1D-FEL Without Approximations

motivation, capabilities

1D theory $\rightarrow$ 1D-solver for waves

implementation (without and with Lorentz transformation)

excitation of waves (single particle)

without self effects

one and few particles with self effects

mystery

Motivation, Capabilities

FEL codes use many approximations, as averaged equation of motion, local periodical approximation, EM-field calculation by paraxial approximation, one or several harmonics

these approximations are questionable for ultra-short bunches or bunches with extreme energy modulation

it is easy to implement complete FEL effects in 1D

1D model can be used to verify approximations

capabilities of 1D model:

- ultra broadband (not split into harmonics, excitation and radiation)
- no local periodic approximation
- complete 1D field computation
- seems possible: particle = macro particle
- LT method can be tested

1D Theory

EM Fields

a) undulator field

b) external wave

c) longitudinal self field $\quad \varepsilon \partial E_z / \partial z = \rho(z,t) \leftarrow \frac{q}{A} \sum_\nu \delta(z - z_\nu)$

no principle problem, but neglected

d) transverse self field $\quad J_x(z,t) \leftarrow \frac{q}{A} \sum_\nu v_{x,\nu} \delta(z - z_\nu)$

$$E_{x,z} = -B_{y,t}$$
$$-B_{y,z} = \mu J_x + c^{-2} E_{x,t}$$

$$\downarrow$$

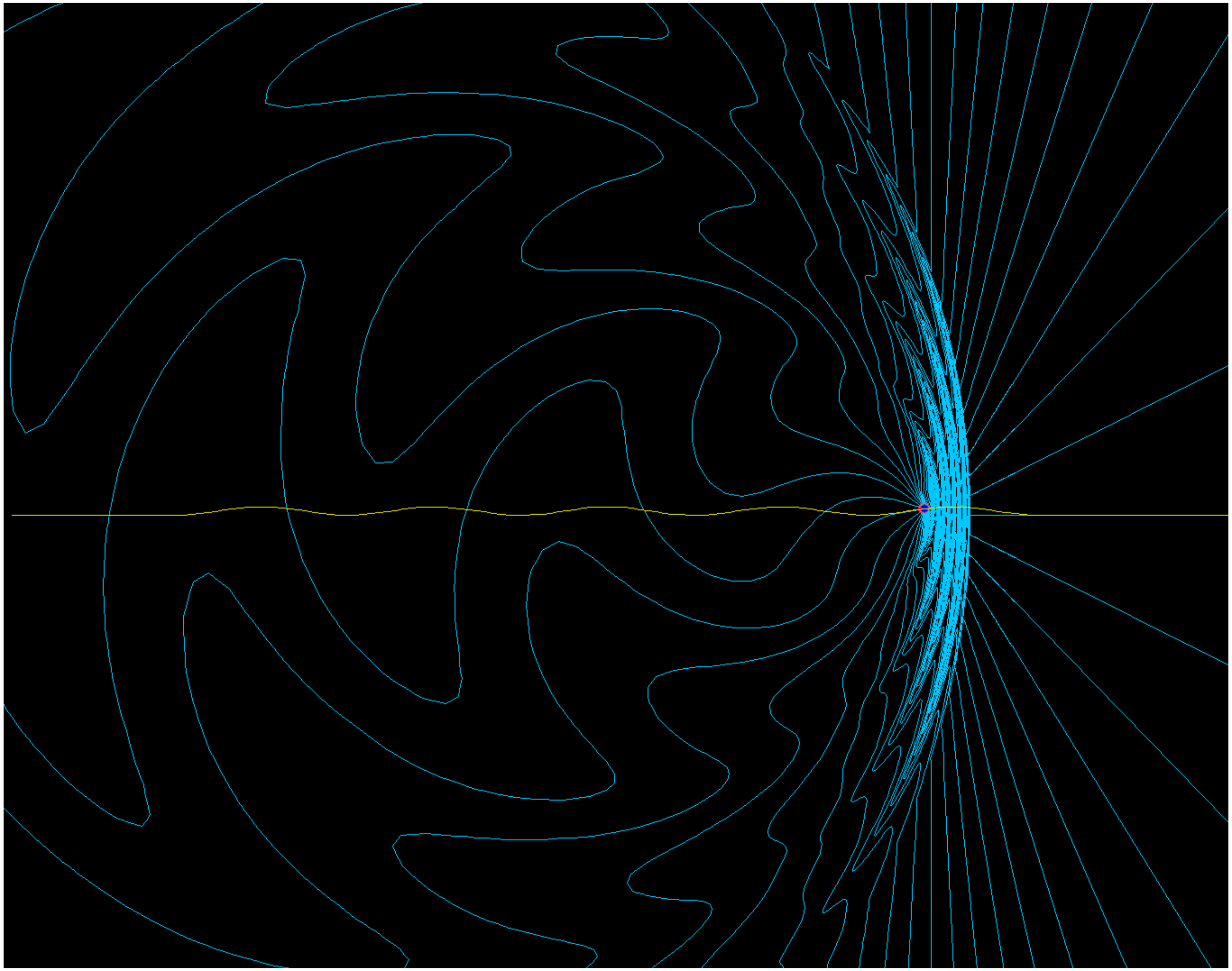
$$E_x = (L + R)/2$$
$$cB_y = (-L + R)/2$$

L and R are waves to the left and to the right

$$\downarrow$$

$$R(U + ct, t) = \tilde{R}(U, t) \quad \text{with} \quad \partial_t \tilde{R} = -\varepsilon^{-1} J(U + ct, t)$$
$$L(V - ct, t) = \tilde{L}(V, t) \qquad\qquad \partial_t \tilde{L} = -\varepsilon^{-1} J(V - ct, t)$$

waves to the left and to the right



Implementation

Equation of Motion

$$\frac{d}{dt}\begin{bmatrix}\mathbf{r}_\nu \\ \mathbf{p}_\nu\end{bmatrix} = \begin{bmatrix}\mathbf{v}_\nu \\ \mathbf{f}_\nu\end{bmatrix}$$

with

$$\mathbf{v}_\nu = \frac{\mathbf{p}_\nu}{m_0\gamma} = c\frac{\mathbf{p}_\nu}{\sqrt{p_\nu^2 + (m_0 c)^2}}$$

$$\mathbf{f}_\nu = q_0\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

horizontal plane, only x and z components:

$$\begin{pmatrix}F_x \\ F_z\end{pmatrix}_\nu = \underbrace{q_0\begin{pmatrix}E_{x,\mathrm{e}} - v_z B_{y,\mathrm{e}} \\ v_x B_{y,\mathrm{e}}\end{pmatrix}_\nu}_{\text{external}} + \underbrace{\frac{q_0}{2}\begin{pmatrix}\tilde{R} + \tilde{L} - \beta_z\left(\tilde{R} - \tilde{L}\right) \\ \beta_x\left(\tilde{R} - \tilde{L}\right)\end{pmatrix}_\nu}_{\text{self}}$$

