Center for the Advancement of Natural
Discoveries using Light Emission

Transverse Wakes for XFEL Database

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DESY – S2E Meeting
25 November 2013

Motivation

$$W_{//}^{(\lambda)}(s) = \int_{-\infty}^{s} w_{//}(s-s')\lambda(s')\,ds'$$

How to obtain ?

Use developed analytical models for short-range wake function

Singular

Wake potential is not-singular

Use wake potential of much shorter bunch than the bunch one need to model

Not singular

Difficult to obtain

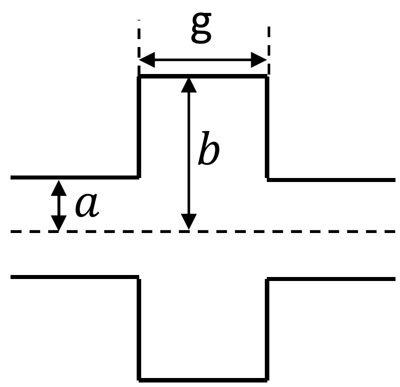
To tabulate the wake function there is need of a model that will solve singularity problem!

Such approach is used, for example, in:

- A. Novokhatsky, M. Timm, and T. Weiland, Single bunch energy spread in the TESLA cryomodule, Tech. Rep. DESY-TESLA-99-16
- T. Weiland, I. Zagorodnov, The Short-Range Transverse Wake Functionfor **TESLA** Accelerating Structure, DESY-TESLA-03-23

More Examples

Pillbox Cavity



Diffraction model $\Rightarrow$ $\sigma_b < 2\sqrt{(b-a)^2 + (g/2)^2} - g$

$$w_{//}^{(m)}(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}}$$

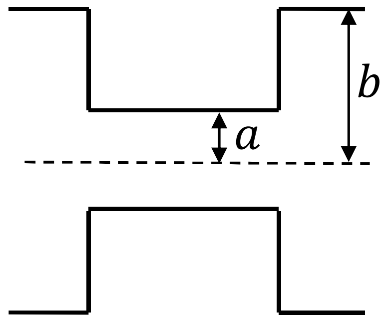
$$w_{//}^{(d)}(s) = \frac{2}{a^2} w_{//}^{(m)}(s)$$

$\Rightarrow$ $$w_{//}^{(m,d)}(s) = c\frac{\partial}{\partial s} w_{//,m,d}^{(-1)}(s)$$ $\Rightarrow$

$$w_{//,m}^{(-1)}(s) = \frac{2 \cdot Z_0}{\sqrt{2}\pi^2 a}\sqrt{g\, s}$$

$$w_{//,d}^{(-1)}(s) = \frac{2}{a^2} w_{//,m}^{(-1)}(s)$$

Step Collimator & Step-out transition



$$w_{//}^{(m)}(s) = \frac{Z_0 c}{\pi}\ln\left(\frac{b}{a}\right)\delta(s)$$

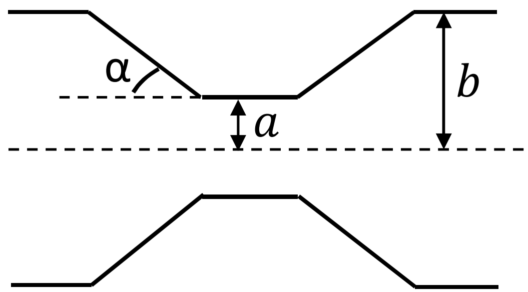
$$w_{//}^{(d)}(s) = \frac{Z_0 c}{\pi}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\delta(s)$$

