

# First global NNPDF analysis

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& Université Catholique de Louvain

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"A first unbiased global NLO determination of parton distribution functions"  
[arXiv:1002.4407](https://arxiv.org/abs/1002.4407)



The NNPDF Collaboration

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# Outline

1 Introduction

2 NNPDF method

3 NNPDF2.0: a global fit

4 Conclusions and outlook

# Motivation

Historical overview

Monte Carlo representation of the probability measure in the space of functions

Use of neural network as redundant ([259 pars](#)) and unbiased parametrization

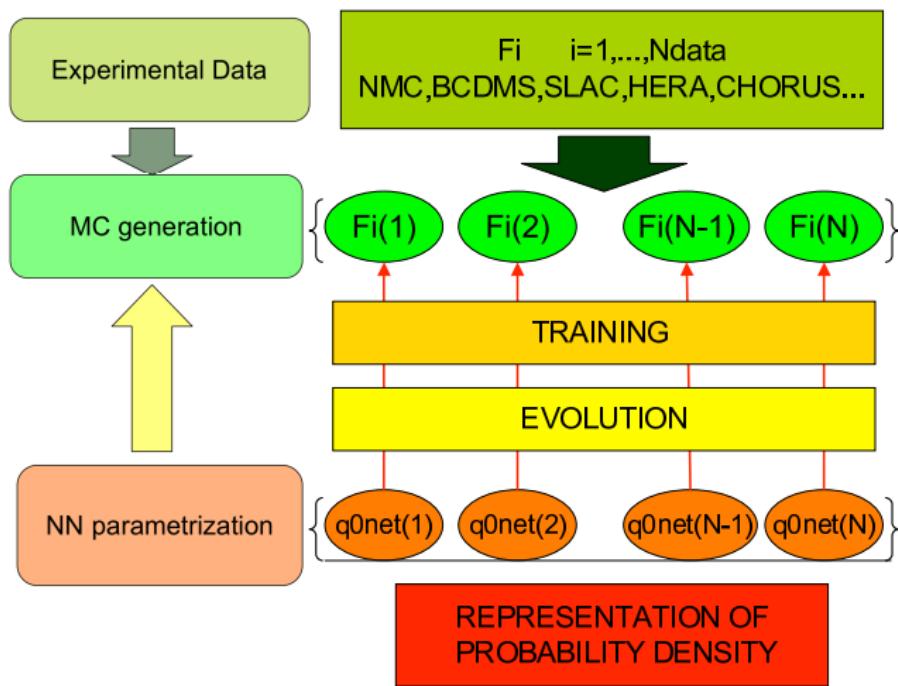
- Structure functions [\[hep-ph/0501067\]](#)
- Non-singlet PDF  $q^- = u + d - (\bar{u} + \bar{d})$  [\[hep-ph/0701127\]](#)
- DIS global analysis: NNPDF1.0 [\[arXiv:0808.1231\]](#)
- Determination of the strange content: NNPDF1.2 [\[arXiv:0906.1958\]](#)
- Global (DIS+DY+JET) analysis: NNPDF2.0 [\[arXiv:1002.4407\]](#)

All sets are available in the LHAPDF interface

# NNPDF approach

## General scheme

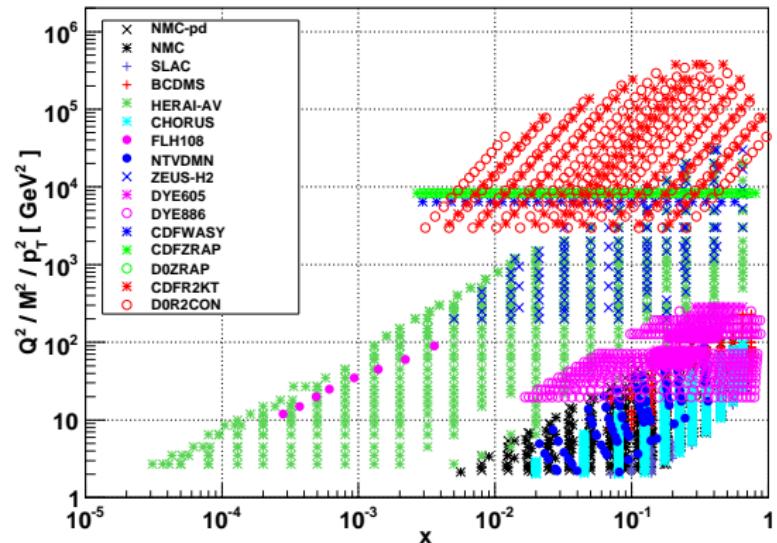
$$F_i^{\text{rep}}(k) = S_{i,N}^{(k)} F_i^{\text{exp}} \left( 1 + r_i^{(k)} \sigma_i^{\text{stat}} + \sum_{j=1}^{N_{\text{sys}}} r_{i,j}^{(k)} \sigma_{i,j}^{\text{sys}} \right) \quad r_{i,j}^{(k)} = r_{i',j}^{(k)} \text{ if } i \text{ and } i' \text{ correlated}$$



# NNPDF2.0

## Data sets

### NNPDF2.0 dataset



3415(4520) data points

For comparison MSTW08 includes 2699 data points

OBS	Data sets
$F_2^p$	NMC,SLAC,BCDMS
$F_2^d$	SLAC,BCDMS
$F_2^d / F_2^p$	NMC-pd
$\sigma_{NC}$	HERA-I AV, ZEUS-H2
$\sigma_{CC}$	HERA-I AV, ZEUS-H2
$F_L$	H1
$\sigma_\nu, \sigma_{\bar{\nu}}$	CHORUS
dimuon prod.	NuTeV
$d\sigma^{DY} / dM^2 dy$	E605
$d\sigma^{DY} / dM^2 dx_F$	E886
W asymmetry	CDF
Z rap. distr.	CDF,D0
incl. $\sigma^{(jet)}$	D0(cone) Run II
incl. $\sigma^{(jet)}$	CDF( $k_T$ ) Run II

- Kinematical cuts on DIS data  
 $Q^2 > 2 \text{ GeV}^2$   
 $W^2 = Q^2(1-x)/x > 12.5 \text{ GeV}^2$
- No cuts on hadronic data
- New HERA combined data included

- PDFs are evolved from initial scale to experiment scales

$$f_i(x, Q^2) = \sum_j \Gamma_{ij}(x, \alpha_s, \alpha_s^0) \otimes f_j^0(x)$$

- Observables are convolution of PDFs and Coefficient Functions

$$F_I(x, Q^2) = \sum_{j,k} C_{lj}(x, \alpha_s) \otimes \Gamma_{jk}(x, \alpha_s, \alpha_s^0) \otimes f_k^0(x)$$

$x$ (50 pts)	$e_{\text{rel}}(u_V)$	$e_{\text{rel}}(\Sigma)$	$e_{\text{rel}}(g)$
$1 \cdot 10^{-7}$	$2.1 \cdot 10^{-4}$	$2.7 \cdot 10^{-5}$	$4.7 \cdot 10^{-6}$
$1 \cdot 10^{-6}$	$8.9 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$	$2.1 \cdot 10^{-5}$
$1 \cdot 10^{-5}$	$9.3 \cdot 10^{-5}$	$2.3 \cdot 10^{-5}$	$2.0 \cdot 10^{-5}$
$1 \cdot 10^{-4}$	$4.5 \cdot 10^{-5}$	$4.4 \cdot 10^{-5}$	$4.2 \cdot 10^{-5}$
$1 \cdot 10^{-3}$	$3.0 \cdot 10^{-5}$	$4.0 \cdot 10^{-5}$	$3.5 \cdot 10^{-5}$
$1 \cdot 10^{-2}$	$7.9 \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$	$5.8 \cdot 10^{-5}$
$1 \cdot 10^{-1}$	$1.7 \cdot 10^{-4}$	$1.6 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$
$3 \cdot 10^{-1}$	$9.1 \cdot 10^{-6}$	$1.1 \cdot 10^{-5}$	$1.9 \cdot 10^{-7}$
$5 \cdot 10^{-1}$	$2.4 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$
$7 \cdot 10^{-1}$	$9.1 \cdot 10^{-5}$	$7.8 \cdot 10^{-5}$	$1.2 \cdot 10^{-4}$
$9 \cdot 10^{-1}$	$1.0 \cdot 10^{-3}$	$8.0 \cdot 10^{-4}$	$2.8 \cdot 10^{-3}$

We want: Mellin space evolution

$$K_{Ik}(N, \alpha_s, \alpha_s^0) = \sum_j C_{lj}(N, \alpha_s) \Gamma_{jk}(N, \alpha_s, \alpha_s^0)$$

We do not want: Complex NNs

$$F_I(x, Q^2) = \sum_k \int_x^1 \frac{dy}{y} K_{Ik}(x/y, \alpha_s, \alpha_s^0) f_k^0(y)$$

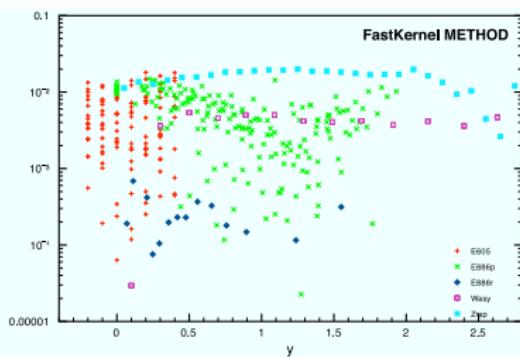
- New strategy for PDFs evolution based on use of **high-orders polynomial** interpolation
- Implementation benchmarked against LH tables: much faster and accurate

$$F_I(x, Q^2) = \sum_{\alpha, \beta=1}^{N_X} \sum_k f_k^0(x_\alpha) \int_x^1 \frac{dy}{y} K_{Ik}(x/y, \alpha_s, \alpha_s^0) \mathcal{I}^{(\alpha)}(y)$$

- NLO computation of hadronic observables too slow for parton global fits.
- MSTW08 and CTEQ include Drell-Yan NLO as (local) K factors rescaling the LO cross section
- K-factor depends on PDFs and it is not always a good approximation.

