Theoretical perspective

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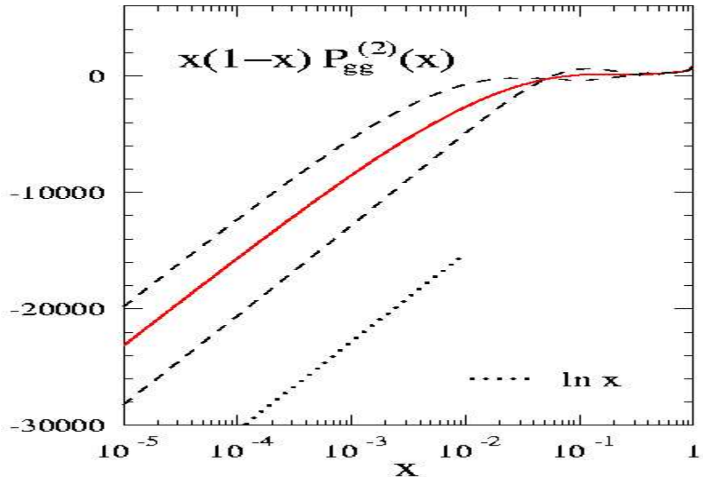
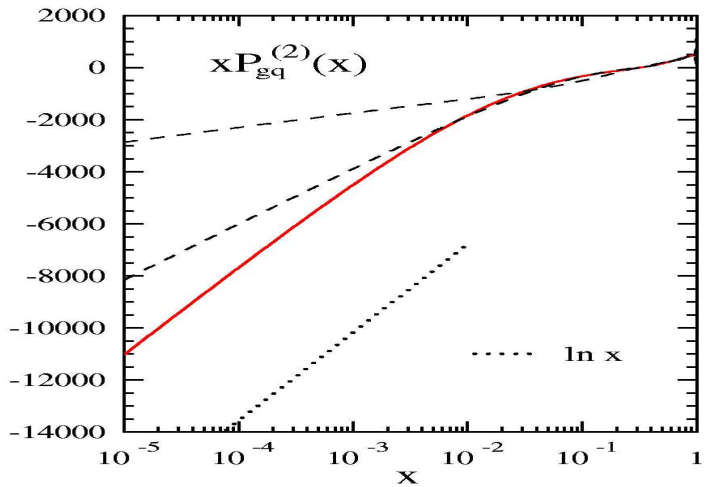
Topics

- High-order QCD corrections
- Heavy quarks in the DIS
- PDFs in the global fits
- Definition of the heavy-quark mass
- DIS and other processes (α_s, gluons, ...)

The high-order corrections do the PDF evolution





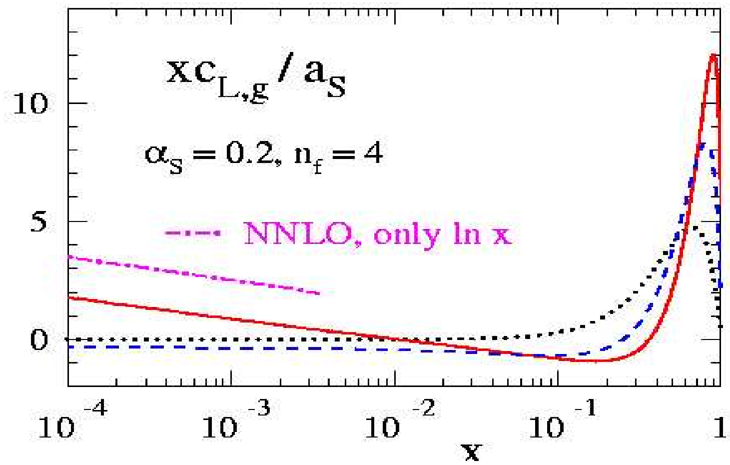
- The accurate kernels up to the 3-loop (NNLO) accuracy are know

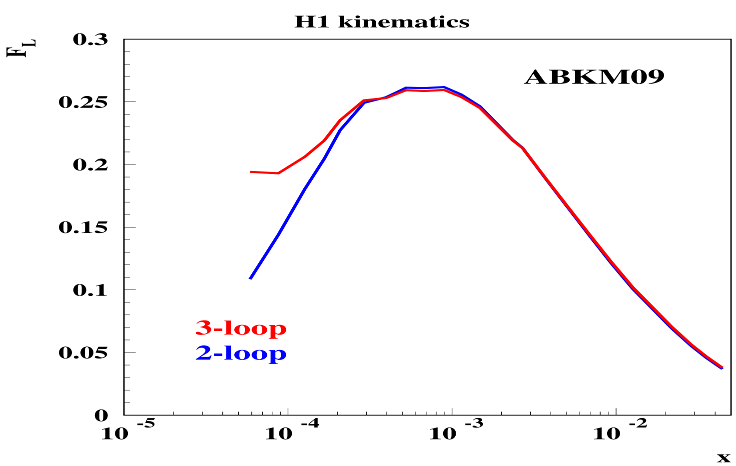
  (Altarelli-Parisi 77)
  (Furmanski-Petrozio 81)
  (Moch-Vermasseren-Vogt 04)

- The small-x resummation (Catani-Hautmann 94, Fadin-Lipatov 98) overestimates the NNLO kernels.
- The 4-loop non-singlet kernels are partially known (Baikov-Chetyrkin 06)

The 3-loop corrections to the coefficient functions

(Moch-Vermasseren-Vogt 04)





