DCE-APU 97-01
AUE 97/01
DPNU-97-24
May 1997

New Possibility of Solving the Problem of Lifetime Ratio $\tau(\Lambda_b)/\tau(B_d)$

Toshiaki Ito[†] , Masahisa Matsuda[*‡] and Yoshimitsu Matsui[**§]

Department of Childhood Education, Aichi Prefectural University, Nagoya 467, Japan
**Department of Physics and Astronomy,*
Aichi University of Education, Kariya 448, Japan
***Department of Physics, Nagoya University, Nagoya 464-01, Japan*

Abstract

We discuss the problem of the large discrepancy between the observed lifetime ratio of B_d to Λ_b and the theoretical prediction obtained by the heavy quark effective theory. A new possibility of solving this problem is proposed from the viewpoint of operator product expansion and the lifetime ratio of Λ_b to Ω_b is predicted.

The heavy quark effective theory (HQET) is successful in explaining various nature of the hadrons containing a heavy quark. The HQET has been extensively applied to the meson system containing a heavy quark and has brought about many remarkable results. The HQET has been also applied to the system of baryons containing a heavy quark and has led to some interesting results. However, when applying the HQET to the heavy baryon systems, two problems are encountered. One is that the experimental value of the mass difference between Σ_b and Σ_b^* is quite large in comparison with the predicted value from the mass relation in the HQET as

$$\begin{aligned} m_{\Sigma_b^*} - m_{\Sigma_b} &= 56 \pm 13\,\mathrm{MeV} \quad (Exp.)\,[1], \\ &= 15.8 \pm 3.3\,\mathrm{MeV} \quad (Theory)\,[2]. \end{aligned} \tag{1}$$

[†] toshiaki@eken.phys.nagoya-u.ac.jp
[‡] mmatsuda@auecc.aichi-edu.ac.jp
[§] matsui@eken.phys.nagoya-u.ac.jp

The other one is that the experimental lifetime ratio of B_d to Λ_b [3]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.78 \pm 0.05 \tag{2}$$

is quite small compared with the theoretical predictions[4]–[7].

In this paper, we concentrate our attention to the latter problem and propose a possible solution to this problem from the viewpoint of operator product expansion(OPE). Moreover, we point out the importance of measuring the lifetime of Ω_b to select out the models.

The inclusive decay width of a hadron H_b containing a b-quark can be written as [5]

$$\Gamma(H_b \to X_f) = \frac{1}{m_{H_b}} \mathrm{Im}\,\langle H_b|\,\hat{T}\,|H_b\rangle, \tag{3}$$

where the transition operator $\hat{T}$ is given by

$$\hat{T} = i\int d^4x\, \mathrm{T}\{\,\mathcal{L}_W(x)\,\mathcal{L}_W(0)\,\}\,. \tag{4}$$

The effective Lagrangian $\mathcal{L}_W$ for the weak decay $H_b \to X_f$ is

$$\begin{aligned}\mathcal{L}_W \;=\; -\frac{4G_F}{\sqrt{2}}\,V_{cb}\Big\{ & c_1(m_b)\left[\bar{d}'_L\gamma_\mu u_L\,\bar{c}_L\gamma^\mu b_L + \bar{s}'_L\gamma_\mu c_L\,\bar{c}_L\gamma^\mu b_L\right] \\ & + c_2(m_b)\left[\bar{c}_L\gamma_\mu u_L\,\bar{d}'_L\gamma^\mu b_L + \bar{c}_L\gamma_\mu c_L\,\bar{s}'_L\gamma^\mu b_L\right] \\ & + \sum_{\ell=e,\mu,\tau}\bar{\ell}_L\gamma_\mu\nu_\ell\,\bar{c}_L\gamma^\mu b_L\Big\} + \mathrm{h.c.}\,,\end{aligned} \tag{5}$$

where q_L denotes a left-handed quark field and d' and s' stand for the weak eigenstates. We neglect the CKM suppressed transitions $b \to u$, because this effect is negligibly small in the present analyses. Up to the leading order, the combinations $c_\pm = c_1 \pm c_2$ are given by

$$c_\pm(m_b) = \left(\frac{\alpha_s(m_W)}{\alpha_s(m_b)}\right)^{a_\pm}, \qquad a_- = -2a_+ = -\frac{12}{33-2n_f}\,. \tag{6}$$