- \* NNPDF2.0 includes full NLO calculation of hadronic observables.
- \* Use available fastNLO interface for jet inclusive cross-sections. [[hep-ph/0609285](#)]
- \* Built up our own **FastKernel** computation of DY observables.

$$\int_{x_{0,1}}^1 dx_1 \int_{x_{0,2}}^1 dx_2 f_a(x_1) f_b(x_2) C^{ab}(x_1, x_2) \rightarrow \sum_{\alpha, \beta=1}^{N_X} f_a(x_1, \alpha) f_b(x_2, \beta) \int_{x_{0,1}}^1 dx_1 \int_{x_{0,2}}^1 dx_2 \mathcal{I}^{(\alpha, \beta)}(x_1, x_2) C^{ab}(x_1, x_2)$$



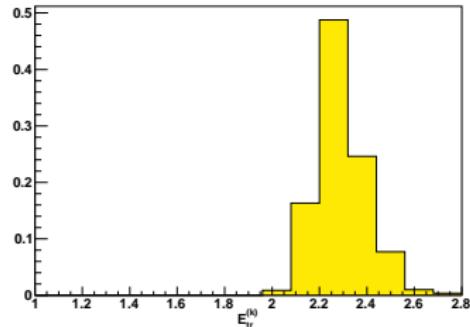
- Both PDFs evolution and double convolution sped up by
  - Use high-orders polynomial interpolation
  - Precompute all Green Functions

A truly NLO analysis

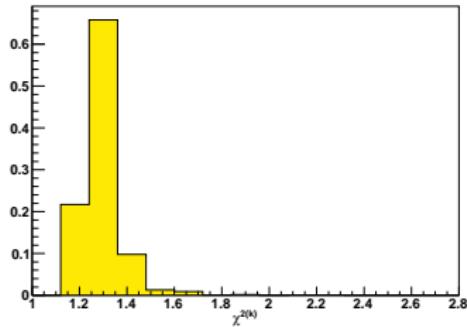
# Results

Statistical features: global  $\chi^2$

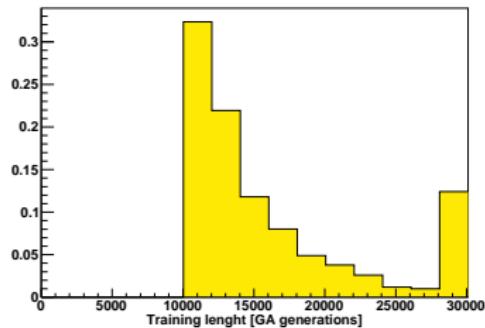
$E_{tr}$  distribution for MC replicas



$\chi^{2(k)}$  distribution for MC replicas



Distribution of training lengths

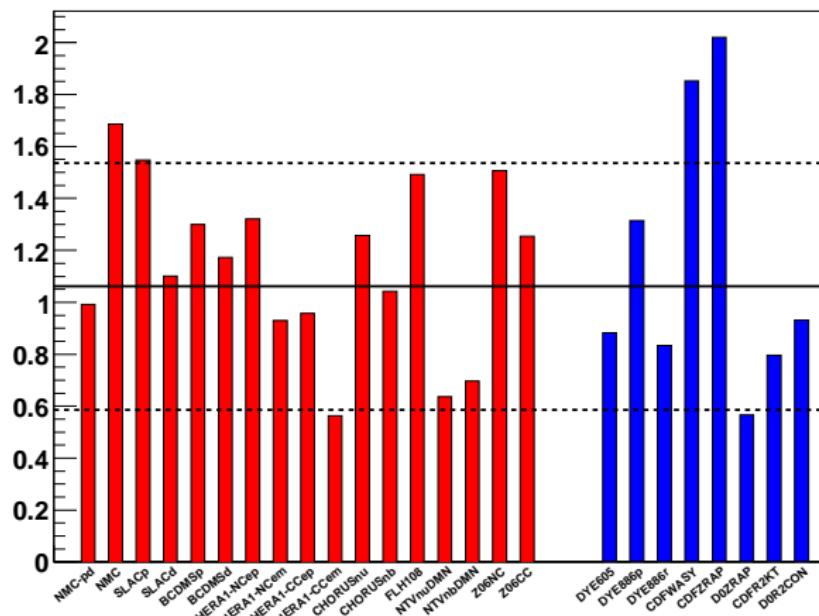


	$\chi^2_{tot}$
$\langle E \rangle \pm \sigma_E$	$2.32 \pm 0.10$
$\langle E_{val} \rangle \pm \sigma_{E_{val}}$	$2.29 \pm 0.11$
$\langle E_{val} \rangle \pm \sigma_{E_{val}}$	$2.35 \pm 0.12$
$\langle TL \rangle \pm \sigma_{TL}$	$16175 \pm 6275$
$\langle \chi^{2(k)} \rangle \pm \sigma_{\chi^2}$	$1.29 \pm 0.09$

# Results

Statistical features: individual experiments

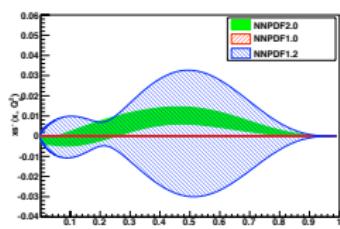
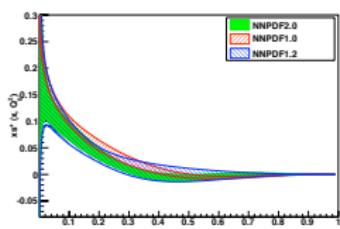
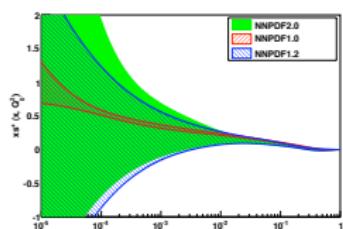
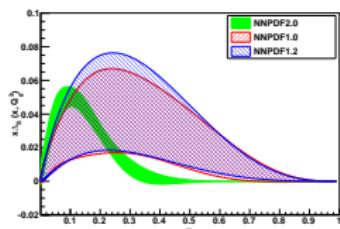
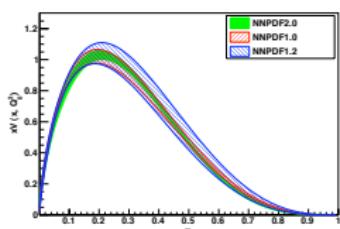
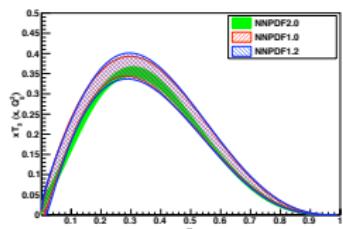
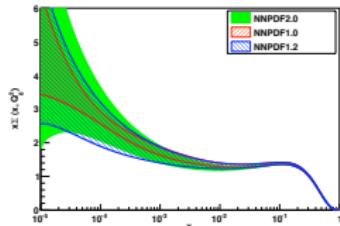
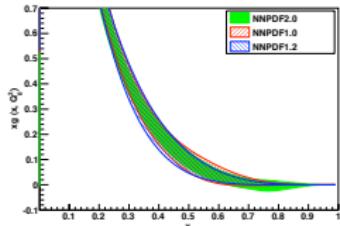
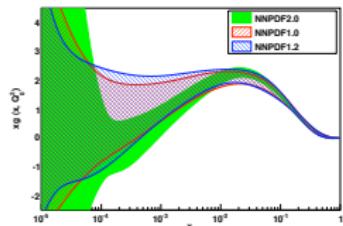
Distribution of  $\chi^2$  for sets



No obvious tension between hadronic and DIS data

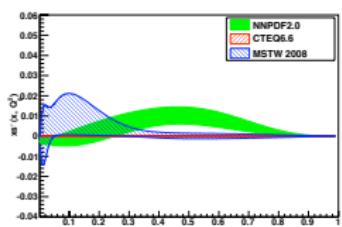
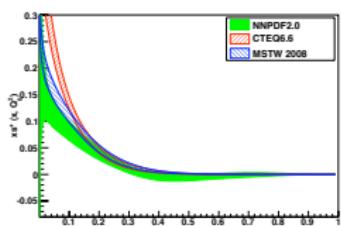
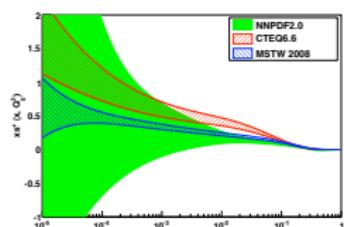
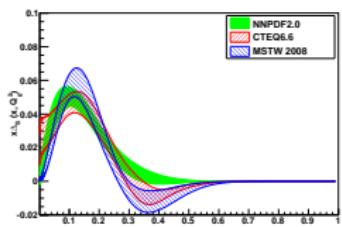
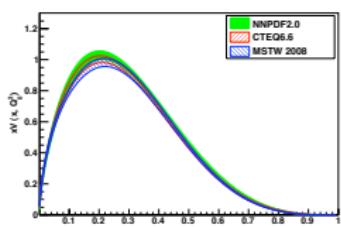
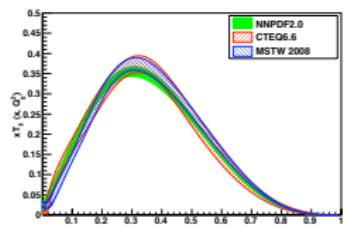
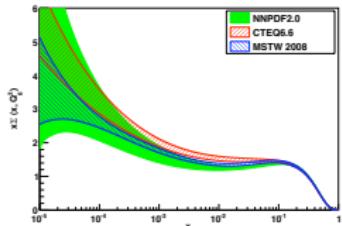
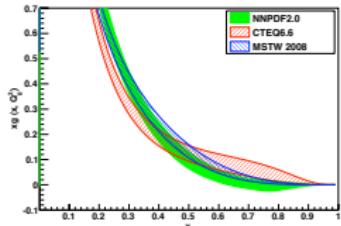
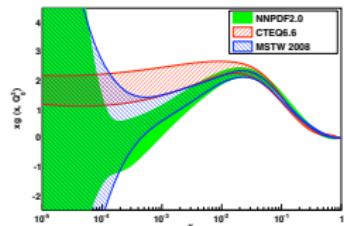
# Results