Coupled Problem

$$J_x(z,t) \leftarrow \frac{q}{A}\sum_\nu v_{x,\nu}\,\delta(z - z_\nu)$$

← binning and smoothing (on equi. mesh) needs spatial resolution

$$\frac{d}{dt}\begin{bmatrix}\mathbf{r}_\nu \\ \mathbf{p}_\nu \\ \tilde{L}(V,t) \\ \tilde{R}(U,t)\end{bmatrix} = \begin{bmatrix}\mathbf{v}_\nu \\ \mathbf{f}_\nu \\ -\varepsilon^{-1}J(V - ct, t) \\ -\varepsilon^{-1}J(U + ct, t)\end{bmatrix}$$

← PDE solver (f.i. RK4) needs time resolution

without Lorentz transformation

integration of PDE (by rk4) needs small time step for left wave

criterion: slip between source and wave < dz/c

$\rightarrow$ dt <≈ dz/c, number of time steps ~ undulator length / photon wavelength !!!

solution a: neglect L, $\max(L) <\approx \dfrac{\max(R)}{4\gamma^2}$

solution b: the part of the left wave, seen by the bunch, is determined by near interaction; use $J_x(z,t) \approx J_x(z+\bar{v}\tau, t+\tau)$

solution c:
with Lorentz transformation

differenced between length scales are shrunk

huge external fields (from undulator) $\begin{pmatrix} E_{x,u}' \\ B_{y,u}' \end{pmatrix} = \gamma_{\mathrm{LT}} \begin{pmatrix} -c\beta_{\mathrm{LT}} \\ 1 \end{pmatrix} B_{y,u}$

same magnitude of left and right wave

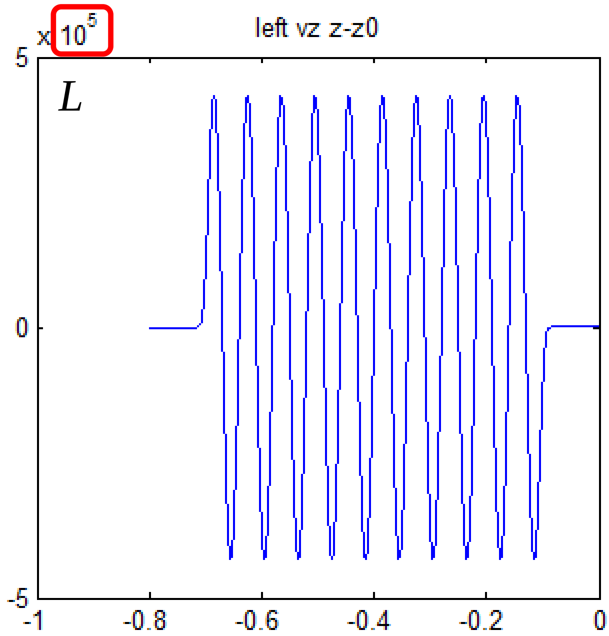
it is possible, it is applicable even in 3D!

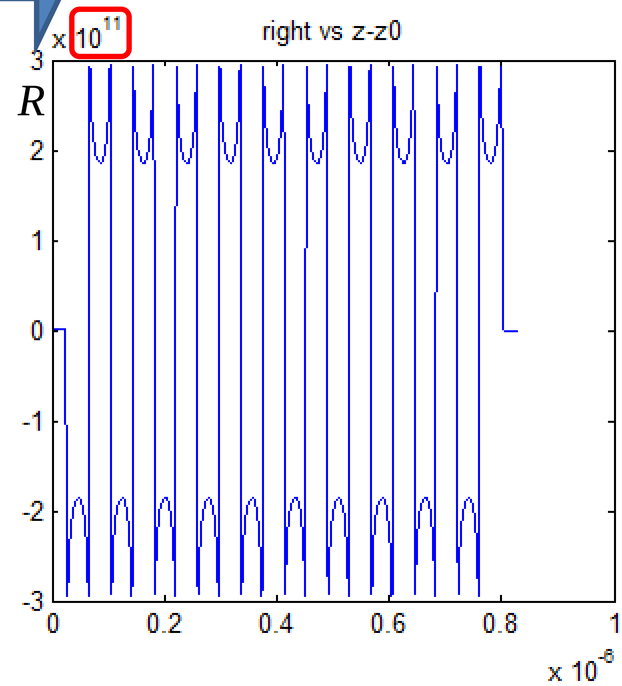
Excitation of Waves (single particle)

left and right wave

weak backward wave

intense fwd. wave !



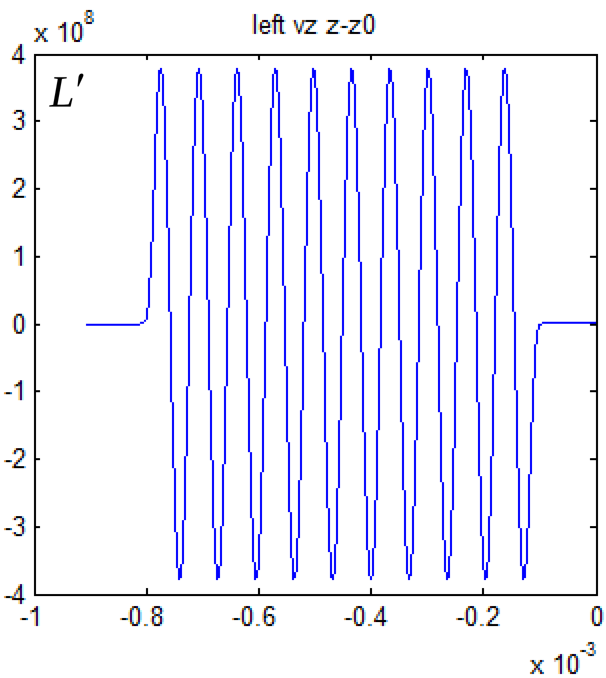


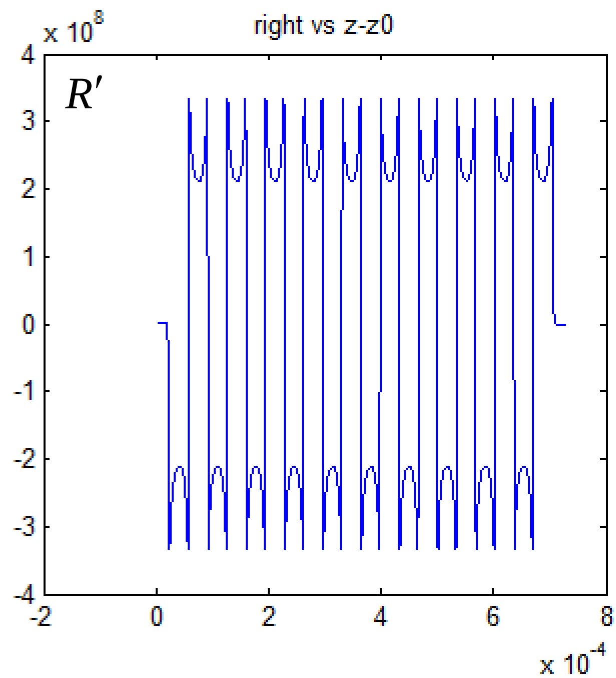
$\lambda_w = 6\,\mathrm{cm}$

$\lambda_w = 77\,\mathrm{nm}, 3\times 77\,\mathrm{nm}, 5\times 77\,\mathrm{nm} \cdots$

left and right wave **with Lorentz transformation**

less power in fwd. direction !





