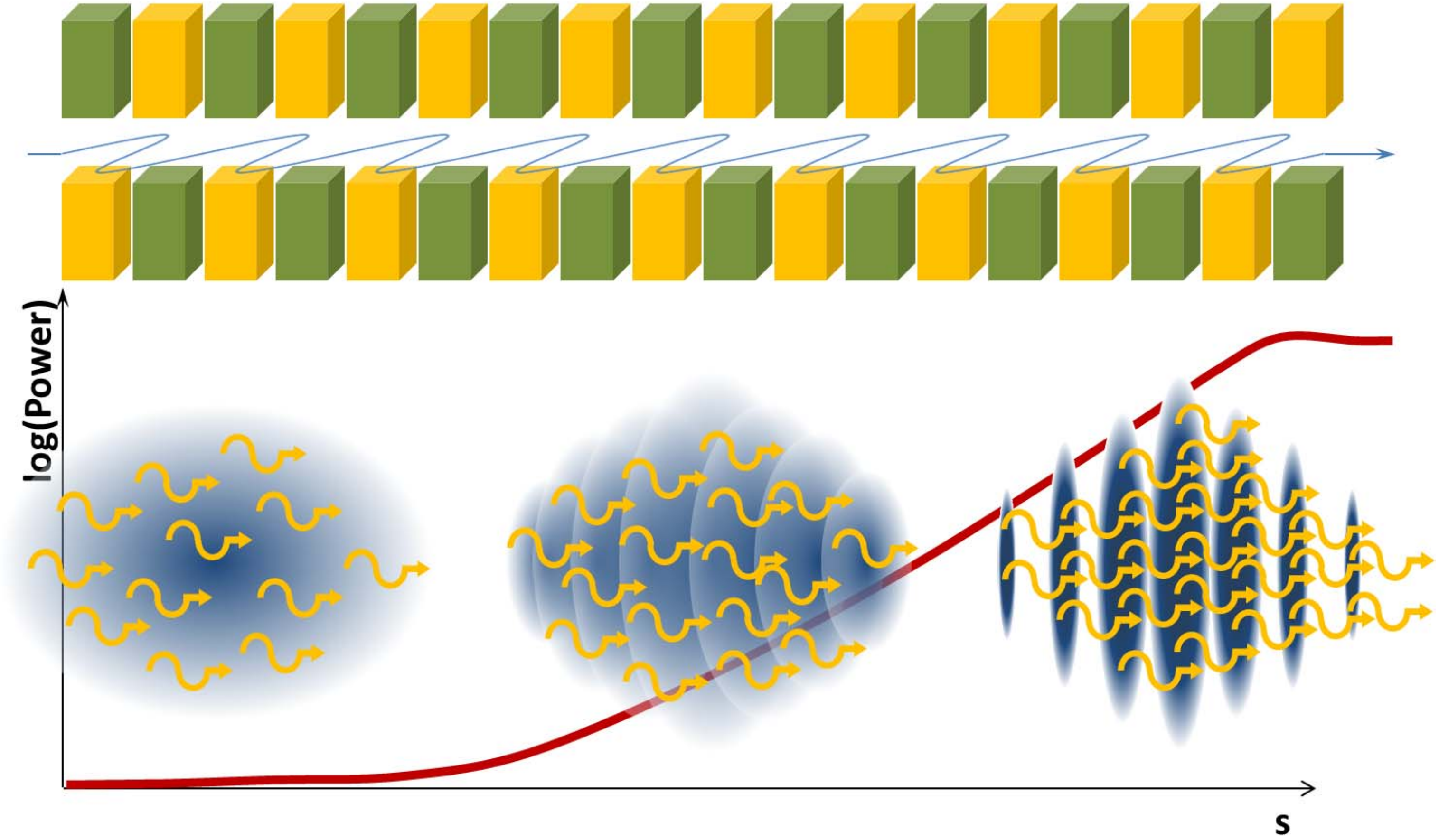
Coherent Synchrotron Radiation and Beam Interaction

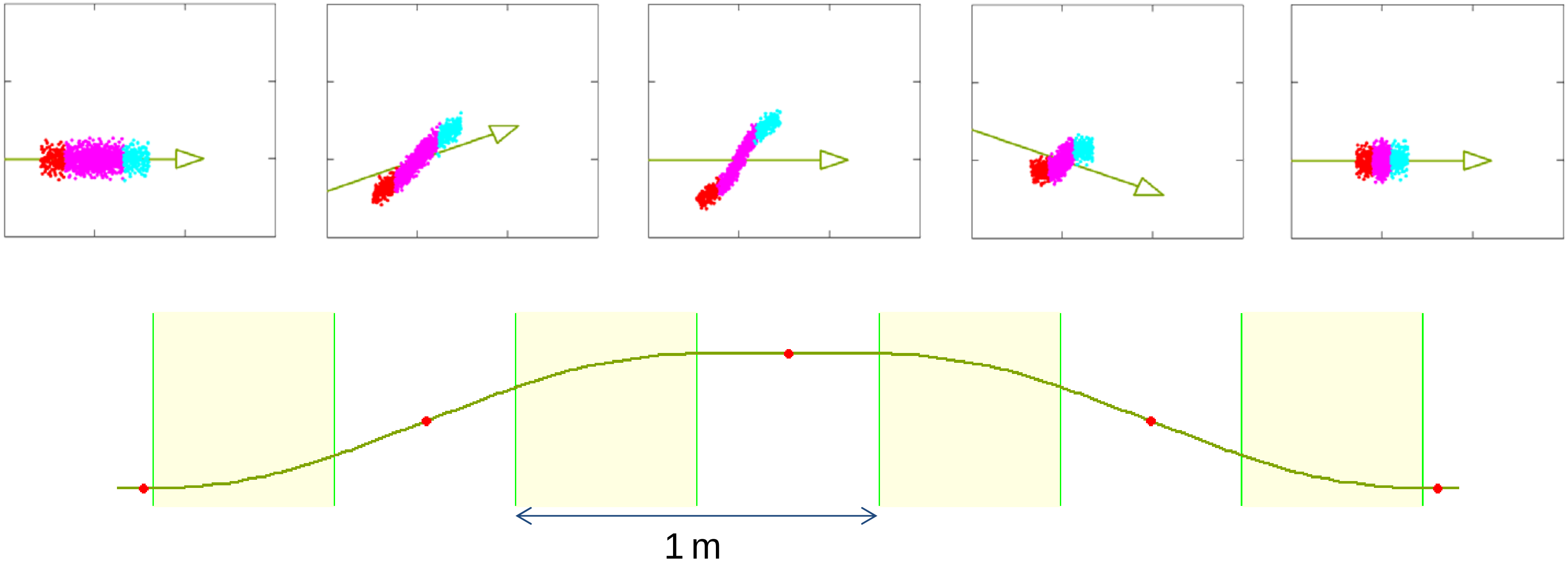
3rd ARD ST3 workshop
Martin Dohlus
16th July 2015

FEL process

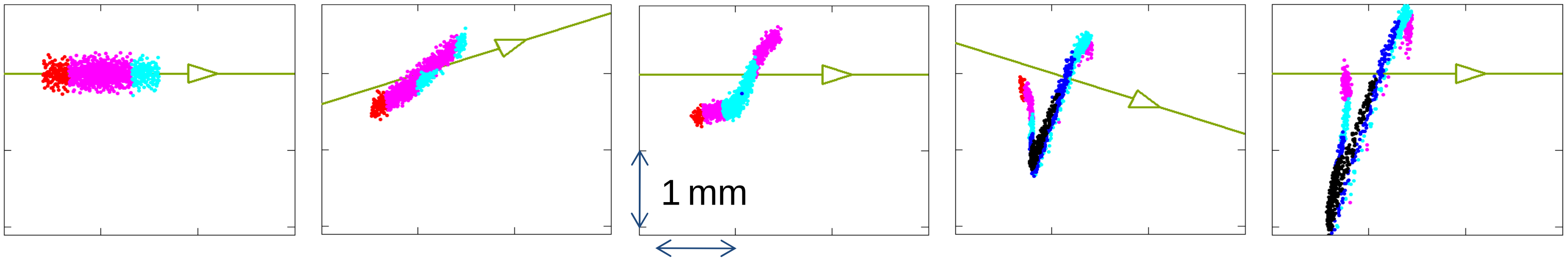


beam preparation: bunch compressor

without self-interaction



with self-interaction



1 - one particle

2 - near field

3 - multiple particles

4 - circular motion & shielding

5 - general trajectories

6 - projected model

7 - bunch compressor

8 - other forces / effects

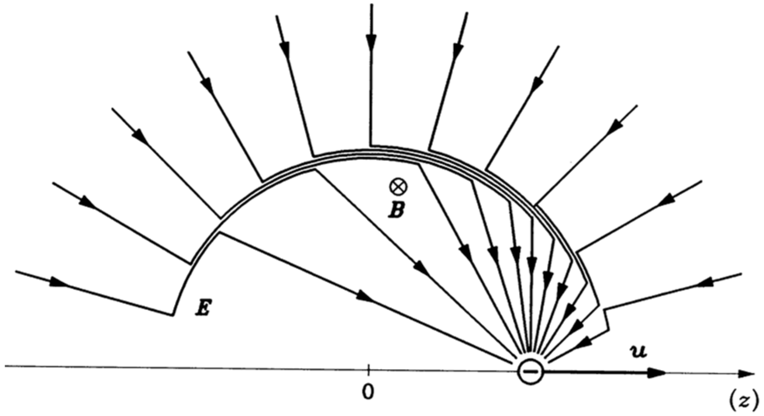
9 - transverse effects

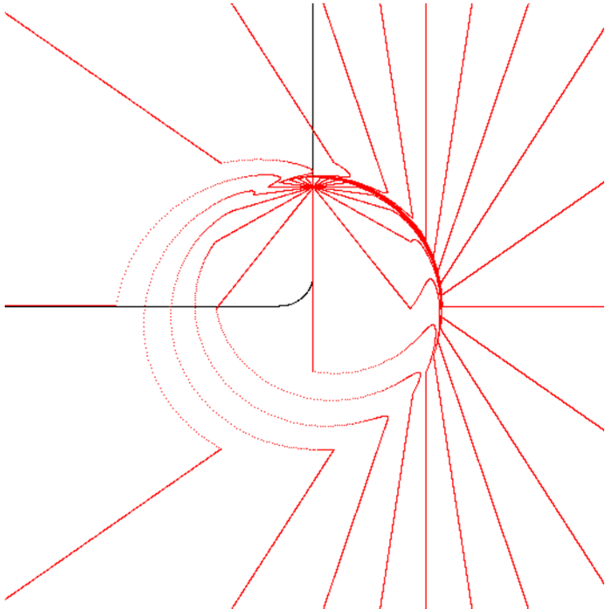
one particle

one particle

instantaneously

http://www.shintakelab.com/en/enEducationalSoft.htm
→ Radiation 2D Simulator Free Download

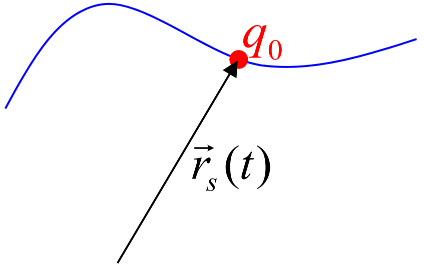




one particle

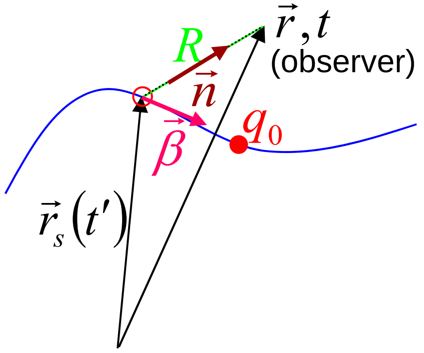
Lienert-Wiechert

charge q_0 on trajectory $\vec{r}_s(t)$



retarded time

$$c_0\left(t-t'\right)=\left\|\vec{r}-\vec{r}_s(t')\right\|$$

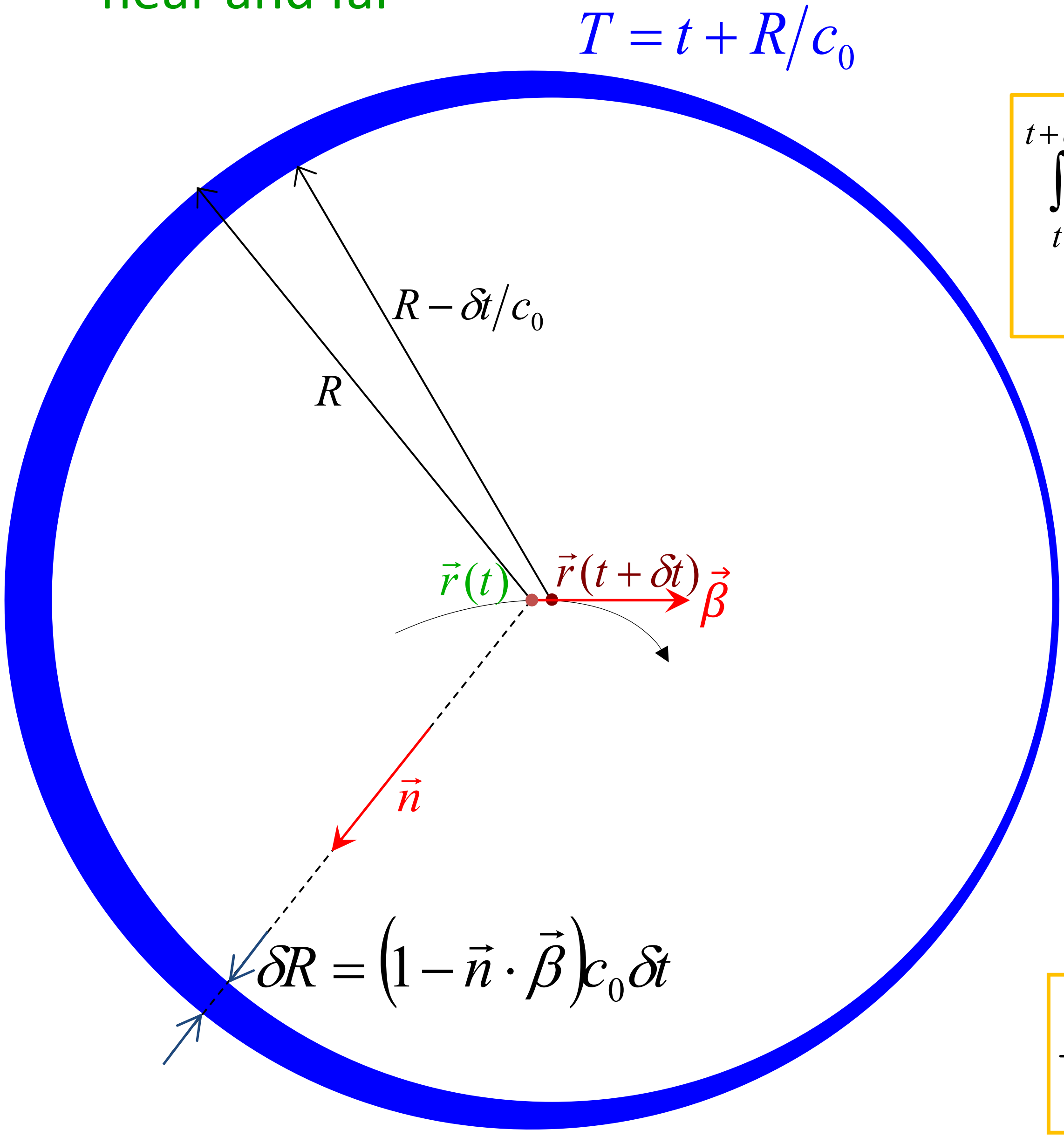


fields

$$\boxed{\begin{aligned}\vec{E}&=\frac{q_0}{4\pi\varepsilon_0}\left(\frac{\vec{n}-\vec{\beta}}{\left(1-\vec{\beta}\cdot\vec{n}\right)^3\gamma^2R^2}+\underbrace{\frac{\vec{n}\times\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}}{c_0\left(1-\vec{\beta}\cdot\vec{n}\right)^3R}}_{\text{radiation term}}\right)_{t'}\\ \vec{B}&=\frac{1}{c_0}\vec{n}\times\vec{E}\end{aligned}}$$