In order to pursue the calculation of the inclusive decay width in eq.(3), we must evaluate the matrix element of the nonlocal operator $\hat{T}$ in eq.(4). Although we don't know how to evaluate such the matrix element of the nonlocal operator directly, it is efficient for us to use the method of the OPE for the $\hat{T}$. This is because the energy release

is quite large compared with the QCD scale in the b-quark decay. Using the OPE, the total decay width of a hadron H_b can be written in the form [8]

$$\Gamma(H_b \to X_f) = \frac{G_F^2 m_b^5}{192\pi^3} \frac{1}{2m_{H_b}} \left\{ c_3(f) \langle H_b | \bar{b} b | H_b \rangle + c_5(f) \frac{\langle H_b | \bar{b}\, g_s \sigma_{\mu\nu} G^{\mu\nu} b | H_b \rangle}{\Delta^2} + \sum_i c_6^i(f) \frac{\langle H_b | (\bar{b}\, \Gamma_i q)(\bar{q} \Gamma_i b) | H_b \rangle}{\Delta^3} + \cdots \right\}, \quad (7)$$

where $c_n(f)$ are dimensionless coefficient functions depending on the quantum numbers of the final state, Γ_i's denote the combinations of γ-matrices and Δ is an expansion parameter with the mass dimension, which should be much larger than the scale of Λ_{QCD} to justify the OPE.

The Dirac spinor b in eq.(7), which is an operator of QCD, can be expressed as

$$b(x) = e^{-im_b v \cdot x} \{ h_b(x) + \chi_b(x) \}, \quad (8)$$

where $h_b(x)$ and $\chi_b(x)$ are the large and small component of the spinor $b(x)$ respectively and v stands for the velocity of the hadron containing the b-quark. The $\chi_b(x)$ can be written in terms of $h_b(x)$ by using the equation of motion. So the first and second matrix elements in eq. (7) are expanded as [4, 5]

$$\frac{1}{2m_{H_b}} \langle H_b | \bar{b} b | H_b \rangle = 1 - \frac{\mu_\pi^2(H_b) - \mu_G^2(H_b)}{4m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right), \quad (9)$$

$$\frac{1}{2m_{H_b}} \langle H_b | \bar{b}\, g_s \sigma_{\mu\nu} G^{\mu\nu} b | H_b \rangle = 2\mu_G^2(H_b) + \mathcal{O}\left(\frac{1}{m_b}\right), \quad (10)$$

with $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ defined by

$$\mu_\pi^2(H_b) \equiv -\frac{1}{2m_{H_b}} \langle H_b | \bar{h}_b\, (iD_\perp)^2\, h_b | H_b \rangle, \quad (11)$$

$$\mu_G^2(H_b) \equiv \frac{1}{4m_{H_b}} \langle H_b | \bar{h}_b\, g_s \sigma_{\mu\nu} G^{\mu\nu} h_b | H_b \rangle, \quad (12)$$

where $D_\perp^\mu = \partial^\mu - v^\mu (v \cdot D)$ and D^μ is the covariant derivative of QCD. The parameters $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ represent the matrix elements of the kinetic energy operator and the chromo-magnetic one, respectively. These parameters can be estimated by using the mass spectra of heavy hadron states and are of $\mathcal{O}(\Lambda_{QCD}^2)$.

The lifetime ratio of two hadrons containing a b-quark which can only decay through the weak interaction is given by

$$\frac{\tau\left(H_b^{(1)}\right)}{\tau\left(H_b^{(2)}\right)} = 1 + \frac{\mu_\pi^2\left(H_b^{(1)}\right) - \mu_\pi^2\left(H_b^{(2)}\right)}{4m_b^2} + \left(\frac{1}{4} + c_M \frac{m_b^2}{\Delta^2}\right) \frac{\mu_G^2\left(H_b^{(2)}\right) - \mu_G^2\left(H_b^{(1)}\right)}{m_b^2} + \mathcal{O}\left(\frac{1}{\Delta^3}\right), \quad (13)$$

where

$$c_M \equiv \frac{2c_5(f)}{c_3(f)} \tag{14}$$

$$\cong -\frac{2\left(N_c A_0 z_0 + 4A_2 z_2 + 2z_0\right)}{N_c A_0 z_0 + 2z_0}. \tag{15}$$

