Some Recipes for Wakefield Calculations

Fast Analytical Estimations and Scaling

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Collaboration Meeting at PAL

Pohang, Korea
2-6. September 2013

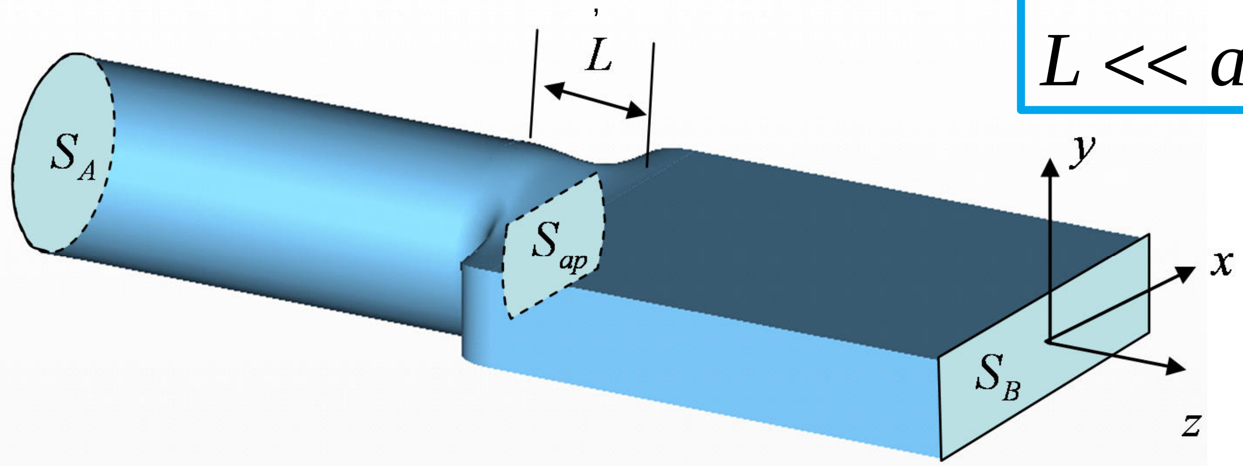
Overview

- Optical regime
  - general case
  - step-out, step-in, iris
  - scaling with bunch length
- Diffractive regime
  - cavity and gap wakes
  - scaling with bunch length
- Inductive regime
- Resistive wake
- Scaling with structure resizing
- Emittance growth

General Case (Optical Regime)

G. Stupakov, K. Bane, and I. Zagorodnov, Phys. Rev. ST Accel. Beams **10**, 054401 (2007).
K. Bane, G. Stupakov, and I. Zagorodnov, Phys. Rev. ST Accel. Beams **10**, 074401 (2007).
M.Dohlus, I. Zagorodnov, O.Zagorodnova, DESY-10-063 (2010)

$$L << a^2 / \sigma$$



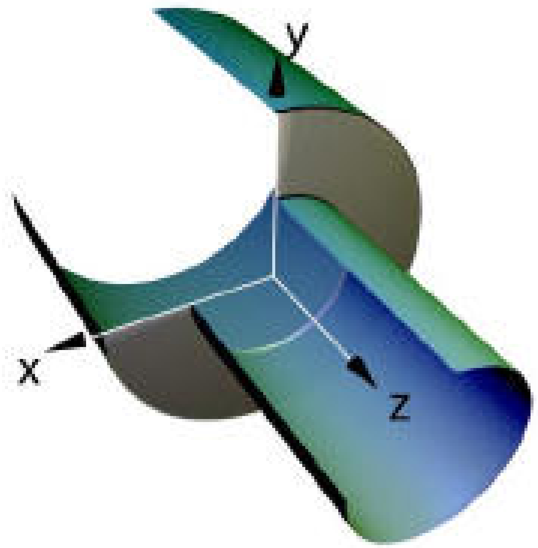
$$Z_{\|}(\mathbf{r}_1,\mathbf{r}_2)=\frac{2\varepsilon_0}{c}\left[\int_{S_B}\nabla\varphi_B(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds-\int_{S_{ap}}\nabla\varphi_A(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds\right]$$