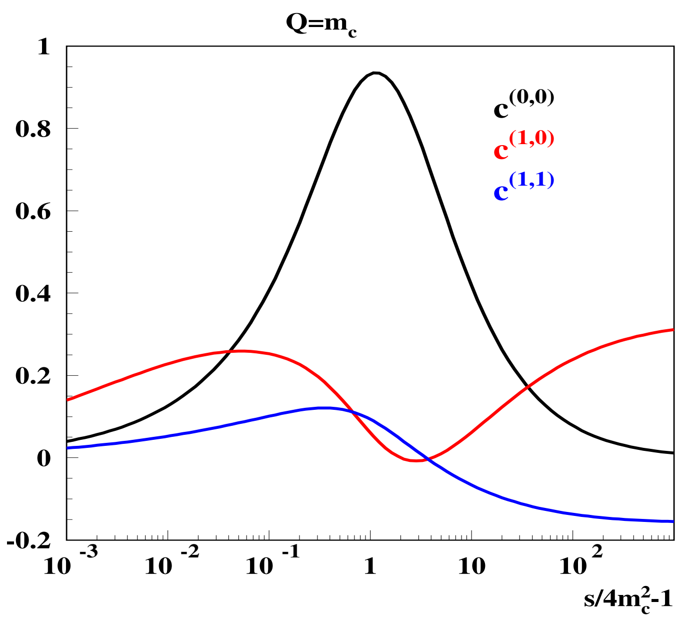
At small x and Q the 3-loop corrections to the massless coefficient functions are smaller that the leading-log term of the small-x resummation result (Catani-Hautmann 94). Nonetheless they are quite important numerically for the realistic H1 kinematics.

The corrections to the heavy-quark production

$$C_{2,g}^{LO} = c^{(0,0)} \qquad O(\alpha_s)$$

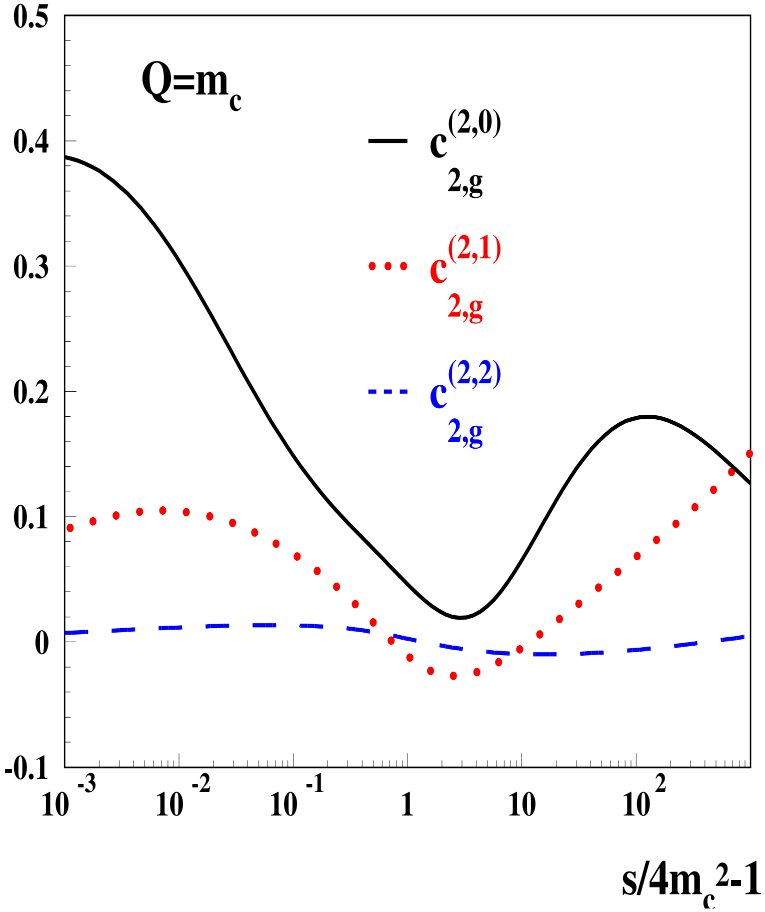
$$C_{2,g}^{NLO} = c^{(1,0)} + c^{(1,1)} \ln(\mu_F^2/m_c^2) \qquad O(\alpha_s^2)$$

$$C_{2,g}^{NNLO} = c^{(2,0)} + c^{(2,1)} \ln(\mu_F^2/m_c^2) + c^{(2,2)} \ln^2(\mu_F^2/m_c^2) \qquad O(\alpha_s^3)$$