Here we use the same definition of A_i and z_i as in Ref.[10]. Eq.(13) is derived from eqs.(7), (9) and (10) by taking account of

$$\mu_\pi^2(H_b^{(i)}),\ \mu_G^2(H_b^{(i)}) \sim \mathcal{O}(\Lambda_{QCD}^2). \tag{16}$$

Thus the lifetime ratio of B_d to Λ_b can be estimated. At the heavy quark limit $m_b \to \infty$, the lifetime ratio (13) approaches unity. However, the recent experiment (2) shows that the large discrepancy exists between the B_d and Λ_b lifetimes. Recently several literatures [4]–[7] have been devoted to study this problem. To seek a solution that can reconcile this discrepancy, the contributions of the $\mathcal{O}(1/\Delta^3)$ term in eq.(7) have been estimated under the condition that the expansion parameter Δ is equal to m_b. This condition might be natural because the physical scales in the system are only Λ_{QCD} and m_b. There are some uncertainties in the correction of $\mathcal{O}(1/\Delta^3)$, which show the effect of weak scattering and Pauli interference, because the estimation of the $\mathcal{O}(1/\Delta^3)$ term is model-dependent. The parameters $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ in eq.(13) can be estimated from the mass formulae of the hadrons containing a b-quark as follows

$$\begin{aligned}
\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d) &= -(0.01 \pm 0.03)\ (\mathrm{GeV})^2, \\
\mu_G^2(B_d) &= \frac{3}{4}(m_{B^*}^2 - m_B^2) \simeq 0.36\ (\mathrm{GeV})^2, \\
\mu_G^2(\Lambda_b) &= 0.
\end{aligned} \tag{17}$$

Although the estimation of $\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d)$ is influenced by the problem of the mass difference between Σ_b and Σ_b^*, this uncertainty hardly influences to the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$. The analyses [6, 7] give the result in the ratio

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \geq 0.94. \tag{18}$$

Thus the problem remains unsolved at this stage.

One possibility of solving this problem was proposed in Ref.[11]. This proposal contains the insistence that the mass m_b in the factor m_b^5 in eq.(7) should be replaced by the mass of the parent hadron m_{B_d} or m_{Λ_b}. If this insistence is correct, the lifetime ratio of Λ_b to B_d is almost determined by the ratio of the decaying hadron masses;

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \sim \left(\frac{m_B}{m_{\Lambda_b}}\right)^5 = 0.73 \pm 0.01. \tag{19}$$