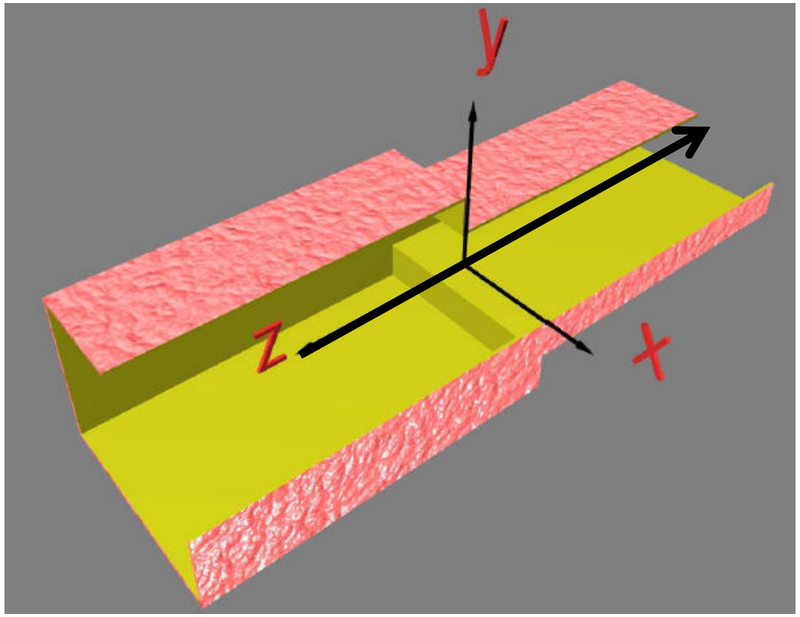
$$\Delta\varphi_A(\mathbf{r}_i,\mathbf{r})=-\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i)$$
$$\mathbf{r}\in S_A$$
$$\varphi_A(\mathbf{r}_i,\mathbf{r})=0$$
$$\mathbf{r}\in\partial S_A$$

$$\Delta\varphi_B(\mathbf{r}_i,\mathbf{r})=-\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i)$$
$$\mathbf{r}\in S_B$$
$$\varphi_B(\mathbf{r}_i,\mathbf{r})=0$$
$$\mathbf{r}\in\partial S_A$$

$$i=1,2$$

Step-In (Optical Regime)

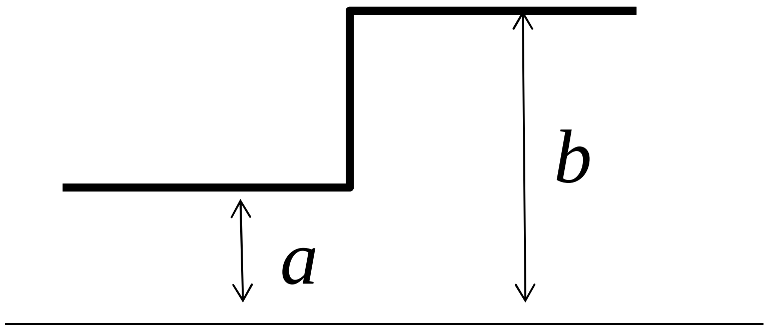


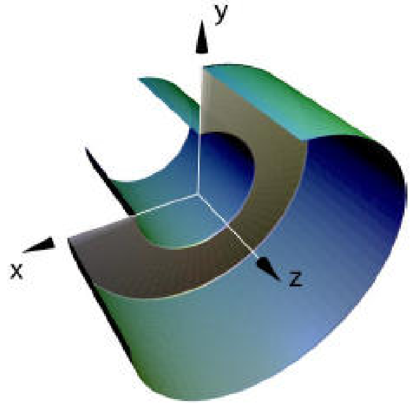


Longitudinal and Transverse Wakes are 0!