$\Rightarrow$ $$w_{//}^{(m,d)}(s) = c\, R_{m,d}\, \delta(s)$$ $\Rightarrow$

$$R_m = \frac{Z_0}{\pi}\ln\left(\frac{b}{a}\right)$$

$$R_d = \frac{Z_0}{\pi}\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$$

Tapered Collimator



Small tapered angle $\Rightarrow$ $\rho = tg\alpha \frac{a}{\sigma_b} < 1$

$$w_{//}^{(m)}(s) = c^2\left(\frac{Z_0}{4\pi c}\int_{Geom.} r'^{\,} dr\right)\frac{\partial}{\partial s}\delta(s)$$

$$w_{//}^{(d)}(s) = c^2\left(2\frac{Z_0}{4\pi c}\int_{Geom.}\frac{r'}{r^2}dr\right)\frac{\partial}{\partial s}\delta(s)$$

$\Rightarrow$ $$w_{//}^{(m,d)}(s) = c^2 L_{m,d}\frac{\partial}{\partial s}\delta(s)$$ $\Rightarrow$

$$L_m = \frac{Z_0}{4\pi c}\int_{Geom.} r' dr$$

$$L_d = 2\frac{Z_0}{4\pi c}\int_{Geom.}\frac{r'}{r^2}dr$$

Wake function model

Longitudinal wake function:

$$\Phi(s) = \begin{cases} 1, s > 0 \\ 0, s = 0 \end{cases}$$

$$\underbrace{w_{||}(s) = w_{||}^{(0)}(s) + \frac{\Phi(s)}{C}}_{\text{Regular part}} + \underbrace{R\,c\,\delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w_{||}^{(-1)}(s)\right]}_{\text{Singular part}}$$

Regular part

Singular part

Wake model:
I. Zagorodnov

$$w(s) = \int Z(\omega)e^{-j\frac{\omega}{c}s}d\omega$$

⇓

$$Z(\omega) = Z^{(0)}(\omega) - \frac{1}{j\omega C} + R + j\omega\left[L + Z^{(-1)}(\omega)\right]$$

Transverse wake function

Panowsky-Wenzel ⇒

$$\vec{w}_{\perp}(\vec{r},\vec{r}_0,s) = -\vec{\nabla}_{\perp,\vec{r}}\int_{-\infty}^{s}\vec{w}_{||}(\vec{r},\vec{r}_0,s')ds' =$$

$$= -\vec{\nabla}_{\perp,\vec{r}}\left[\vec{w}_{\perp}^{(0)}(s) + \frac{\Phi(s)}{C}s + c\,R - Lc^2\delta(s) - cw_{||}^{(-1)}(s)\right]$$

Wake Potential for arbitrary bunch shape λ(s)

Longitudinal Wake Potential

$$W_{\|}(s) = \int_{-\infty}^{s} w_{\|}^{(0)}(s-s')\lambda(s')ds' + \frac{\Phi(s)}{C}\int_{-\infty}^{s}\lambda(s')ds' + R\,c\,\lambda(s) - c^2L\lambda'(s) - c\int_{-\infty}^{s} w_{\|}^{(-1)}(s-s')\lambda'(s')ds'$$

⇒ No singularities!

ASTRA Format for Wake Data
(Taylor Method)

Implemented in ASTRA:
M. Dohlus

Taylor Expansion of wake function
(Test particle coordinates – $\{x_t, y_t\}$)

$$w_{//}(x,x_t,y,y_t,s) = w_0(s) + \begin{pmatrix} w_1(s) \\ w_2(s) \\ w_3(s) \\ w_4(s) \end{pmatrix}^T \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix} + \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}^T \begin{pmatrix} w_{11}(s) & w_{12}(s) & w_{13}(s) & w_{14}(s) \\ w_{12}(s) & w_{22}(s) & w_{23}(s) & w_{24}(s) \\ w_{13}(s) & w_{23}(s) & w_{33}(s) & w_{34}(s) \\ w_{14}(s) & w_{24}(s) & w_{34}(s) & w_{44}(s) \end{pmatrix} \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}$$

In the special case (monopole+dipole wake) non-vanishing coefficients are:

$$w_0(s) = w_{//}^{(monopole)}(s)$$
$$w_{13}(s) = w_{24}(s) = 0.5 \cdot w_{//}^{(dipole)}(s)$$