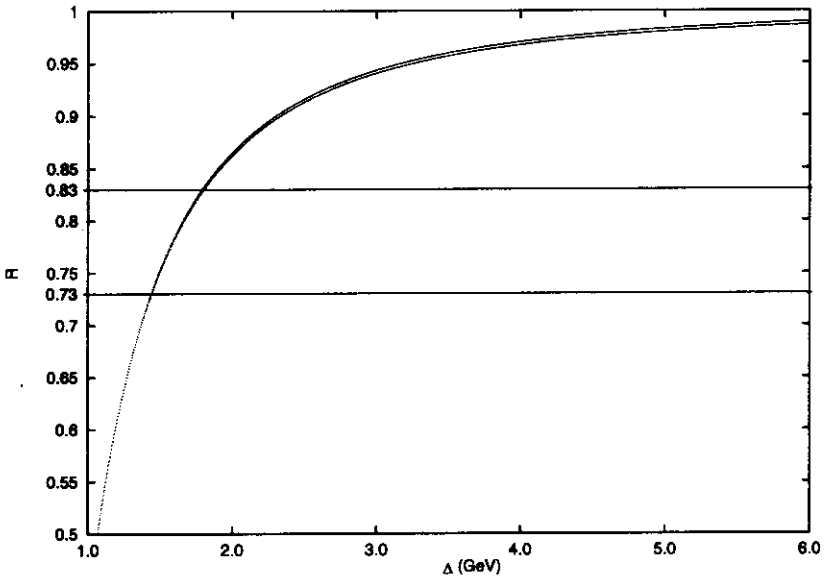

Figure 1: The lifetime ratio $R = \tau(\Lambda_b)/\tau(B)$ versus the expansion parameter Δ. The straight lines indicates the range of the experimental data. The upper and lower curved lines represent the calculated value of R for $m_b = 4.3\,\mathrm{GeV}$ and $m_b = 7.0\,\mathrm{GeV}$, respectively.

When this proposal is applied to charmed hadrons lifetime, the results are in good agreement with the experiment [12].

In this paper, we discuss another possibility of solving the problem of the lifetime ratio of B_d to Λ_b. In the literatures [4]–[7], the expansion parameter Δ in eq.(7) is taken as m_b. However, the physical reason of the setting $\Delta = m_b$ is not so obvious. Moreover, when we take a rather small Δ compared m_b, the calculated ratio $R = \tau(\Lambda_b)/\tau(B_d)$ is consistent with the experimental one (2). In Fig.1, we plot the ratio $R = \tau(\Lambda_b)/\tau(B_d)$ versus Δ for $m_b = 4.3\,\mathrm{GeV}$ ($\mu = m_b$) and $7.0\,\mathrm{GeV}$ ($\mu = 1\,\mathrm{GeV}$) [13, 14]. From this we obtain

$$\Delta = 1.62 \pm 0.18\,(\mathrm{GeV})\,, \tag{20}$$

as the consistent region for Δ, where we use the result of Ref.[6] as for the estimation of $\mathcal{O}(1/\Delta^3)$ term and the parameter c_M. This shows that the parameter Δ is preferred to be much smaller than m_b.

Let us consider the physical meanings of Δ. Here we can point out two viewpoints about the physical interpretation of it.

The first is that the parameter Δ is regarded as the m_c. Blok and Shifman[8] have discussed the role of the expansion parameter Δ to study the effects of the subleading

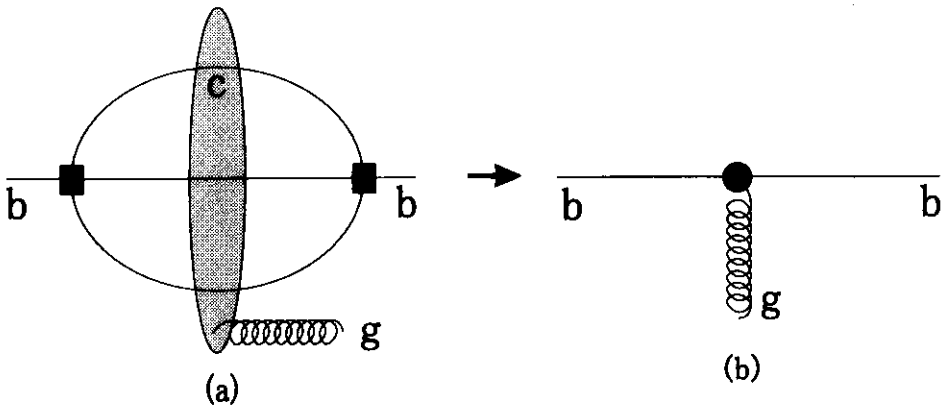

Figure 2: Figure (b) and (a) represent the operator of the second term in eq.(7) and one of the corresponding perturbative contributions to the transition operator $\hat{T}$, respectively.

operators in the inclusive heavy hadrons decays. In Ref.[8], the parameters Δ has been taken as

$$\Delta = m_b - m_c \tag{21}$$

and the analyses have been carried out at large m_b limit

$$\Delta = m_b - m_c \simeq 3\,\mathrm{GeV} \sim m_b \gg \Lambda_{QCD}\,. \tag{22}$$