Step-Out, Collimator (Optical Regime)

Longitudinal Wake





$$Z_{\|} = \frac{Z_0}{\pi} \ln \frac{b}{a} \qquad w_{\|}(s) = Z_{\|} c \delta(s)$$

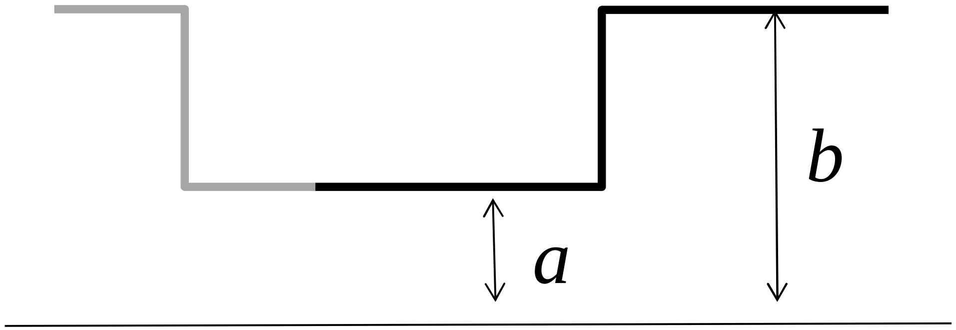
$$W_{\|}(s) = \int_{-\infty}^{s} \lambda(\xi) w_{\|}(s-\xi) d\xi = Z_{\|} c \lambda(s) \qquad k_{\|} = \int_{-\infty}^{\infty} W_{\|}(s)\lambda(s) ds = c Z_{\|} \int_{-\infty}^{\infty} \lambda^2(s) ds$$

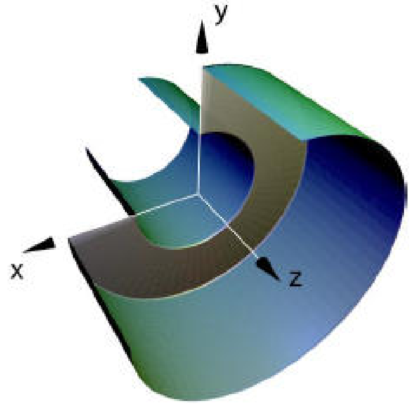
for Gaussian bunch

$$k_{\|} = \frac{c}{2\sqrt{\pi}\sigma} Z_{\|} \qquad k_{\|}^{rms} = \sqrt{\int_{-\infty}^{\infty} \left(W_{\|}(s) - k_{\|}\right)^2 \lambda(s) ds} = k_{\|} \sqrt{\frac{2}{\sqrt{3}} - 1} \approx 0.4 k_{\|}$$

Step-Out, Collimator (Optical Regime)

Transverse Wake





$$Z_{\perp} = \frac{Z_0 c}{\omega \pi}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \qquad w_{\perp}(s) = \theta(s)\omega Z_{\perp}$$

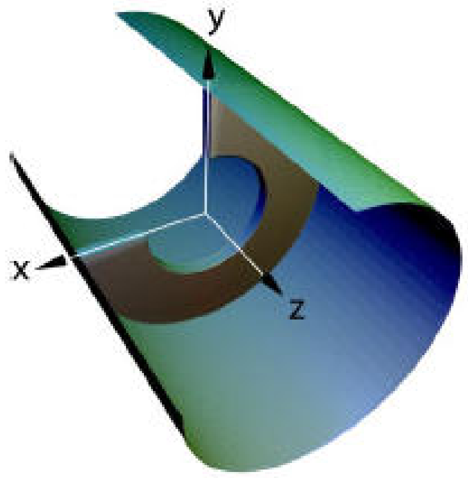
$$W_{\perp}(s) = \int_{-\infty}^{s} \lambda(\xi) w_{\perp}(s-\xi) d\xi = \omega Z_{\perp} \Lambda(s) \qquad \Lambda(s) = \int_{-\infty}^{s} \lambda(\xi) d\xi$$

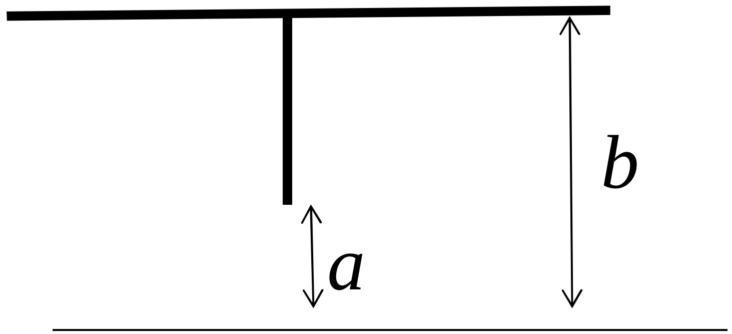
$$k_{\perp} = \omega Z_{\perp} \int_{-\infty}^{\infty} \lambda(s) \int_{-\infty}^{s} \lambda(\xi) d\xi ds = \frac{\omega}{2} Z_{\perp}$$