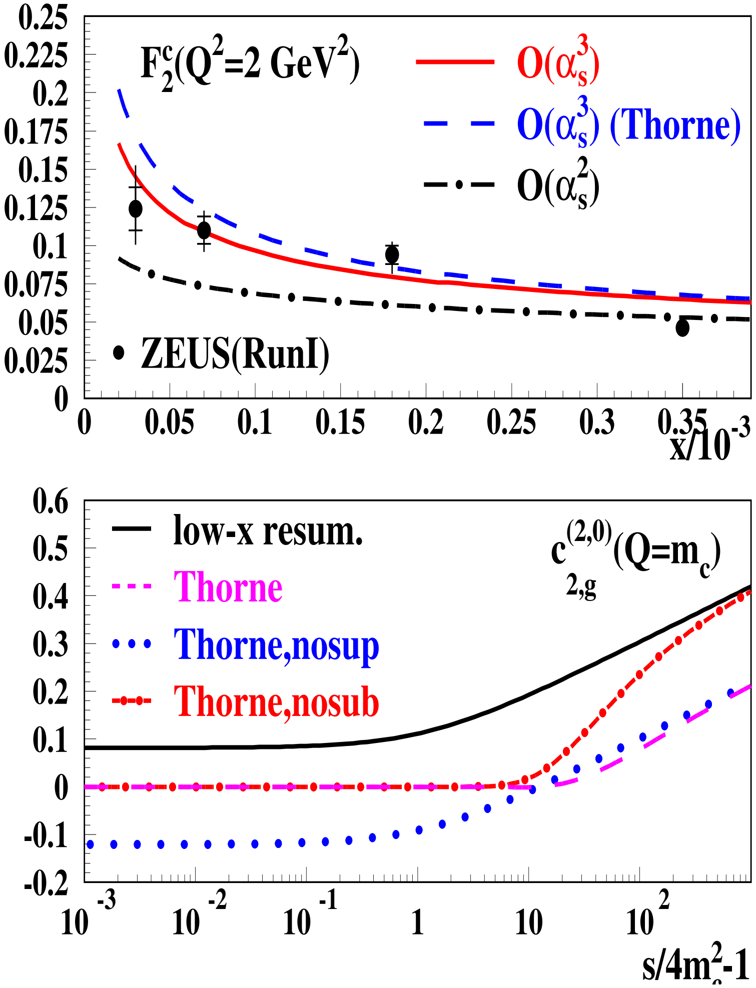


The FFNS corrections up to the NLO are know exactly in the (Witten 76, Laenen *et al* 93), for the NNLO only partial results are available (the threshold soft-gluon resummation term and the limited set of moments at $Q \gg m_c$).

The threshold resummation

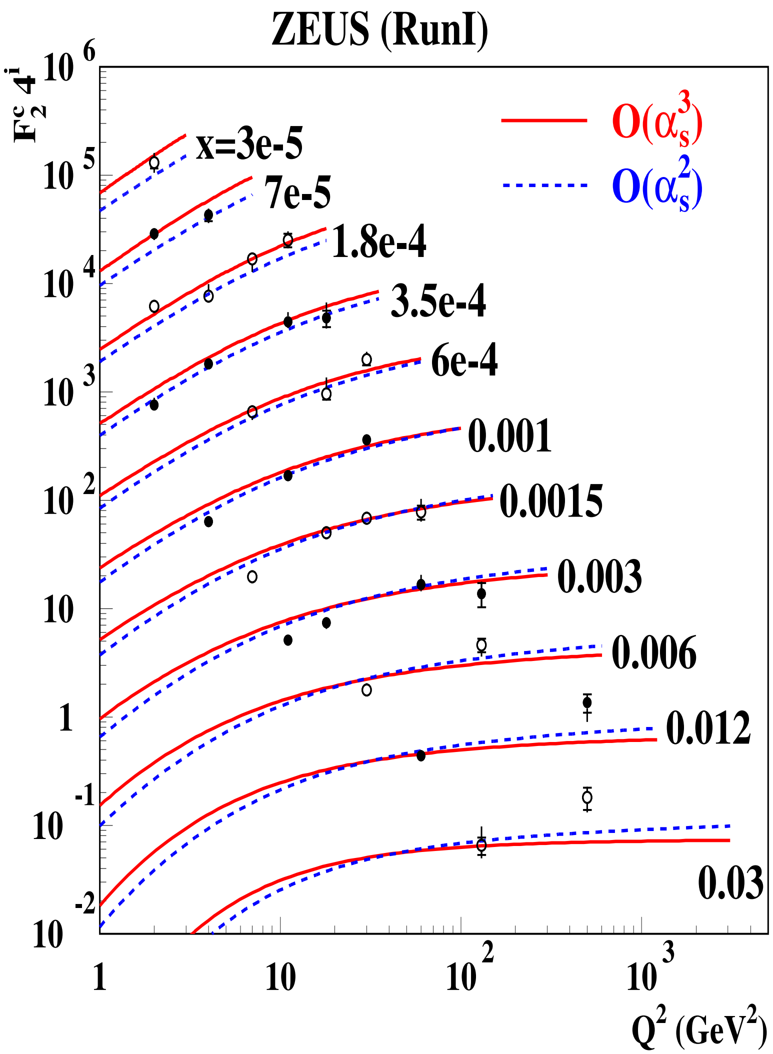


- The coefficients $c_{2,g}^{(2,1)}$ and $c_{2,g}^{(2,2)}$ are known exactly.
- At small heavy-quark velocity β the coefficient $c_{2,g}^{(2,0)}$ is calculated up to the term of $O(\beta)$ (Laenen-Moch 99, sa-Moch 08). At $\beta \gg 1$ the expansion over β is out of control and $c_{2,g}^{(2,0)}$ must be set to 0. Due to gluon distribution spikes at small x the small-β contribution is most important at small x and Q^2.