## Partons



# Results

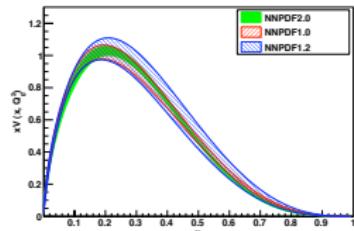
## Partons



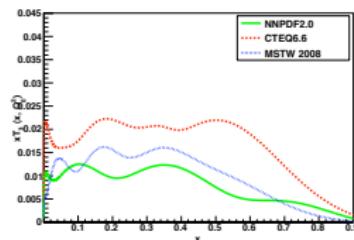
# Results

## Partons

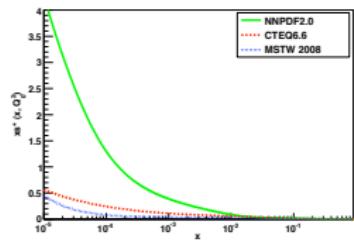
- Reduction of uncertainties with respect to older NNPDF sets due to **inclusion of new data**



- Uncertainties on PDFs competitive with results from other groups. Smaller uncertainty due to **wide set of consistent data**



- In regions where there are little or no experimental constraints, uncertainty larger due to **minimum parametrization bias**



# Impact of modifications

A quantitative assessment

- A quantitative assessment is possible

$$d(q_j) = \sqrt{\left\langle \frac{(\langle q_j \rangle_{(1)} - \langle q_j \rangle_{(2)})^2}{\sigma_1^2[q_j] + \sigma_2^2[q_j]} \right\rangle_{N_{\text{part}}}}$$
$$d(\sigma_j) = \sqrt{\left\langle \frac{(\langle \sigma_j \rangle_{(1)} - \langle \sigma_j \rangle_{(2)})^2}{\sigma_1^2[\sigma_j] + \sigma_2^2[\sigma_j]} \right\rangle_{N_{\text{part}}}}$$

- Comparisons performed in NNPDF2.0 analysis

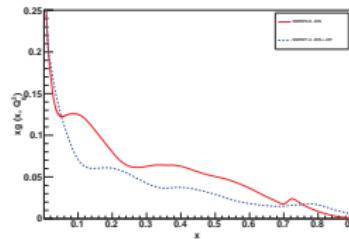
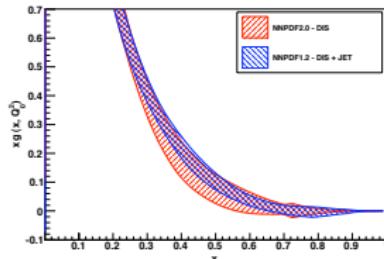
- ① Start from NNPDF1.2
- ② NNPDF1.2 vs. NNPDF1.2 + minimization/training improvements
- ③ Improved NNPDF1.2 vs. Improved NNPDF1.2 +  $t_0$ -method
- ④ Fit to DIS dataset with H1/ZEUS data vs. Fit with HERA-I combined
- ⑤ Fit to DIS dataset vs. Fit to DIS+JET
- ⑥ Fit to DIS+JET vs. NNPDF2.0 final

# Impact of modifications

Tevatron inclusive Jet data

Fit	NNPDF1.2	NNPDF1.2+IGA	NNPDF1.2+IGA+ $t_0$	2.0 DIS	2.0 DIS+JET
$\chi^2_{\text{tot}}$	1.32	1.16	1.12	<b>1.20</b>	<b>1.18</b>
$\langle E \rangle$	2.79	2.41	2.24	<b>2.31</b>	<b>2.28</b>
$\langle \chi^{2(k)} \rangle$	1.60	1.28	1.21	1.29	1.27
CDFR2KT	<b>1.10</b>	<b>0.95</b>	<b>0.78</b>	<b>0.91</b>	<b>0.79</b>
D0R2CON	<b>1.18</b>	<b>1.07</b>	<b>0.94</b>	<b>1.00</b>	<b>0.93</b>

- Tevatron Run-II inclusive jet data provide a valuable constrain on large-x gluon.
- No incompatibility.
- Run-I data not included but compatibility with the outcome of the fit has been checked.

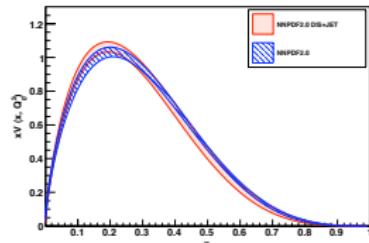
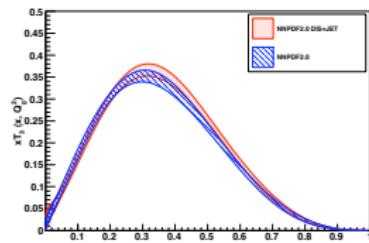


# Impact of modifications

## Drell-Yan and Vector Boson production data

Fit	NNPDF1.2	NNPDF1.2+IGA	NNPDF1.2+IGA+ $t_0$	2.0 DIS	2.0 DIS+JET	NNPDF2.0
$\chi^2_{\text{tot}}$	1.32	1.16	1.12	1.20	<b>1.18</b>	<b>1.21</b>
$\langle E \rangle$	2.79	2.41	2.24	2.31	<b>2.28</b>	<b>2.32</b>
$\langle \chi^2(k) \rangle$	1.60	1.28	1.21	1.29	<b>1.27</b>	<b>1.29</b>
DYE605	<b>11.19</b>	<b>22.89</b>	8.21	7.32	<b>10.35</b>	0.88
DYE866	<b>53.20</b>	4.81	2.46	2.24	<b>2.59</b>	1.28
CDFWASY	<b>26.76</b>	28.22	20.32	13.06	<b>14.13</b>	1.85
CDFZRAP	<b>1.65</b>	<b>4.61</b>	<b>3.13</b>	3.12	<b>3.31</b>	2.02
D0ZRAP	0.56	0.80	0.65	0.65	<b>0.68</b>	0.47

- Good description of fixed target Drell-Yan data (E605 proton and E886 proton and p/d ratio)
- Vector boson production at colliders (CDF W-asymmetry and Z rapidity distribution) harder to fit
- All valence-type PDF combinations are affected by these data
- Sizable reduction in the uncertainty of the strange valence (possible impact on NuTeV anomaly)

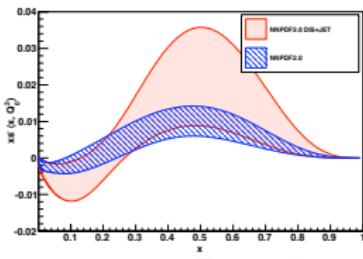
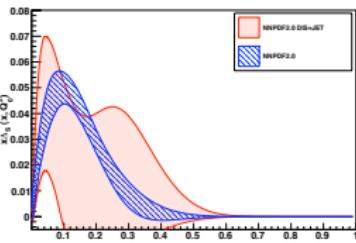


# Impact of modifications

Drell-Yan and Vector Boson production data

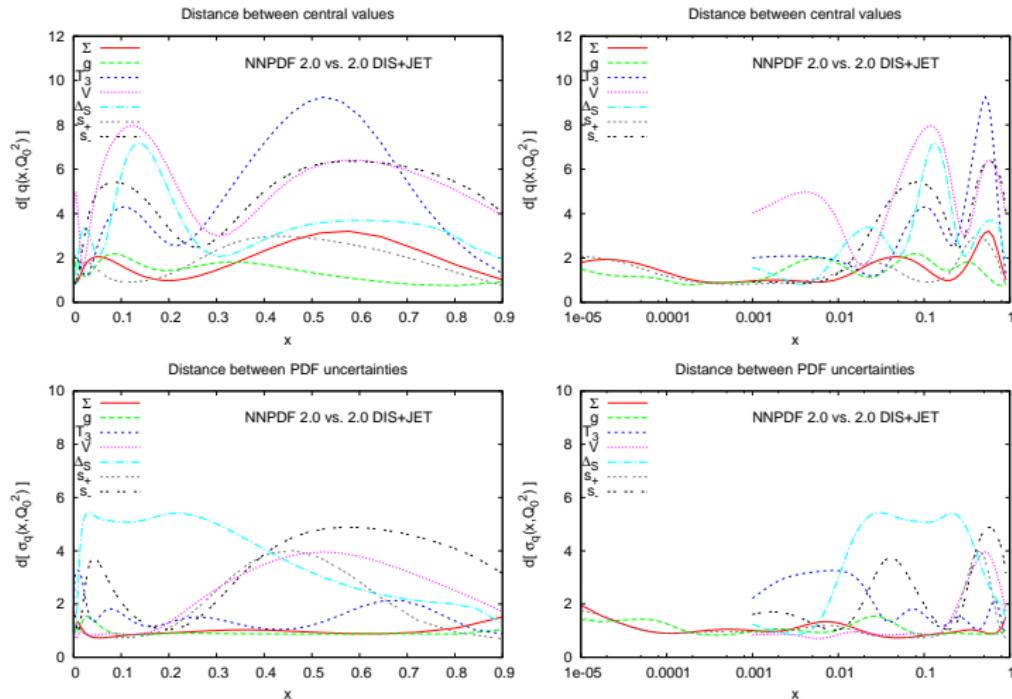
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- All valence-type PDF combinations are affected by these data
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# Impact of modifications

## Drell-Yan and Vector Boson production data



# Phenomenology

The proton strangeness revisited

- Discrepancy  $\geq 3\sigma$  between indirect and direct determination from NuTeV measurement assuming  $[S^-] = 0$  and isospin symmetry solved in NNPDF1.2 analysis