$$\lambda_w = 0.67 \text{ mm}$$

higher harmonics:

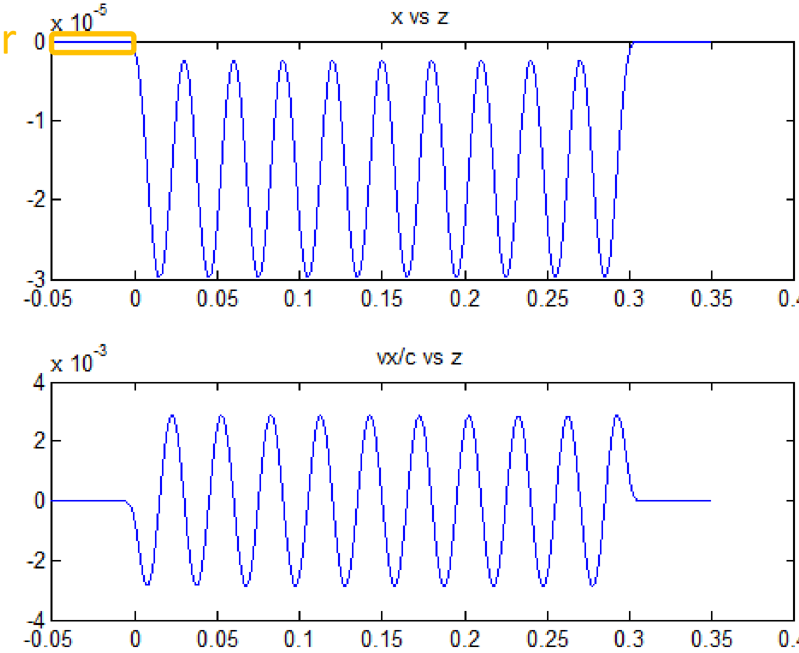
$$m = 2n + 1$$

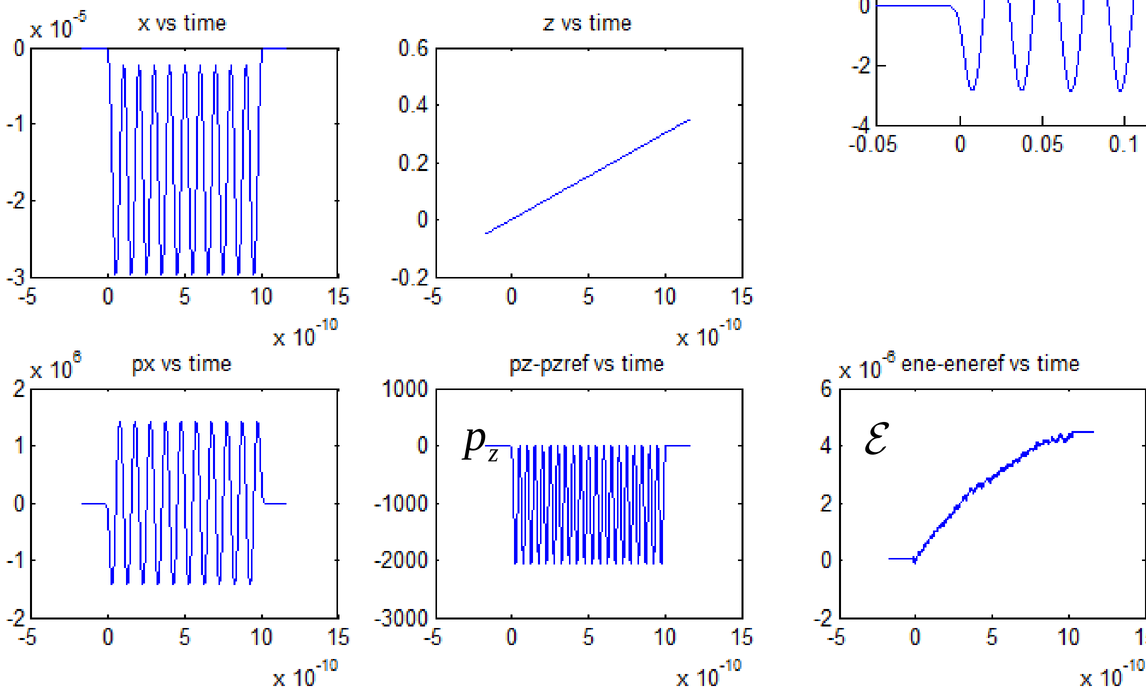
$$JJ = J_n\left(\frac{mK^2}{4 + 2K^2}\right) - J_{n+1}\left(\frac{mK^2}{4 + 2K^2}\right)$$

