now the Dirac's standard procedure [3] in order to build an Hamiltonian framework on the LF. The definition of the canonical momenta leads to two primary constraints

$$\pi^+ \approx 0, \tag{9}$$

$$\Omega_1 \equiv (\pi_\varphi - \varphi') \approx 0 \tag{10}$$

and we derive one secondary constraint

$$\Omega_2 \equiv \partial_- \pi^- + +2e\varphi' + ae^2 A_- \approx 0. \tag{11}$$

This one follows when we require the τ independence (e.g., the persistency) of the primary constraint π^+ with respect to the preliminary Hamiltonian

$$H' = H_c^{\,l.f.} + \int dx\ u_+ \pi^+ + \int dx\ u_1 \Omega_1, \tag{12}$$

where H_c is the canonical Hamiltonian

$$H_c^{\,l.f.} \ = \ \int dx\ \left[\frac{1}{2}(\pi^- + 2e\varphi)^2 + (\pi^- + 2e\varphi)A'_+ - ae^2 A_+ A_- \right], \tag{13}$$

and we employ the standard equal-τ Poisson brackets. The u_+ and u_1 denote the Lagrange multiplier fields. The persistency requirement for Ω_1 give conditions to determine u_1. The Hamiltonian is next extended to include also the secondary constraint

$$H_e^{\,l.f.} = H_c^{\,l.f.} + \int dx\ u_+ \pi^+ + \int dx\ u_1 \Omega_1 + \int dx\ u_2 \Omega_2 \tag{14}$$

and the procedure is now repeated with respect to the extended Hamiltonian. For the case $a > 1$, no more secondary constraints are seen to arise and we are left only with the persistency conditions which determine the multipliers u_1 and u_2 while u_+ is left undetermined. We also find[3] $\{\Omega_i, \Omega_j\} \ = \ D_{ij} \ \ (-2\partial_x \delta(x-y))$ where $i,j = 1,2$ and $D_{11} = 1,\ D_{22} = ae^2,\ D_{12} = \ D_{21} = -e$ and π^+ is shown to have vanishing brackets with $\Omega_{1,2}$. The $\pi^+ \approx 0$ constitutes a first class constraint on the phase space; it generates local transformations of A_+ which leave the H_e invariant, $\{\pi^+, H_e\} = \Omega_2 \approx 0$. The Ω_1, Ω_2 constitute a set of second class constraints and do not involve A_+ or π^+. It is very convenient, though not necessary, to add to the set of constraints on the phase space the (accessible) gauge fixing constraint $A_+ \approx 0$. It is evident from that such a gauge freedom is *not* available at the Lagrangian level. We will also implement (e.g., turn into strong equalities) the (trivial) pair of weak constraints $A_+ \approx 0,\ \pi^+ \approx 0$

[3]We make the convention that the first variable in an equal- τ bracket refers to the longitudinal coordinate $x^- \equiv x$ while the second one to $y^- \equiv y$

by defining the Dirac brackets with respect to them. It is easy to see that for the other remaining dynamical variables the corresponding Dirac brackets coincide with the standard Poisson brackets. The variables A_+, π^+ are thus removed from the discussion, leaving behind a constrained dynamical system with the two second class constraints $\Omega_1,\ \Omega_2$ and the light-front Hamiltonian

$$H^{l.f.} = \frac{1}{2}\int dx\ (\pi^- + 2e\varphi)^2 + \int dx\ u_1\Omega_1 + \int dx\ u_2\Omega_2 \tag{15}$$

which will be now handled by the BFT procedure.

We introduce the following linear combinations $\top_i$, $i = 1, 2$, of the above constraints

$$\begin{aligned} \top_1 &= c_1(\Omega_1 + \frac{1}{M}\Omega_2) \\ \top_2 &= c_2(\Omega_1 - \frac{1}{M}\Omega_2) \end{aligned} \tag{16}$$

where $c_1 = 1/\sqrt{2(1-e/M)}$, $c_2 = 1/\sqrt{2(1+e/M)}$, $M^2 = ae^2$, and $a > 1$. They satisfy

$$\{\top_i, \top_j\} = \delta_{ij}(-2\partial_x\delta(x-y)) \tag{17}$$

and thus diagonalize the constraint algebra.