EW fit

$$\sin^2 \theta_W = 0.2223 \pm 0.0002$$

$$\delta_s \sin^2 \theta_W \sim -0.240 \frac{[S^-]}{[Q^-]}$$

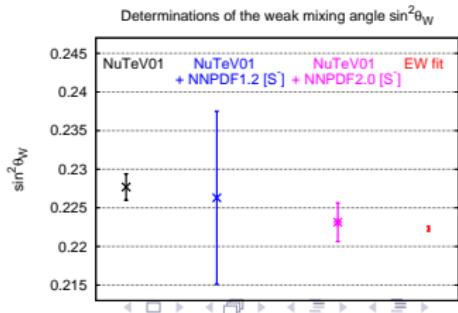
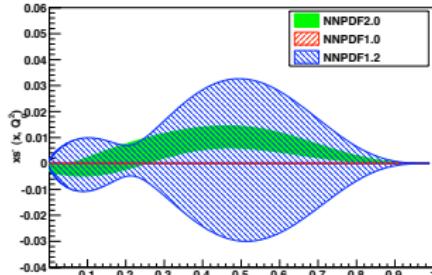
NuTeV

$$\sin^2 \theta_W = 0.2276 \pm 0.0014$$

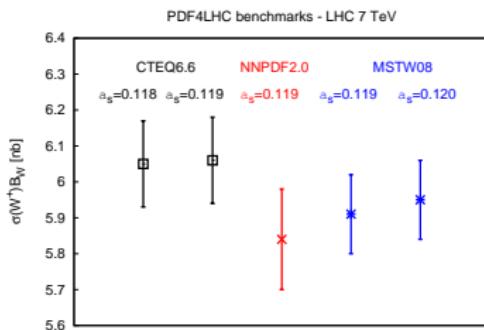
$$[F] = \int_0^1 dx \times f(x, Q^2)$$

- Uncertainty reduced by addition of DY data

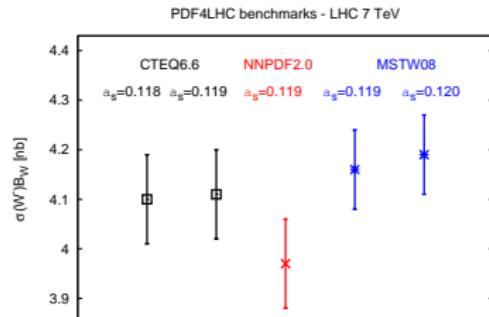
- Striking agreement with EW fits



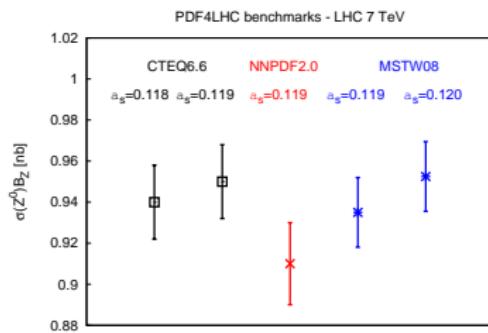
### $W^+$ production



### $W^-$ production



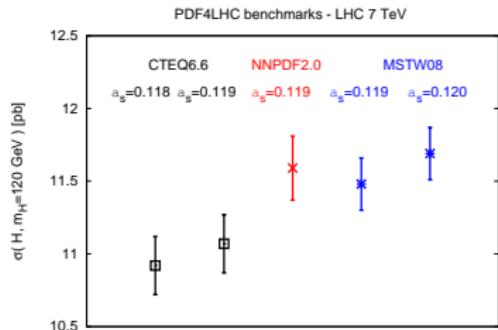
### $Z$ production



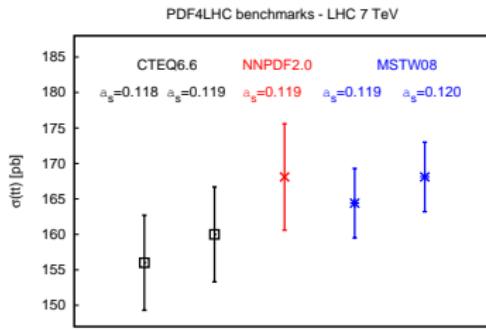
### PDF4LHC benchmark

Evaluate LHC standard candles  
with same value of  $\alpha_s$ .  
Use MCFM with same settings.

### Higgs production



### $t\bar{t}$ production



- When using the same  $\alpha_s(M_Z^2)$  the agreement between different sets improves.
- At LHC, 7 TeV, using the same value of  $\alpha_s(M_Z^2)$ , the impact of different treatments of the HQ masses on central values is within  $1\sigma$  uncertainty.

W. Giele and S. Keller, hep-ph/9803393

- The  $N_{\text{rep}}$  replicas of a NNPDF fit give the probability density in the space of PDFs

$$\langle \mathcal{F}[f_i(x, Q^2)] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{F}\left(f_i^{(\text{net})(k)}(x, Q^2)\right)$$

- We can assess the impact of including new data in the fit updating the probability density distribution taking into account the new data,

$$P_{\text{new}}(\lambda) = P(\lambda|x^e) = \frac{P(x^e|\lambda)P_{\text{init}}(\lambda)}{P(x^e)}, \quad P(x^e|\lambda) = e^{-\frac{\chi_{\text{new}}^2(\lambda)}{2}} \quad \text{Bayes Theorem}$$

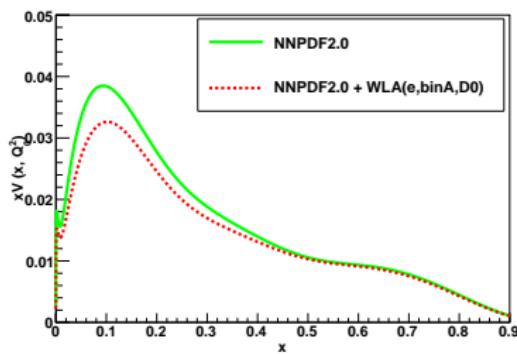
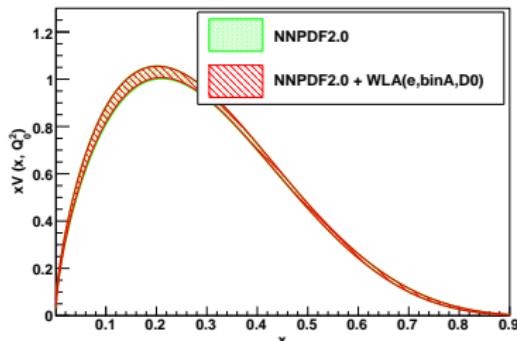
- Monte Carlo integrals are given by weighted sums

$$\langle \mathcal{F}[f_i(x, Q^2)] \rangle = \sum_{k=1}^{N_{\text{rep}}} w_k \mathcal{F}\left(f_i^{(\text{net})(k)}(x, Q^2)\right) \quad w_k = \frac{e^{-\frac{1}{2}\chi_{\text{new}}^2(\lambda^k)}}{\sum_{i=1}^{N_{\text{rep}}} e^{-\frac{1}{2}\chi_{\text{new}}^2(\lambda^i)}}$$

# Phenomenology

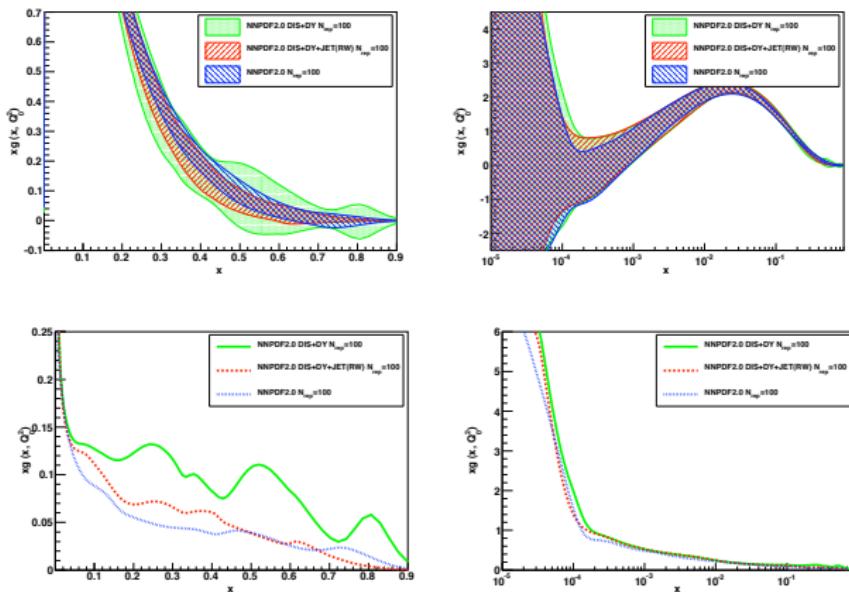
Reweighting real data: W lepton asymmetry

- In the fit only CDF W asymmetry is included [arXiv:0901.216 [hep-ex]]
- We evaluated W electron asymmetry with 1000 replicas of NNPDF20 set using DYNNNLO [arXiv:0903.2120 [hep-ph]].
- ... and included D0 W electron asymmetry data points [arXiv:0807.3367 [hep-ex]] through re-weighting.
- Main impact on reduction of middle- $x$  Valence uncertainty.
- No need of rifitting! Everybody can do it.
- Same method can be applied to pseudo-data to predict future experiments' constrains on PDFs (see J. Rojo's talk).



# Phenomenology

Test the reweighting procedure



- Include JET data through re-fitting and through re-weighting
- Use (DIS+DY) 100 replicas fit as a initial condition
- Re-weighting and re-fitting give same results within statistical fluctuations
- Less fluctuations if one uses 1000 replicas

- Monte Carlo ensemble
  - \* Any statistical property of PDFs can be calculated using standard statistical methods.
  - \* No need of any tolerance criterion.
  - \* Perfectly suitable for Bayesian Reweighting.
- The Neural Network parametrization
  - \* Small uncertainties come from the data, not from bias due to functional form.
  - \* Inconsistent data or underestimated uncertainties do not require a separate treatment and are automatically signalled by a larger value of the  $\chi^2$ .
- The NNPDF2.0 is the first unbiased global NLO fit [FastKernel].
- Same consistent statistical behaviour under addition of hadronic data.
- Little tension between hadronic and DIS data.
- Predictions for standard candles close to those of other collaborations.
- Available on the common LHAPDF interface (<http://projects.hepforge.org/lhapdf>)
- NNPDF2.X with better treatment of HQ masses, FONLL (see Ref. [ArXiv:1001.2312](#)), and NNPDF2.Y NNLO fit are work in progress.

THANK YOU!!