one particle

near and far



$$\boxed{\int_t^{t+\delta t} P_{\text{rad}}\, dt \to \int_V \vec{S}(\vec{r},T)\cdot\vec{e}_r c^{-1} dV}$$

with Poynting flux in far zone

$$\vec{S} \to \frac{\vec{n}}{c_0\varepsilon_0}\left(\frac{q_0}{4\pi R}\right)^2 \frac{\left\|\vec{n}\times\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right\|^2}{\left(1-\vec{\beta}\cdot\vec{n}\right)^6}$$

from radiation term

$$\boxed{\frac{dP_{\text{rad}}}{d\Omega} = R^2\left(1-\vec{n}\cdot\vec{\beta}\right)S(\vec{r},T)}$$

one particle

power loss

$$P_{\text{rad}} = \frac{q_0^2}{6\pi\varepsilon_0 c_0}\gamma^6\left(\dot{\vec{\beta}}^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right) \qquad = q_0 \boxed{E_\parallel} v_\parallel$$

effective longitudinal field (self effect)

---

linear acceleration $\vec{\beta}\times\dot{\vec{\beta}} = \vec{0}$

$$P_{\text{rad}} = \frac{q_0^2}{6\pi\varepsilon_0 c_0}\gamma^6\dot{\vec{\beta}}^2 = \frac{q_0^2}{6\pi\varepsilon_0 c_0}\left(\frac{\dot{\gamma}}{\beta}\right)^2$$

$$\frac{dP_{\text{rad}}}{d\Omega} \propto \frac{\sin^2\vartheta}{(1-\beta\cos\vartheta)^5}$$

$$\boxed{\vartheta = \sphericalangle\vec{\beta},\vec{n} \qquad \phi = \sphericalangle\dot{\vec{\beta}},\vec{n}}$$

---

circular motion $\vec{\beta}\cdot\dot{\vec{\beta}} = 0$

$$\boxed{P_{\text{rad}} = \frac{q_0^2}{6\pi\varepsilon_0 c_0}\gamma^4\dot{\vec{\beta}}^2 = \frac{q_0^2 c_0}{6\pi\varepsilon_0}\frac{\beta^4\gamma^4}{R^2}}$$

$$\frac{dP_{\text{rad}}}{d\Omega} \propto \frac{1}{(1-\beta\cos\vartheta)^3}\left(1-\frac{\sin^2\vartheta\cos^2\phi}{\gamma^2(1-\beta\cos\vartheta)^2}\right)$$

near field

near field

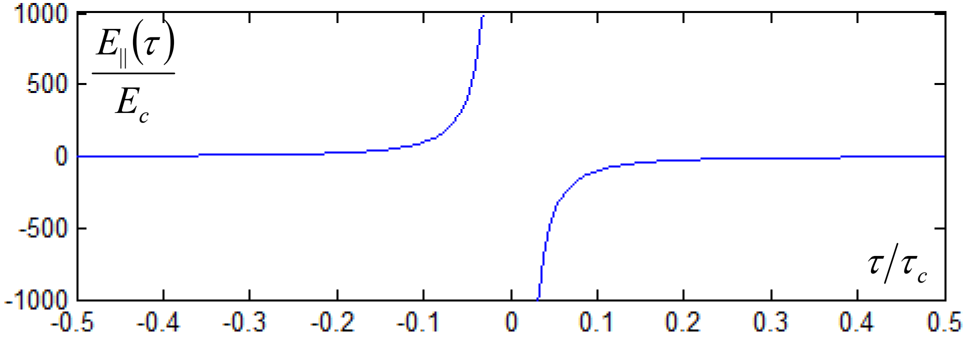
circular motion

longitudinal component on arc R_0,
test particle at $s=vt$, source particle at $s=v(t+\tau)$

$$E_\parallel(\tau) = \vec{e}_\parallel \cdot \vec{E} = \frac{q_0}{4\pi\varepsilon_0}\vec{e}_\parallel \cdot \left(\frac{\vec{n}-\vec{\beta}}{\left(1-\vec{\beta}\cdot\vec{n}\right)^3\gamma^2R^2} + \frac{\vec{n}\times\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}}{c_0\left(1-\vec{\beta}\cdot\vec{n}\right)^3 R}\right)$$

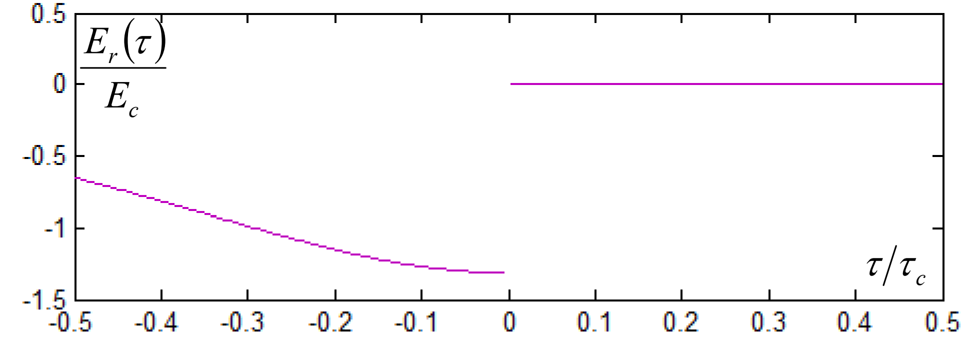
$$E_\parallel(\tau) = \boxed{\frac{q_0}{4\pi\varepsilon_0}\frac{\operatorname{sgn}(-\tau)}{(\gamma\beta c\tau)^2}} + \boxed{E_r(\tau)}$$

singular part as for linear motion
residual part



with $E_c = \frac{q_0\gamma^4}{4\pi\varepsilon_0 R_0^2}$

$\tau_c = R_0/c\gamma^3$

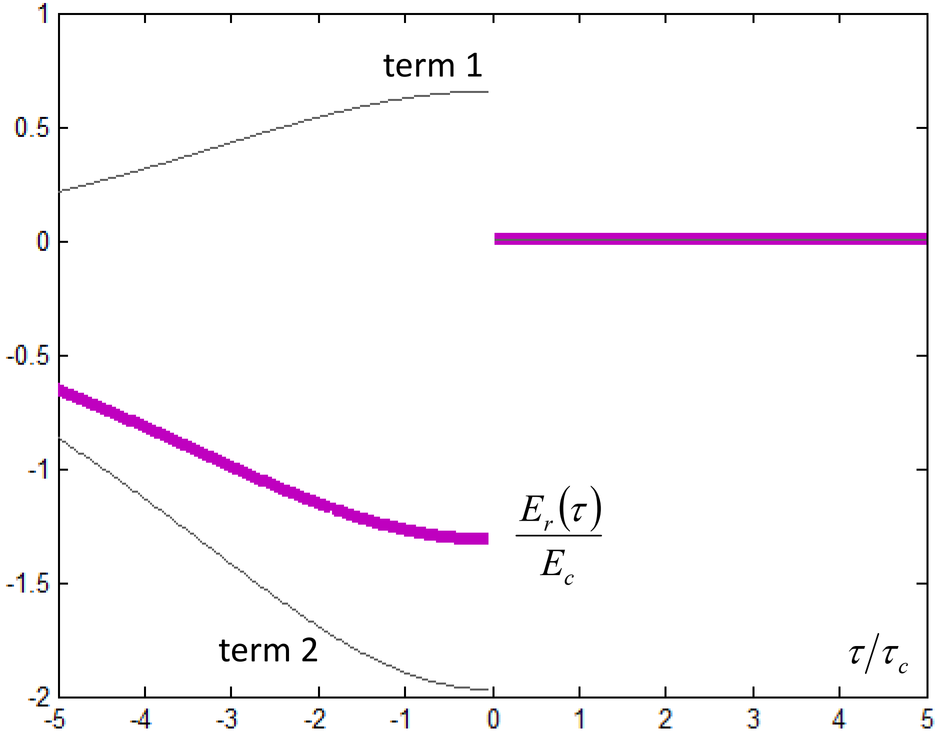


for $R_0 = 1\mathrm{m}$, $\gamma = 10$

near field

residual part

$$E_r(\tau)=\overset{\text{term 1}}{\boxed{\frac{q_0}{4\pi\varepsilon_0}\left\{\left(\frac{\vec{e}_{\parallel}\cdot\left(\vec{n}-\vec{\beta}\right)}{\left(1-\vec{\beta}\cdot\vec{n}\right)^3\gamma^2R^2}\right)_{t'}-\frac{\mathrm{sgn}(-\tau)}{(\gamma\beta c\tau)^2}\right\}}}+\overset{\text{term 2}}{\boxed{\frac{q_0}{4\pi\varepsilon_0}\vec{e}_{\parallel}\cdot\left(\frac{\vec{n}\times\left(\vec{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}}{c_0\left(1-\vec{\beta}\cdot\vec{n}\right)^3R}\right)_{t'}}}$$

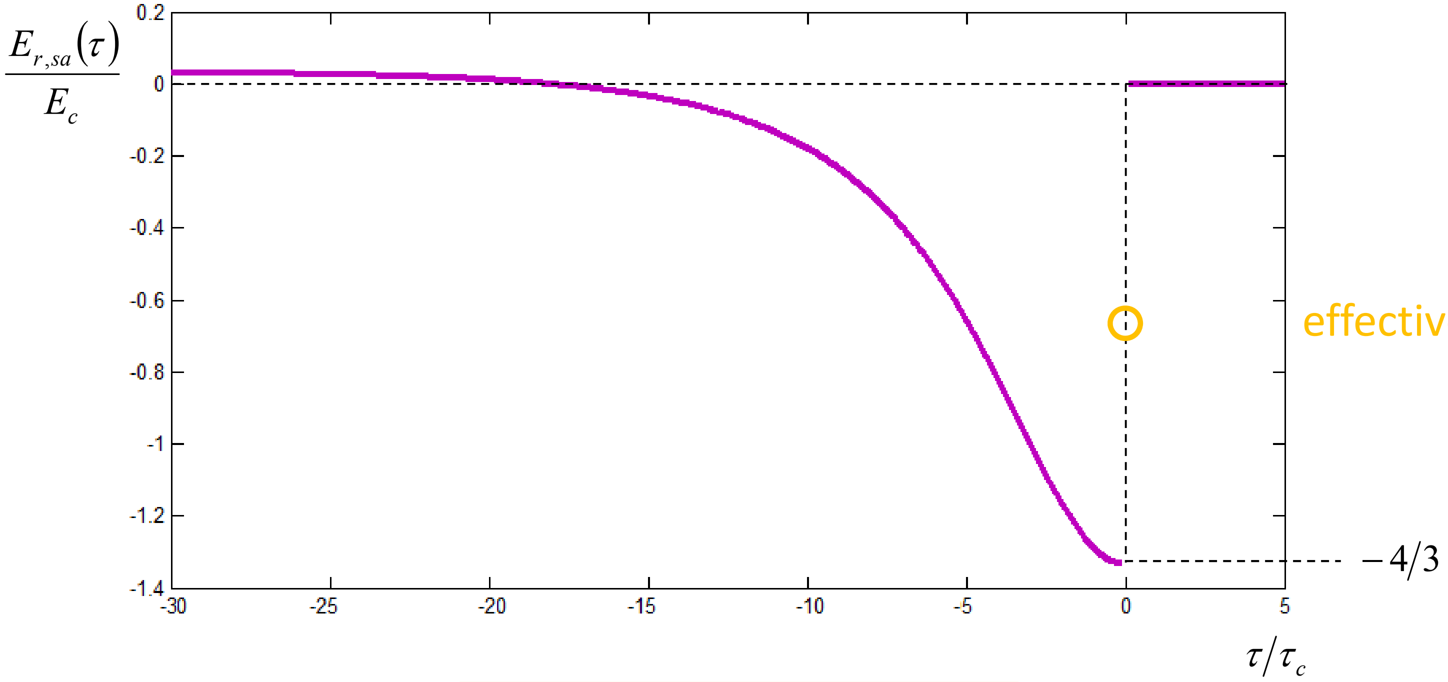


both terms contribute to near-interaction!

small angle approximation of residual part

$$E_{r,sa}\left(\tau<0\right)=\frac{q_0\gamma^4}{4\pi\varepsilon R_0^2}\frac{-32}{4+\phi^2}\frac{\partial}{\partial\phi}\left(\frac{\phi(8+\phi^2)}{(4+\phi^2)(12+\phi^2)}\right)$$

with $\tau=\frac{R_0}{c\gamma^3}\left(\phi/2+\phi^3/24\right)$



effective self field

power loss of one particle:

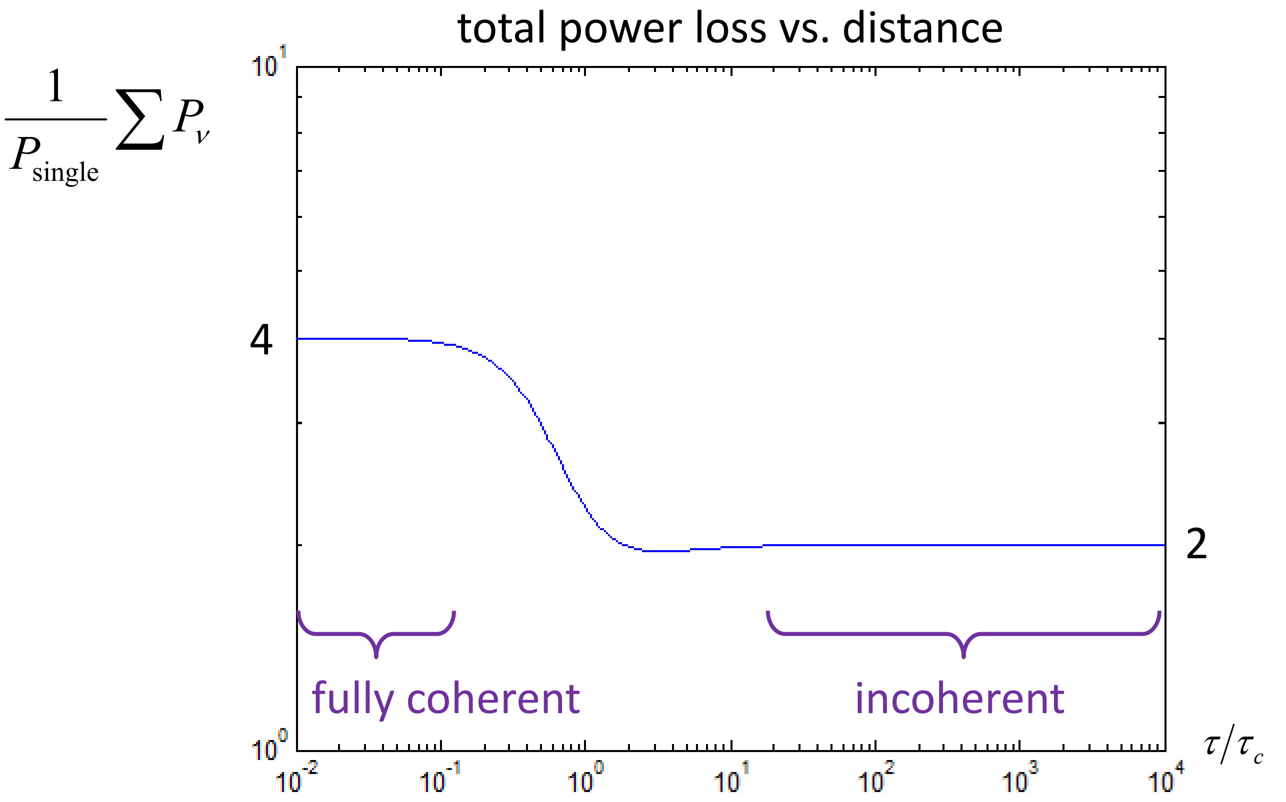
$$\boxed{qc_0\frac{E_{r,sa}\left(0-\right)+E_{r,sa}\left(0+\right)}{2}=P_{rad}}$$

far field radiation

power loss of two particles

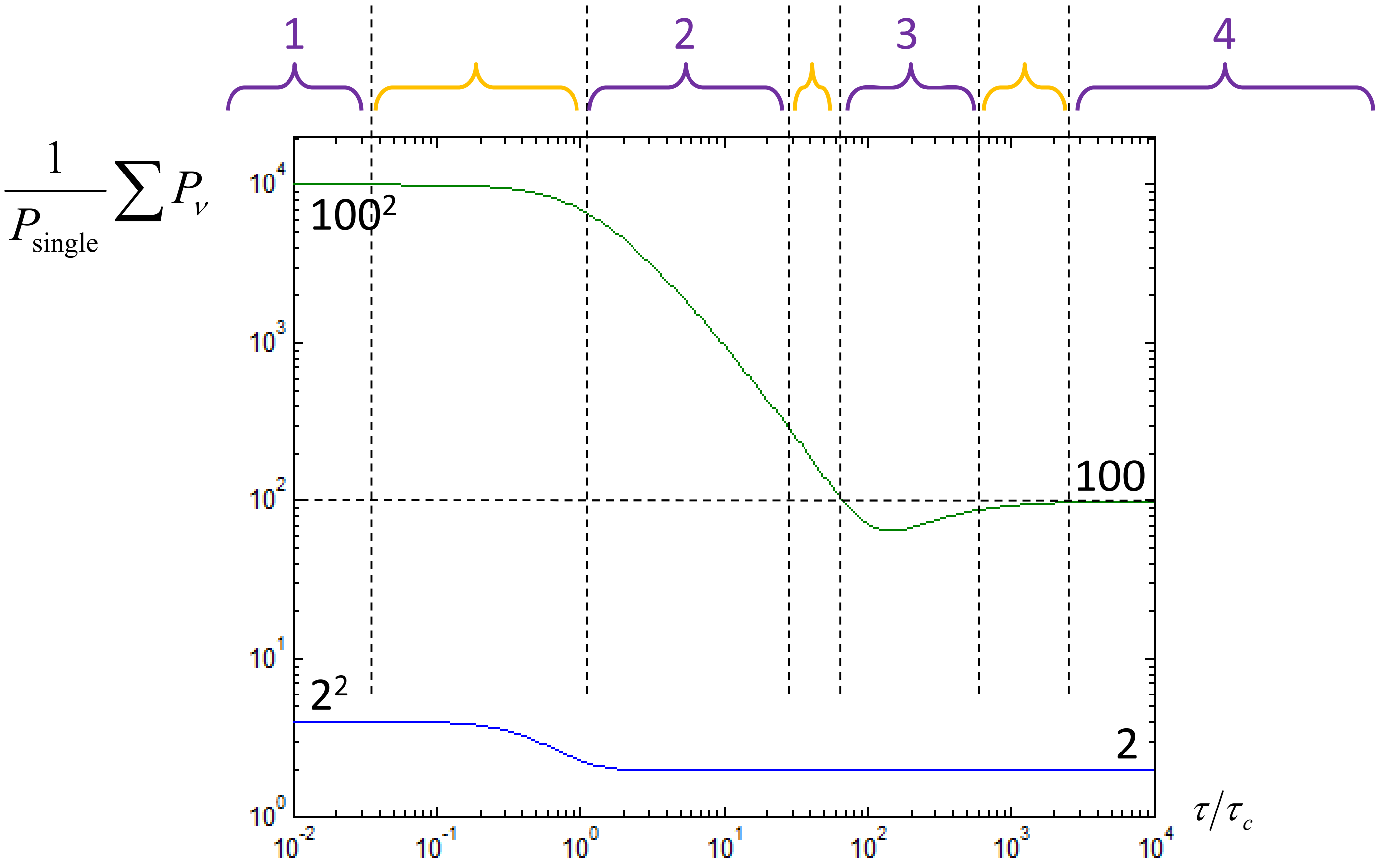
$$P_1 = q_0 c\left(E_r(-\tau) + E_r(0)\right) \qquad \text{head particle}$$

$$P_2 = q_0 c E_r(0) \qquad \text{tail particle}$$



100 particles: total power loss vs. distance (first-last)

1) fully coherent
2) energy independent
3) cool beam
4) incoherent
x) transition



multiple particles

multiple particles

superposition

for instance all (N) particles on the same trajectory $\vec{r}_{s,\nu}(t) = \vec{r}_s(t-\tau_\nu)$

$$\vec{E}(\vec{r},t) = \sum \vec{E}_0(\vec{r},t-\tau_\nu)$$

$$\vec{B}(\vec{r},t) = \sum \vec{B}_0(\vec{r},t-\tau_\nu)$$

random time delay

probability distribution of delay: $p(\tau_1,\tau_2,\cdots,\tau_N)$

independent delay: $p(\tau_1,\tau_2,\cdots,\tau_N) = \prod p_0(\tau_\nu)$

(delay is not independent for systems with longitudinal dispersion + self effects!)

expectation of spectral power density (in principle)

$$\left\{\tilde{S}(\omega)\right\} = \underbrace{\left|F_0(i\omega)\right|^2}_{\text{one particle}} \times \underbrace{\left\{\underbrace{N}_{\text{white}} + \underbrace{N(N-1)\left|P_0(i\omega)\right|^2}_{\text{“coherent”}}\right\}}_{\text{“collective term”}} \quad \text{with} \quad P_0(i\omega) = \int p_0(t)\exp(-i\omega t)dt$$

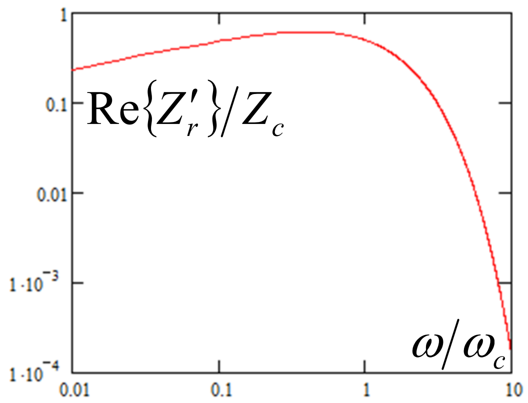
spectral power density (loss) for circular motion

power loss of all particles

$$E_{r,\Sigma}(\tau)=\sum_{\nu}E_r(\tau-\tau_\nu)$$

$$P_{r,\Sigma}=q_0c\sum_{\mu}E_{r,\Sigma}(\tau_\mu)=q_0c\sum_{\nu,\mu}E_r(\tau_\mu-\tau_\nu)$$

impedance of residual part $E_r(\tau)=\frac{q_0}{2\pi}\int Z_r'(i\omega)e^{i\omega\tau}d\omega$



$$\boxed{\{S(\omega)\}=\frac{q_0^2c}{\pi}\mathrm{Re}\{Z_r'(i\omega)\}\times\left\{N+N(N-1)\left|P_0(i\omega)\right|^2\right\}}$$

notation $P=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{S}(\omega)d\omega=\int_{0}^{\infty}S(\omega)d\omega$

impedance of residual part

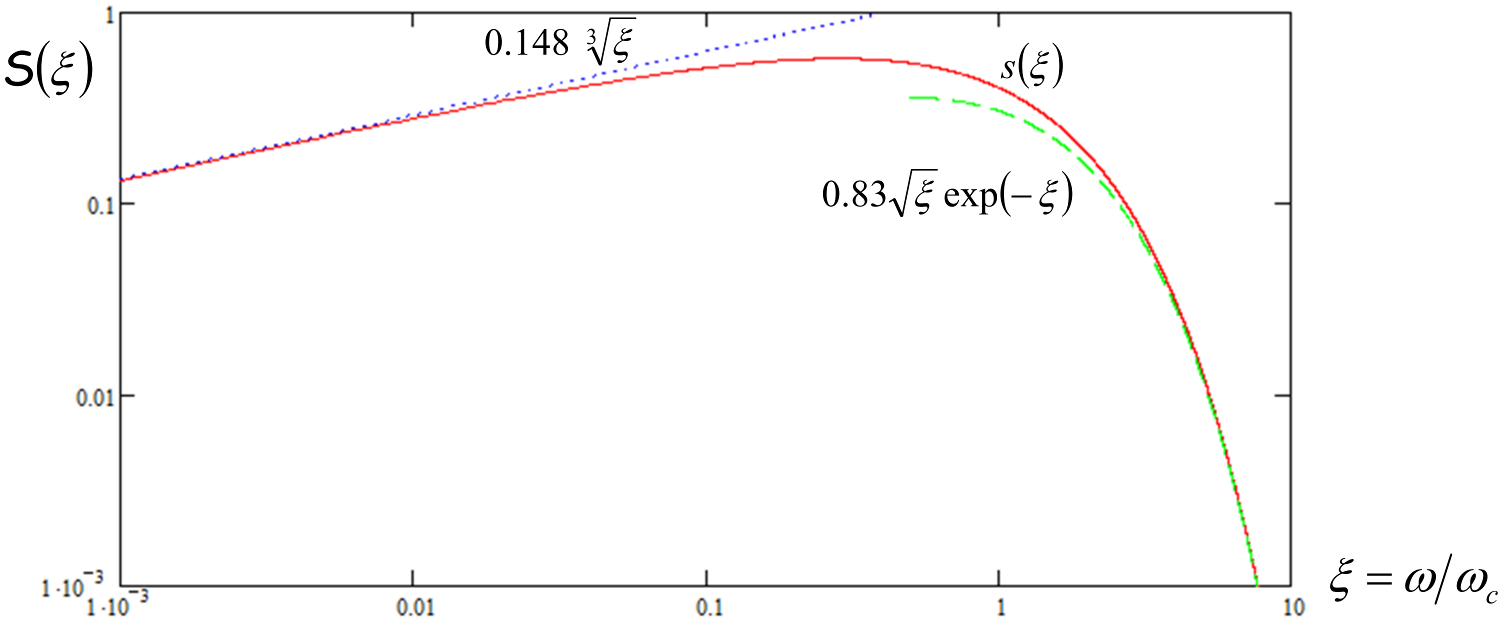
$$\mathrm{Re}\{Z_r'(i\omega)\} = Z_c S\left(\frac{\omega}{\omega_c}\right)$$

with $S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_{\xi}^{\infty} K_{5/3}(x)dx$

and $\int_{0}^{\infty} S(\xi)d\xi = 1$

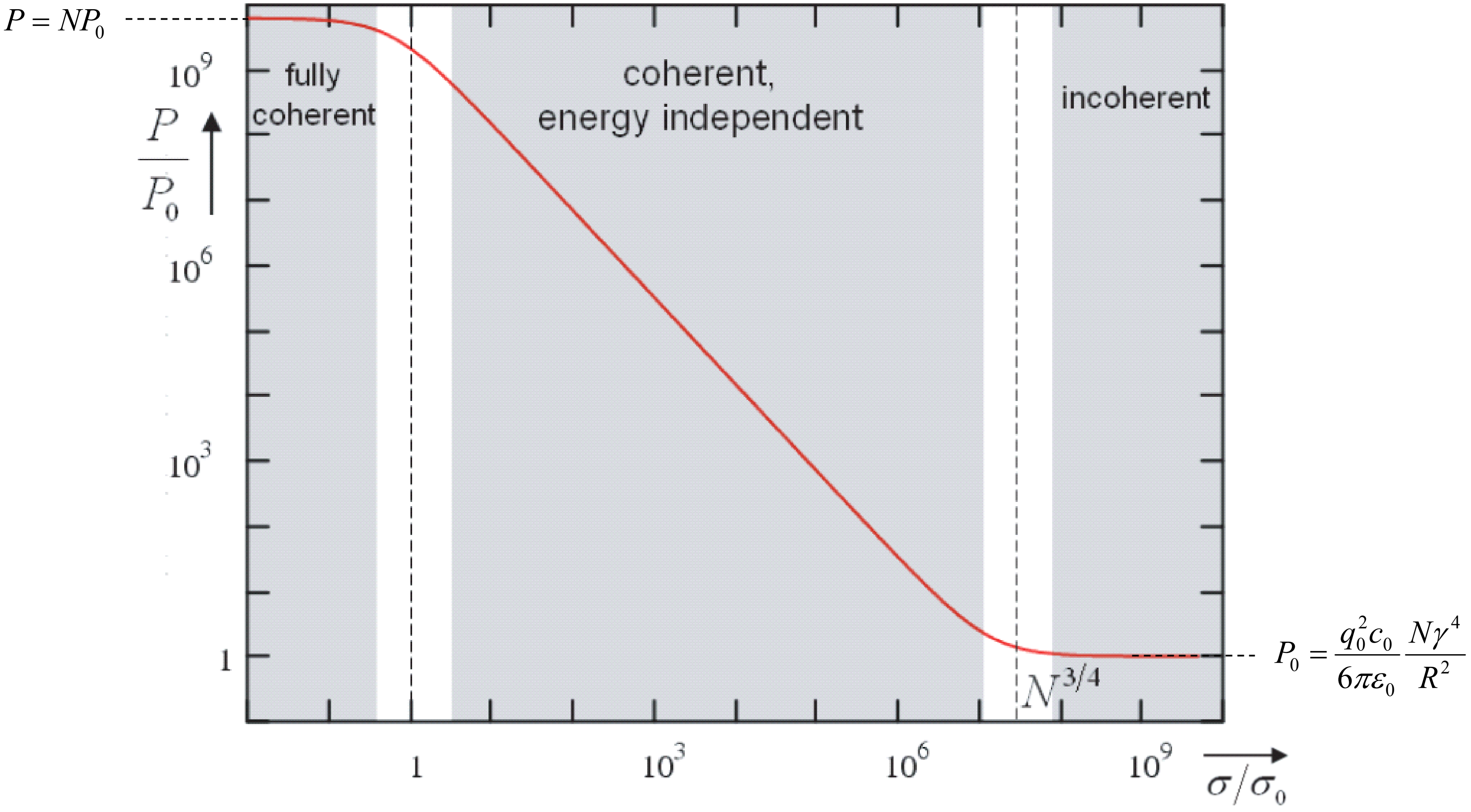
$\omega_c = \frac{3}{2}\frac{\gamma^3 c}{R_0}$ critical frequency