We now introduce new auxiliary fields Φ^i in order to convert the second class constraint $\top_i$ into first class ones in the extended phase space. Following BFT [5,6], we require these fields to satisfy

$$\begin{aligned} \{A^\mu(\text{or } \pi_\mu), \Phi^i\} &= 0, \quad \{\varphi(\text{or } \pi_\varphi), \Phi^i\} = 0, \\ \{\Phi^i(x), \Phi^j(y)\} &= \omega^{ij}(x,y) = -\omega^{ji}(y,x), \end{aligned} \tag{18}$$

where ω^{ij} is a constant and antisymmetric matrix. The strongly involutive modified constraints $\widetilde{\top}_i$ satisfying the abelian algebra

$$\{\widetilde{\top}_i, \widetilde{\top}_j\} = 0 \tag{19}$$

as well as the boundary conditions, $\widetilde{\top}_i \mid_{\Phi^i=0} = \top_i$ are then postulated to take the form of the following expansion

$$\widetilde{\top}_i(A^\mu, \pi_\mu, \varphi, \pi_\varphi; \Phi^j) = \top_i + \sum_{n=1}^{\infty} \widetilde{\top}_i^{(n)}, \qquad \top_i^{(n)} \sim (\Phi^j)^n. \tag{20}$$

The first order correction terms in the infinite series (20) are written as [3,4]

$$\widetilde{\top}_i^{(1)}(x) = \int dy X_{ij}(x,y)\Phi^j(y). \tag{21}$$

The first class constraint algebra (19) of $\widetilde{\top}_i$ then leads to the following condition :

$$\{\top_i, \top_j\} + \{\widetilde{\top}_i^{(1)}, \widetilde{\top}_i^{(1)}\} = 0 \tag{22}$$

or

$$(-2\partial_x\delta(x-y))\delta_{ij} + \int dw\ dz\ X_{ik}(x,w)\omega^{kl}(w,z)X_{jl}(y,z) = 0. \tag{23}$$

There is clearly some arbitrariness [9,11] in the choice of ω^{ij} and X_{ij} which corresponds to the canonical transformation in the extended phase space [5,6]. We can take without any loss of generality the simple solutions,

$$\begin{aligned} \omega^{ij}(x,y) &= -\delta^{ij}\epsilon(x-y) \\ X_{ij}(x,y) &= \delta_{ij}\partial_x\delta(x-y), \end{aligned} \tag{24}$$

Their inverses are easily shown to be

$$\begin{aligned} {\omega^{-1}}_{ij}(x,y) &= -\frac{1}{2}\delta_{ij}\partial_x\delta(x-y) \\ (X^{-1})^{ij}(x,y) &= \frac{1}{2}\delta^{ij}\epsilon(x-y), \end{aligned} \tag{25}$$

With the above choice, we find up to the first order

$$\begin{aligned} \widetilde{\top}_i &= \top_i + \widetilde{\top}_i^{(1)} \\ &= \top_i + \partial\Phi^i, \end{aligned} \tag{26}$$

and a strongly first class constraint algebra

$$\{\top_i + \widetilde{\top}_i^{(1)}, \top_j + \widetilde{\top}_j^{(1)}\} = 0. \tag{27}$$

The higher order correction terms in Eq. (20) (suppressing the integration operation) []

$$\widetilde{\top}_i^{(n+1)} = -\frac{1}{n+2}\Phi^l {\omega^{-1}}_{lk}(X^{-1})^{kj}B_{ji}^{(n)} \qquad (n \geq 1) \tag{28}$$

with

$$B_{ji}^{(n)} \equiv \sum_{m=0}^{n}\{\widetilde{\top}_j^{(n-m)}, \widetilde{\top}_i^{(m)}\}_{(A,\pi,\varphi,\pi_\varphi)} + \sum_{m=0}^{n-2}\{\widetilde{\top}_j^{(n-m)}, \widetilde{\top}_i^{(m+2)}\}_{(\Phi)} \tag{29}$$

automatically vanish as a consequence of the proper choice (24). The Poisson brackets are to be computed here using the standard canonical definition for A_μ and φ, as well as Eq. (18). We have now only the first class constraints in the extended phase space and in view of the proper choice (24) only $\widetilde{\top}_i^{(1)}$ contributes in the series (20).