Example: Without Self Effects

B_u = 1T
λ_u = 3 cm
$\mathcal{E}$ = 500 MeV
N_u = 10
↓
K = 2.80
λ_w = 77 nm

before undulator



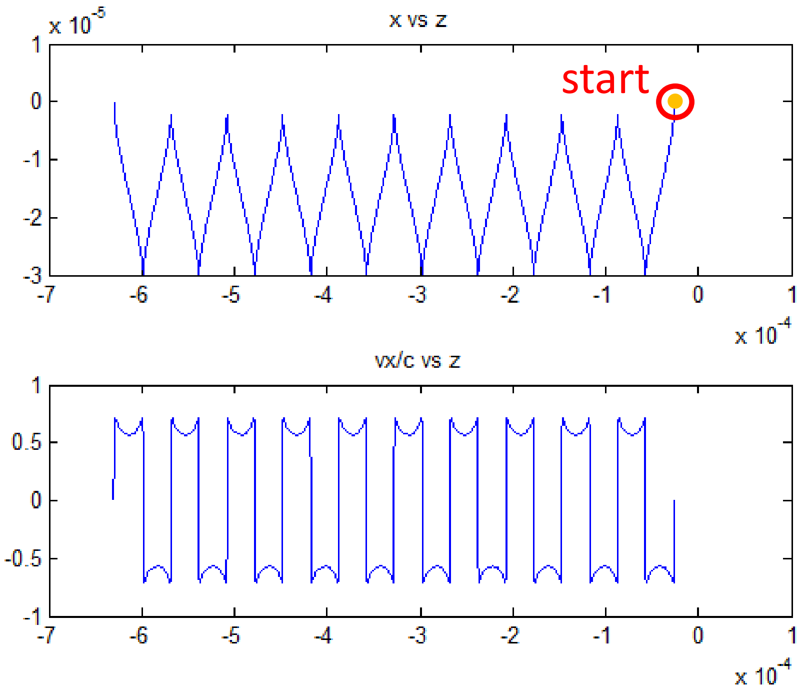


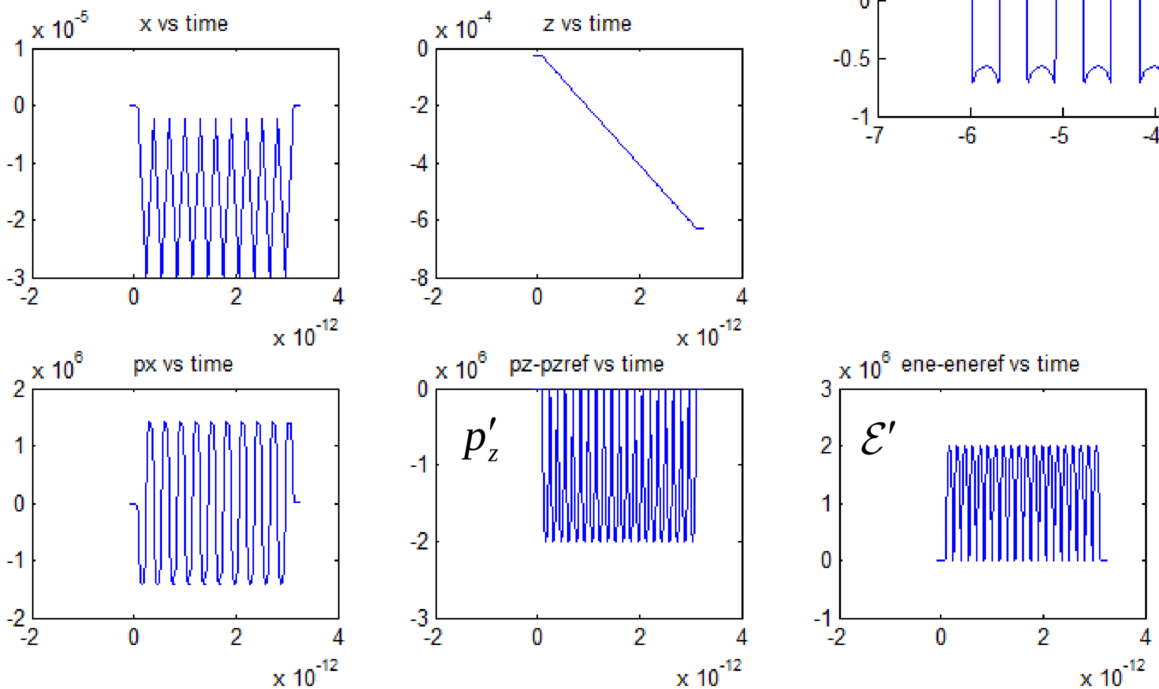
same parameters,

with Lorentz transformation

frame = initial velocity

$\mathcal{E}/\mathcal{E}_0 = 978 = \gamma_{LT}$



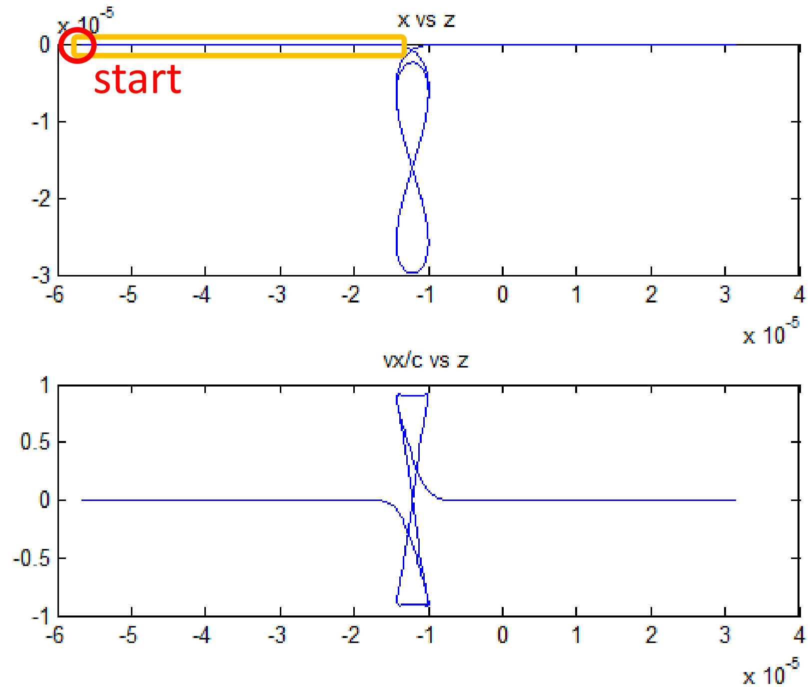