$Z_c = \gamma\frac{Z_0}{R_0}\frac{1}{9}$



circular motion & shielding

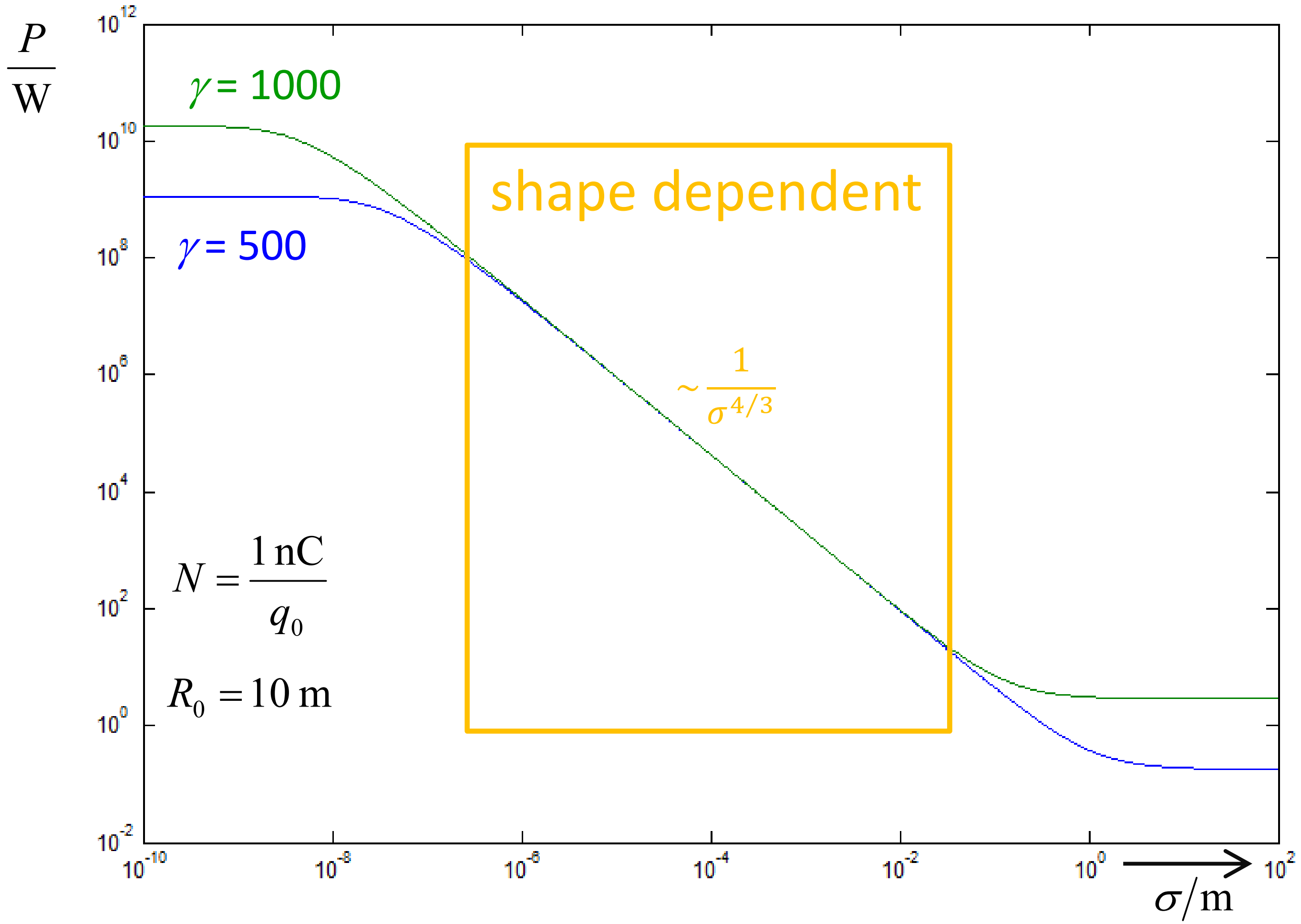
radiated power of Gaussian bunch in circular motion



with $\sigma_0 = \frac{R_0}{\gamma^3}$

energy independent regime

$$1 << \frac{\sigma}{\sigma_0} << N^{3/4} \quad \text{with} \quad \sigma_0 = \frac{R_0}{\gamma^3}$$

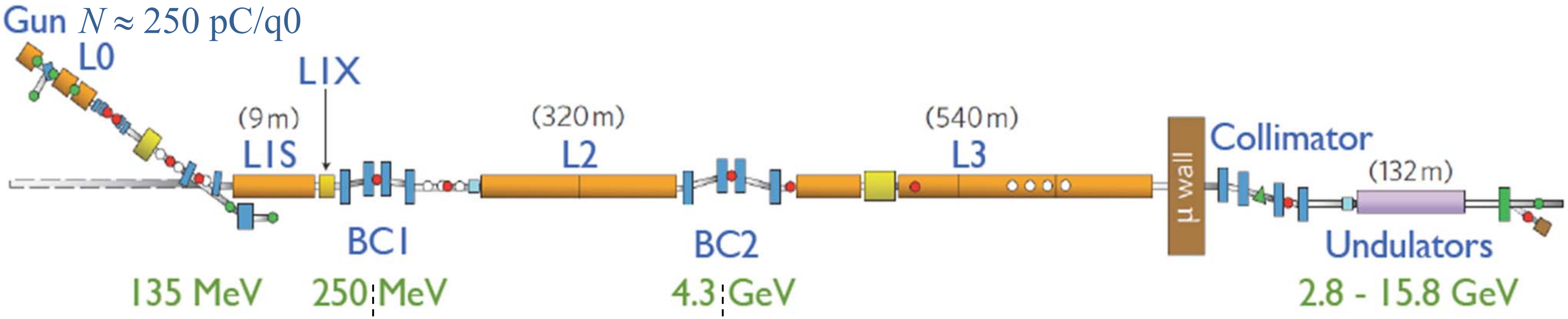


Gaussian shape: $P_{\mathrm{CSR}} = N^2 x \frac{q_0^2 c_0}{\varepsilon_0} \frac{1}{R_0^{2/3} \sigma^{4/3}}$ with $x = \Gamma(5/6) / \left(4\pi^{3/2} 6^{1/3}\right) \approx 0.0279$

energy independent regime $1 << \frac{\sigma}{\sigma_0} << N^{3/4}$ with $\sigma_0 = \frac{R_0}{\gamma^3}$

f.i. LCLS 2009

curvature in BC magnets $R_0 \sim 10\,\mathrm{m}$



$\sigma_{\mathrm{bunch}} \approx 0.83\,\mathrm{mm}$

$\sigma_{\mathrm{bunch}} \approx 0.19\,\mathrm{mm}$

$\sigma_{\mathrm{bunch}} \approx 22\,\mu\mathrm{m}$

$$\frac{\sigma_{\mathrm{bunch}}}{\sigma_0} \approx \boxed{10^4 \quad 2.2\times10^3} \qquad \boxed{11\times10^6 \quad 1.3\times10^6}$$

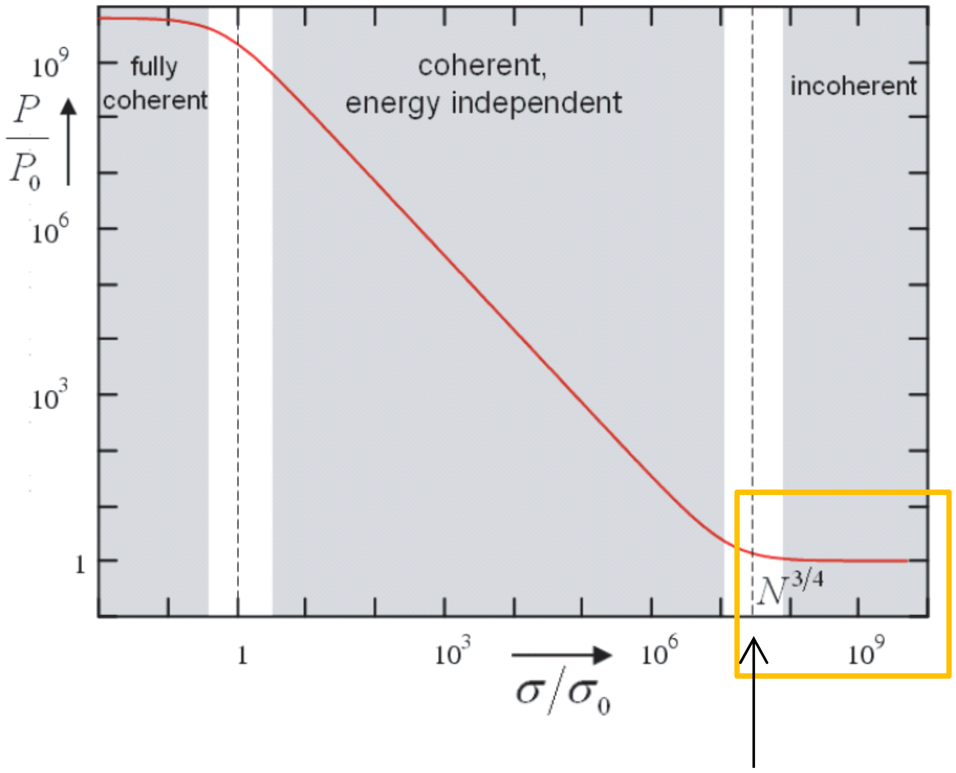
$N^{3/4} \approx 8\times10^6$

$\uparrow$

coherent "≈" incoherent

circular motion & shielding

incoherent regime $N^{3/4} << \dfrac{\sigma}{\sigma_0}$ with $\sigma_0 = \dfrac{R_0}{\gamma^3}$



$$\sigma_i = N^{3/4}\frac{R_0}{\gamma^3}$$

shape independent

$$P_{\text{ISR}} = P_0 = \frac{q_0^2 c_0}{6\pi\varepsilon_0}\frac{N\gamma^4}{R_0^2}$$

f.i. HERA-E

$$\left.\begin{aligned} \gamma &\approx 50000 \\ R_0 &\approx 500 \\ N &\approx 10^{10} \end{aligned}\right\} \sigma_i \approx 0.13\,\text{mm}$$

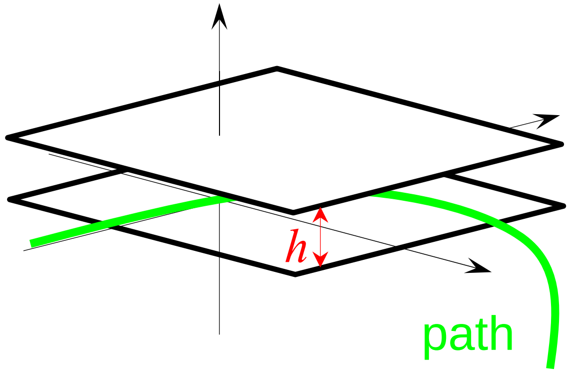
$$\sigma_{\text{bunch}} > 3\,\text{mm}$$

f.i. PETRA

$$\left.\begin{aligned} \gamma &\approx 12000 \\ R_0 &\approx 200 \\ N &\approx 10^{10} \end{aligned}\right\} \sigma_i \approx 3.7\,\text{mm}$$

$$\sigma_{\text{bunch}} \approx 13\,\text{mm}$$

a simple shielding model = parallel conducting planes



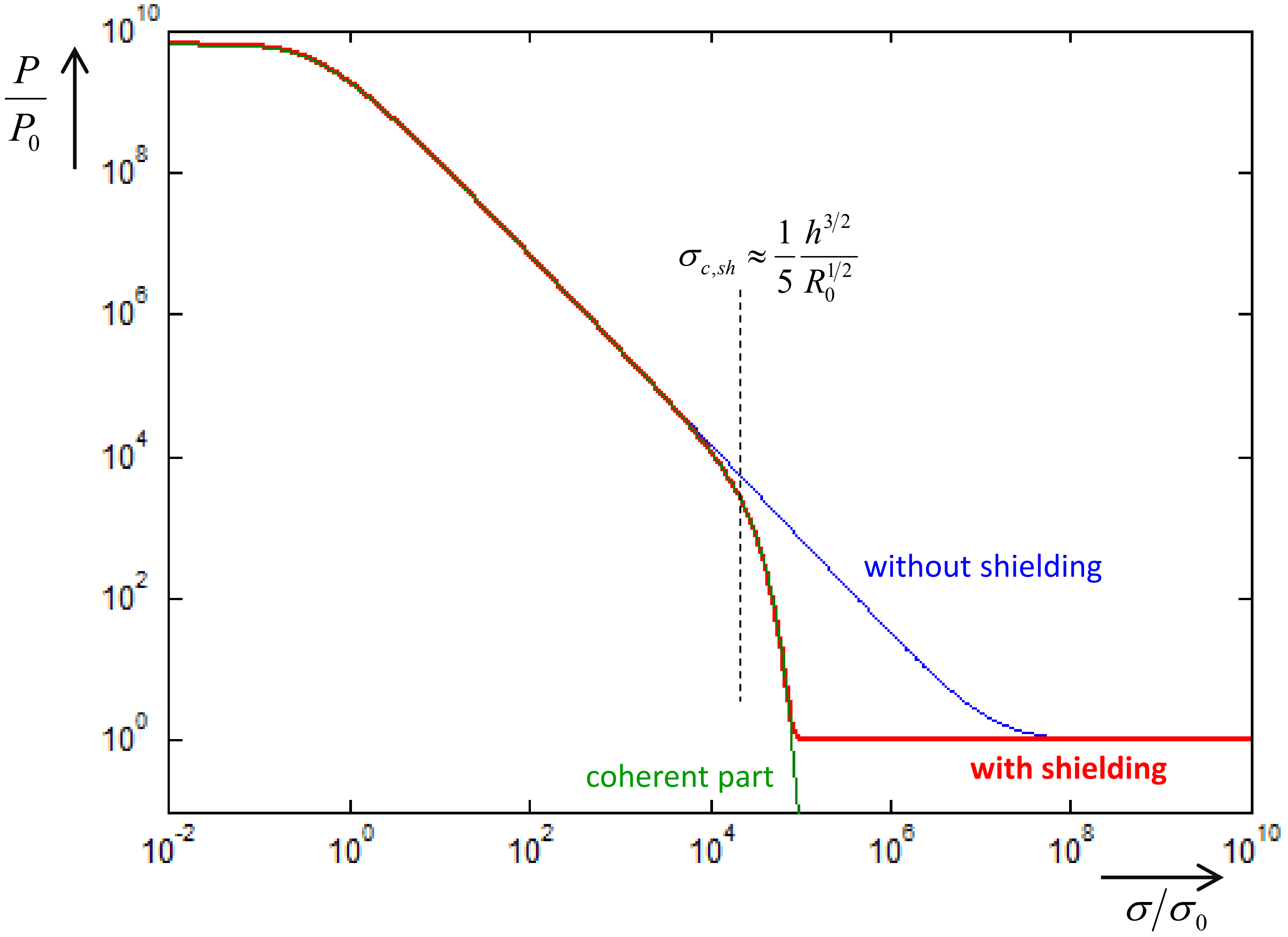
real part of impedance is short-circuited below a cutoff frequency $\omega_{c,sh} \approx 3c\sqrt{\frac{R_0}{h^3}}$

$$\mathrm{Re}\left\{Z_r'\left(\omega < \omega_{c,sh}\right)\right\} \approx 0$$

spectral power density:

$$\left\{S(\omega)\right\} = \frac{q_0^2 c}{\pi} \mathrm{Re}\left\{Z_r'(i\omega)\right\} \times \left\{N + N(N-1)\left|P_0(i\omega)\right|^2\right\}$$