for Gaussian bunch

$$k_{\perp}^{rms} = \frac{k_{\perp}}{\sqrt{3}} \approx 0.58 k_{\perp}$$

Iris (Optical Regime)





$$Z_{\parallel} = \frac{Z_0}{\pi} \ln \frac{b}{a}$$

$$Z_{\perp} = \frac{Z_0 c}{2\omega\pi} \left(\frac{1}{a^2} - \frac{a^2}{b^4} \right)$$

Scaling with Bunch Length (Optical)

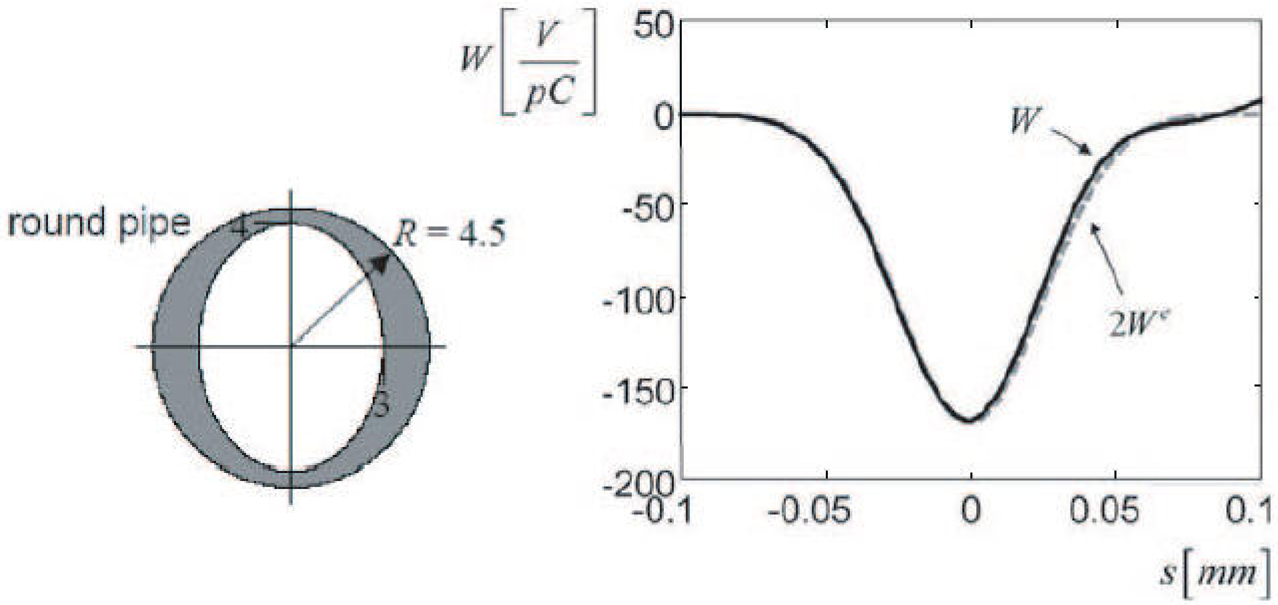