- The threshold resummation contribution is quite significant at small x and improves agreement with the data.
- The coefficient $c_{2,g}^{(2,0)}$ at large β was modeled by Thorne using the small-x resummation results (Catani-Hautmann 94), however uncertainty in the model is quite big.

$$c^{(2,0)}_{2,g,\mathrm{Thorne}}(\beta,Q^2) = \frac{3}{(2\pi)^3}\,\beta\,\left(\ln(z/x) - 4\right)\,(1 - ax/z)^{20}\,\frac{\kappa_2(Q^2)}{Q^2}$$



- At large Q^2 the FFNS with a partial account of the NNLO corrections undershoots the data, might be due to negative contribution from $c_{2,g}^{(2,1)}$.
- The first moments of $c_{2,g}^{(2,0)}$ calculated for $Q \gg m_c$ (Blümlein *et al* 09) can be matched with the threshold resummation results in order to improve agreement at large Q^2 (work in progress).

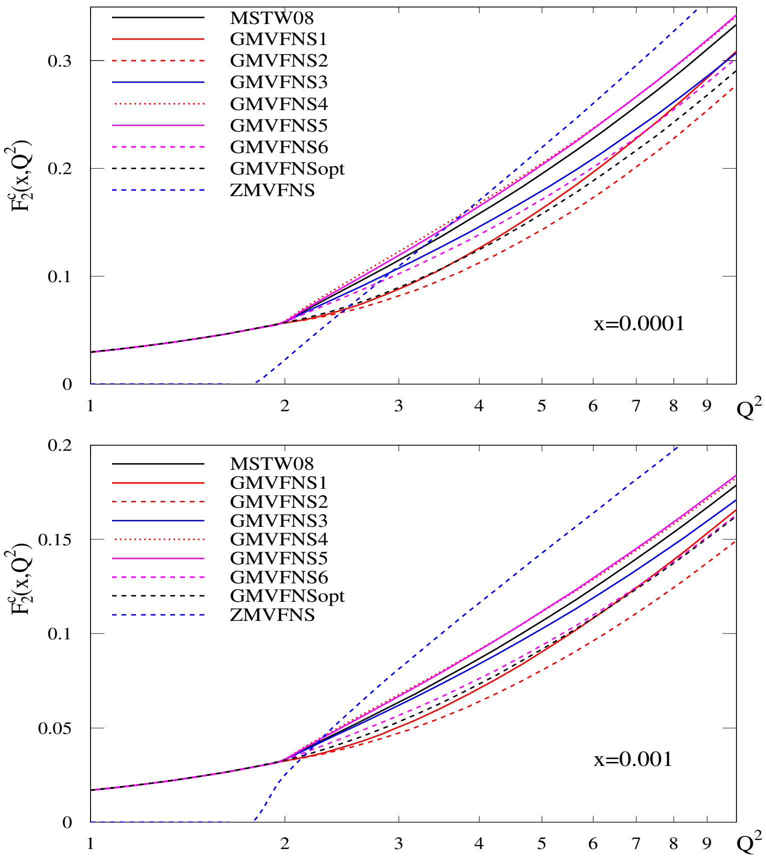
The VFNS

+ In the VFNS the big logs $\sim \alpha_{\mathrm{s}}^m \ln^n(Q/m_c)$ are resummed in the QCD evolution of the massless heavy quarks, this improves the fixed-order FFNS results.

+ Convolution of the massless coefficient functions with the heavy-quark PDFs is quite simple, it gives $F_{2,c}$ in the zero-mass variable-flavor-number scheme, $F_{2,c}^{\mathrm{ZMVFNS}}$.

– A complete definition of the VFNS should include a matching between $F_{2,c}^{\mathrm{FFNS}}$ at small Q^2 and $F_{2,c}^{\mathrm{ZMVFNS}}$ at large Q^2. This matching cannot be derived from the first principles and has to be modeled. Number of VFNS variants were suggested in last years (ACOT, Thorne-Roberts, Thorne, ACOT(χ), and modifications of these) for the use in global PDFs fits including the DIS data.