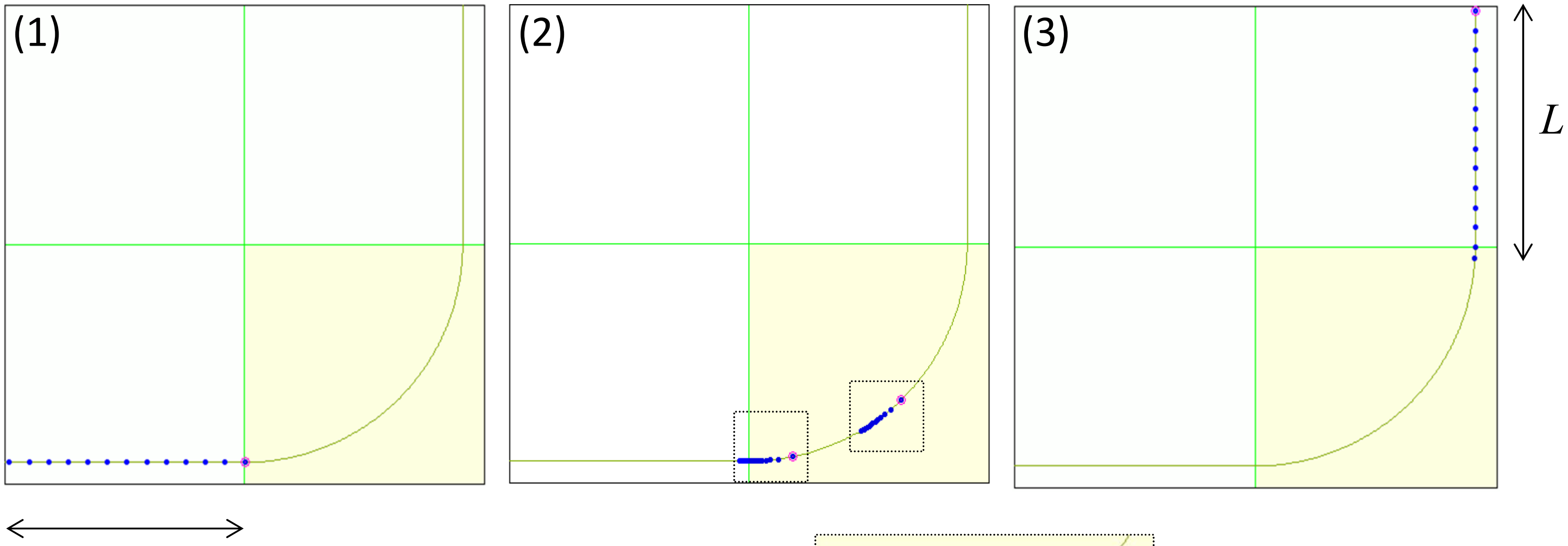
a simple shielding model = parallel conducting planes



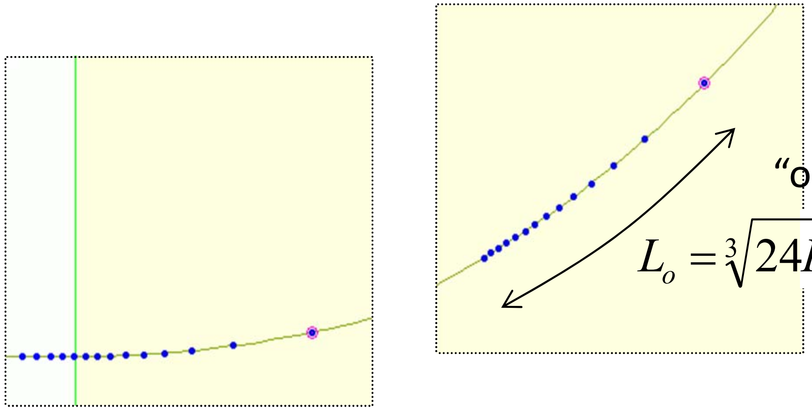
general trajectories and transients

retarded particles, seen from head particle

seen from the head (1) before, (2) in and (3) after a 90 degree bending magnet



$$L = \frac{l}{1-\beta} \approx 2\gamma^2 l$$



"overtaking length"

$$L_o = \sqrt[3]{24R_0^2 l} \quad \text{for} \quad l >> R_0/\gamma^3$$

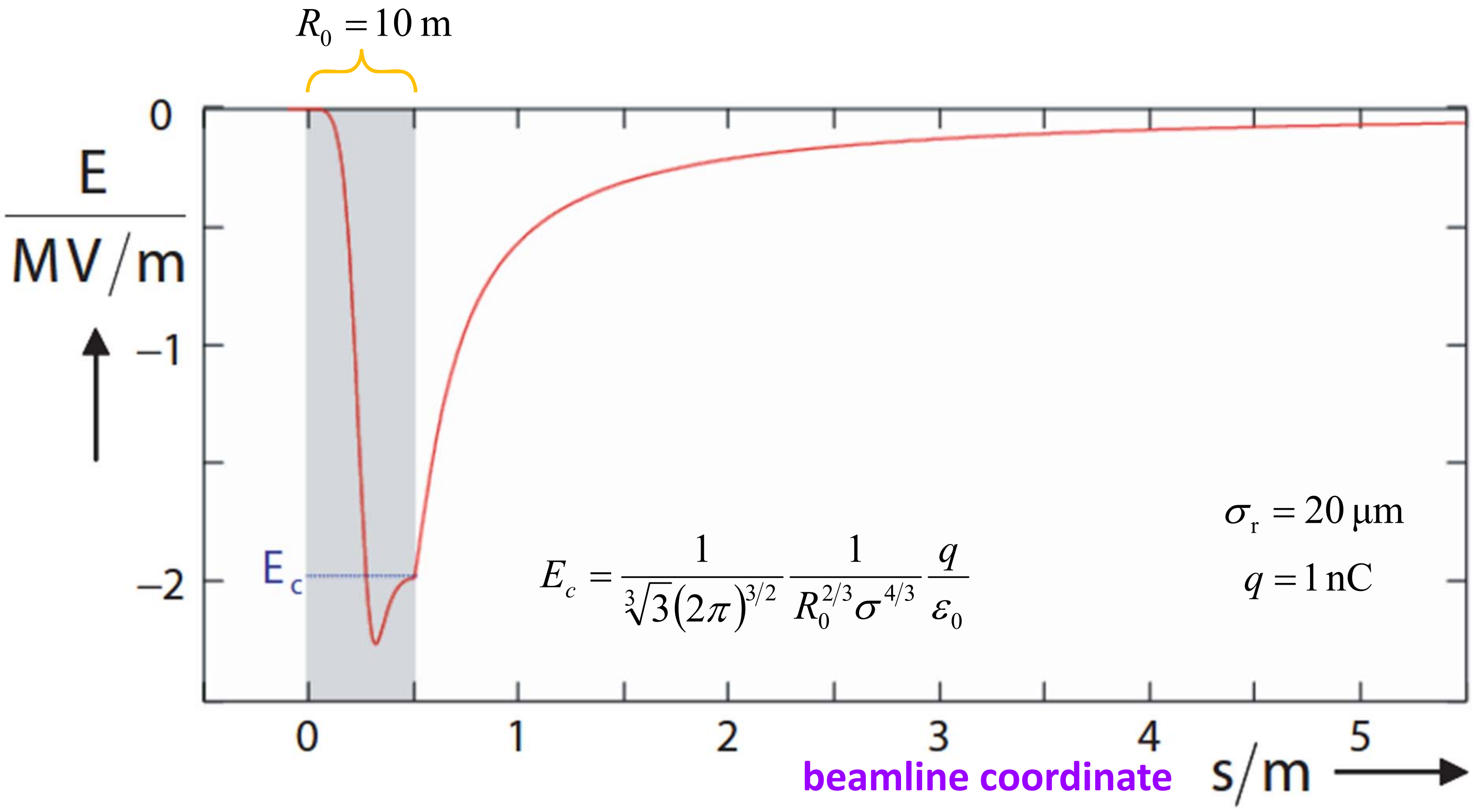
for instance:

$l = 10\ \mu\text{m},\ \gamma = 1000 \rightarrow L = 20\ \text{m}$

$l = 10\ \mu\text{m},\ \gamma = 1000,\ R_0 = 10\ \text{m} \rightarrow L_o = 29\ \text{cm},\ l >> R_0/\gamma^3 = 10\ \text{nm}$

longitudinal field in the center of a Gaussian spherical bunch

that travels through a bending magnet



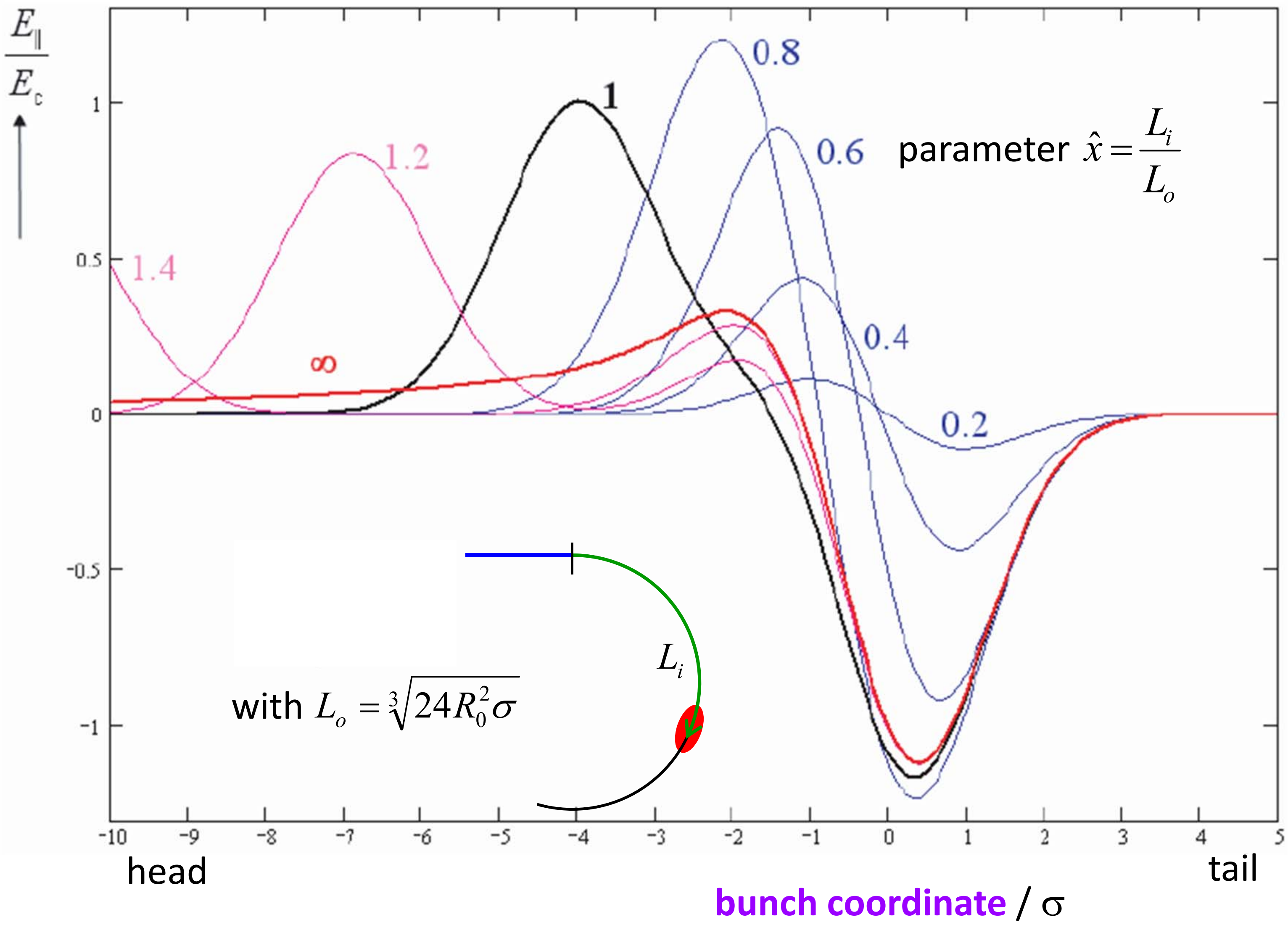
$$E_c = \frac{1}{\sqrt[3]{3}(2\pi)^{3/2}} \frac{1}{R_0^{2/3}\sigma^{4/3}} \frac{q}{\varepsilon_0}$$

$\sigma_\mathrm{r} = 20\,\mu\mathrm{m}$

$q = 1\,\mathrm{nC}$

note: $E \equiv E_r$ (all $\frac{\vec{e}_r}{r^2}$ contributions cancel for the center of the distribution)