Wake file is in ASCII format and is a “multi-table” describing up to 14 coefficient functions.
Each function is described by following model:

$$w_{ij}(s) = w^{(0)}(s) + \frac{\Phi(s)}{C} + R\, c\, \delta(s) - c \frac{\partial}{\partial s}\left[Lc\delta(s) + w_{||}{}^{(-1)}(s)\right]$$

The format is:

K	0
Table 1	
Table 2	
…	
Table K	

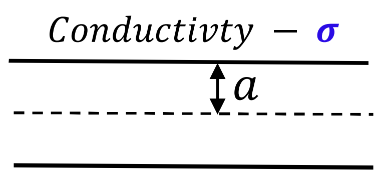
Each Table format

N_0	N_1
R	L
$\tilde{C}$	$i \, or \, i+10j$
$s_1^{(0)}$	$w^{(0)}\left(s_1^{(0)}\right)$
$\vdots$	$\vdots$
$s_{N_0}^{(0)}$	$w^{(0)}\left(s_{N_0}^{(0)}\right)$
$s_1^{(-1)}$	$w^{(-1)}\left(s_1^{(-1)}\right)$
$\vdots$	$\vdots$
$s_{N_1}^{(-1)}$	$w^{(-1)}\left(s_{N_1}^{(-1)}\right)$

Here $\{i \; or \; i + 10\, j\}$ is a number that indicates the wake coefficient indexes [$\boldsymbol{w_i}$ or $\boldsymbol{w_{i+10\,j}}$]

Special cases	
Mon.+Dip.	K=3
Mon.	K=1
Dip.	K=2

Resistive wakes (per unit length)



$$Z_{//}^{(m)}(\omega) = \frac{Z_s(\omega)}{2\pi a}\left[1 + j\frac{\omega}{c}\frac{a}{2}\frac{Z_s(\omega)}{Z_0}\right]^{-1}$$

$$w_{//}^{(d)}(s) = \frac{2}{a^2} w_{//}^{(m)}(s)$$

$$Z_s(\omega) = Z_s^{\sigma}(\omega) + Z_s^{L}(\omega)$$

$$Z_s^{\sigma}(\omega) \approx \sqrt{\frac{j\omega\mu_0}{\sigma(\omega)}}$$

$$\sigma(\omega) \approx \frac{\sigma_0}{1 + j\omega\tau}$$

$$Z_s^{L}(\omega) \approx j\omega L$$

$$L \approx \mu_0\left[\frac{\varepsilon_r - 1}{\varepsilon_r}\cdot\Delta_{oxide} + 0.01\cdot\Delta_{rough}\right]$$