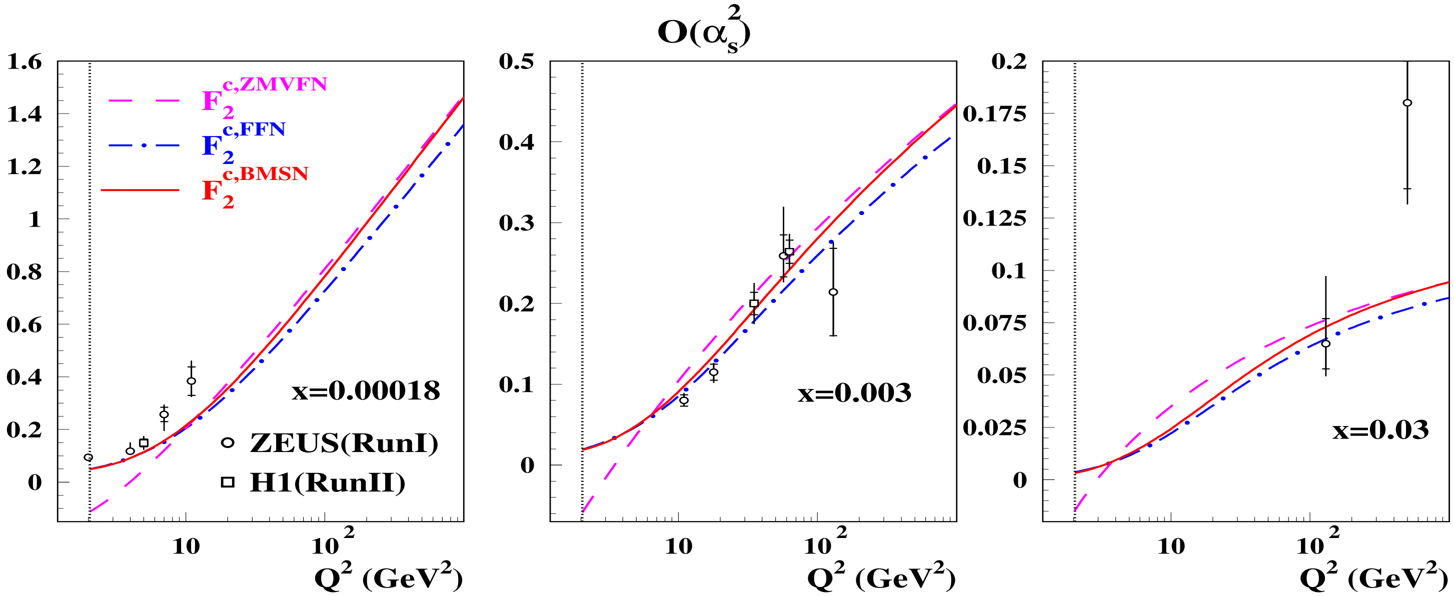
The VFNS modeling



(Thorne 10)

+ The VFNS behavior at small Q is defined by 4 parameters. Such flexibility is an advantage of the model, which can easily provide a smooth transition from the FFNS at small Q to ZMVFNS at large Q.

– It is not clear what is the corresponding factorization scheme employed (evidently not commonly used $\overline{\mathrm{MS}}$).



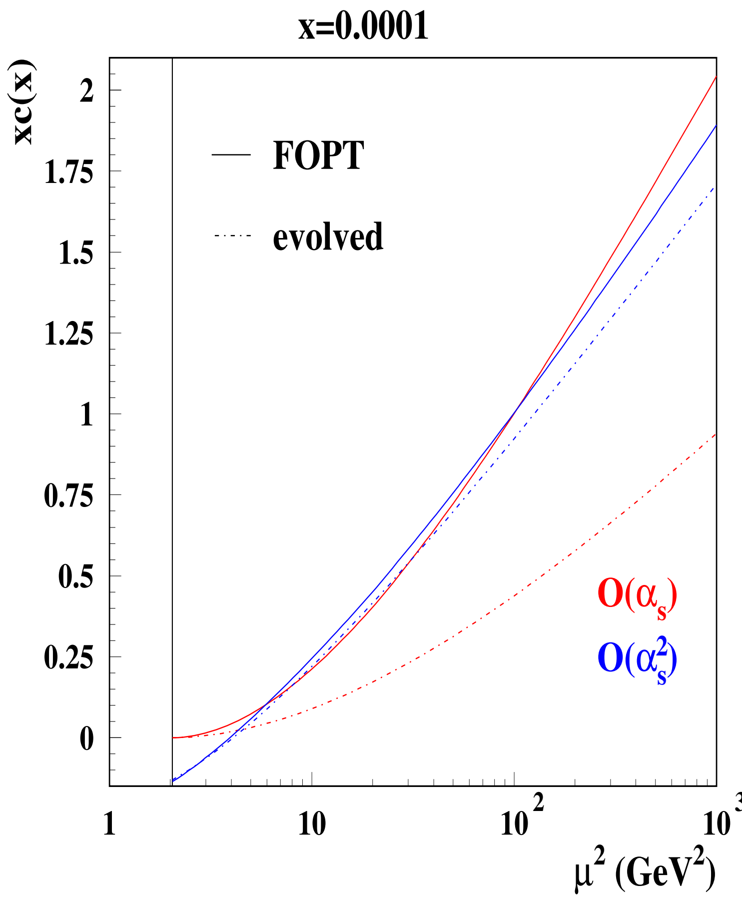
In the $O(\alpha_s)^2$ *the VFNS cannot improve the agreement with the NC DIS data, if the smooth matching with FFNS at small Q is provided (i.e. BMSN prescription or FONLL, reincarnation of BMSN)*

$$F_{2,c}^{\mathrm{BMSN}} = F_{2,c}^{\mathrm{FFNS}}(N_f = 3) + F_{2,c}^{\mathrm{ZMVFNS}}(N_f = 4) - F_{2,c}^{\mathrm{ASYMP}}(N_f = 3)$$

(Buza *et al* 96),(Forte *et al* 10)

$$A_{H,i}^{(1)} = a_{H,i}^{(1,1)} \ln(\mu^2/m_H^2) \qquad\qquad O(\alpha_s)$$

$$A_{H,i}^{(2)} = a_{H,i}^{(2,0)} + a_{H,i}^{(2,1)} \ln(\mu^2/m_H^2) + a_{H,i}^{(2,2)} \ln^2(\mu^2/m_H^2) \qquad O(\alpha_s^2)$$