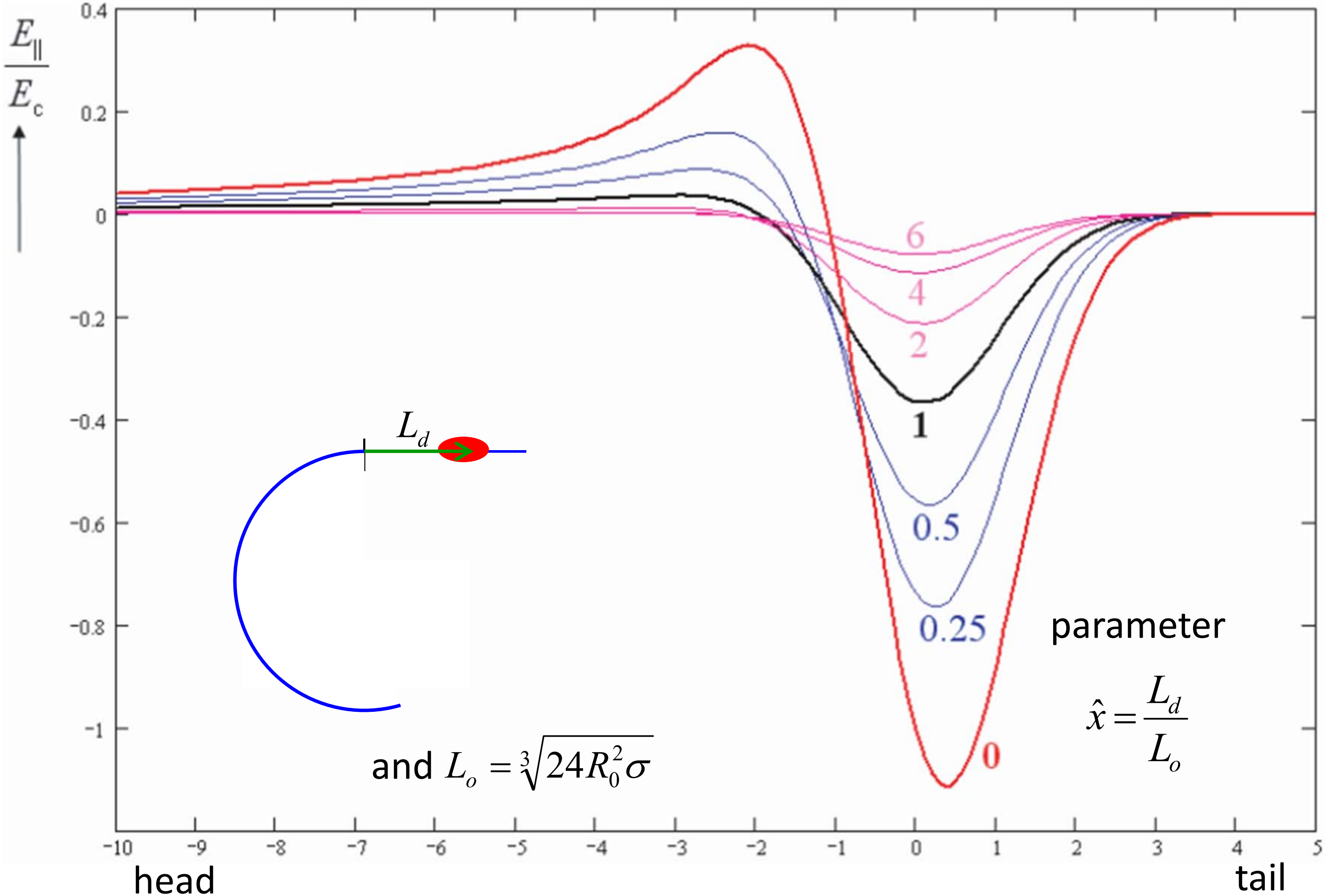
transient CSR field, injection of a Gaussian bunch



parameter $\hat{x} = \frac{L_i}{L_o}$

with $L_o = \sqrt[3]{24 R_0^2 \sigma}$

transient CSR field, ejection of a Gaussian bunch



and $L_o = \sqrt[3]{24R_0^2\sigma}$

parameter

$$\hat{x} = \frac{L_d}{L_o}$$

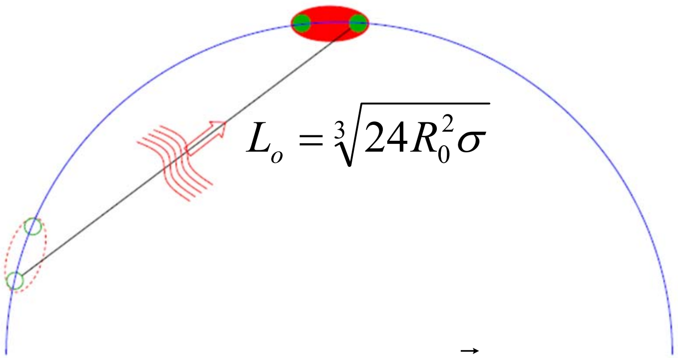
bunch coordinate / σ

some remarks

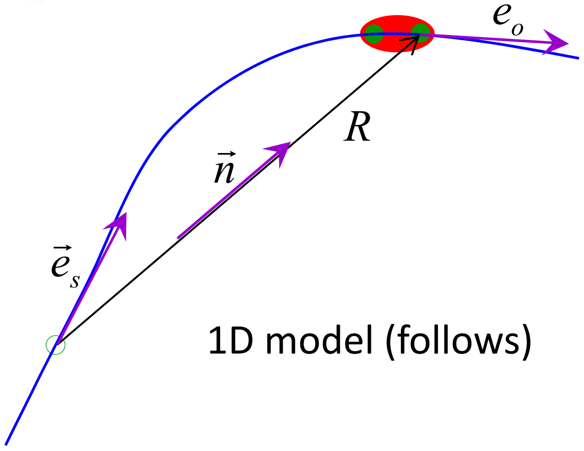
sloppy notation: “residual” part (of longitudinal E-field) is called CSR-field

for free space: interaction by CSR-field (CSR-interaction) is tail-to-head interaction

significant part of interaction is over long distance
→ weak sensitivity on transverse offset
→ 1D model

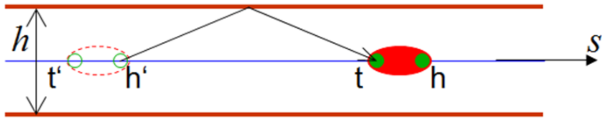


energy independent model for $\sigma >> R_0 / \gamma^3$

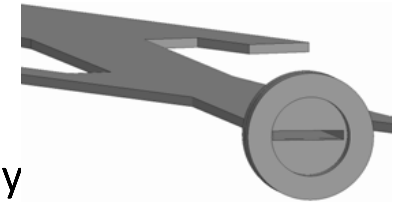


1D model (follows)

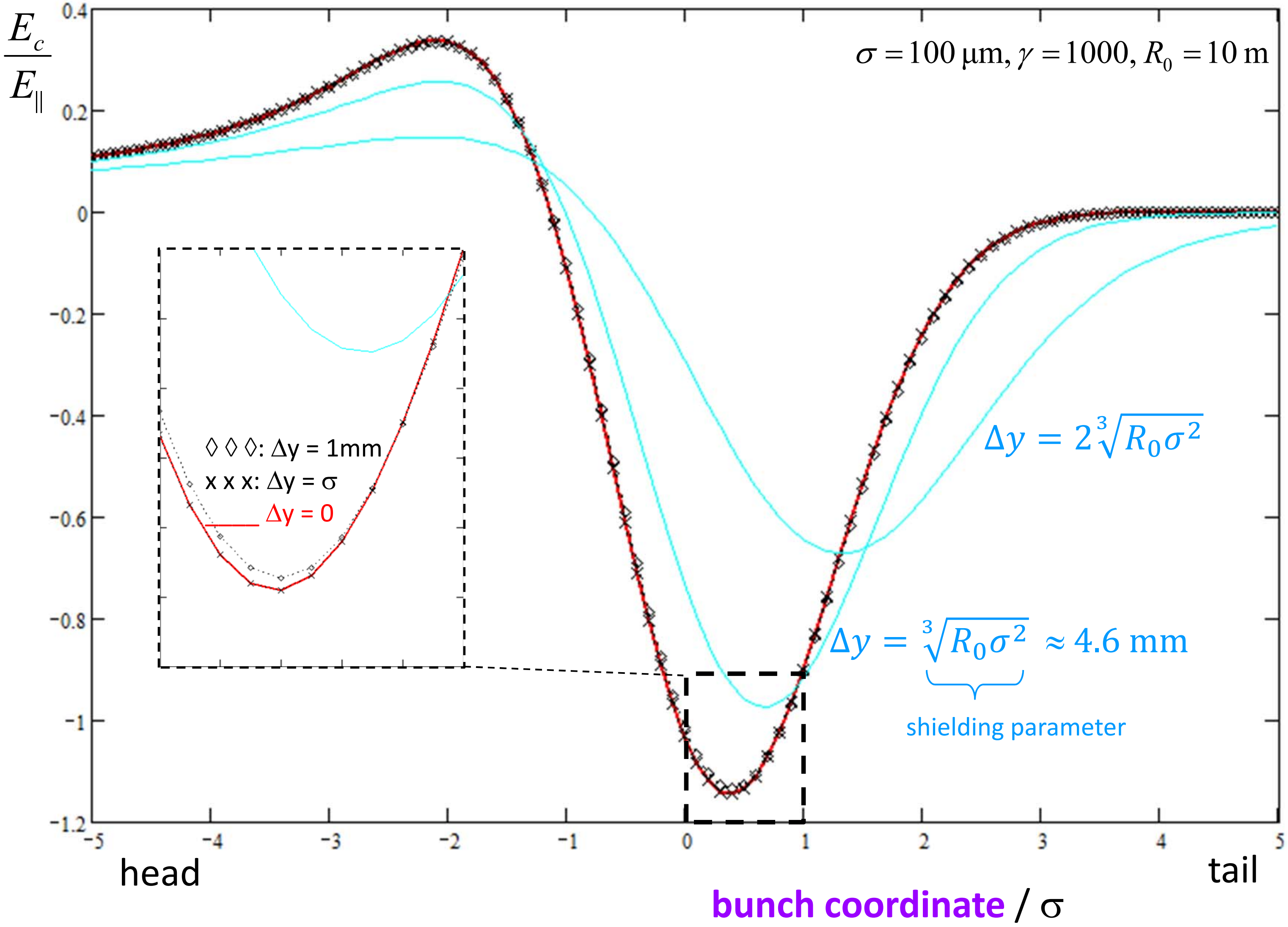
closed environment (metallic chambers):
tail-to-head and head-to-tail interaction



influence of finite conductivity

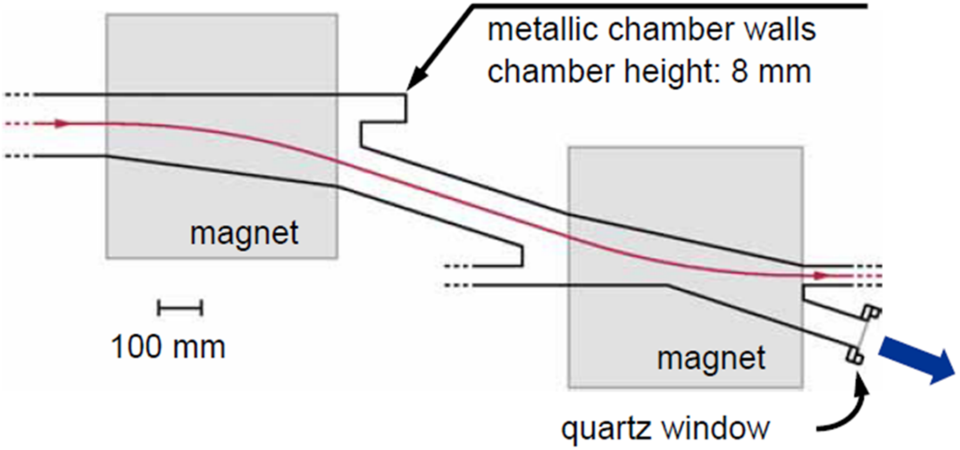


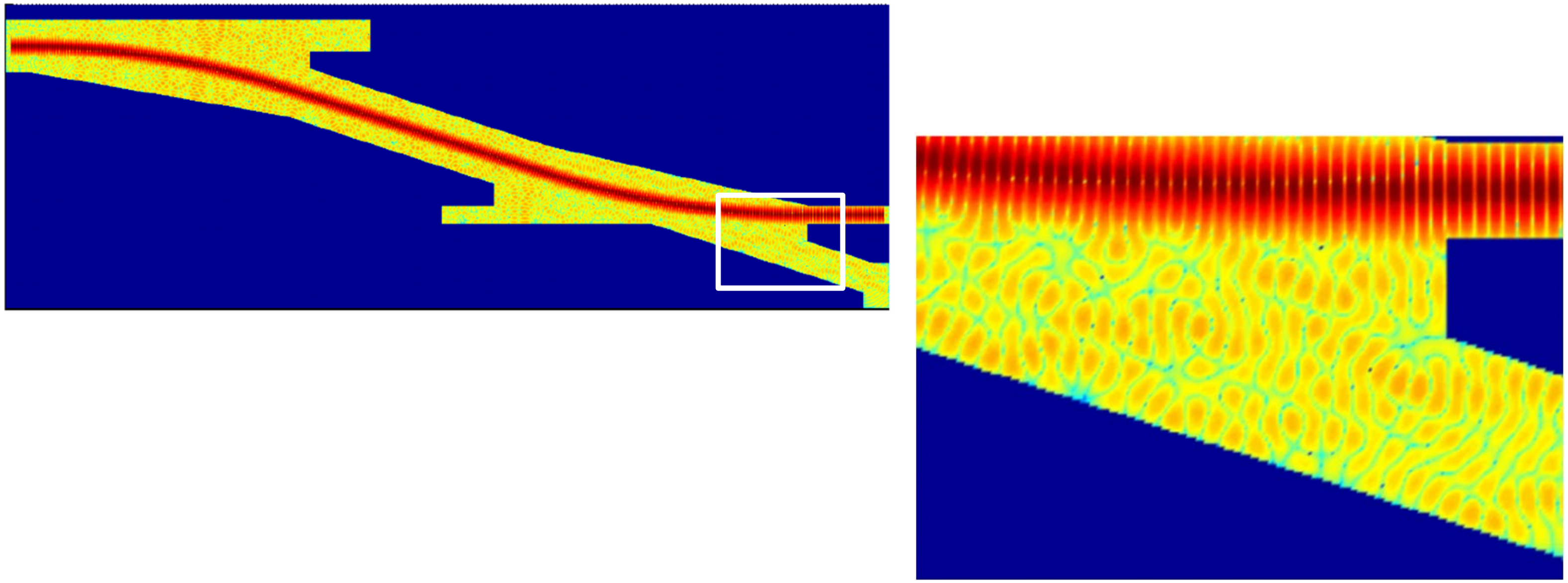
vertical offset dependency: Gaussian bunch in circular motion



bunch coordinate / σ

closed environment, long range





projected model

projected model

phase space

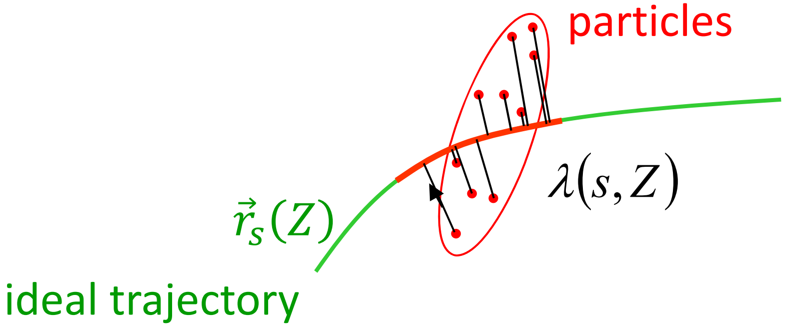
$$\mathbf{X}(\underset{\substack{\uparrow\\ \text{beamline coordinate}}}{Z}) = \begin{bmatrix} x \\ x' \\ y \\ y' \\ s \\ E \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} x \\ x' \\ y \\ y' \end{matrix}}\right\} \text{transverse coordinates (local system)} \\ \left.\vphantom{\begin{matrix} s \\ E \end{matrix}}\right\} \text{longitudinal coordinates (bunch coordinate} \sim \text{time, energy)} \end{matrix}$$

projected model, equation of motion

$$\frac{d}{dZ}\mathbf{X}_{\overset{\substack{\text{particle index}\\ \downarrow}}{\nu}} = \underbrace{f_{\text{ext}}(\mathbf{X}_\nu, Z)}_{\text{external forces}} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \nu \boxed{F_{\text{CSR}}(s_\nu, Z)} \end{bmatrix} + \underbrace{\mathbf{F}_{\text{SC}}(\mathbf{X}_\nu)}_{\text{collective SC-self-force}} + \cdots$$

collective longitudinal CSR-self-force

line charge density



projected particles to ideal trajectory (neglect all coordinates but s)
generate continuous function $\lambda(s,Z)$ by binning and smoothing techniques

collective longitudinal self-force

$$F(s_\nu, Z) = \int \lambda(s_\nu - x, Z) \underbrace{K(x, Z)}_{\text{CSR kernel}} dx$$

in principle

$$F(s_\nu, Z) = \int \lambda'(s_\nu - x(\tilde{x}), Z) \tilde{K}(\tilde{x}, Z) d\tilde{x}$$