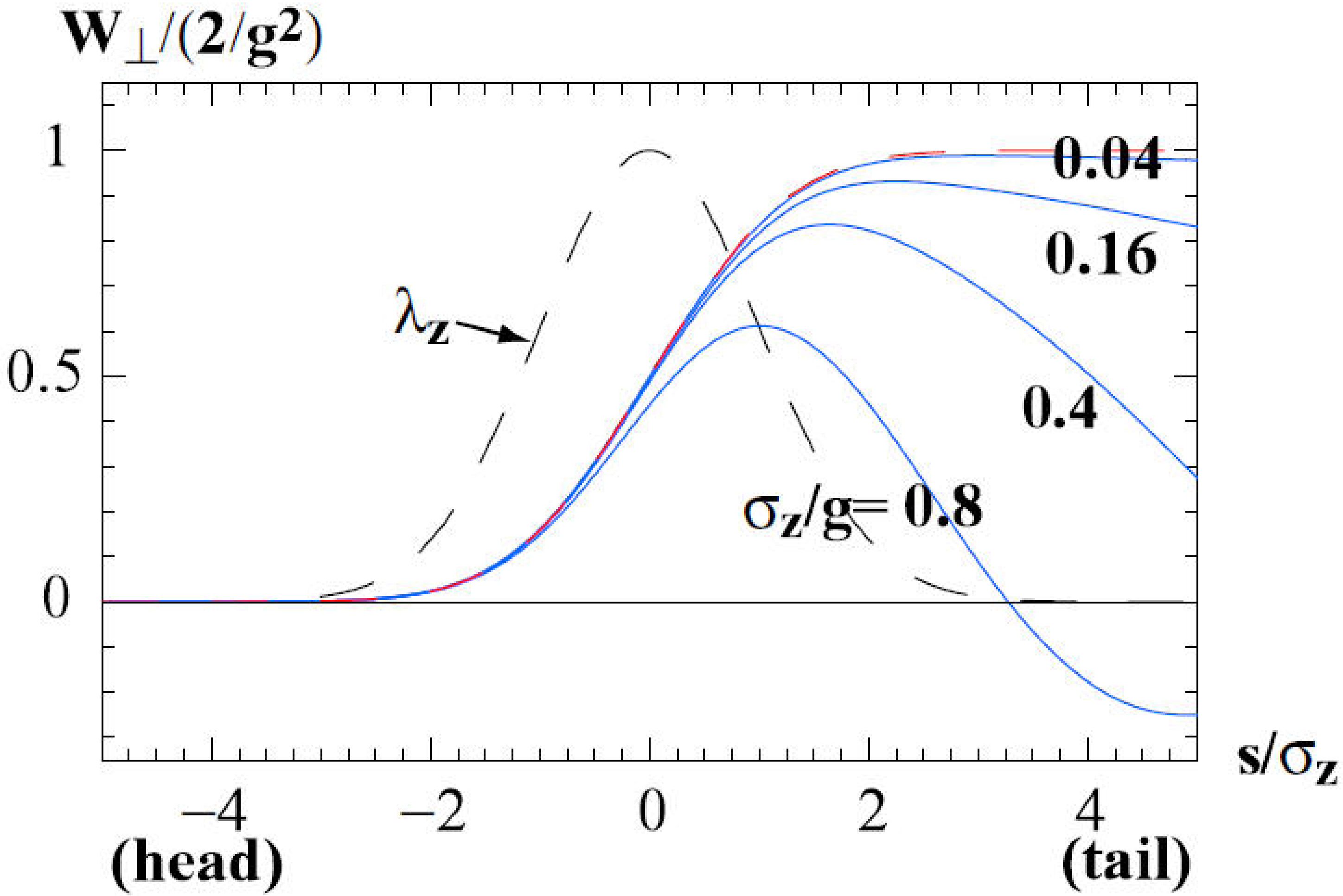
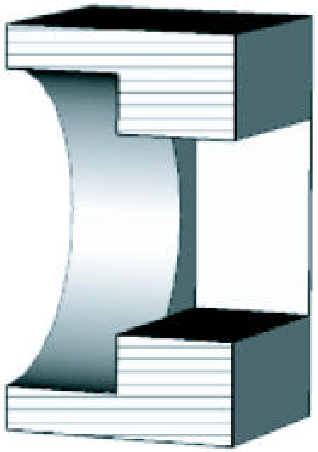


FIG. 5: Wake of a $\sigma_z = 25\ \mu$m Gaussian bunch in the elliptical collimator (head is to the left): comparison of the indirect numerical result (solid line) with the analytical potential energy calculation (dashes).

