**uB in XFEL**

one stage, rigid beam approximation
shot noise and rms current fluctuation
induced energy spread

→ BC2 exit

better models
LGM
1d particle model
3d periodic

→ BC0 exit

XFEL with 1d particle model

→ BC1 exit

XFEL with 3d periodic
comparison with non-periodic
increased initial energy spread (gaussian)
increased initial energy spread (LH)

summary/conclusion
beam current and wave number before compression

one stage, rigid beam approximation

\[ G = \left( 1 - i \frac{C r_{56}}{\varepsilon_{\text{ref}}/e I_1 k_1 Z} \right) \exp \left( -\frac{(C k_1 r_{56} \sigma_0)^2}{2} \right) \]

round beam space charge impedance:

\[ Z = \int_{S_1}^{S_2} Z'(\sigma_r(S), \gamma(S))dS \]

"effective" beam size:

\[ \sigma_r(Z) = \left( \varepsilon_x(Z) \beta_x(Z) \varepsilon_y(Z) \beta_y(Z) \right)^{1/4} \]

multi stage gain (pre LGM!)

\[ G = G_1 G_2 G_3 \]

for large one stage gains
energy profile and optic (normalized emittance = 0.2 um)
integrated impedance per length

1m to start LH
end LH to start DOGLEG
end DOGLEG to start BC0
end BC0 to start BC1
end BC1 to start BC2
\( \sigma_{E_1} = 450 \text{ eV} \)

\[ G_1 = \text{one stage gain after BC0} \]

\( G_2 = \text{one stage gain after BC1} \)

\( G_3 = \text{one stage gain after BC2} \)

\[ \max \{G\} > 10^4 \]
\[ \sigma_{E1} = 450 \text{ eV} \]
\[ \rightarrow \sigma_{E4} = C_{tot} \sigma_{E1} = 155 \text{ keV} \]

\[ \sigma_{E1} = 850 \text{ eV} \]
\[ \rightarrow \sigma_{E4} = C_{tot} \sigma_{E1} = 285 \text{ keV} \]
shot noise and rms current fluctuation

\[ I_1 \quad \text{initial coasting beam} \]

\[ I_{\text{rms, in}} = \sqrt{\frac{eI_1}{\pi} |\omega_1 - \omega_2|} \]

white shot noise $\rightarrow$

rms current fluctuation of initial coasting beam in frequency range between $\omega_1$ and $\omega_2$

\[ \rightarrow I_{\text{rms, out}} = C \sqrt{\frac{eI_1}{\pi}} \int_{\omega_1}^{\omega_2} |G(\omega)|^2 \, d\omega \]

linear amplified noise (from the initial frequency range, transformed to $C\omega_1$ and $C\omega_2$)

this does not consider the shot noise of the compressed beam nor shot noise after intermediate stages

assumption: shot noise of later stages is negligible compared to amplified initial noise

c. b. \hline
\begin{array}{cccc}
I_1 & I_2 = C_1 I_1 & I_3 & I_4 \\
\begin{array}{c}
\begin{array}{c}
\text{white shot noise} \\
\text{particle start times are independent}
\end{array}
\end{array} & \begin{array}{c}
\begin{array}{c}
\text{amplified } I_1 \text{ shot noise} \\
\text{other noise}
\end{array}
\end{array} & \begin{array}{c}
\begin{array}{c}
\text{amplified } I_1 \text{ shot noise} \\
\text{amplified other noise + further noise}
\end{array}
\end{array} & \begin{array}{c}
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\text{amplified other + further noise}
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\end{array}
\end{array}

... correlated

... correlated
induced energy spread: linear regime + rigid beam

effect of impedance after compressor, range from $S_1$ to $S_2$

\[ C \left( S_1 \leq S < S_2 \right) = C \]

\[ G(\omega, S_1 \leq S < S_2) \approx G(\omega) \quad Z(\omega) = \int_{S_1}^{S_2} Z'(\omega, S) dS \]

\[ \Delta E_{rms} \approx eC \sqrt{\frac{e^2 l_1}{\pi}} \int |Z(\omega) G(\omega)|^2 d\omega \approx eI_{rms, out} |Z_{eff}| \]
linear regime

\[ \frac{I_{\text{rms,out}}}{CI_1} \ll 1 \rightarrow e^{\int_{\omega}^{\omega_2} |G(\omega)|^2 d\omega} \ll I_1 \]

\[ I_1 = 5.8 \text{ A} \]

multiplicative one stage rigid beam approximation:

\[ \sigma_{E0} = 450 \text{ eV} \]
\[ \downarrow \]
\[ \frac{e^{\int_{\omega}^{\omega_2} |G_{1\rightarrow2}(\omega)|^2 d\omega}}{\pi^{\omega_2}} \approx 4.3 \text{ A} \]

\[ \sigma_{E0} = 830 \text{ eV} \]
\[ \downarrow \]
\[ \frac{e^{\int_{\omega}^{\omega_2} |G_{1\rightarrow2}(\omega)|^2 d\omega}}{\pi^{\omega_2}} \approx 0.52 \text{ A} \]