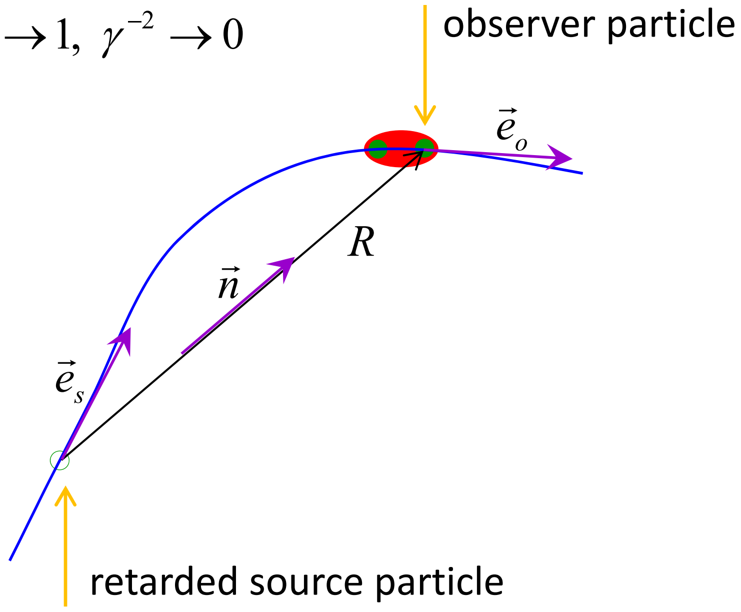
in practice, with retarded source position $\tilde{x}$

projected model

kernel function

$$4\pi\varepsilon\tilde{K}(x,Z)=\frac{\beta\vec{n}\cdot(\vec{e}_s-\vec{e}_o)-\beta^2(1-\vec{e}_s\vec{e}_o)-\gamma^{-2}}{R}-\gamma^{-2}\frac{1-\beta\vec{e}_s\cdot\vec{n}}{s+\beta R}$$

energy independent approximation with $\beta\rightarrow 1,\ \gamma^{-2}\rightarrow 0$



approximations

no transverse forces

no transverse beam dimensions

} partial compensation of transverse effects, see SLAC-PUB-7181

local rigid bunch approximation $\lambda(s,Z)\equiv\lambda(s)$

only residual part

??? add collective SC forces

implementations

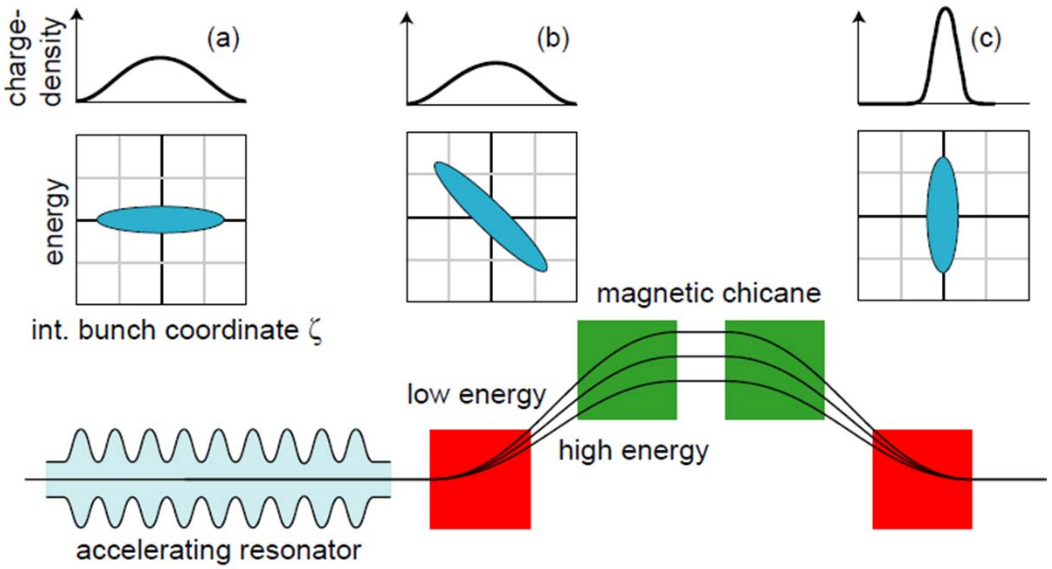
(incomplete list)

Elegant	only one magnet
CSRtrack	projected model, alternatively 2.5D model
Impact-T	+ collective SC forces
Bmad	
GPT	

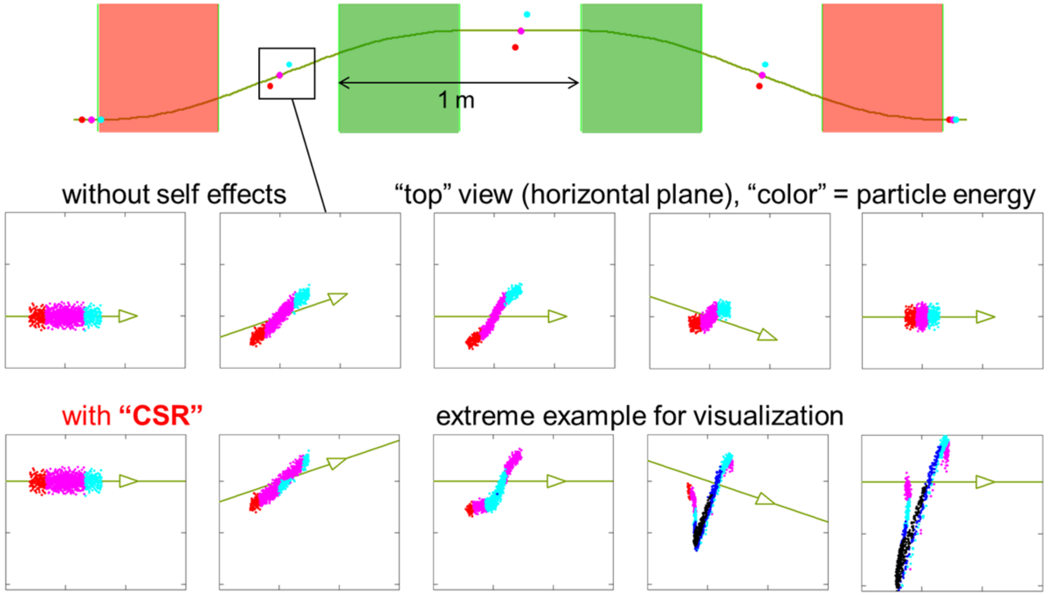
bunch compressor

4 magnet bunch compressor

in principle

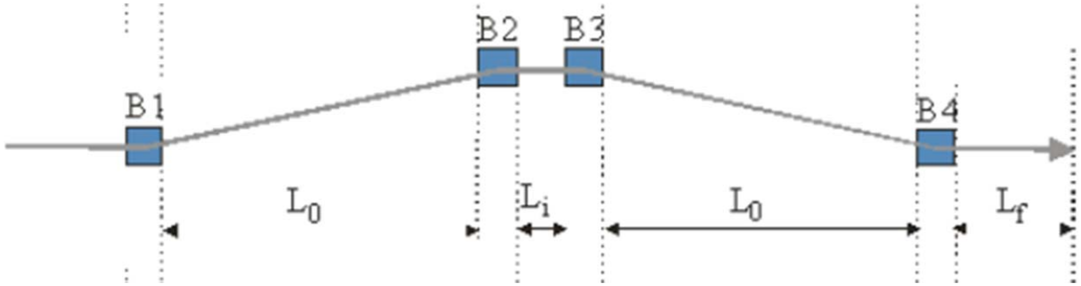


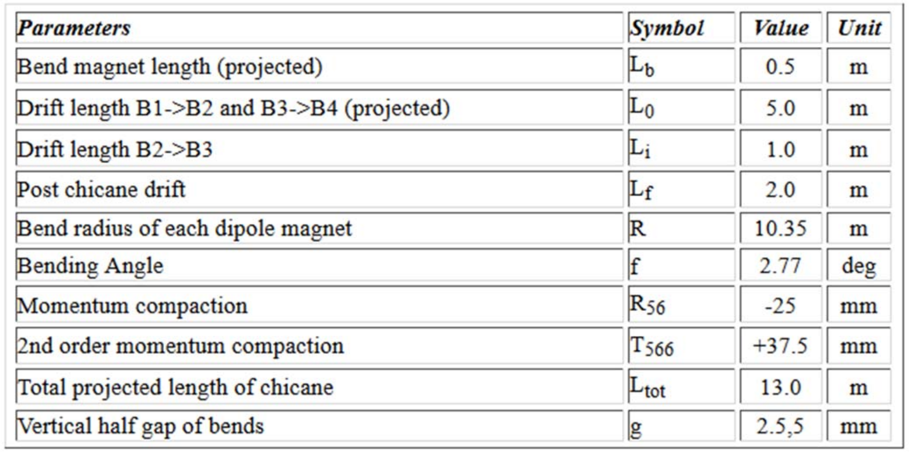
growth of emittance and energy spread



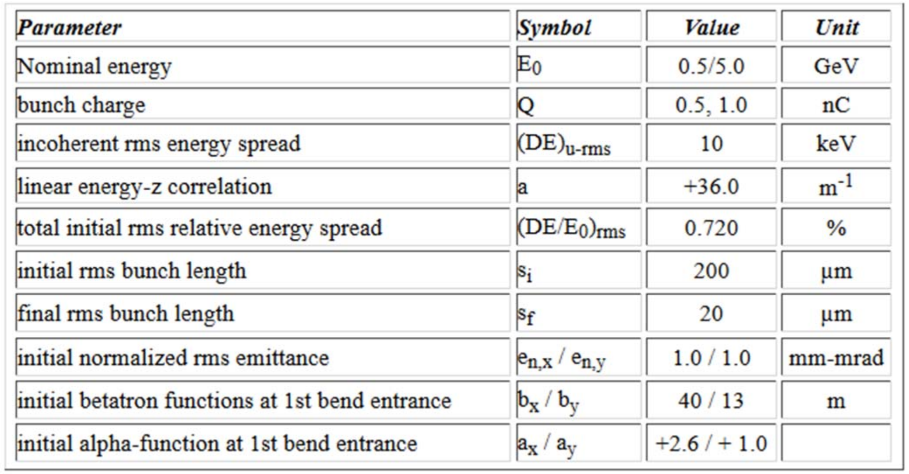
“typical” beam dimensions in a “typical” bunch compressor

example: benchmark BC from CSR workshop 2002
http://www.desy.de/csr/csr_workshop_2002/csr_workshop_2002_index.html



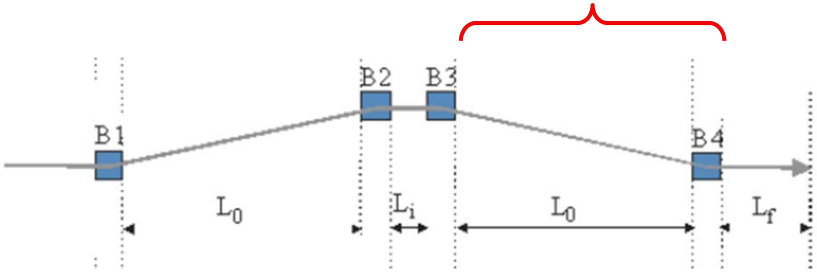


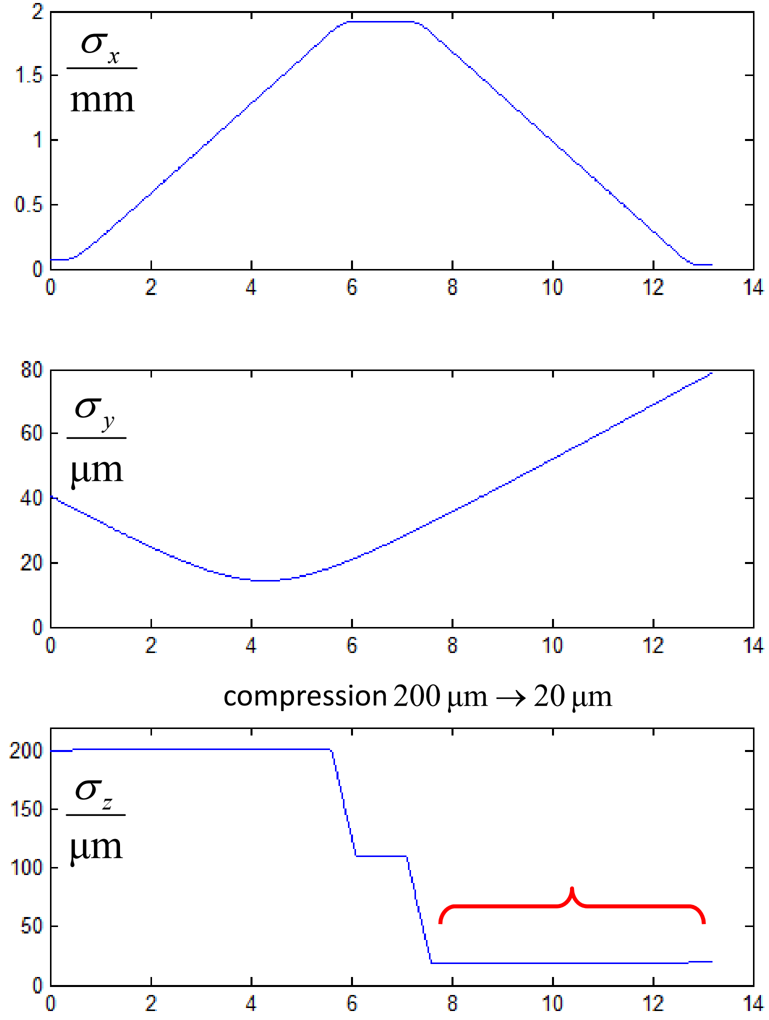
Parameters	*Symbol*	*Value*	*Unit*
Bend magnet length (projected)	L_b	0.5	m
Drift length B1->B2 and B3->B4 (projected)	L_0	5.0	m
Drift length B2->B3	L_i	1.0	m
Post chicane drift	L_f	2.0	m
Bend radius of each dipole magnet	R	10.35	m
Bending Angle	f	2.77	deg
Momentum compaction	R_{56}	-25	mm
2nd order momentum compaction	T_{566}	+37.5	mm
Total projected length of chicane	L_{tot}	13.0	m
Vertical half gap of bends	g	2.5,5	mm