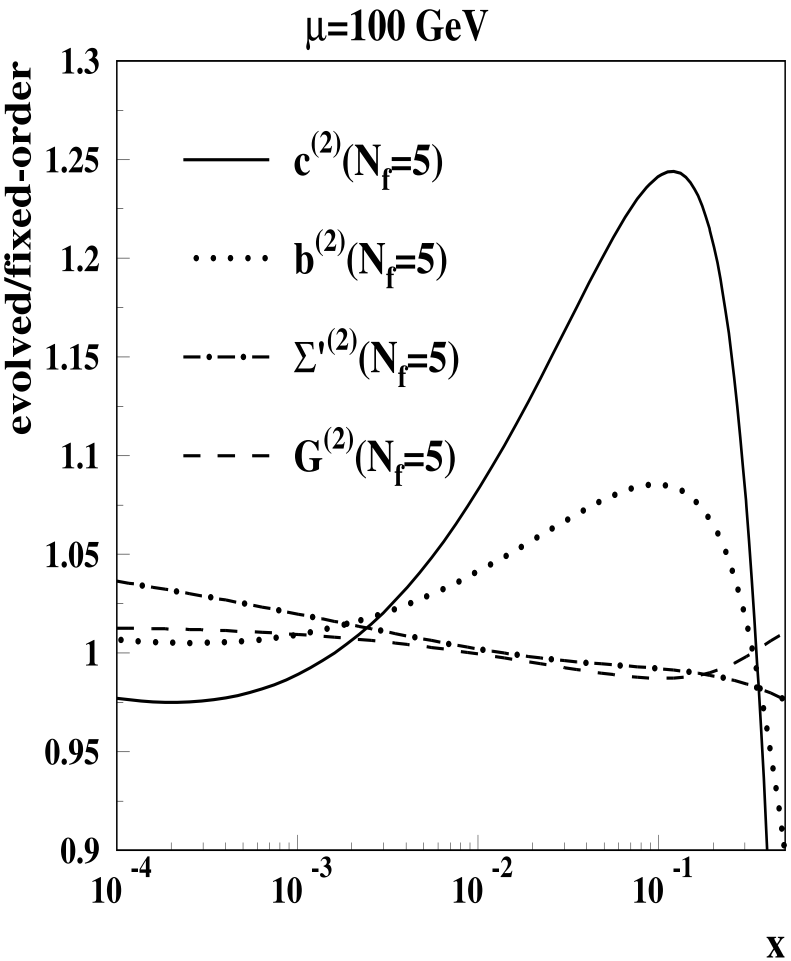


At $O(\alpha_\mathrm{s})$ the difference between the evolved and fixed-order-perturbative theory (FOPT) PDFs is sizable due to resummation of the big logs in the case of evolution. In the $O(\alpha_\mathrm{s}^2)$ these big-logs appear in the coefficient functions, the difference is greatly reduced.

(Glück-Reya-Stratmann 94)

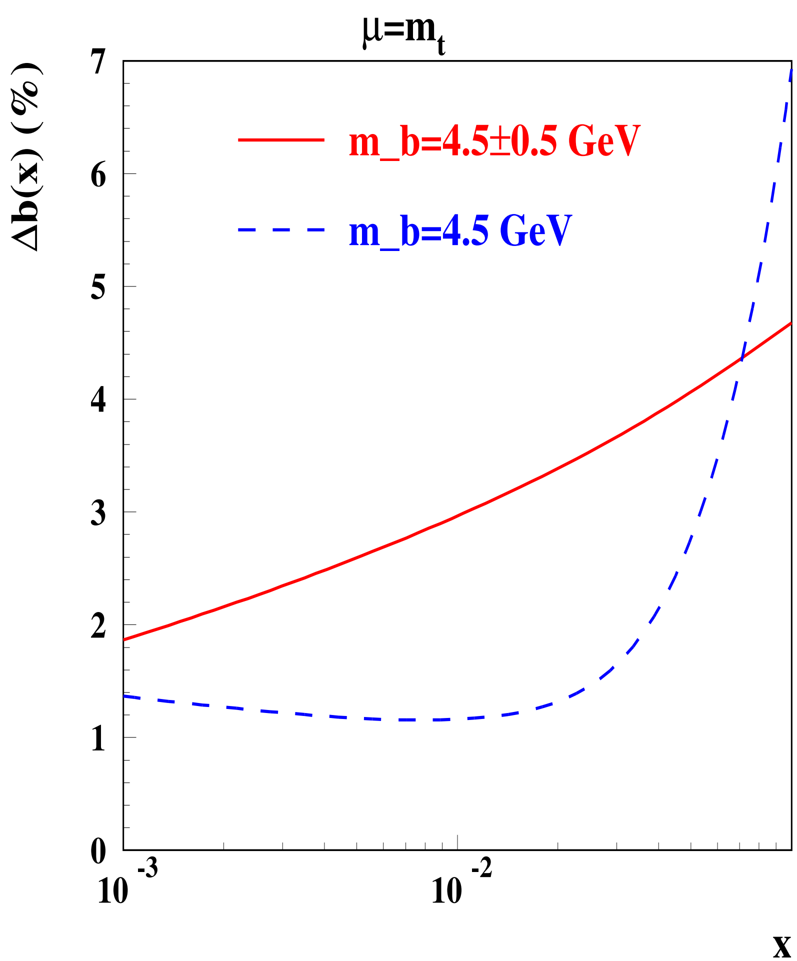
In the $O(\alpha_\mathrm{s}^3)$ it should be even smaller.

The PDFs in the global fit



- For the processes with a really big factorization scale (Higgs, top-quark, ...) one has to employ the 4- or 5-flavor PDFs. At this point we have to admit that the PDFs cannot be fully universal.
- The difference between the evolved and generated PDFs gives an estimate of theoretical uncertainty due to the missed high orders.