However, the result (20) shows that the preferred value of Δ is very close to the region of m_c [13, 14]

$$m_c(\,\mu = 1\,\mathrm{GeV}\,) = 1.49 \pm 0.04\,\mathrm{GeV} \tag{23}$$

rather than $m_b - m_c \simeq 3\,\mathrm{GeV}$. The $\mathcal{O}(1/\Delta^2)$ term in eq.(7) is induced from the chromomagnetic interaction as shown in Fig.2. This contribution directly describes the effect of the final state interaction including a c-quark. Since the final state is a hybrid system consisting of a heavy quark (c-quark) and the light degrees of freedom, it is reasonable that the expansion parameter $1/\Delta$ of the system is taken as $1/m_c$. As for the $\mathcal{O}(1/\Delta^3)$ term of four-quark operators in eq.(7), we could apply the same arguments as mentioned above, because the final state consists of a c-quark and the light degrees of freedom. The setting $\Delta \simeq m_c$ corresponds to the inclusion of the effects of the final state interactions.

The second is that the parameter Δ^2 is regarded as the averaged value of released momentum squared q^2 in transition of b into c. By using the spectator model for the exclusive semileptonic decay $b \to c\ell\bar{\nu}_\ell$, we can estimate the averaged value of q^2 as

$$\sqrt{< q^2 >} \cong 1.87\,\mathrm{GeV} \tag{24}$$

with $m_b = 4.3\,\mathrm{GeV}$ and $m_c = 1.3\,\mathrm{GeV}$. If this averaged value (24) is regarded as Δ, this is approximately consistent with the value obtained in eq.(20).

If the lifetime difference of B_d and Λ_b can be explained on the basis of the present approach, its applicability to other heavy hadrons should be investigated. Here we take the baryon Ω_b as a good candidate to implement our purpose. The baryon Ξ_b is also a candidate which decays only through the weak interaction. For Ξ_b, however, we have to solve the mixing problem between Ξ_b and Ξ_b' [15]. Thus it is difficult to expect that we obtain the meaningful result for Ξ_b.

The ratio $\tau(\Lambda_b)/\tau(\Omega_b)$ will give the important clue to the lifetime problem in B_d-meson and Λ_b-baryon. In order to estimate $\tau(\Lambda_b)/\tau(\Omega_b)$, we need to know the values of

$$\begin{aligned} &\mu_\pi^2(\Omega_b) - \mu_\pi^2(\Lambda_b), \\ &\mu_G^2(\Omega_b) - \mu_G^2(\Lambda_b)\,. \end{aligned} \tag{25}$$

Since the baryon Ω_b has not been found yet experimentally, we take the relations

$$\begin{aligned} &\mu_\pi^2(\Omega_b) \simeq \mu_\pi^2(\Sigma_b), \\ &\mu_G^2(\Omega_b) \simeq \mu_G^2(\Sigma_b)\,, \end{aligned} \tag{26}$$

which are followed by $SU(3)$ light flavor symmetry. The mass formulae of the HQET are translated into the forms

$$\begin{aligned} &\left\{\frac{1}{3}\left(2m_{\Sigma_b^*} - m_{\Sigma_b}\right) - \frac{1}{3}\left(2m_{\Sigma_c^*} - m_{\Sigma_c}\right)\right\} - \left(m_{\Lambda_b} - m_{\Lambda_c}\right) \\ &\quad = \left\{\mu_\pi^2(\Lambda_b) - \mu_\pi^2(\Sigma_b)\right\}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \end{aligned} \tag{27}$$

and

$$\mu_G^2(\Sigma_b) \simeq \frac{1}{6}\left(m_{\Sigma_b^*}^2 - m_{\Sigma_b}^2\right)\,. \tag{28}$$

From eqs.(26) to (28), we obtain

$$\begin{aligned} &\mu_\pi^2(\Omega_b) - \mu_\pi^2(\Lambda_b) \sim 0.04\,(\mathrm{GeV})^2, \\ &\mu_G^2(\Omega_b) - \mu_G^2(\Lambda_b) \sim 0.1\,(\mathrm{GeV})^2\,. \end{aligned} \tag{29}$$