3. First Class Hamiltonian and Dirac Brackets

We next introduce modified ("gauge invariant") dynamical variables $\widetilde{F} \equiv (\widetilde{A}_\mu, \widetilde{\pi}^\mu, \widetilde{\varphi}, \widetilde{\pi}_\varphi)$ corresponding to $F \equiv (A_\mu, \pi^\mu, \varphi, \pi_\varphi)$ over the phase space by requiring the the following strong involution condition for $\widetilde{F}$ with the first class constraints in our extended phase space, viz,

$$\{\widetilde{\top}_i, \widetilde{F}\} = 0 \tag{30}$$

with

$$\widetilde{F}(A_\mu, \pi^\mu, \varphi, \pi_\varphi; \Phi^j) \;=\; F + \sum_{n=1}^{\infty} \widetilde{F}^{(n)}, \qquad \widetilde{F}^{(n)} \sim (\Phi^j)^n \tag{31}$$

and which satisfy the boundary conditions, $\widetilde{F}\mid_{\Phi^i=0}= F$.

The first order correction terms are easily shown to be given by

$$\widetilde{F}^{(1)}(x) = -\int du\, dv\, dz\; \Phi^j(u) {\omega^{-1}}_{jk}(u,v) {X^{-1}}^{kl}(v,z)\; \{\top_l(z), F(x)\}_{(A,\pi,\varphi,\pi_\varphi)}. \tag{32}$$

We find

$$\begin{aligned}
\widetilde{A}_-^{(1)} &= \frac{1}{2M}\partial(c_1\Phi^1 - c_2\Phi^2) \\
\widetilde{\pi}^{-(1)} &= \frac{M}{2}(c_1\Phi^1 - c_2\Phi^2) \\
\widetilde{\varphi}^{(1)} &= -\frac{1}{2}(c_1\Phi^1 + c_2\Phi^2), \\
\widetilde{\pi}_\varphi^{(1)} &= \frac{1}{2}\partial\left[c_1(1-\frac{2e}{M})\Phi^1 + c_2(1+\frac{2e}{M})\Phi^2\right]
\end{aligned} \tag{33}$$

where only the combinations $(c_1\Phi^1 \pm c_2\Phi^2)$ of the auxiliary fields are seen to occur. Furthermore, since the modified variables $\widetilde{F} = F + \widetilde{F}^{(1)} + ...$, up to the first order corrections, are found to be strongly involutive as a consequence of the proper choice (24), the higher order correction terms

$$\widetilde{F}^{(n+1)} \;=\; -\frac{1}{n+1}\Phi^j \omega_{jk} X^{kl} G_l^{(n)}, \tag{34}$$

with

$$G_l^{(n)} \;=\; \sum_{m=0}^{n}\{\top_i^{(n-m)}, \widetilde{F}^{(m)}\}_{(A,\pi,\phi,\pi_\phi)} + \sum_{m=0}^{n-2}\{\top_i^{(n-m)}, \widetilde{F}^{(m+2)}\}_{(\Phi)} + \{\top_i^{(n+1)}, \widetilde{F}^{(1)}\}_{(\Phi)} \tag{35}$$

again vanish. In principle we may follow similar procedure for any functional of the phase space variables; it may get, however, involved.

We make a side remark on the Dirac formulation for dealing with the systems with second class constraints by using the Dirac bracket (DB), rather than extending the phase space. In fact, the Poisson brackets of the modified (gauge invariant) variables $\widetilde{F}$ in the BFT formalism are related [8,9] to the DB, which implement the constraints $\top_i \approx 0$ in the problem under discussion, by the relation $\{f,g\}_D = \{\widetilde{f},\widetilde{g}\} \mid_{\Phi^i=0}$. In view of only the linear first order correction in CSM the computation of the right hand side is quite simple. We list some of the Dirac brackets

$$\begin{aligned}
\{\pi^-,\pi^-\}_D &= \{\widetilde{\pi^-},\widetilde{\pi^-}\}|_{\Phi=0} \\
&= \{\widetilde{\pi^-}^{(1)},\widetilde{\pi^-}^{(1)}\} = \frac{a^2e^2}{(a-1)}(-\frac{1}{4}\epsilon(x-y)), \\
\{\varphi,\varphi\}_D &= \{\widetilde{\varphi},\widetilde{\varphi}\}|_{\Phi=0} \\
&= \{\widetilde{\varphi}^{(1)},\widetilde{\varphi}^{(1)}\} = \frac{a}{(a-1)}(-\frac{1}{4}\epsilon(x-y)) \\
\{\varphi,\pi^-\}_D &= \{\widetilde{\varphi}^{(1)},\widetilde{\pi^-}^{(1)}\} = \frac{ae}{(a-1)}(-\frac{1}{4}\epsilon(x-y)).
\end{aligned} \tag{36}$$