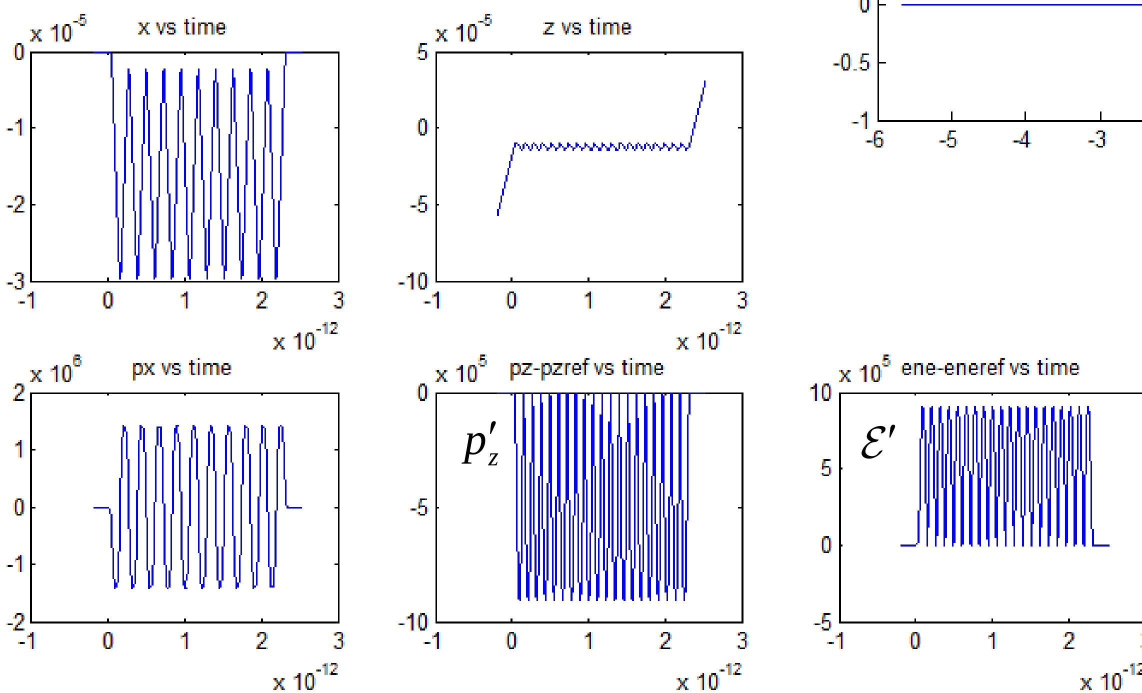
same parameters,

with Lorentz transformation

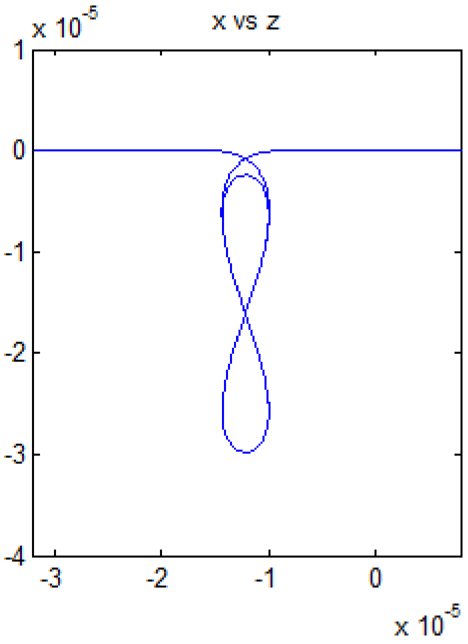
frame = av. velocity

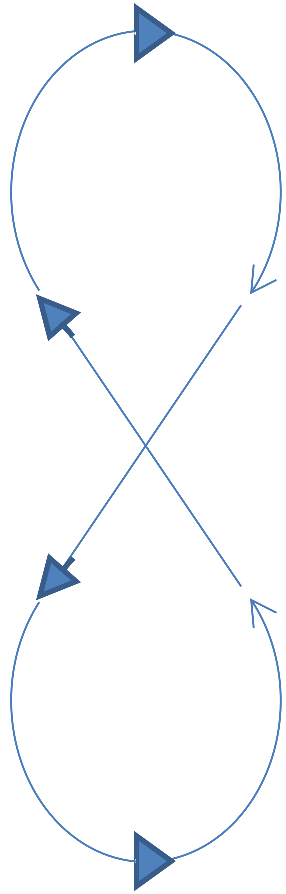
$\gamma_{av} = 441 = \gamma_{LT}$

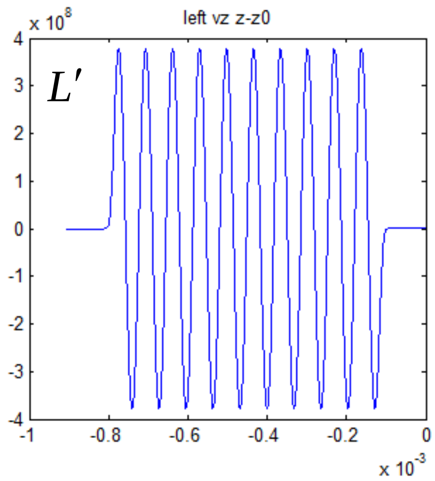


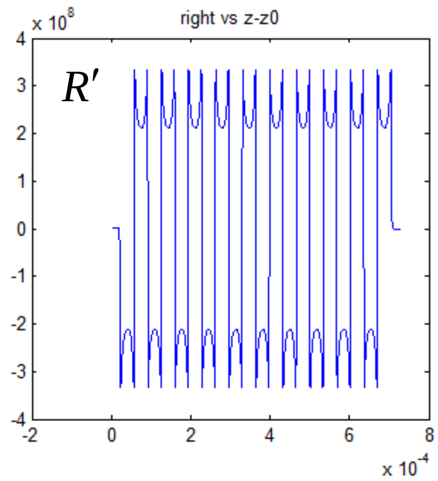


Why are left and right waves asymmetric?