## BACK-UP

# NNPDF approach

## Ingredient #1: Monte Carlo Errors

Generate a  $N_{\text{rep}}$  Monte Carlo sets of artificial data, or "pseudo-data" of the original  $N_{\text{data}}$  data points

$$F_i^{(\text{art})(k)}(x_p, Q_p^2) \equiv F_{i,p}^{(\text{art})(k)} \quad i = 1, \dots, N_{\text{data}} \\ k = 1, \dots, N_{\text{rep}}$$

Multi-gaussian distribution centered on each data point:

$$F_{i,p}^{(\text{art})(k)} = S_{p,N}^{(k)} F_{i,p}^{\text{exp}} \left( 1 + r_p^{(k)} \sigma_p^{\text{stat}} + \sum_{j=1}^{N_{\text{sys}}} r_{p,j}^{(k)} \sigma_{p,j}^{\text{sys}} \right)$$

If two points have correlated systematic uncertainties

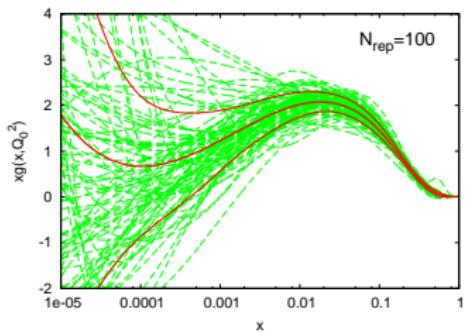
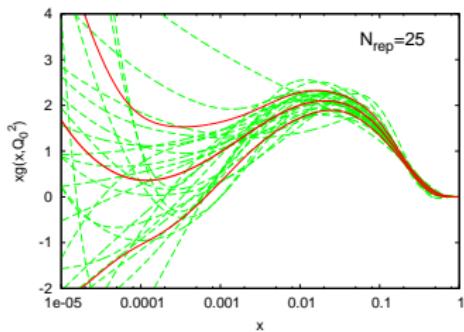
$$r_{p,j}^{(k)} = r_{p',j}^{(k)}$$

Correlations are properly taken into account.

# NNPDF approach

## Ingredient #1: Monte Carlo Errors

$$\langle \mathcal{F}[f(x)] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{F}[f^{(k)(\text{net})}(x)]$$
$$\sigma_{\mathcal{F}[f(x)]} = \sqrt{\langle \mathcal{F}[f(x)]^2 \rangle - \langle \mathcal{F}[f(x)] \rangle^2}$$

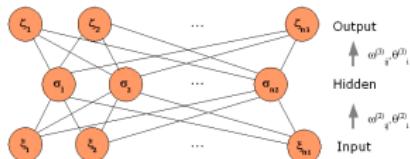


Even though individual replicas may fluctuate significantly, average quantities such as central values and error bands are smooth inasmuch as stability is reached due to the dimension of the ensemble increasing.

# NNPDF approach

Ingredient #2: Neural Network as unbiased parametrization

Each independent PDF at the initial scale  $Q_0^2 = 2\text{GeV}^2$  is parameterized by an individual NN.



- \* Each neuron receives input from neurons in preceding layer.
- \* Activation determined by weights and thresholds according to a non linear function:

$$\xi_i = g(\sum_j \omega_{ij} \xi_j - \theta_i), \quad g(x) = \frac{1}{1 + e^{-x}}$$

In a simple case (1-2-1) we have,

$$\xi_1^{(3)} = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - \xi_1^{(1)} \omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - \xi_1^{(1)} \omega_{21}^{(1)}}}}}$$

7 parameters

...Just a convenient functional form which provides a **redundant** and flexible parametrization.

We want the best fit to be independent of any assumption made on the parametrization.

# NNPDF approach

Ingredient #2: Neural Network as unbiased parametrization

Basis set:  $Q_0^2 = 2 \text{ GeV}^2$

Singlet : $\Sigma(x)$	$\longmapsto \text{NN}_\Sigma(x)$	2-5-3-1 37 pars
Gluon : $g(x)$	$\longmapsto \text{NN}_g(x)$	2-5-3-1 37 pars
Total valence : $V(x) \equiv u_V(x) + d_V(x)$	$\longmapsto \text{NN}_V(x)$	2-5-3-1 37 pars
Non-singlet triplet : $T_3(x)$	$\longmapsto \text{NN}_{T_3}(x)$	2-5-3-1 37 pars
Sea asymmetry : $\Delta_S(x) \equiv \bar{d}(x) - \bar{u}(x)$	$\longmapsto \text{NN}_\Delta(x)$	2-5-3-1 37 pars
Total strangeness : $s^+(x) \equiv (s(x) + \bar{s}(x))/2$	$\longmapsto \text{NN}_{(s+)}(x)$	2-5-3-1 37 pars
Strangeness valence : $s^-(x) \equiv (s(x) - \bar{s}(x))/2$	$\longmapsto \text{NN}_{(s-)}(x)$	2-5-3-1 37 pars

259 parameters

# NNPDF approach

## Ingredient #3: Training and dynamical stopping

Our fitting strategy is very different from that used by other collaborations: instead of a set of basis functions with a small number of parameters, we have an unbiased basis of functions parameterized by a very large and redundant set of parameters.

CTEQ,MSTW,AL

$\mathcal{O}(20)$  parm

NNPDF

$\mathcal{O}(200)$  parm

Not trivial because ...

A redundant parametrization might adapt not only to physical behavior but also to random statistical fluctuations of data.

### Ingredients of fitting procedure

- ① Flexible and redundant parametrization
- ② Genetic Algorithm minimization
- ③ Dynamical stopping criterion: cross-validation technique

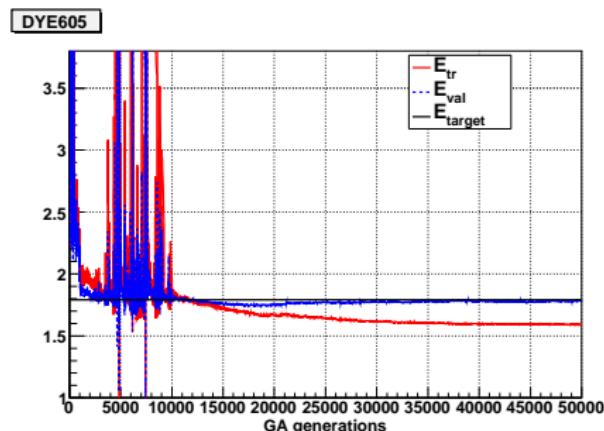
# NNPDF approach

## Ingredient #3: Dynamical stopping

- \* GA is monotonically decreasing by construction.
- \* The best fit is not given by the absolute minimum.
- \* The best fit is given by an optimal training beyond which the figure of merit improves only because we are fitting statistical noise of the data.

### Cross-validation method

- \* Divide data in two sets: training and validation.
- \* Random division for each replica ( $f_t = f_v = 0.5$ ).
- \* Minimisation is performed only on the training set. The validation  $\chi^2$  for the set is computed.
- \* When the training  $\chi^2$  still decreases while the validation  $\chi^2$  stops decreasing  $\rightarrow$  STOP.

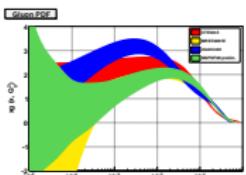
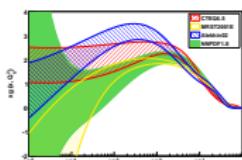
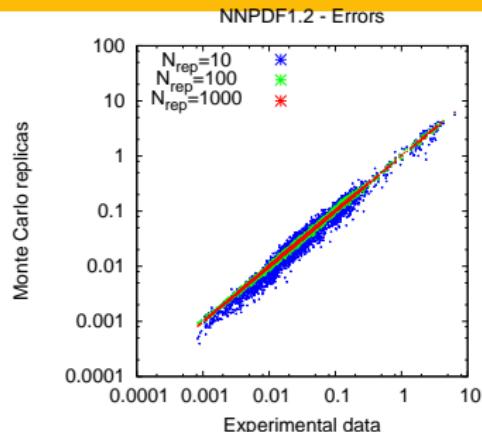


# Motivation

What's nice about NNPDF #1

## 1) Monte Carlo behaves in a statistically consistent way

- Generate a Monte Carlo ensemble in the space of data
- $N$  pseudo-data reproduce the probability distribution of the original experimental data.
- Precision of estimators scales as the size of the sample.



## 2) Results are shown to be independent of the parametrization and even stable upon the addition of independent PDFs parametrizations

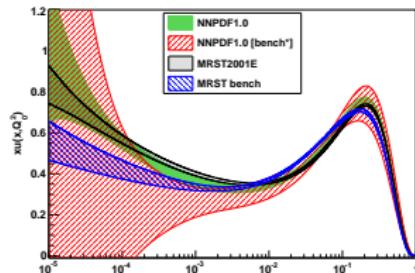
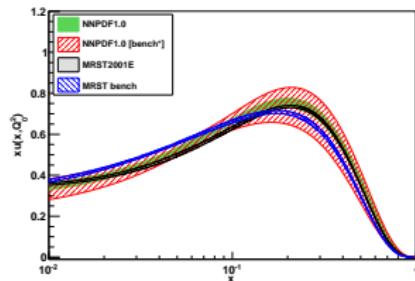
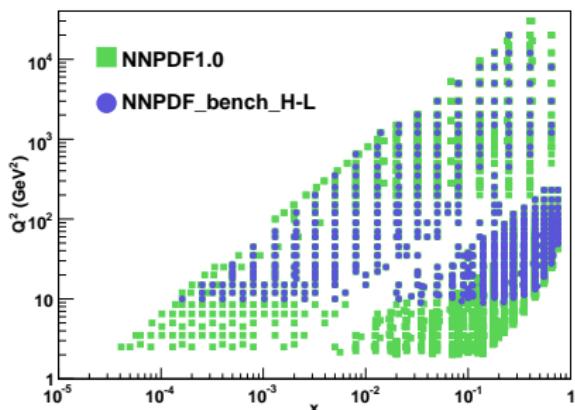
7x37 pars → 7x31 pars

# Motivation

What's nice about NNPDF #2

- 3) PDFs behave as expected upon the addition of new data

**HERA-LHC benchmark:** DIS data



- 4) Control on PDFs uncertainties: NuTeV anomaly solved AND  $V_{cs}$  precise determination at the same time  
[NNPDF1.2 [[arXiv:0906.1958](https://arxiv.org/abs/0906.1958)]]

# Motivation

What's nice about NNPDF #5

- Define second momentum of PDFs  $f$ :  $[F] = \int_0^1 dx x f(x, Q^2)$ .
- Discrepancy  $\geq 3\sigma$  between indirect and direct determination from NuTeV measurement assuming  $[S^-] = 0$  and isospin symmetry.