Scaling with Bunch Length (Optical)



Scaling with Bunch Length (Optical)



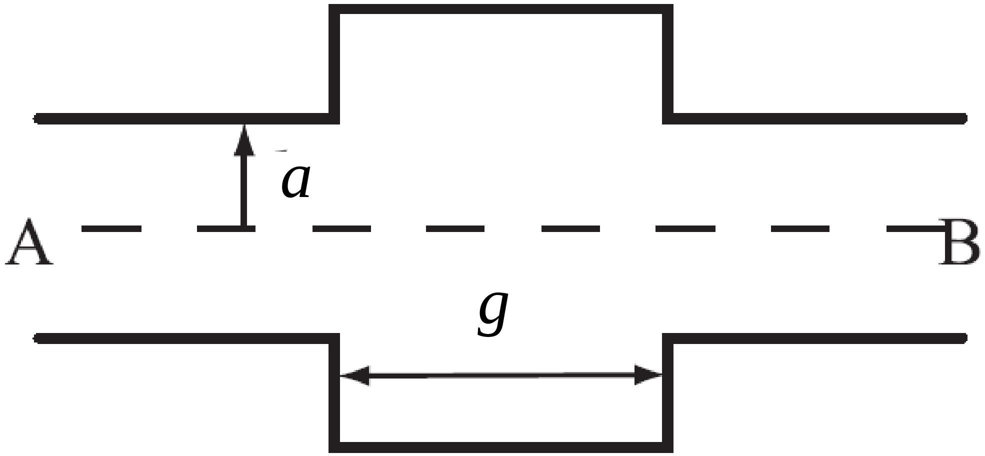
$$Z_{\|} = k_{\|}\left(\frac{c}{2\sqrt{\pi}\sigma}\right)^{-1} = const$$

$$k_{\|}(\sigma_{short}) = k_{\|}(\sigma_{long})\frac{\sigma_{long}}{\sigma_{short}}$$

$$\omega Z_{\perp} = 2k_{\perp} = const$$

$$k_{\perp}(\sigma_{short}) = k_{\perp}(\sigma_{long})$$

Cavity and Gap Wakes (Diffractive Regime)



SLAC – PUB – 4441
November 1987
(A)

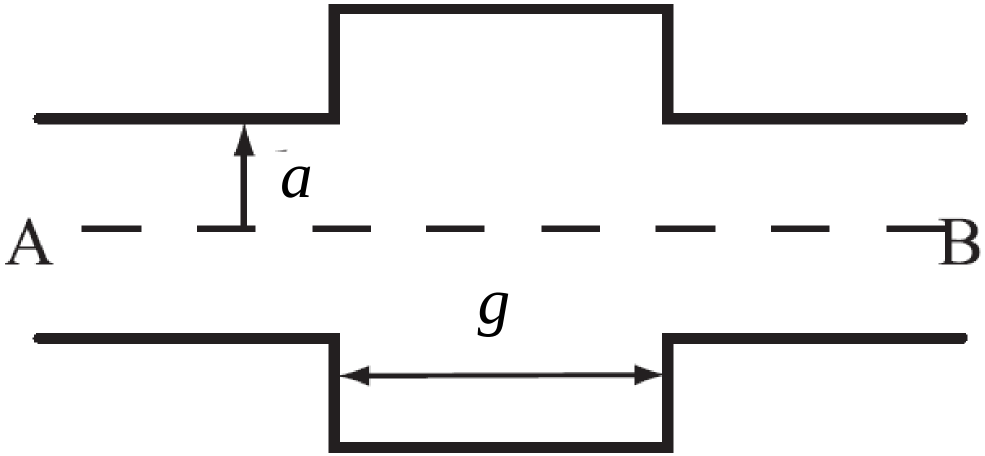
WAKEFIELDS OF VERY SHORT BUNCHES
IN AN ACCELERATING CAVITY*

KARL BANE AND MATTHEW SANDS

$$w_{\parallel}(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}} \qquad k_{\parallel} = \frac{Z_0 c}{4a\pi^{2.5}}\Gamma\left(\frac{1}{4}\right)\sqrt{\frac{g}{\sigma}} \qquad k_{\parallel}^{rms} \approx 0.4 k_{\parallel}$$

$$w_{\perp}(s) = \frac{2}{a^2}\frac{\sqrt{2}Z_0 c}{\pi^2 a}\sqrt{gs} \qquad k_{\perp} = \frac{2}{a^3}\frac{Z_0 c}{\pi^{2.5}}\Gamma\left(\frac{3}{4}\right)\sqrt{g\sigma} \qquad k_{\perp}^{rms} \approx 0.77 k_{\perp}$$