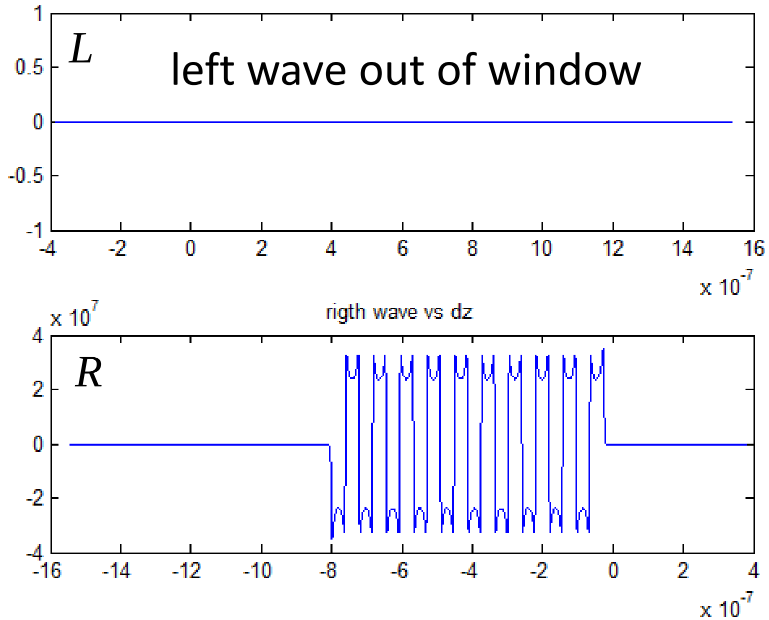


One and Few Particles with Self effects

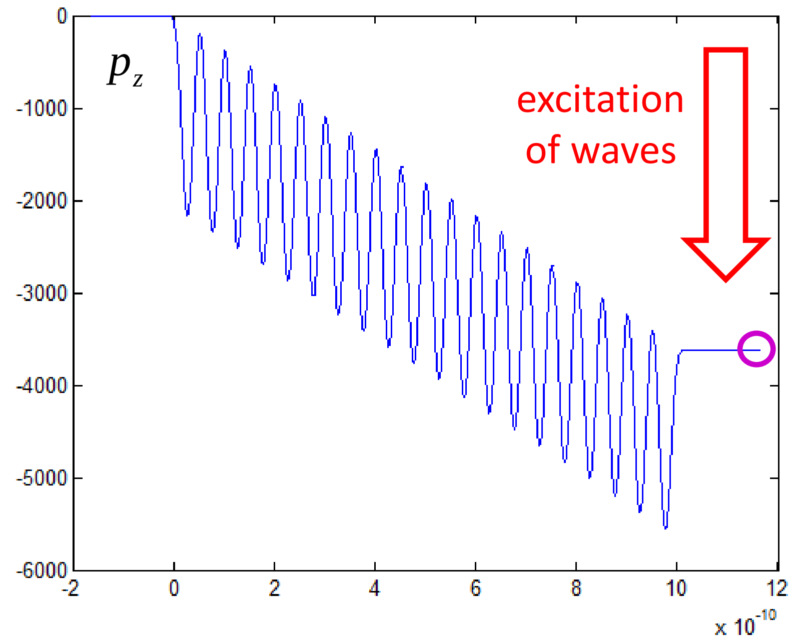
parameters as before

- ○ particle **after** undulator

left and right waves



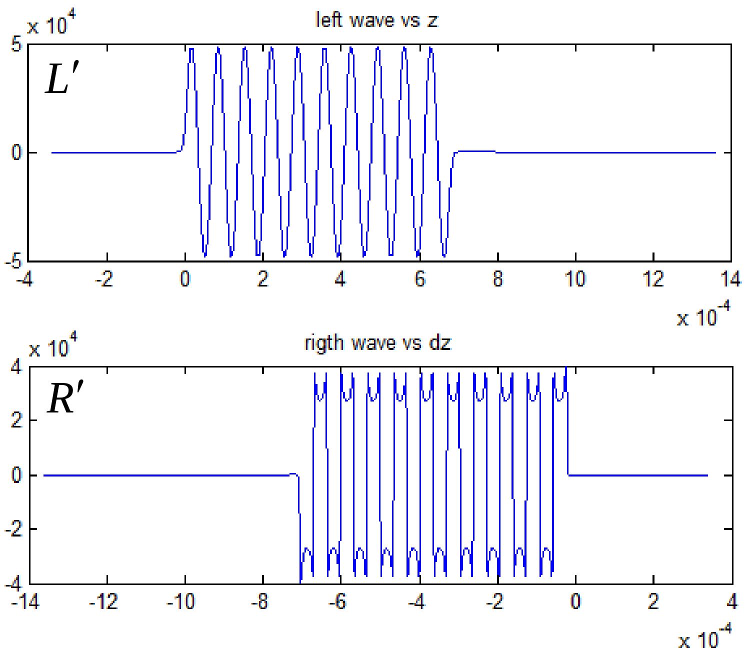
longitudinal momentum

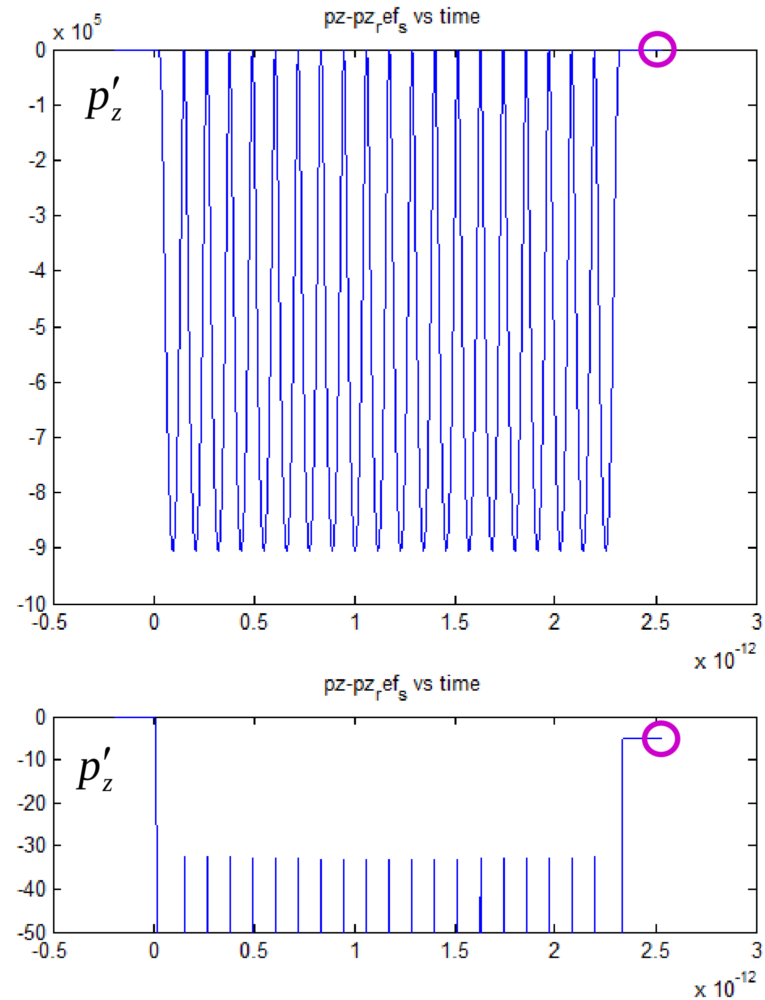


pictures from solutions (a) and (b) cannot be distinguished by eye!

parameters as before, with Lorentz transformation to frame = av. velocity

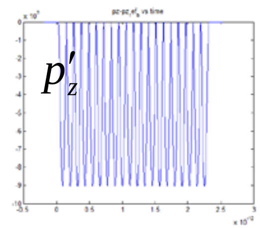
- particle **after** undulator

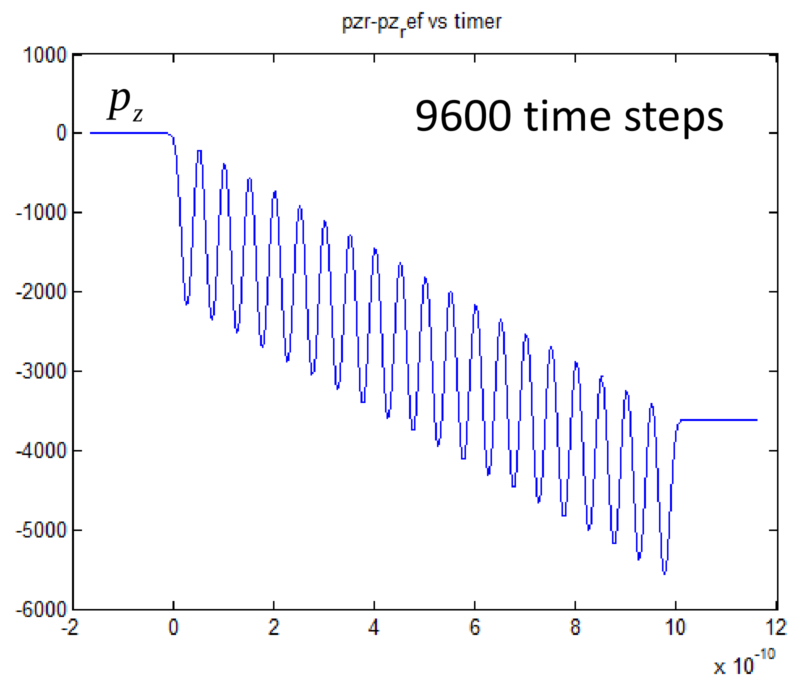




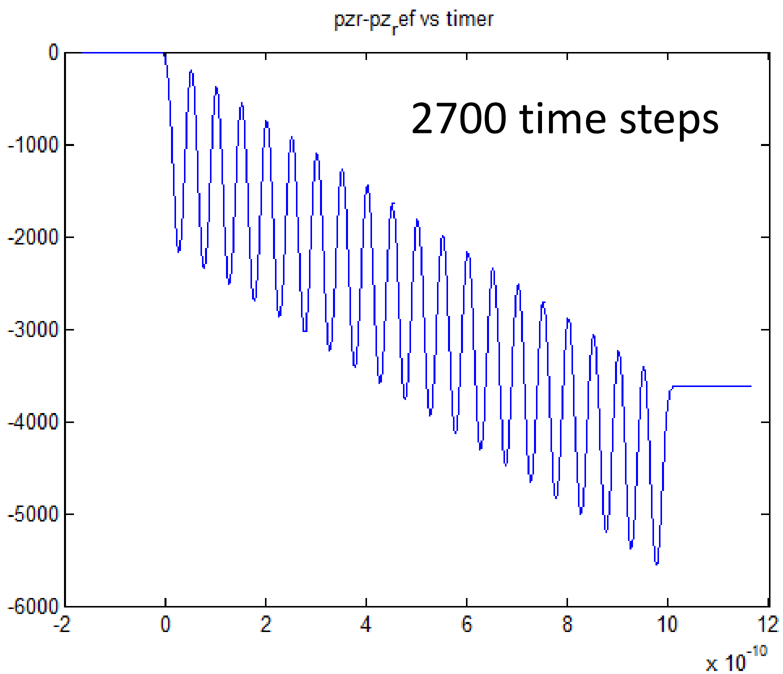
parameters as before, with Lorentz transformation to frame = av. velocity