EW fit

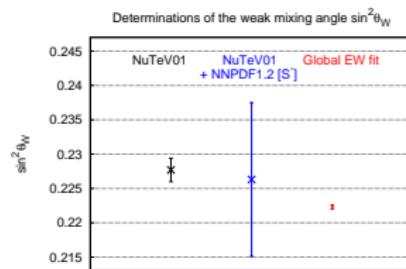
$$\sin^2 \theta_W = 0.2223 \pm 0.0002$$

NuTeV

$$\sin^2 \theta_W = 0.2276 \pm 0.0014$$

$$\delta_s \sin^2 \theta_W \sim -0.240 \frac{[S^-]}{[Q^-]}$$

$$\delta_s \sin^2 \theta_W = -0.0005 \pm 0.0096^{\text{PDFs}} \pm \text{sys}$$

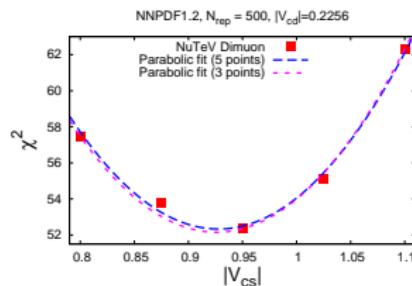


# Motivation

What's nice about NNPDF #5

Control on PDFs uncertainties: NuTeV anomaly solved AND precision studies at the same time

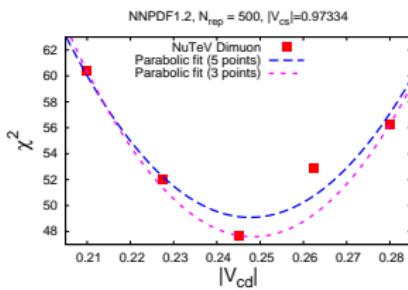
## CKM fit



$$V_{cs} = 0.97334 \pm 0.00023, \Delta V_{cs} \sim 0.02\%$$

## Direct Determination

$$\begin{aligned} V_{cs} &= 1.04 \pm 0.06, \Delta V_{cs} \sim 6\% && \text{D/B decays} \\ V_{cs} &> 0.59 && \text{DIS fit} \end{aligned}$$



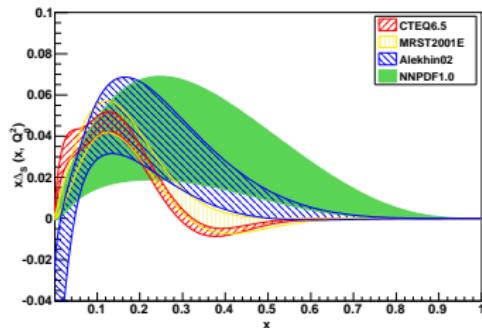
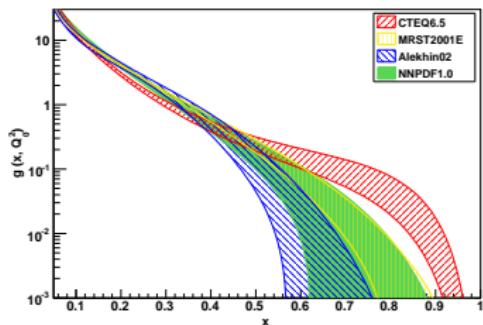
## NNPDF1.2 analysis

$$V_{cs} = 0.97 \pm 0.07, \Delta V_{cs} \sim 6\%$$

- DIS data are insufficient to determine accurately PDFs.
- Flavor decomposition of quark–antiquark sea and large- $x$  gluon distribution.

$$R^{pd} = \frac{d\sigma^d/dM^2 dx_F}{d\sigma^p/dM^2 dx_F} \propto (1 + \bar{d}/\bar{u})$$

$$A^W = \frac{d\sigma_W^+/dy - d\sigma_W^-/dy}{d\sigma_W^+/dy + d\sigma_W^-/dy} \propto \frac{u\bar{d} - d\bar{u}}{u\bar{d} + d\bar{u}}$$



- **NNPDF2.0** includes most of the available hadronic data.

- ➊ GLOBAL: Includes fixed target Drell-Yan, Tevatron jets and weak bosons data.
- ➋ Improvements in training/stopping
  - Target Weighted Training
  - Improved stopping
- ➌ Improved treatment of normalization errors ( $t_0$  method)
  - For details see [[arXiv:0912.2276](#)]
- ➍ Fast DGLAP evolution based on higher-order interpolating polynomials
- ➎ NLO: Fast computation of Drell-Yan observables based on higher-order interpolating polynomials.
- ➏ Enforced positivity constraints

# Impact of modifications

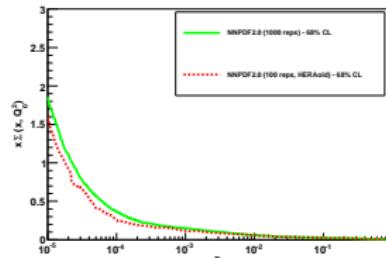
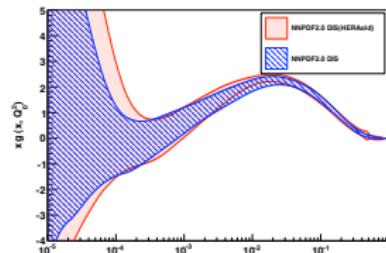
Fit	NNPDF1.2	NNPDF1.2+IGA	NNPDF1.2+IGA+ $t_0$	2.0 DIS	2.0 DIS+JET	NNPDF2.0
$\chi^2_{\text{tot}}$	1.32	1.16	1.12	1.20	1.18	1.21
$\langle E \rangle$	2.79	2.41	2.24	2.31	2.28	2.32
$\langle E_{\text{tr}} \rangle$	2.75	2.39	2.20	2.28	2.24	2.29
$\langle E_{\text{val}} \rangle$	2.80	2.46	2.27	2.34	2.32	2.35
$\langle \chi^2(k) \rangle$	1.60	1.28	1.21	1.29	1.27	1.29
NMC-pd	1.48	0.97	0.87	0.85	0.86	0.99
NMC	1.68	1.72	1.65	1.69	1.66	1.69
SLAC	1.20	1.42	1.33	1.37	1.31	1.34
BCDMS	1.59	1.33	1.25	1.26	1.27	1.27
HERAI	1.05	0.98	0.96	1.13	1.13	1.14
CHORUS	1.39	1.13	1.12	1.13	1.11	1.18
FLH108	1.70	1.53	1.53	1.51	1.49	1.49
NTVDMN	0.64	0.81	0.71	0.71	0.75	0.67
ZEUS-H2	1.52	1.51	1.49	1.50	1.49	1.51
DYE605	11.19	22.89	8.21	7.32	10.35	0.88
DYE866	53.20	4.81	2.46	2.24	2.59	1.28
CDFWASY	26.76	28.22	20.32	13.06	14.13	1.85
CDFZRAP	1.65	4.61	3.13	3.12	3.31	2.02
D0ZRAP	0.56	0.80	0.65	0.65	0.68	0.47
CDFR2KT	1.10	0.95	0.78	0.91	0.79	0.80
D0R2CON	1.18	1.07	0.94	1.00	0.93	0.93

# Impact of modifications

HERA-I combined dataset

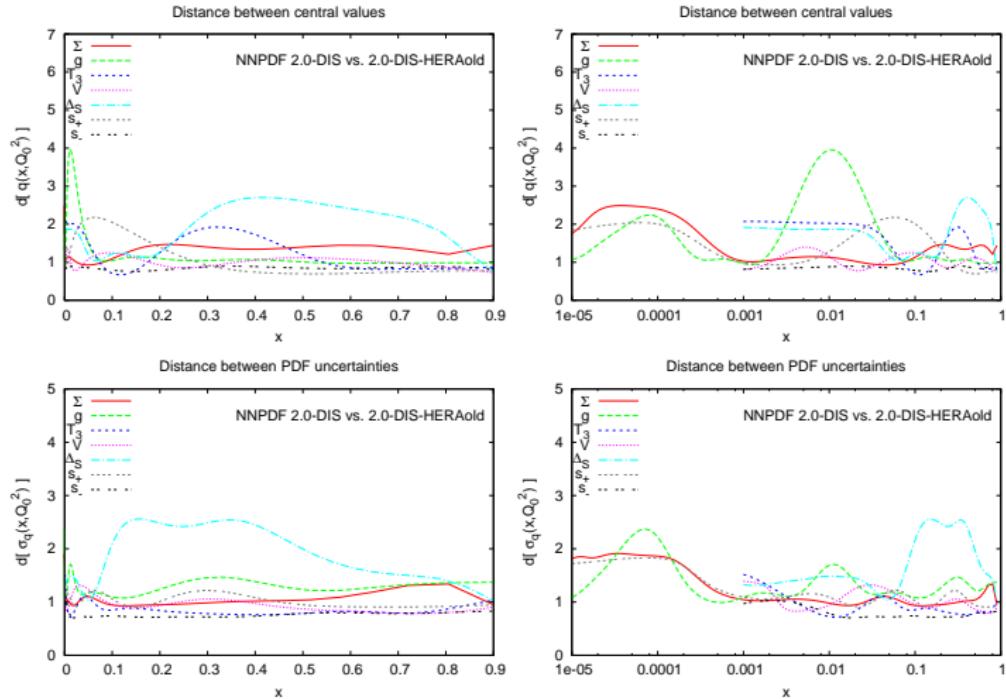
Fit	NNPDF1.2	NNPDF1.2+IGA	NNPDF1.2+IGA+ $t_0$	2.0 DIS
$\chi^2_{\text{tot}}$	1.32	1.16	<b>1.12</b>	<b>1.20</b>
$\langle E \rangle$	2.79	2.41	<b>2.24</b>	<b>2.31</b>
$\langle \chi^{2(k)} \rangle$	1.60	1.28	<b>1.21</b>	<b>1.29</b>
HERAI	1.05	0.98	<b>0.96</b>	<b>1.13</b>