The other ones follow on using the now strong relations $\Omega_1 = \Omega_2 = 0$ with respect to $\{,\}_D$ and from (15) it follows that the LF Hamiltonian reduces effectively to

$$H_D^{l.f.} = \frac{1}{2}\int dx\,(\pi^- + 2e\varphi)^2. \tag{37}$$

The first class LF Hamiltonian $\widetilde{H}$ which satisfies the boundary condition $\widetilde{H} \mid_{\Phi^i=0}= H_D^{l.f.}$ and is in strong involution with the constraints $\widetilde{\top}_i$, e.g., $\{\widetilde{\top}_i,\widetilde{H}\} = 0$, may be constructed following the BT procedure [] or simply guessed for the CSM. It is given by

$$\widetilde{H} = \frac{1}{2}\int dx(\widetilde{\pi}^- + 2e\widetilde{\varphi})^2 \tag{38}$$

which is just the expression in (37) with field variables F replaced by the $\widetilde{F}$ variables, which already commute with the constraints $\widetilde{T}_i$. We do also check that $\{\widetilde{H},\widetilde{H}\} = 0$ and we may identify $\widetilde{H}$ with the BRS Hamiltonian. This completes the operatorial conversion of the original second class system with the Hamiltonian H_c and constraints Ω_i into the first class one with the Hamiltonian $\widetilde{H}$ and (abelian) constraints $\widetilde{T}_i$.

4. First Class Lagrangian

We consider now the partition function of the model in order to construct the Lagrangian corresonding to $\widetilde{H}$ in the canonical Hamiltonian formulation discussed above.

We start by representing each of the auxiliary field Φ^i by a pair of fields $\pi^i, \theta^i,\ i = 1,2$ defined by

$$\Phi^i = \frac{1}{2}\pi^i - \int du\ \ \epsilon(x-u)\,\theta^i(u) \tag{39}$$

such that π^i, θ^i satisfy

$$\{\pi^i,\theta^j\} = \delta^{ij}\delta(x-y) \quad etc., \tag{40}$$

e.g., the (standard Heisenberg type) canonical Poisson brackets.

Then, the phase space partition function is given by the Faddeev formulae

$$Z = \int \mathcal{D}A_- \mathcal{D}\pi^- \mathcal{D}\varphi \mathcal{D}\pi_\varphi \mathcal{D}\theta^1 \mathcal{D}\pi^1 \mathcal{D}\theta^2 \mathcal{D}\pi^2 \prod_{i,j=1}^{2} \delta(\widetilde{\top}_i)\delta(\Gamma_j) \det \mid \{\widetilde{\top}_i, \Gamma_j\} \mid e^{iS}, \qquad (41)$$

where

$$S = \int d^2x \left(\pi^- \dot{A}_- + \pi_\varphi \dot{\varphi} + \pi^1 \dot{\theta}^1 + \pi^2 \dot{\theta}^2 - \widetilde{\mathcal{H}}\right) \equiv \int d^2x\, \mathcal{L}, \qquad (42)$$

with the Hamiltonian density $\widetilde{\mathcal{H}}$ corresponding to the Hamiltonian $\widetilde{H}$ which is now expressed in terms of (θ^i, π_i) rather than in terms of Φ^i. The gauge-fixing conditions Γ_i are chosen such that the determinants occurring in the functional measure are non-vanishing. Moreover, Γ_i may be taken to be independent of the momenta so that they correspond to the Faddeev-Popov type gauge conditions.

We will now verify in the *unitary gauge* defined by the original second class constraints: $\Gamma_i \equiv \Omega_i = 0$, i=1,2 applied to (41-42) do in fact lead to the initial Lagrangian (4). We check that the determinants in the functional measure are non-vanishing and field independent while the product of delta functionals reduces to

$$\delta(\pi_\varphi - \varphi')\delta(\pi^{-'} + 2e\varphi' + M^2 A_-)\delta(\pi^{1'} - 4\theta^1)\delta(\pi^{2'} - 4\theta^2) \qquad (43)$$