\[ \frac{e^{\int_{\omega}^{\omega_2} |G_{1\rightarrow3}(\omega)|^2 d\omega}}{\pi^{\omega_2}} \approx 420 \text{ A} \]

\[ \frac{e^{\int_{\omega}^{\omega_2} |G_{1\rightarrow3}(\omega)|^2 d\omega}}{\pi^{\omega_2}} \approx 2.3 \text{ kA} \]

\[ \frac{e^{\int_{\omega}^{\omega_2} |G_{1\rightarrow3}(\omega)|^2 d\omega}}{\pi^{\omega_2}} \approx 3.8 \text{ A} \]

no!
better models

1d impedance or wake
- one stage, rigid beam linear gain model
- (2x)1d particle model

3d periodic space charge
- (2x)3d particle tracking

discrete
- linear dynamic
- non-linear dynamic

3d linear optic
example: **LGM vs 1d particles**

![Graph showing effect of plasma oscillations on LGM particles and non-linear effects at other working points](image-url)

**DESY-FLASH**

- start
- before BC2
- after BC2
- before BC3
- after BC3
- end

**LGM particles**

- other working point → non-linear effects
XFEL with 1d particle model

back to Igor’s problem, only 1st stage
but with $r_{56}$ of LH and DOGLEG

\[
\begin{align*}
C_1 &= 3.5 \\
I_1 &= 5.8 \text{ Å} \\
\xi_{562} &= 0.075 \\
\xi_{563} &= 0.053 \\
\xi_{563} &= 0.018 \\
\beta &= 0.45 \text{ keV} \\
\beta_2 &= 1.5 \text{ keV} \\
\beta_3 &= 10 \text{ keV} \\
E_1 &= 830 \text{ MeV} \\
E_2 &= 900 \text{ MeV} \\
E_3 &= 2400 \text{ MeV} \\
\varepsilon_1 &= 0.18 \text{ μm} \\
\varepsilon_2 &= 0.2 \text{ μm}
\end{align*}
\]
**simplified 1d particle model**

**continuous drifts:**

\[
\frac{dz_v}{dS} = \frac{1}{\gamma^2} \delta_v
\]

\[
\frac{d\delta_v}{dS} \sim \text{Re}\{\tilde{I}Z' \exp(-ikz_v)\}
\]

\[
\tilde{I} \sim \sum_v \exp(ikz_v)
\]

**discrete compression stages:**

\[
z_v^{\text{(out)}} = z_v^{\text{(in)}} + r_s \delta_v^{\text{(in)}}
\]

\[
\delta_v^{\text{(out)}} = \delta_v^{\text{(in)}}
\]
after BC0: current and slice energy

uncorrelated energy spread

\[ \sigma_{E_0} = 450 \text{ eV} \]

rms current fluctuation

different qualitative behavior than expected for rigid beam approximation!!!

150 keV was expected after BC2 for 2kA beam!!!
comparison for simulation with two different band widths
same initial particle distribution!

non linear mixing of phase space:
before BC0: similar time signals
after BC0: different time signals but similar rms- and similar frequency-structure

current fluctuation can be decreased
**XFEL with 3d periodic**

**comparison with non-periodic**

March 27, 2019

**beam parameters:**
- initial beam current 5.8A;
- normalized emittance after A1 0.188 \( \mu m \) (in both planes);
- initial energy 6.65 MeV;
- initial beamline coordinate 3.3 m;
- uncorrelated energy spread 450 eV, gaussian;
- bunch is generated by random generator; horizontal, vertical and longitudinal phase spaces are decoupled; the initial beam is round, all transverse density functions are gaussian; twiss parameters \( \alpha_x = \alpha_y \) and \( \beta_x = \beta_y \) are chosen according to a Astra simulation; the emittances are chosen \( \varepsilon_x = \varepsilon_y \) to obtain a normalized emittance of about 0.2 \( \mu m \) after A1;
- the chirp of A1 is chosen for an compression \( C_0 = 3.5 \) after bunch compressor BC0;

**dispersive things and compression:**
- \( r_{56LH}/mm = 4.7 \)
- \( r_{56DOLEG}/mm = 30.8 \)
- \( r_{56BC0}/mm = 54.2 \)
- \( r_{56BC1}/mm = 51.7 \);
- beam current after BC0 is 20.3A;
- beam current after BC1 is 162A;

**resolution:**
- charge of macroparticles is elementary charge;
- longitudinal density is uniform;
- length is 0.2 mm for nonperiodic or 0.1 mm for periodic simulations;

laser heater off !!!
before and after LH

energy [eV]

non-periodic “before”
non-periodic “after”
periodic “after”

current [A]

bunch coordinate [m]
only periodic model:
only periodic model:

<table>
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<th>rms noise (A)</th>
<th>$\sigma_E$ (eV)</th>
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<td>6.0</td>
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</tr>
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<td>20.3</td>
<td>1.05</td>
<td>10350</td>
</tr>
<tr>
<td>before BC1</td>
<td>20.6</td>
<td>0.50</td>
<td>22170</td>
</tr>
<tr>
<td>after BC1</td>
<td>172</td>
<td>18.8</td>
<td>190E3</td>
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periodic model
1d particles, discrete stages (LH, DOGLEG, BC0), BW=1THz

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1d model severely overestimates effects
**XFEL with 3d periodic**

**Gaussian “laser heater”:**

April 9, 2019

**beam parameters:**
all beam parameters are as before with exception of the uncorrelated energy spread;
start distribution with an arbitrary gaussian spread of 450 eV, 1000 eV, 1500 eV ... 6000 eV;

**dispersive things and compression:**
r56 values and compression as before;

**resolution:**
charge of macroparticles is elementary charge;
the resolution to the exit of BC0 is as before (0.1µm);
the longitudinal resolution from BC0 exit to BC1 exit is enhanced to 0.03µm;
initial period length 0.1 mm
the extra point (x) is calculated with a period length of 0.3 mm
expected slice energy spread after compression in BC2 to 5 kA

with an effective of about 14 kOhm (for the wavelength of maximal micro-bunching)
the extra point (x) is calculated with a period length of 0.3 mm
yes, I did simulations to BC2 exit: initial $\sigma_E = 4$ keV (gaussian)

wavelength $\sim 1 \mu$m is determined by period length of simulation !!!

$\sigma_I \approx 33.5$A
* $Z$ (main linac) $\approx 0.5$ MeV

I=2 kA, $\sigma_E = 2.1$MeV
an other case: initial period length might be too short!
XFEL with 3d periodic

real laser heater

April 17, 2019

**beam parameters:**
all beam parameters are as before, initial gaussian spread of 450 eV;

**LH:**
matching to design optic in LH
*different LH working points* → *same rms spread of 7 keV*

**dispersive things and compression:**
$r56$ values and compression as before;

**resolution:**
charge of macroparticles is elementary charge;
the resolution to the exit of BC0 is as before (0.1µm);
the longitudinal resolution from BC0 exit to BC1 exit is enhanced to 0.03µm;
*initial period length 0.3 mm*
working points for “5.8 A beam” → 4 keV
Gaussian (0) and LH working points (1-6)

- **RMS Current Fluctuation [A]**
  - Purple C1=8
  - Gray C1=5.5

- **RMS Energy Spread [eV]**
  - Purple C1=8
  - Gray C1=5.5
summary/conclusion

gain models:

one stage, rigid beam approximation
not applicable before BC0: beam is not rigid
not applicable after BC0: non linear effects

LGM useful in linear domain; needs transverse optic for dispersive sections

simulation of real particle noise:

1d particle model
with non-linear effects
without transverse optic: discrete dispersive sections
to be developed: real dispersive sections with effective optic

full 3d CPU cluster: full bunch simulation; moderate resolution
single PC: reduced bunch length; interference with macro effects

3d periodic
linear optic; very high spatial resolution
no macro effects; limited period length
problem: “non linear phase space mixing”

??? how to generate an effective new distribution
no CSR → implement simple periodic CSR model
summary/conclusion

**XFEL:** earlier investigations did not consider LH and DOGLEG 
µB effects are significantly increased

discrete model for DOGLEG is not appropriate

LH: 450 eV → 5 keV
after BC1: $\sigma_I \approx 10$ A, $I \approx 10$ A, $\sigma_E \approx 100$ keV
after BC2: $\sigma_I \approx 33$ A, $I \approx 2$ kA, $\sigma_E \approx 2.1$ MeV

5 kA $\rightarrow \sigma_E \approx 5$ MeV

need to be investigated: collimator and beam distribution system

3D periodic simulations with increased period length & more random seeds !!!
induced energy spread: linear regime + rigid beam

\[ i(t, S) = \int I_0(\omega)G(\omega, S)\exp(j\omega C(S))d\omega \]

\[ E_z(t, S) = \int Z'(\omega, S)I_0(\omega)G(\omega, S)\exp(j\omega C(S))d\omega \]

effect of impedance after compressor, range from \( S_1 \) to \( S_2 \)

\[ C(S_1 \leq S < S_2) = C \]

\[ G(\omega, S_1 \leq S < S_2) \approx G(\omega) \quad \quad Z(\omega) = \int_{S_1}^{S_2} Z'(\omega, S)dS \]

\[ \Delta E(t) = e\int_{S_1}^{S_2} I_0(\omega)\exp(j\omega C)\int Z'(\omega, S)G(\omega, S)dSd\omega \]

\[ = e\int I_0(\omega)\exp(j\omega C)G(\omega)Z(\omega)d\omega \]

\[ \Delta E_{rms} = \sqrt{\frac{1}{T}\int_0^T(\Delta E(t))^2dt} \approx eC\sqrt{\frac{eI_1}{\pi}}\int|Z(\omega)G(\omega)|^2d\omega \]

\[ \approx eI_{rms, out}|Z_{eff}| \]