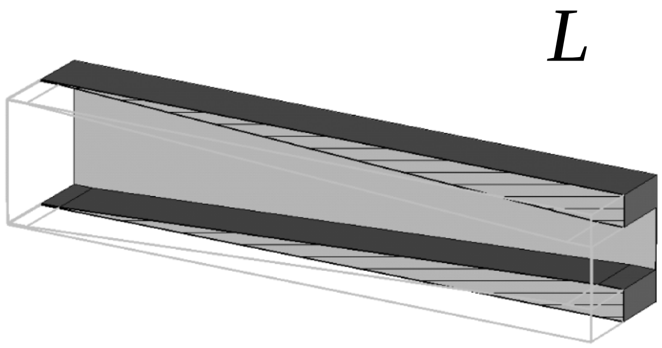
Scaling with Bunch Length (Diffractive)

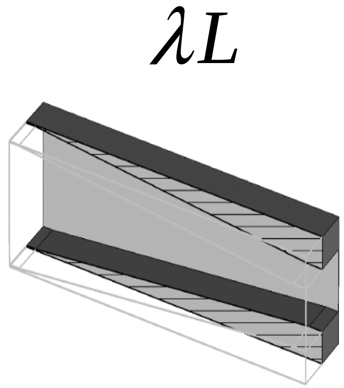


$$k_{\parallel}(\sigma_{short}) = k_{\parallel}(\sigma_{long})\sqrt{\frac{\sigma_{long}}{\sigma_{short}}}$$

$$k_{\perp}(\sigma_{short}) = k_{\perp}(\sigma_{long})\sqrt{\frac{\sigma_{short}}{\sigma_{long}}}$$

Inductive Regime





$$W(s,\sigma_z) = \frac{1}{\lambda} W_\lambda\left(\frac{s}{\lambda}, \frac{\sigma_z}{\lambda}\right) \qquad \mathbf{W}_\perp(s,\sigma_z) = \mathbf{W}_{\perp\lambda}\left(\frac{s}{\lambda}, \frac{\sigma_z}{\lambda}\right)$$

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Impedance scaling for small angle transitions

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(Received 24 September 2010; published 10 January 2011)

Resistive Wake (Round Pipe)

$$Z(\omega) = \frac{Z_s(\omega)}{2\pi R}\left[1 + i\frac{\omega}{c}\frac{R}{2}\frac{Z_s(\omega)}{Z_0}\right]^{-1}$$

$$Z_s(\omega) = Z_s^{\kappa}(\omega) = \sqrt{\frac{j\omega\mu_0}{\kappa(\omega)}} \qquad \kappa(\omega) = \frac{\kappa_0}{1 + j\omega\tau}$$

The effect of the oxide layer and the roughness can be taken into account through the inductive surface impedance

$$Z_s(\omega) \approx Z_s^{\kappa}(\omega) + j\omega L \qquad L = \mu_0\left(\frac{\varepsilon_r - 1}{\varepsilon_r} d_{oxid} + 0.01 d_{rough}\right) \qquad \varepsilon_r \sim 2$$

$$Z_{\perp} = \frac{2}{a^2}\frac{c}{\omega} Z_{\parallel}$$

M.Dohlus. TESLA 2001-26, 2001
A.Tsakanian et al, TESLA-FEL 2009-05

Scaling with Structure Sizing

$$w_{\parallel} \sim \lambda^{-2}, \qquad w_{\perp} \sim \lambda^{-3}$$

Bane K.L.F., *Wakefield effects in a linear collider*, SLAC-PUB-4169, 1986

Emittance Growth

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = \sqrt{1 + S^2 \frac{\beta}{3\varepsilon_{0x}}} - 1 \approx S^2 \frac{\beta}{6\varepsilon_{0x}}$$

$$S = \frac{eQk_x}{\beta_z^2 E}$$

Dohlus M., Limberg T., Impact of optics on CSR-related emittance growth in bunch compressor chicanes, PAC 2005, p. 1015

I. Zagorodnov at al., Mirror kick, EPAC 2008