- HERA-I combined more precise.
- Quality of other data unchanged.
- Overall fit quality to the whole dataset is good
  - $\sigma_{NC}^+$  dataset has relatively high  $\chi^2 \sim 1.3$
  - $\sigma_{CC}^-$  dataset has very low  $\chi^2 \sim 0.55$
- Same  $\chi^2$ -pattern observed in the HERAPDF1.0 analysis
- Impact on PDFs, mainly Gluon at small/medium-x



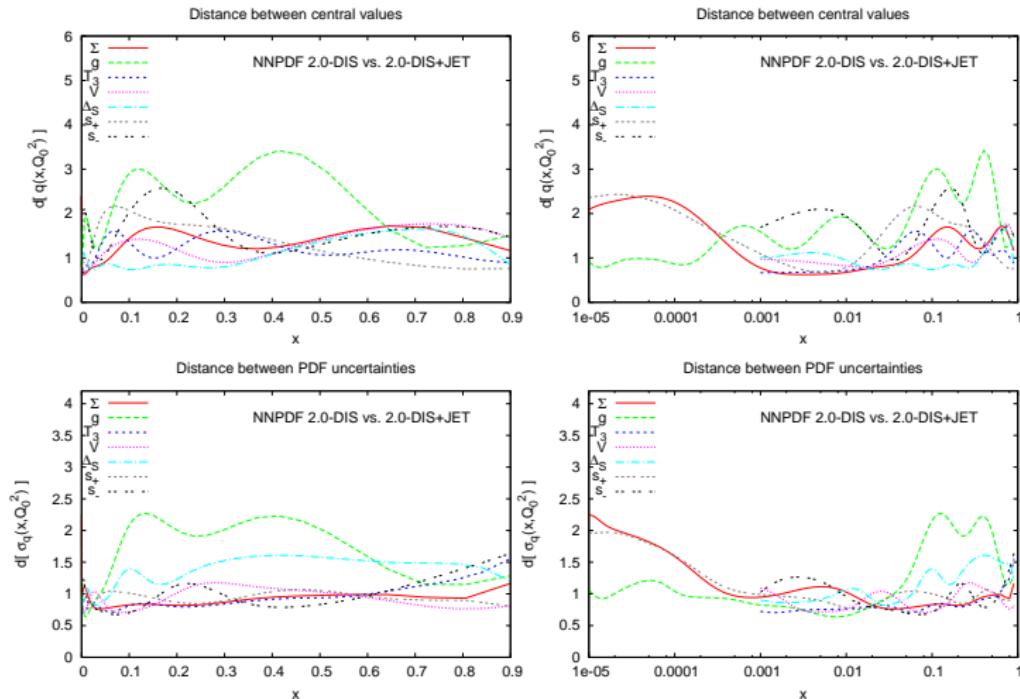
# Impact of modifications

## HERA-I combined dataset



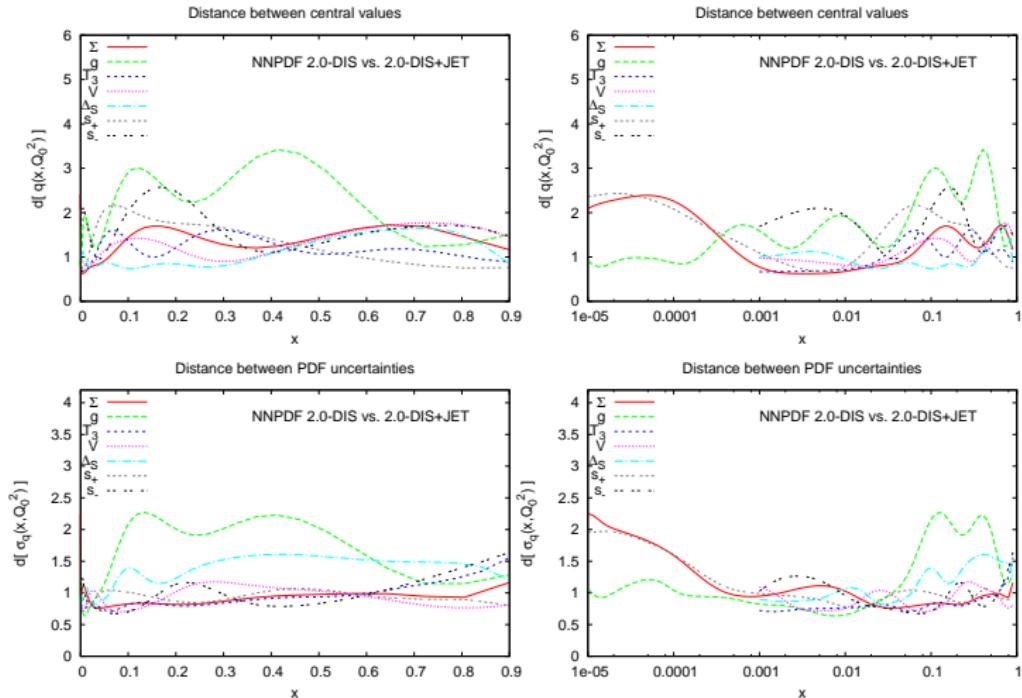
# Impact of modifications

Tevatron inclusive Jet data



# Impact of modifications

## Drell-Yan and Vector Boson production data



## New Features

### Positivity constraints

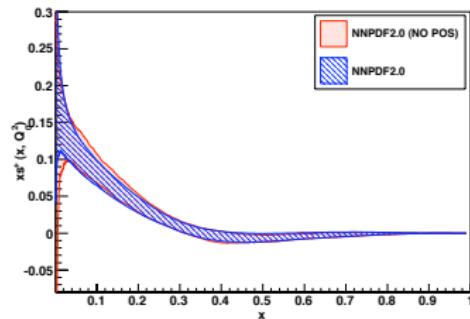
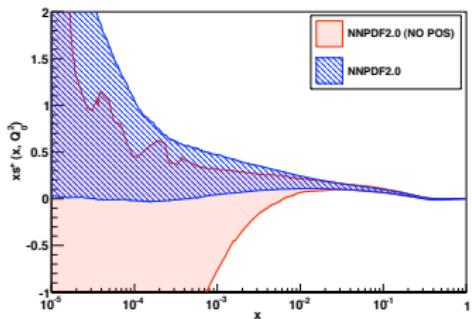
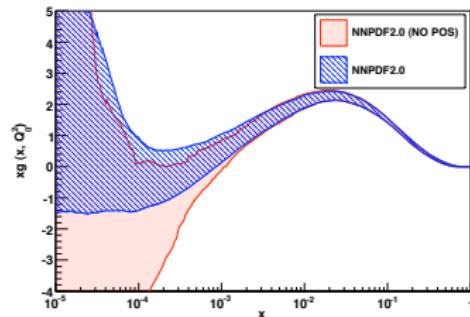
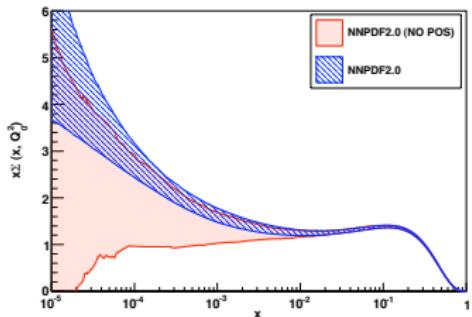
- We want the fitting procedure to explore only the subspace of acceptable physical solutions.
- We want
  - $F_L$  positive.
  - Dimuon cross-section positive.
  - Momentum and valence sum rules.
- Modify the training function with addition of a Lagrangian multiplier:

$$E^{(k)} \longrightarrow E^{(k)} - \lambda_{\text{pos}} \sum_{I=1}^{N_{\text{dat, pos}}} \Theta(F_I^{(\text{net})(k)}) F_I^{(\text{net})(k)}$$

- $N_{\text{dat, pos}}$  : number of pseudodata points used to implement positivity constraints.
- $\lambda_{\text{pos}}$  : associated Lagrangian multiplier ( $10^{10}$ )

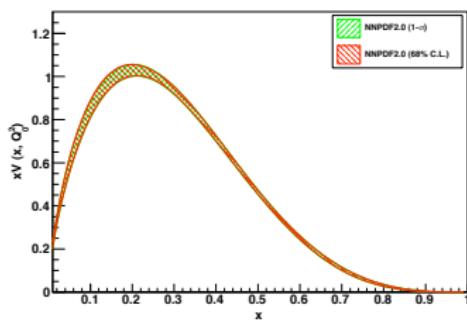
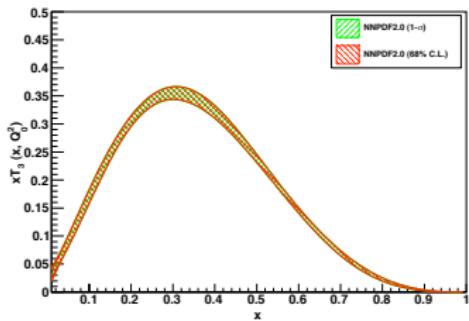
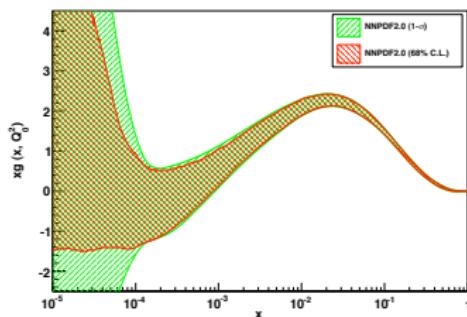
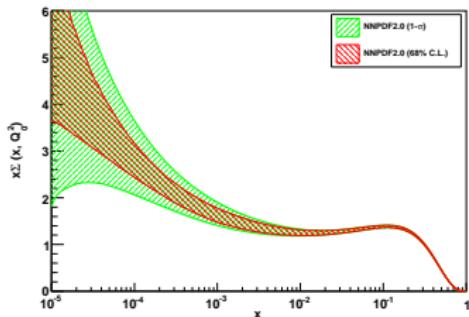
# Results