transformation back to lab-frame



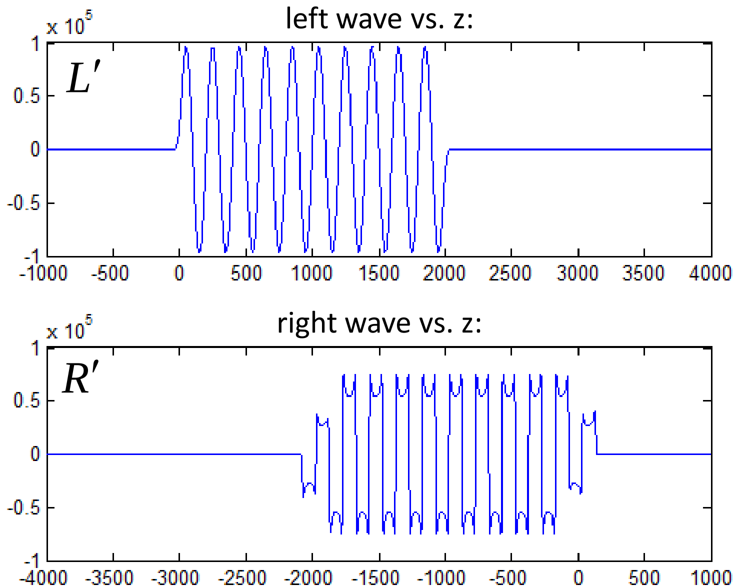


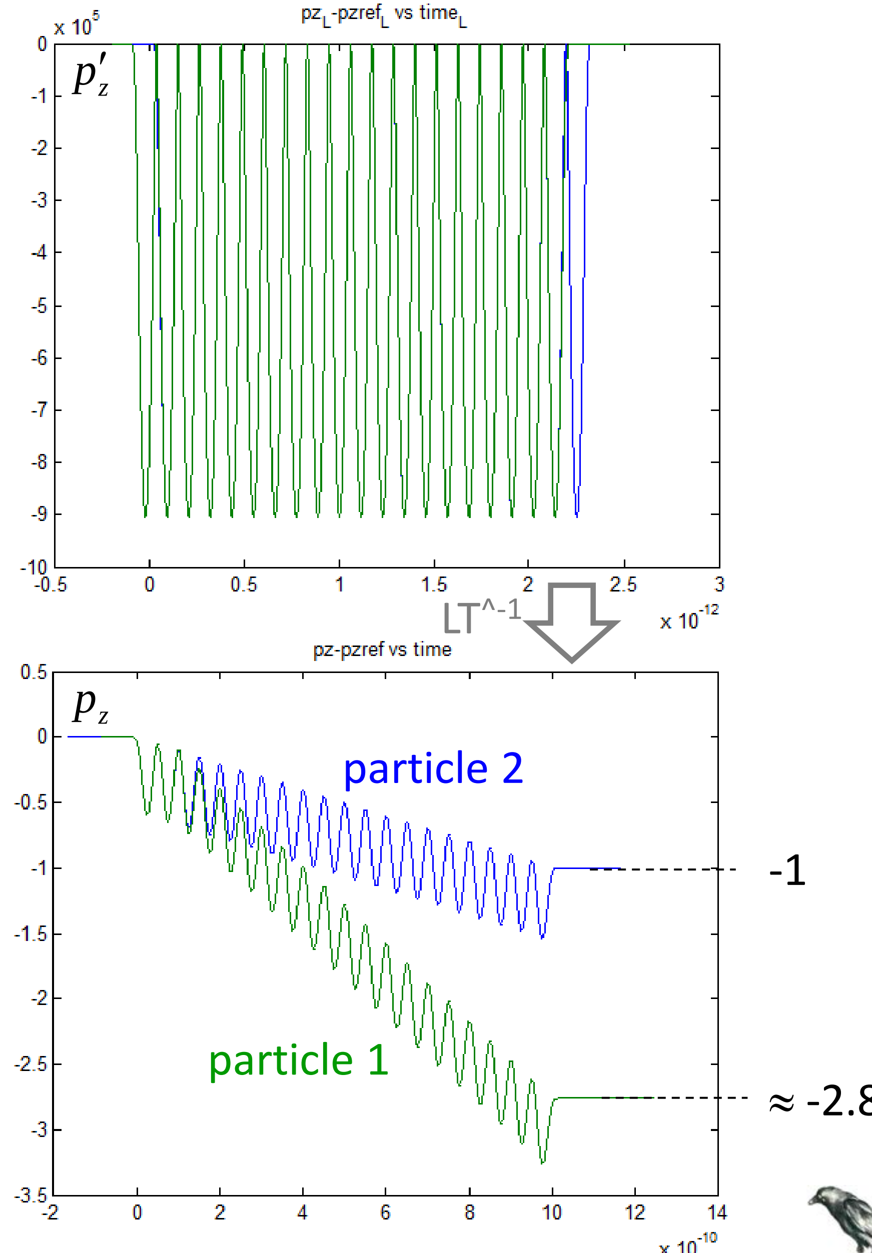
direct calculation in lab frame:



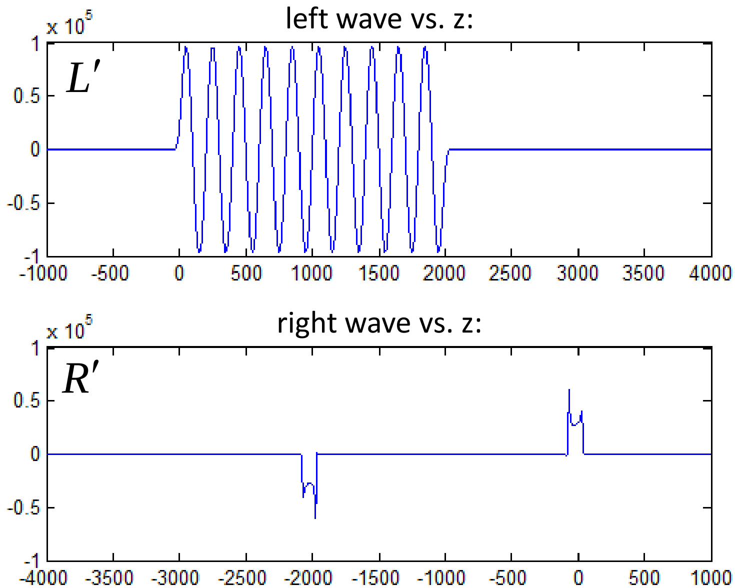
parameters as before, with Lorentz transformation to frame = av. velocity