The uncertainties due the quark masses



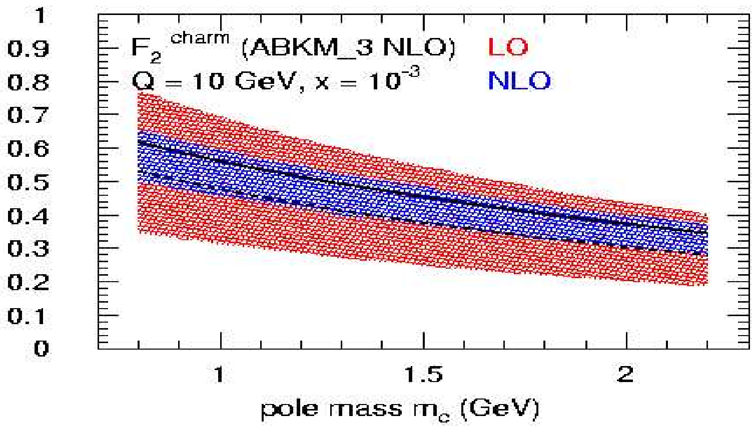
- In the $O(\alpha_s^2)$ at large scales μ_F the heavy-quark distribution $h \sim \ln^2(\mu_F/m_h)$ and the uncertainty in h due to the heavy-quark mass variation is

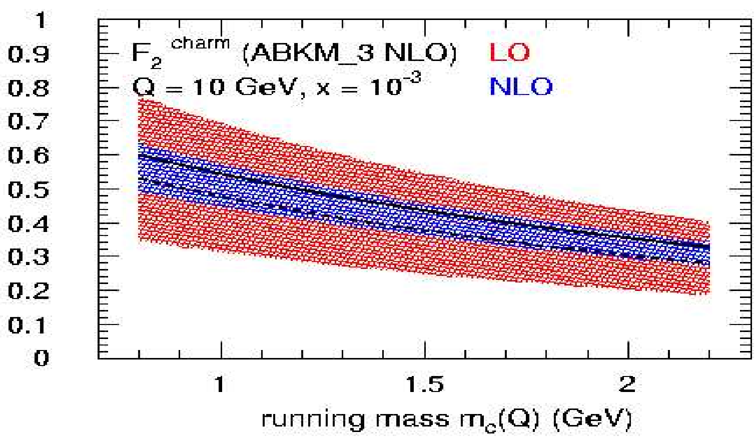
$$(\Delta h/h)_M \sim \frac{\Delta m_h/m_h}{\ln(\mu_F/m_h)}$$

- The uncertainties due to the quark masses are sizable for the W/Z c.s.

(Cooper-Sarkar 10)

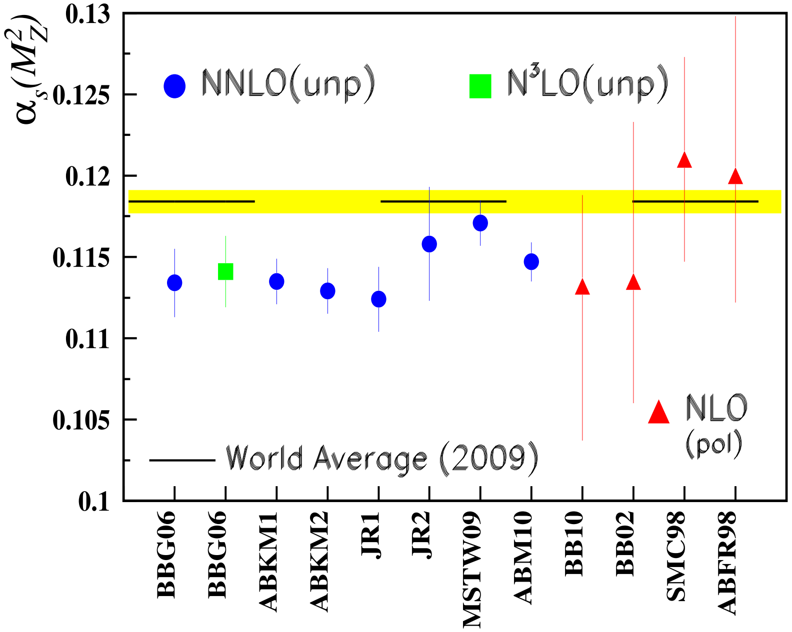
Definition of the quark mass





- The pole mass is usually used in the calculation, however this not an optimal choice due to the big radiative corrections (Bigi *et al* 94).
- The $\overline{\mathrm{MS}}$ mass definition $m(m_h)$ provides better perturbative stability for the t-quark production (Moch-Uwer 09)
- For the heavy-quark electroproduction perturbative stability is also improved for the $\overline{\mathrm{MS}}$ definition (sa-Moch in progress)