Since π_φ is absent from $\widetilde{H}$ we can perform functional integration over it using the first delta functional. The second delta functional is exponentiated as usual and we name the integration variable as A_+ for convenience. The functional integral over θ^1 and θ^2 are easily performed due to the presence of the delta functionals and it also reduces $\widetilde{\mathcal{H}}$ to $(\pi^- + 2e\varphi)^2/2$. The functional integrations over the then decoupled variables π^1 and π^2 give rise to constant factors which are absorbed in the normalization. The partition function in the unitary gauge thus becomes

$$Z = \int \mathcal{D}A_- \mathcal{D}\pi^- \mathcal{D}\varphi \mathcal{D}A_+ e^{iS}, \qquad (44)$$

with

$$S = \int d^2x \left[\pi^- \dot{A}_- + \varphi' \dot{\varphi} + (\pi^{-'} + 2e\varphi' + M^2 A_-)A_+ - \frac{1}{2}(\pi^- + 2e\varphi)^2\right], \qquad (45)$$

Performing the shift $\pi^- \to \pi^- - 2e\varphi$ and doing subsequently a Gaussian integral over π^- we obtain the initial Lagrangian as given in Eq. (4). It is interesting to recall that while constructing the LF Hamiltonian framework we eliminated the variable A_+ making use of the gauge freedom on the LF phase space and it gave rise to appreciable simplification. However, on going over to the first class Lagrangian formalism using the partition functional this variable reappears as it should, since (1) is not gauge invariant due to the presence of the mass term for the gauge field. Making other acceptable choices for gauge-functions we can arrive at different effective Lagrangians for the system under consideration.

It is also clear from our discussion that $\widetilde{H}$ given in (38) is not unique and we could modify it so that it still lead to (4) in the unitary gauge. The corresponding first class Lagrangian would produce still other gauge-fixed effective Lagrangians.

Conclusion

The bosonized CSM with $a > 1$ is quantized on the LF employing the BFT-BFV formalism. There is much simplification due to the the gauge freedom available on the LF phase space. The first class constraints are systematically constructed and the first class Lagrangian obtained. In the *unitary gauge* it is verified to lead to the initial (second class) Lagrangian. The Dirac brackets defined on the original second class system can be easily computed using Poisson brackets defined on the extended LF phase space. It will be interesting to study non-abelian models on the LF using the BFT-BFV formalism.

References

[1] P.A.M. Dirac, Rev. Mod. Phys. **21** (1949) 392.

[2] P.P. Srivastava, *Lightfront quantization of field theory* in *Topics in Theoretical Physics*, *Festschrift for Paulo Leal Ferreira*, eds., V.C. Aguilera-Navarro et. al., pgs. 206-217, IFT-São Paulo, SP, Brasil (1995); hep-th@xxx.lanl.gov/9610044; see also 9312064.

[3]P.A.M. Dirac, *Lectures in Quantum Mechanics*, Benjamin, New York, 1964.

[4]S. Weinberg, Phys. Rev. **150** (1966) 1313.

[5]J.B. Kogut and D.E. Soper, Phys. Rev. **D 1** (1970) 2901.

[6]S.J. Brodsky and H.C. Pauli, *Light-cone Quantization and QCD*, Lecture Notes in Physics, vol. 396, eds., H. Mitter et. al., Springer-Verlag, Berlin, 1991.

[7] K.G. Wilson, Nucl. Phys. B (proc. Suppl.) **17** (1990). R.J. Perry, A. Harindranath, and K.G. Wilson, Phys. Rev. Lett. **65** (1990) 2959.

[8] Batalin I A and Fradkin E S 1987 *Nucl. Phys.* **B279** 514

[9] Batalin I A and Tyutin I V 1991 *Int. J. Mod. Phys.* **A6** 3255

Fradkin E S 1988 *Lecture of the Dirac Model of ICTP* Miramare-Trieste

[10] Faddeev L D and Shatashvili S S 1986 *Phys. Lett.* **B167** 225

[11] Jackiw R and Rajaraman R 1985 *Phys. Rev. Lett.* **54** 1219; 1985 *Phys. Rev. Lett.* **54** 2060(E)

[12] Schaposnik F and Viallet C 1986 *Phys. Lett.* **B177** 385

Harada K and Tsutsui I 1987 *Phys. Lett.* **B183** 311

[13] W.T. Kim, Y. Kim, M. Park, and Y. Park, S.J. Yoon, Preprint SOGANG-HEP 209/96: discusses CSM using BFT-BFV Formalism in the conventional equal-time framework. See for other earlier references.

[14] P.P. Srivastava, θ-Vacua in the Light-Front Quantized Schwinger Model, preprint IF-UERJ 034/96, CBPF-NF-063/96; hep-th@xxx.lanl.gov

[15] J. Kogut and L. Susskind, Phys. Rev. D11, 3594 (1975).