For all elements except Und. Bempipe

$$\varepsilon_r = 10$$
$$\Delta_{oxide} = 7\,nm$$
$$\Delta_{rough} = 500\,nm$$

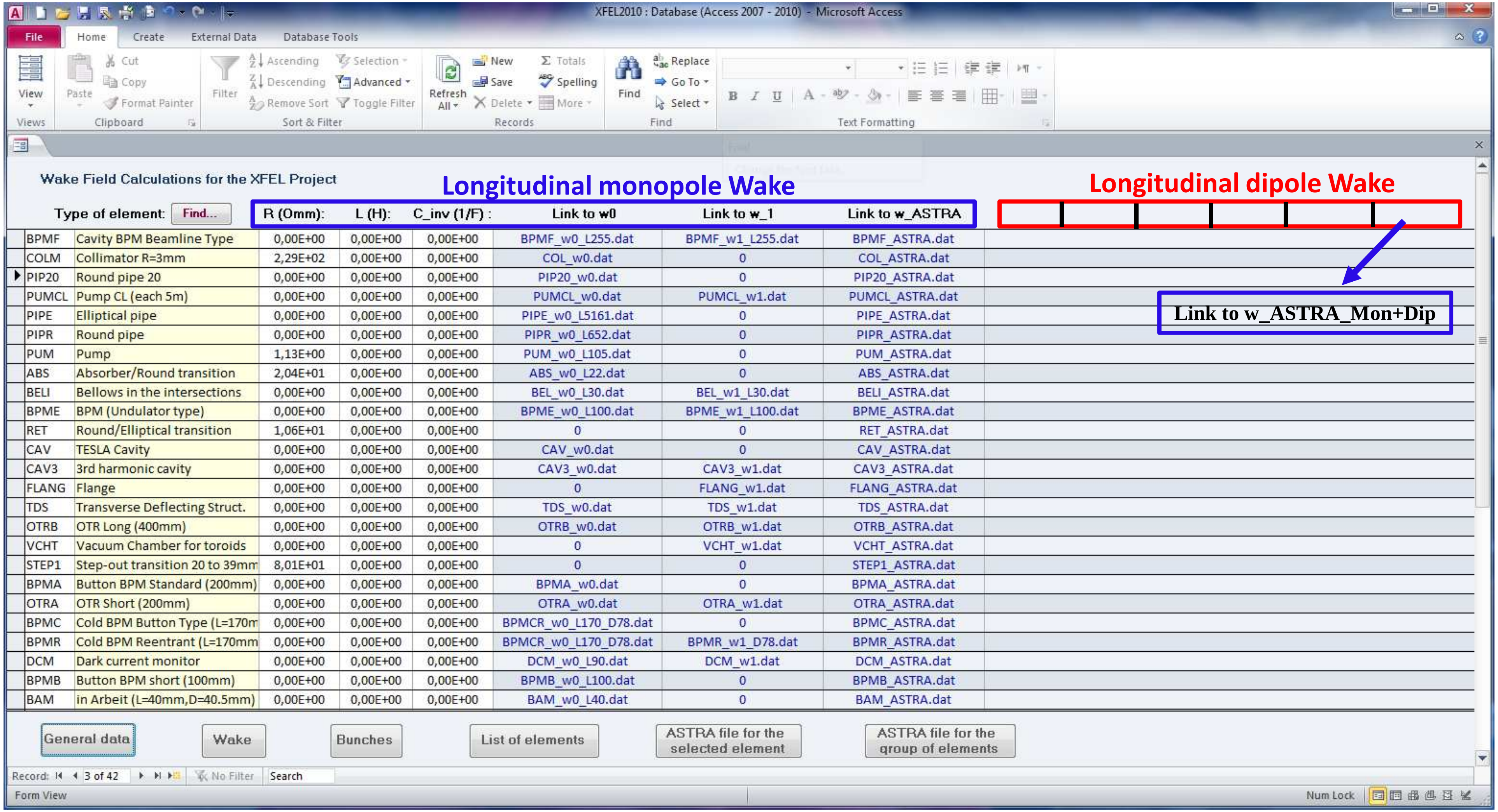
Undulator beampipe

$$\varepsilon_r = 10$$
$$\Delta_{oxide} = 5\,nm$$
$$\Delta_{rough} = 300\,nm$$

- M.Dohlus. TESLA 2001-26, 2001
- K.L.F.Bane, G.V.Stupakov, SLAC-PUB-10707, 2004
- A. Tsakanian, M. Dohlus, I. Zagorodnov, TESLA-FEL-2009-05, 2009