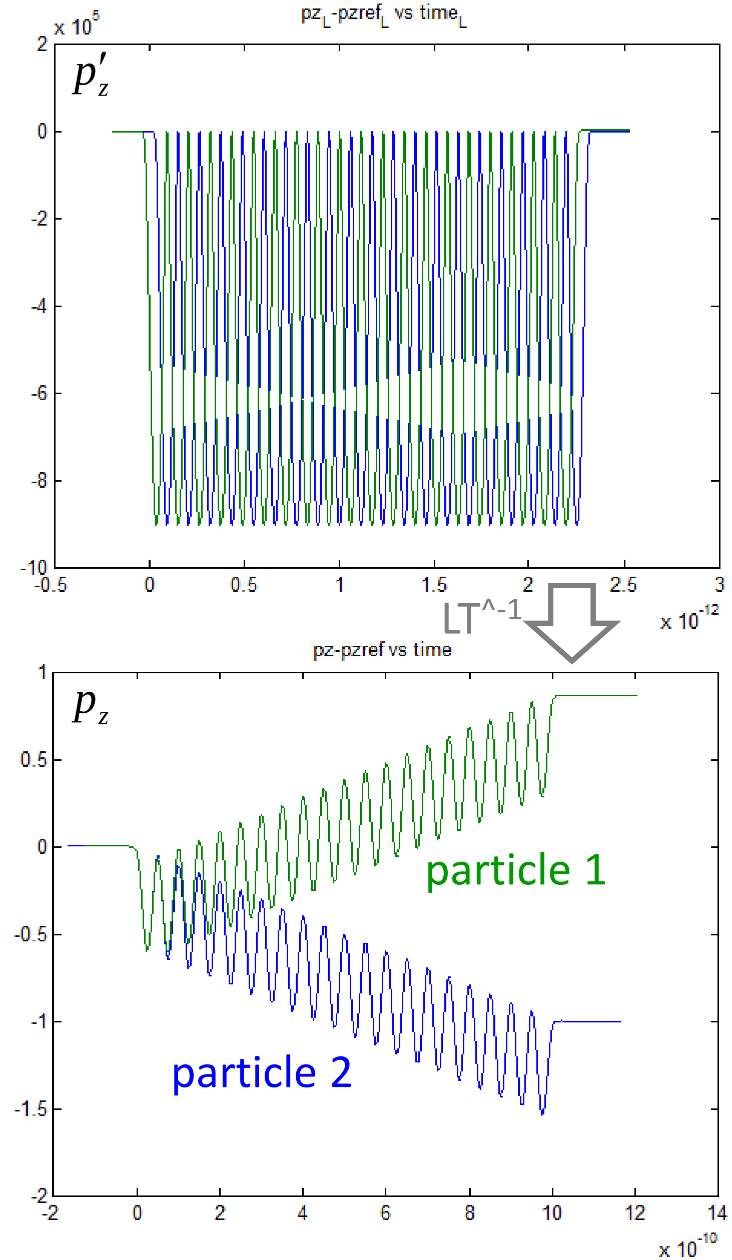
two particles, separated by one photon wavelength





parameters as before, with Lorentz transformation to frame = av. velocity
two particles, separated by half photon wavelength





Mystery

In the frame "av. undulator velocity" the energy loss to both waves (left and right) is about equal.
It seems the effect from both waves to the one-particle dynamic is similar.

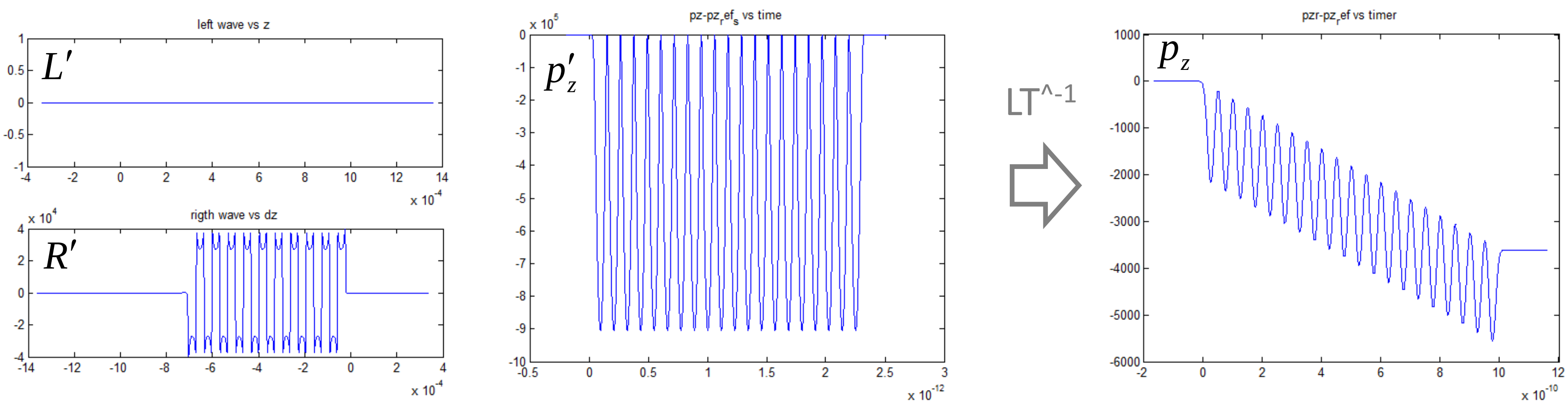
In the rest frame the effect of the left wave seems negligible.

What happens if we neglect the left wave in the frame "av. undulator velocity"?

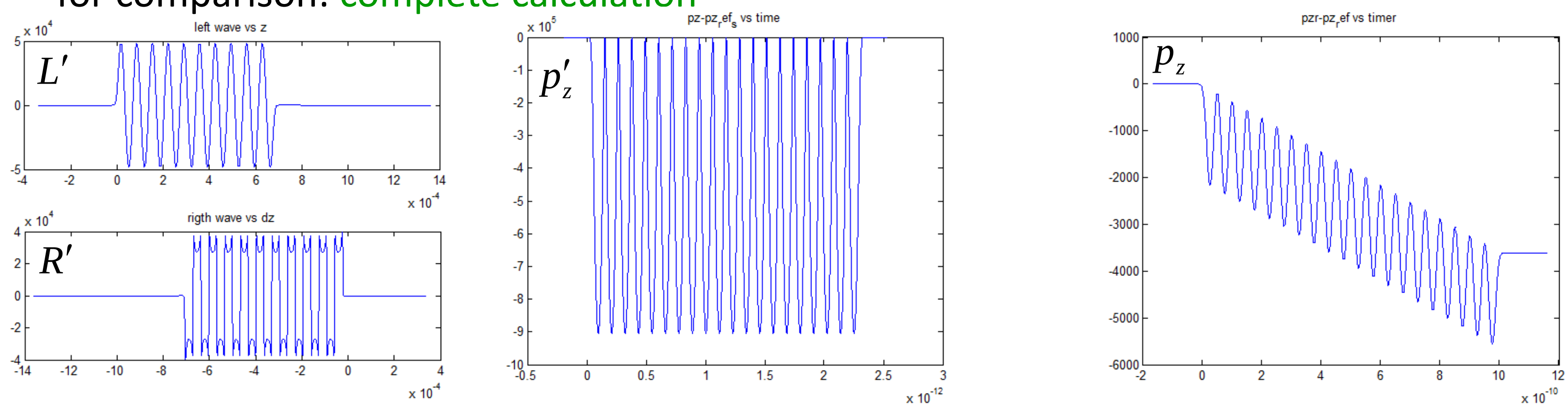
example as before, with Lorentz transformation to frame = av. velocity

no stimulation of left wave:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{r}_\nu\\ \mathbf{p}_\nu\\ \tilde{L}(V,t)\\ \tilde{R}(U,t)\end{bmatrix} = \begin{bmatrix}\mathbf{v}_\nu\\ \mathbf{f}_\nu\\ \boxed{0}\\ -\varepsilon^{-1}J(U+ct,t)\end{bmatrix}$$



for comparison: complete calculation



Why is L negligible in the frame “av. undulator velocity”?