Here we take $m_b = 4.3\,\mathrm{GeV}$, $m_c = 1.3\,\mathrm{GeV}$ and the values of the hadron masses given by Ref.[1] for m_{Σ_b} and $m_{\Sigma_b^*}$ and Ref.[2, 16, 17] for the others. By substituting these values and the Δ given in eq.(20) to eq.(13), we have the ratio

$$\frac{\tau(\Lambda_b)}{\tau(\Omega_b)} = 0.93 \pm 0.02\,. \tag{30}$$

If we take the method in Ref.[11, 12] that m_b^5 in front of eq.(7) is replaced with $m_{H_b}^5$ while keeping $\Delta = m_b$, the ratio becomes

$$\frac{\tau(\Lambda_b)}{\tau(\Omega_b)} = 1.41 \pm 0.03\,, \tag{31}$$

where we use the mass $m_{\Omega_b} = 6.06\,\mathrm{GeV}$ which is derived from the mass relations given by Ref.[2].

The discrepancy between the two predictions of the lifetime ratio $\tau(\Lambda_b)/\tau(\Omega_b)$ is quite large in contrast with that in the ratio $\tau(\Lambda_b)/\tau(B_d)$. The present approach implies

$$\tau(\Lambda_b) < \tau(\Omega_b)\,, \tag{32}$$

whereas the approach of Ref.[11, 12] leads to

$$\tau(\Lambda_b) > \tau(\Omega_b)\,. \tag{33}$$

It should be emphasized that the hierarchy of the lifetime becomes entirely opposite among the both approaches. The hierarchy obtained from the conventional approach [4]–[7] is the same as the present result (32).

In this short note, we proposed a new approach to the problem of the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$. The main point of this approach is that the expansion parameter Δ of OPE is taken to be m_c or $\sqrt{< q^2 >}$ i.e., numerically $1.5 \sim 1.8$GeV. This approach could well reproduce the experimental lifetime ratio $\tau(\Lambda_b)/\tau(B_d) \cong 0.78$.

As for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$, the present approach and the one of Ref.[11] lead to almost the same result. To discriminate the approaches, it is important to measure the lifetime of Ω_b since the prediction of the hierarchy of the lifetimes of Λ_b and Ω_b is opposite between these approaches. So the future experiment of the lifetime of Ω_b will be able to test the models clearly.

Acknowledgment

The authors would like to give thanks to Professors T. Matsuoka and M. Tanimoto for the encouraging discussions and for reading of the manuscript.

References

[1] DELPHI Collaboration, P. Abreu et al., DELPHI 95-107 PHYS 542 (1995).

[2] E. Jenkins, Phys. Rev. **D54** (1996) 4515 ; *ibid.* **D55** (1997) 10.

[3] For updated world averages of B hadron lifetimes, see J. Alcavaz et al. (LEP B Lifetime Group), http://wwwcn.cern.ch/~claires/lepblife.html.

[4] I.I. Bigi, B. Blok, M. Shifman, N. Uraltsev, and A. Vainshtein, in *B Decays*, edited by S. Stone, Second Edition (World Scientific, Singapore, 1994), p. 132.

[5] M. Neubert and C.T. Sachrajda, Nucl. Phys. **B483** (1997) 339.

[6] J.L. Rosner, Phys. Lett. **B379** (1996) 267.

[7] P. Colangelo, hep-ph/9611236.

[8] B. Blok and M. Shifman, Nucl. Phys. **B389** (1993) 534; *ibid.* **B399** (1993) 441, 459.

[9] M. Neubert, Phys. Rep. **245** (1994) 259.

[10] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. **B293** (1992) 430.

[11] G. Altarelli, G. Martinelli, S. Petrarca, and F. Rapuano, Phys. Lett.**B382** (1996) 409; G. Martinelli, Nucl. Instrum. Meth. **A384** (1996) 241.

[12] H.Y. Cheng, hep-ph/9704260.

[13] S. Titard and F.J. Ynduráin, Phys. Rev. . **D49** (1994) 6007.

[14] Y. Koide, hep-ph/9410270.

[15] T. Ito and Y. Matsui, Prog. Theor. Phys. **96** (1996) 659.

[16] A. Falk, Phys. Rev. Lett. **77** (1996) 223.

[17] Particle Data Group, Phys. Rev. **D54** (1996) 1.