XFEL Database Overview

Wake data format for ASTRA will be created for ASTRA taylor_methode_F



Type of element:		R (Omm):	L (H):	C_inv (1/F) :	Link to w0	Link to w_1	Link to w_ASTRA
BPMF	Cavity BPM Beamline Type	0,00E+00	0,00E+00	0,00E+00	BPMF_w0_L255.dat	BPMF_w1_L255.dat	BPMF_ASTRA.dat
COLM	Collimator R=3mm	2,29E+02	0,00E+00	0,00E+00	COL_w0.dat	0	COL_ASTRA.dat
PIP20	Round pipe 20	0,00E+00	0,00E+00	0,00E+00	PIP20_w0.dat	0	PIP20_ASTRA.dat
PUMCL	Pump CL (each 5m)	0,00E+00	0,00E+00	0,00E+00	PUMCL_w0.dat	PUMCL_w1.dat	PUMCL_ASTRA.dat
PIPE	Elliptical pipe	0,00E+00	0,00E+00	0,00E+00	PIPE_w0_L5161.dat	0	PIPE_ASTRA.dat
PIPR	Round pipe	0,00E+00	0,00E+00	0,00E+00	PIPR_w0_L652.dat	0	PIPR_ASTRA.dat
PUM	Pump	1,13E+00	0,00E+00	0,00E+00	PUM_w0_L105.dat	0	PUM_ASTRA.dat
ABS	Absorber/Round transition	2,04E+01	0,00E+00	0,00E+00	ABS_w0_L22.dat	0	ABS_ASTRA.dat
BELI	Bellows in the intersections	0,00E+00	0,00E+00	0,00E+00	BEL_w0_L30.dat	BEL_w1_L30.dat	BELI_ASTRA.dat
BPME	BPM (Undulator type)	0,00E+00	0,00E+00	0,00E+00	BPME_w0_L100.dat	BPME_w1_L100.dat	BPME_ASTRA.dat
RET	Round/Elliptical transition	1,06E+01	0,00E+00	0,00E+00	0	0	RET_ASTRA.dat
CAV	TESLA Cavity	0,00E+00	0,00E+00	0,00E+00	CAV_w0.dat	0	CAV_ASTRA.dat
CAV3	3rd harmonic cavity	0,00E+00	0,00E+00	0,00E+00	CAV3_w0.dat	CAV3_w1.dat	CAV3_ASTRA.dat
FLANG	Flange	0,00E+00	0,00E+00	0,00E+00	0	FLANG_w1.dat	FLANG_ASTRA.dat
TDS	Transverse Deflecting Struct.	0,00E+00	0,00E+00	0,00E+00	TDS_w0.dat	TDS_w1.dat	TDS_ASTRA.dat
OTRB	OTR Long (400mm)	0,00E+00	0,00E+00	0,00E+00	OTRB_w0.dat	OTRB_w1.dat	OTRB_ASTRA.dat
VCHT	Vacuum Chamber for toroids	0,00E+00	0,00E+00	0,00E+00	0	VCHT_w1.dat	VCHT_ASTRA.dat
STEP1	Step-out transition 20 to 39mm	8,01E+01	0,00E+00	0,00E+00	0	0	STEP1_ASTRA.dat
BPMA	Button BPM Standard (200mm)	0,00E+00	0,00E+00	0,00E+00	BPMA_w0.dat	0	BPMA_ASTRA.dat
OTRA	OTR Short (200mm)	0,00E+00	0,00E+00	0,00E+00	OTRA_w0.dat	OTRA_w1.dat	OTRA_ASTRA.dat
BPMC	Cold BPM Button Type (L=170m	0,00E+00	0,00E+00	0,00E+00	BPMCR_w0_L170_D78.dat	0	BPMC_ASTRA.dat
BPMR	Cold BPM Reentrant (L=170mm	0,00E+00	0,00E+00	0,00E+00	BPMCR_w0_L170_D78.dat	BPMR_w1_D78.dat	BPMR_ASTRA.dat
DCM	Dark current monitor	0,00E+00	0,00E+00	0,00E+00	DCM_w0_L90.dat	DCM_w1.dat	DCM_ASTRA.dat
BPMB	Button BPM short (100mm)	0,00E+00	0,00E+00	0,00E+00	BPMB_w0_L100.dat	0	BPMB_ASTRA.dat
BAM	in Arbeit (L=40mm,D=40.5mm)	0,00E+00	0,00E+00	0,00E+00	BAM_w0_L40.dat	0	BAM_ASTRA.dat

General data | Wake | Bunches | List of elements | ASTRA file for the selected element | ASTRA file for the group of elements

Record: 3 of 42

Summary

- Wake model to resolve singularities of Green function
- ASTRA with wake field option
- Transverse wakes in XFEL Database (available soon)

Thank You for Attention