α_s from DIS



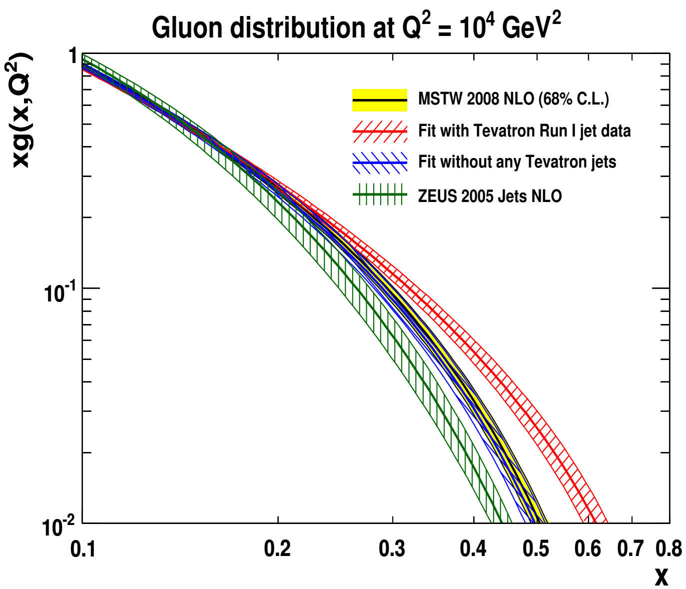
- HERAPDF (prel.): 0.1145 (NNLO) and 0.1166 (NLO)
- ABKM (upd.): 0.1147 ± 0.0012 (NNLO)
- MSTW (prel.): 0.1178 (NNLO) and 0.1215 (NLO)

The inclusive DIS data prefer the small value of α_s in contrast with e^+e^- data. However the NNLO analysis of the global data on the trust distribution with power corrections taken into account gives

$$\alpha_\mathrm{s}(M_Z) = 0.1135 \pm (0.0002)_{\mathrm{exp.}} \pm (0.0005)_{\mathrm{hadr.}} \pm (0.0009)_{\mathrm{pert.}}$$

(Abbate *et al* 10,Weinzierl 08,Gehrman *et al* 07)

α_s from jets



The Run I Tevatron jet data were well above the DIS prediction. This tension lead to the enhanced gluon distribution at large x and bigger value of α_s. With the Run II D0 data

$$\alpha_s^{NLO}(M_Z) = 0.1161^{+0.0041}_{-0.0048}$$

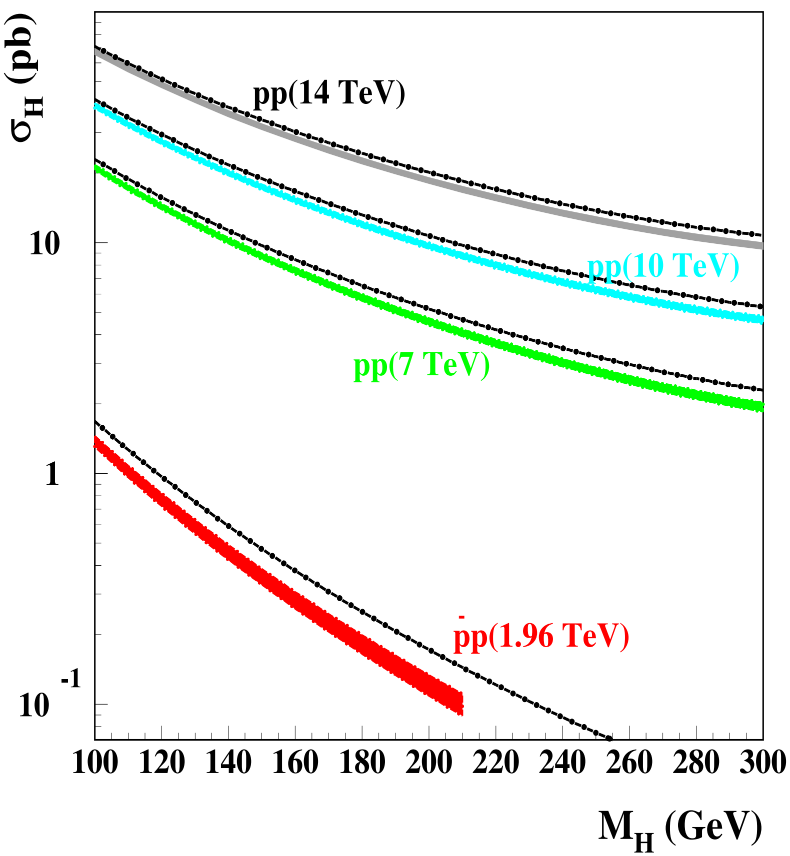
For the *ep* jet data

$$\alpha_s^{NLO}(M_Z) = 0.1161 \pm 0.0007(\text{exp.})^{+0.0041}_{-0.0048}(\text{th.}) \qquad (\text{H1 10})$$

$$\alpha_s^{NLO}(M_Z) = 0.1208^{+0.0037}_{-0.0032}(\text{exp.}) \pm 0.0022(\text{th.}) \qquad (\text{ZEUS 10})$$

The theoretical uncertainties are mainly due to the missed NNLO terms (*power corrections?*).

The NNLO Higgs production rates



- The Higgs NNLO rate predictions are sensitive to the value of α_s and the gluon distributions. The calculations based on the ABKM09 and MSTW09 PDFs are significantly different for the Tevatron case (the same for the $t\bar{t}$ production at LHC).
- Taking this difference as an uncertainty, some 40%, one has to release the constraint on the Higgs mass obtained on Tevatron

(Baglio-Djouadi 10)

Summary of the QCD corrections for DIS

In general the theory accuracy is worse than the experimental one.

Unpolarized

- Massless evolution: NNLO and partial N^3LO
- Massless SFs: N^3LO
- Heavy-flavour SFs: NLO and partial NNLO (*improvements in the NNLO term and the mass definition are foreseen*).
- Jets: NLO (*slow progress*).

Polarized

- Massless evolution: NLO and partial N^3LO
- Massless SFs: NLO
- Heavy-flavour SFs: LO and partial NLO