## Impact of positivity



# Results

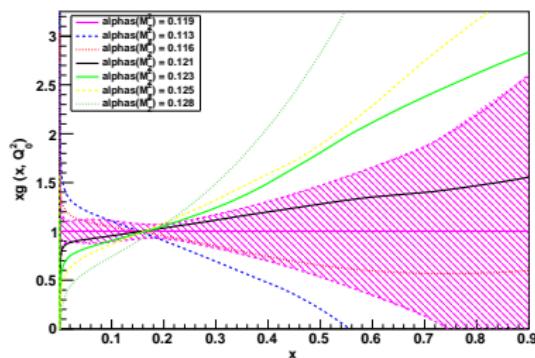
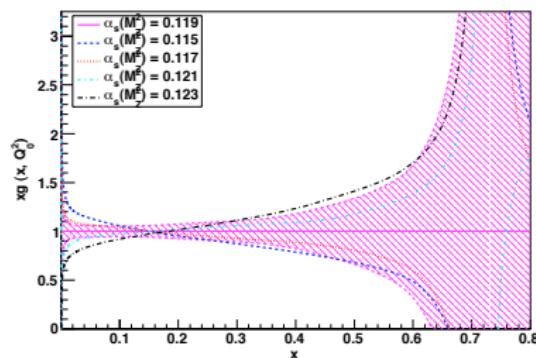
## Gaussian behaviour



# Results

## Impact of $\alpha_s$

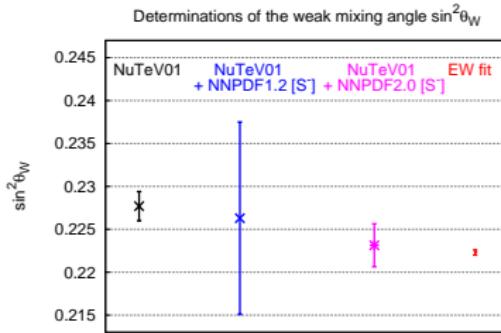
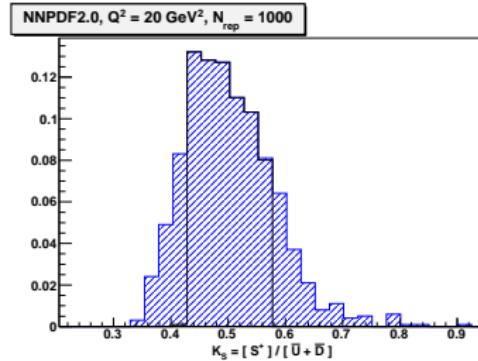
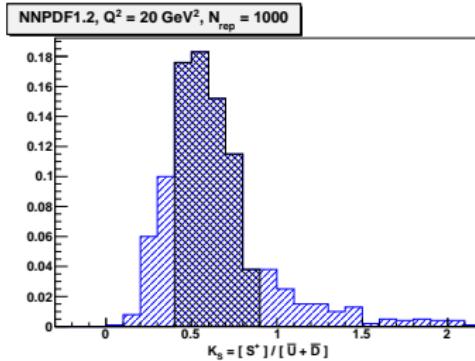
$\alpha_s$  dependence in NNPDF2.0



- Greater sensitivity to  $\alpha_s$  than NNPDF1.2
- Due to large NLO corrections in Drell-Yan observables.

# Phenomenology

## The proton strangeness revisited



$$K_S = \begin{cases} 0.71^{+0.19}_{-0.31} \text{ stat} \pm 0.26 \text{ syst} & (\text{NNPDF1.2}) \\ 0.503 \pm 0.075 \text{ stat}; & (\text{NNPDF2.0}) \end{cases}$$
$$R_S = \begin{cases} 0.006 \pm 0.045 \text{ stat} \pm 0.010 \text{ syst} & (\text{NNPDF1.2}) \\ 0.019 \pm 0.008 \text{ stat} & (\text{NNPDF2.0}) \end{cases}$$

- Uncertainty reduced by addition of DY data
- Striking agreement with EW fits

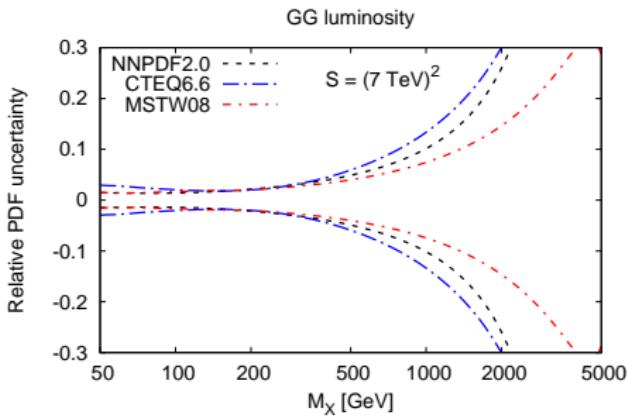
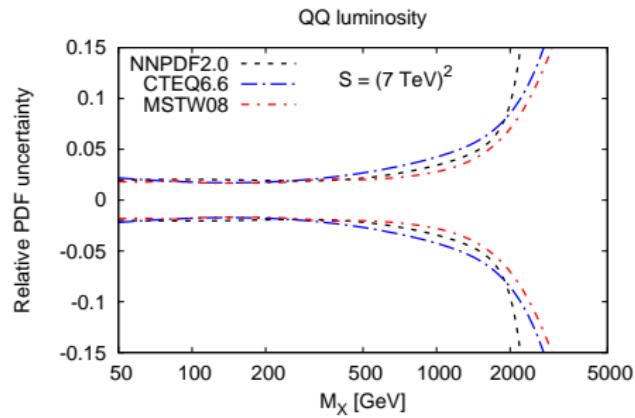
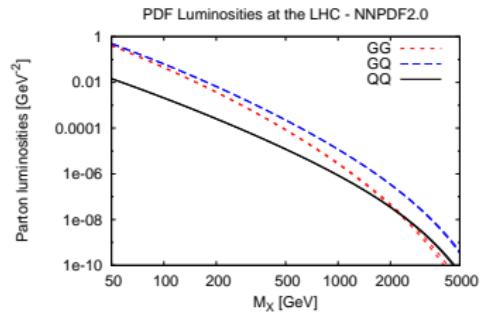
## Results

Parton Luminosities 7 TeV

$$\Phi_{gg} \left( M_X^2 \right) = - \frac{1}{s} \int_\tau^1 \frac{dx_1}{x_1} g \left( x_1, M_X^2 \right) g \left( \tau/x_1, M_X^2 \right)$$

$$\Phi_{gq} \left( M_X^2 \right) = \frac{1}{s} \int_{\tau}^1 \frac{dx_1}{x_1} \left[ g \left( x_1, M_X^2 \right) \Sigma \left( \tau/x_1, M_X^2 \right) + (1 \rightarrow 2) \right]$$

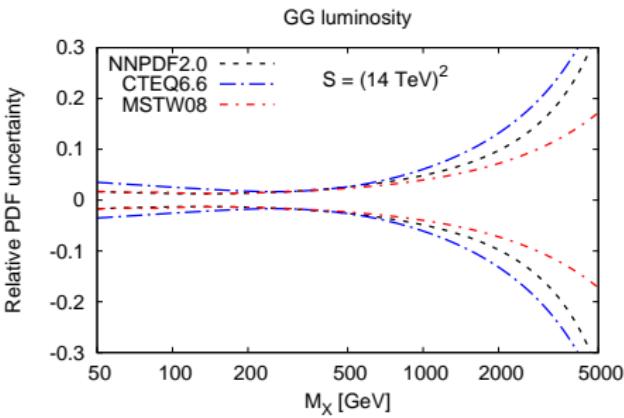
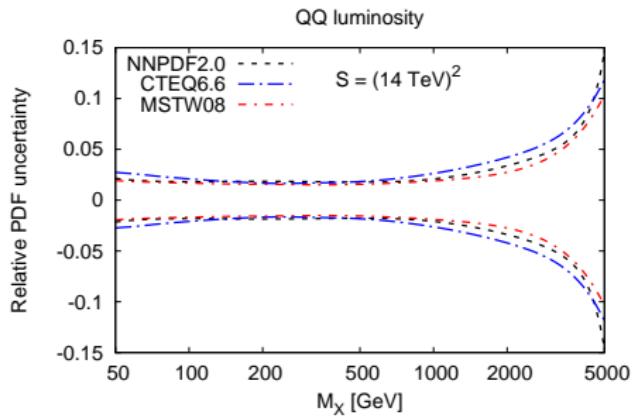
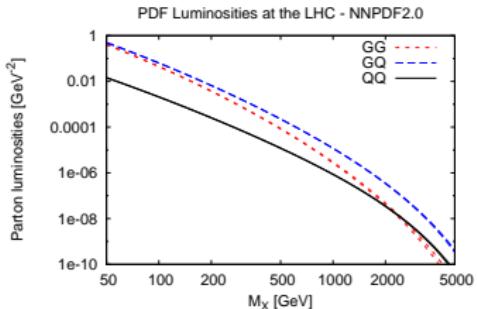
$$\Phi_{qq} \left( M_X^2 \right) = \frac{1}{s} \int_{\tau}^1 \frac{dx_1}{x_1} \sum_{i=1}^{N_f} \left[ q_i \left( x_1, M_X^2 \right) \bar{q}_i \left( \tau/x_1, M_X^2 \right) + (1 \rightarrow 2) \right]$$



# Results

## Parton Luminosities 14 TeV

$$\begin{aligned}\Phi_{gg}(M_X^2) &= \frac{1}{s} \int_{\tau}^1 \frac{dx_1}{x_1} g(x_1, M_X^2) g(\tau/x_1, M_X^2) \\ \Phi_{gq}(M_X^2) &= \frac{1}{s} \int_{\tau}^1 \frac{dx_1}{x_1} [g(x_1, M_X^2) \Sigma(\tau/x_1, M_X^2) + (1 \rightarrow 2)] \\ \Phi_{qq}(M_X^2) &= \frac{1}{s} \int_{\tau}^1 \frac{dx_1}{x_1} \sum_{i=1}^{N_f} [q_i(x_1, M_X^2) \bar{q}_i(\tau/x_1, M_X^2) + (1 \rightarrow 2)]\end{aligned}$$



# Some phenomenology

## LHC standard candles at 14 TeV