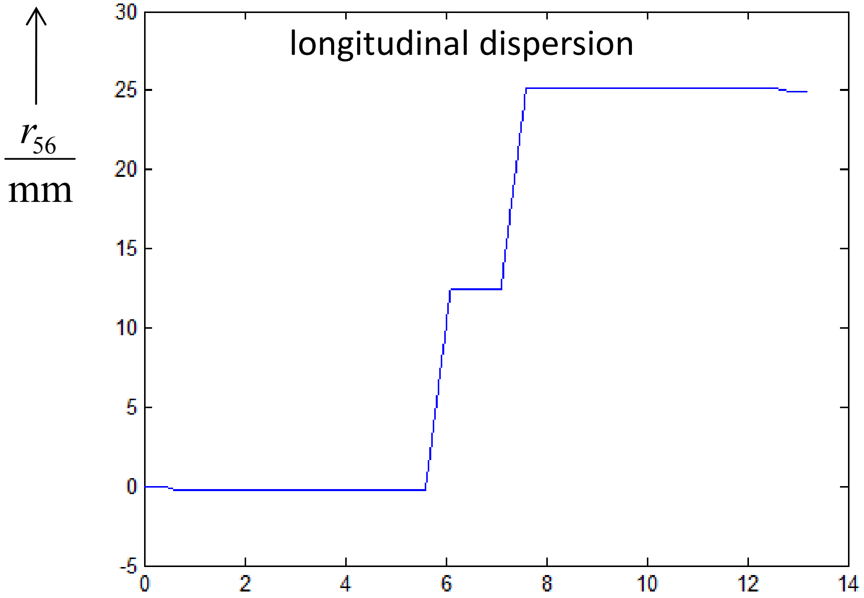
Parameter	*Symbol*	*Value*	*Unit*
Nominal energy	E_0	0.5/5.0	GeV
bunch charge	Q	0.5, 1.0	nC
incoherent rms energy spread	$(DE)_{u\text{-}rms}$	10	keV
linear energy-z correlation	a	+36.0	m^{-1}
total initial rms relative energy spread	$(DE/E_0)_{rms}$	0.720	%
initial rms bunch length	s_i	200	µm
final rms bunch length	s_f	20	µm
initial normalized rms emittance	$e_{n,x}$ / $e_{n,y}$	1.0 / 1.0	mm-mrad
initial betatron functions at 1st bend entrance	b_x / b_y	40 / 13	m
initial alpha-function at 1st bend entrance	a_x / a_y	+2.6 / + 1.0	

"typical" beam dimensions in a "typical" bunch compressor





compression 200 µm → 20 µm



beamline coordinate / m →

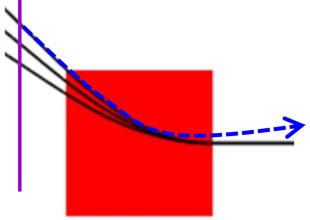
beamline coordinate / m →

usually (but not always) the bunch is short after magnet3

simple model for emittance growth in last magnet

assumption: (1) neglect all self effects before last magnet
(2) represent total energy loss in chicane by discrete loss *ΔE* before magnet

$$X = [x \quad x' \quad y \quad y' \quad s \quad E]^t \rightarrow [x \quad x' \quad y \quad y' \quad s \quad E + \Delta E]^t$$



growth of emittance $\varepsilon = \sqrt{\varepsilon_0^2 + \varepsilon_0 \beta \left(\phi\, \Delta E_{rms} / E_{\mathrm{ref}}\right)}$

with $\varepsilon_0, \varepsilon$	emittance before / after magnet
β	beta function at magnet (lattice)
ϕ	deflection angle
ΔE_{rms}	energy spread of particle bunch (slice or full bunch)
E_{ref}	energy of particle bunch

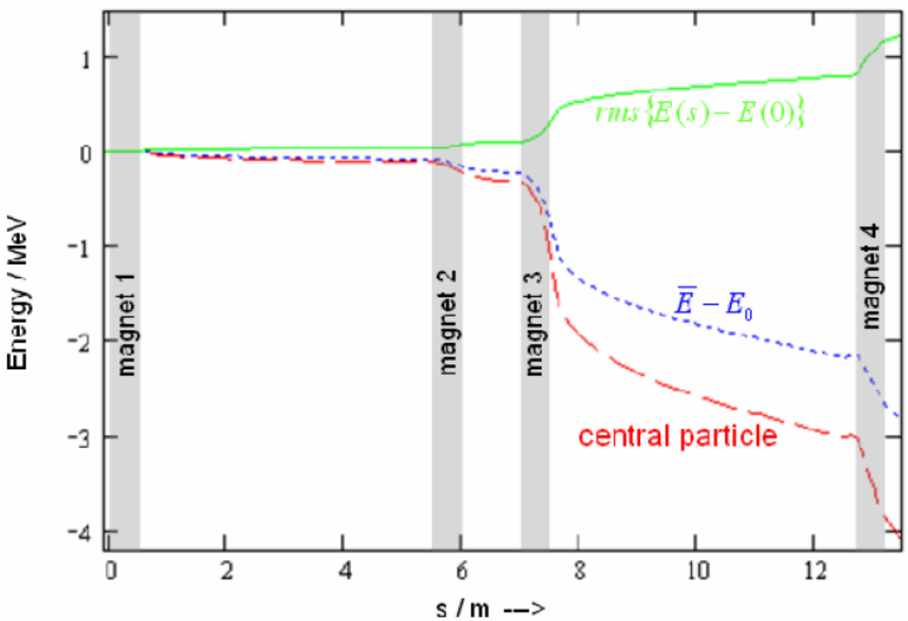
ΔE_{rms} depends weak on energy (energy independent CSR regime)

therefore: β → small; focus of lattice function in last magent
E_{ref} → high

again the “typical” bunch compressor

compression 600 A (1 nC) → 6 kA at 5 GeV

CSR induced energy change along BC



rms energy spread of bunch

mean energy of bunch

energy of particle in bunch center

the rms energy is created essentially: end of magnet 3, drift m3→m4 and magnet 4

$$\left.\begin{array}{r} \text{rough estimation of steady state field in magnet} \quad |E| \propto E_c = \frac{1}{\pi}\frac{Z_0\hat{I}}{L_o} \\ \text{and transient in drift} \quad E \approx -\frac{1}{2\pi}\frac{Z_0 I(s)}{(0.5L_o+\Delta S)} \end{array}\right\} \rightarrow \Delta E_{rms} \rightarrow \varepsilon > \varepsilon_0$$

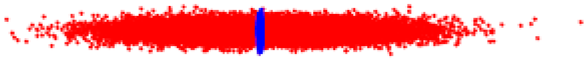
projected emittance and slice emittance

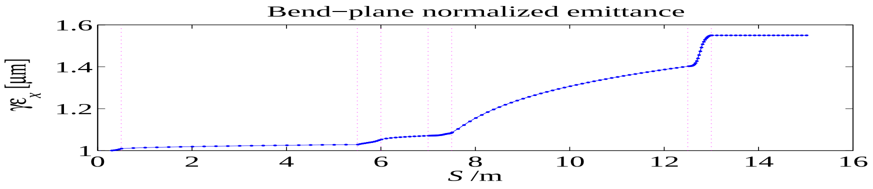
$$\varepsilon = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} \qquad \text{with} \qquad \langle a\rangle = \frac{1}{N}\sum a_n$$

again the “typical” bunch compressor

initial emittance: $\gamma\varepsilon_0$ = 1.00 μm

projected emittance: use all particles: $\gamma\varepsilon_x \approx$ 1.52 μm

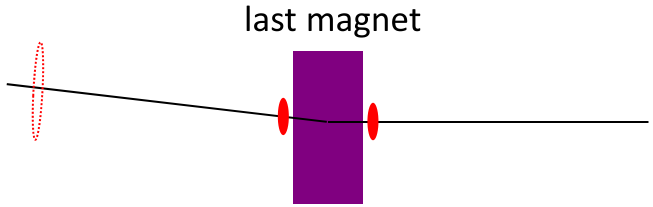




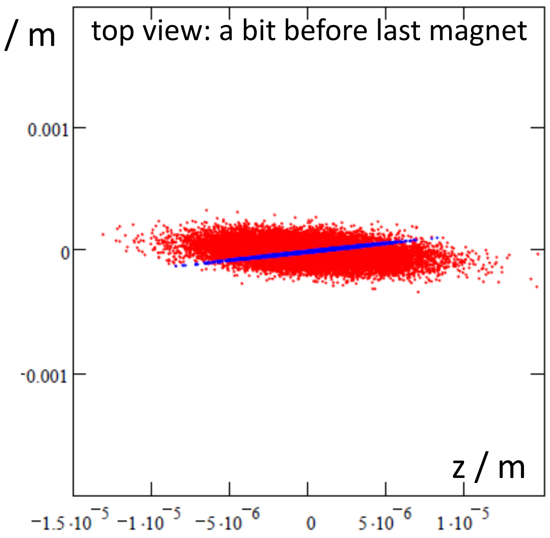
from P. Emma

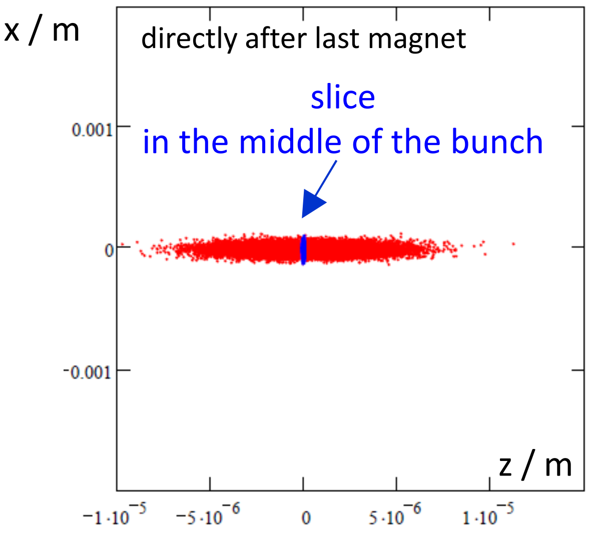
slice emittance: use particles of a certain slice: few percent growth

growth of slice emittance (in principle)



blue: particles $z_v \approx const \pm small$ in one slice **before** the chicane,
they are **not** in the same slice in the chicane, …

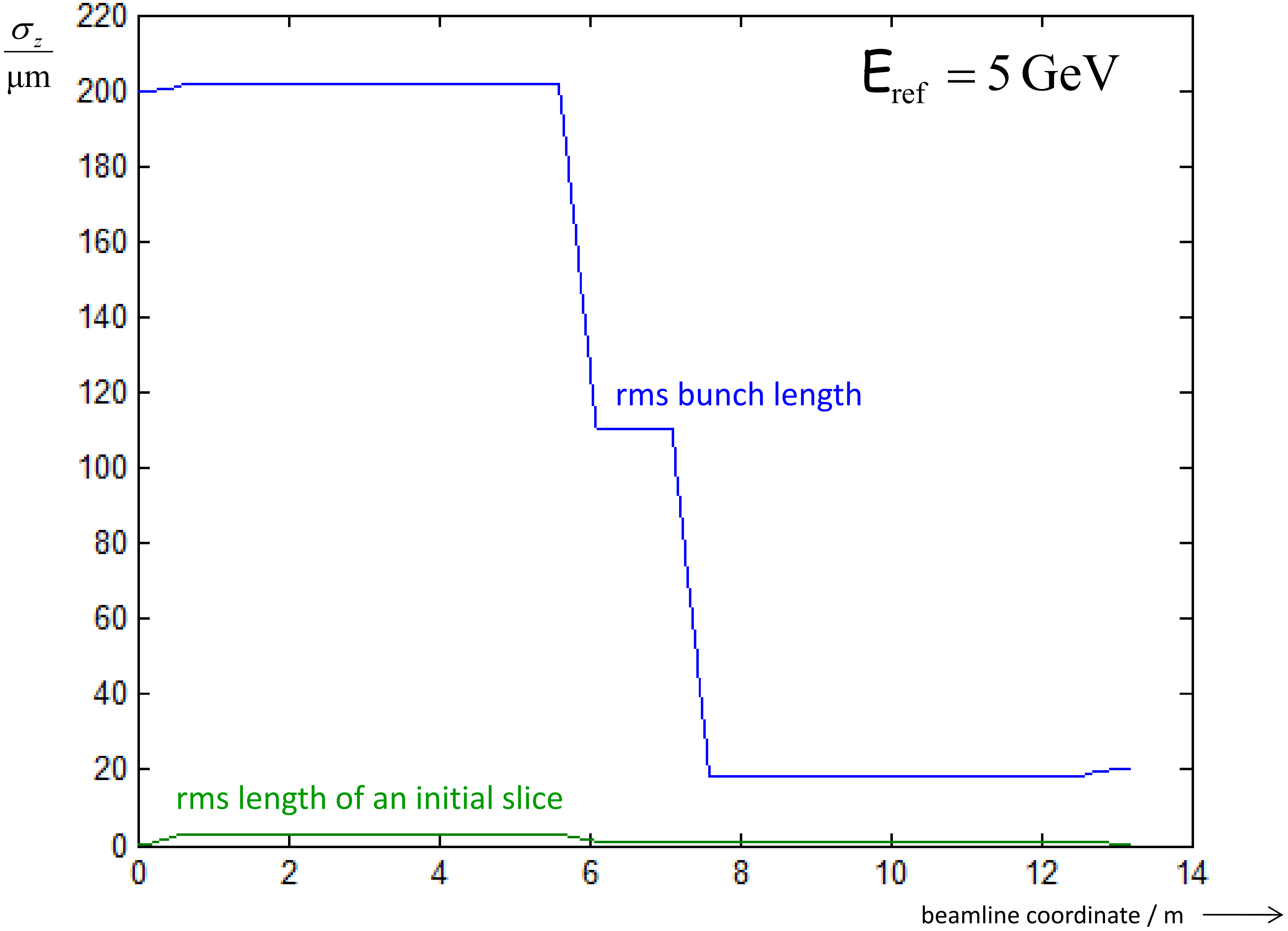




… but they come to the same slice after the chicane

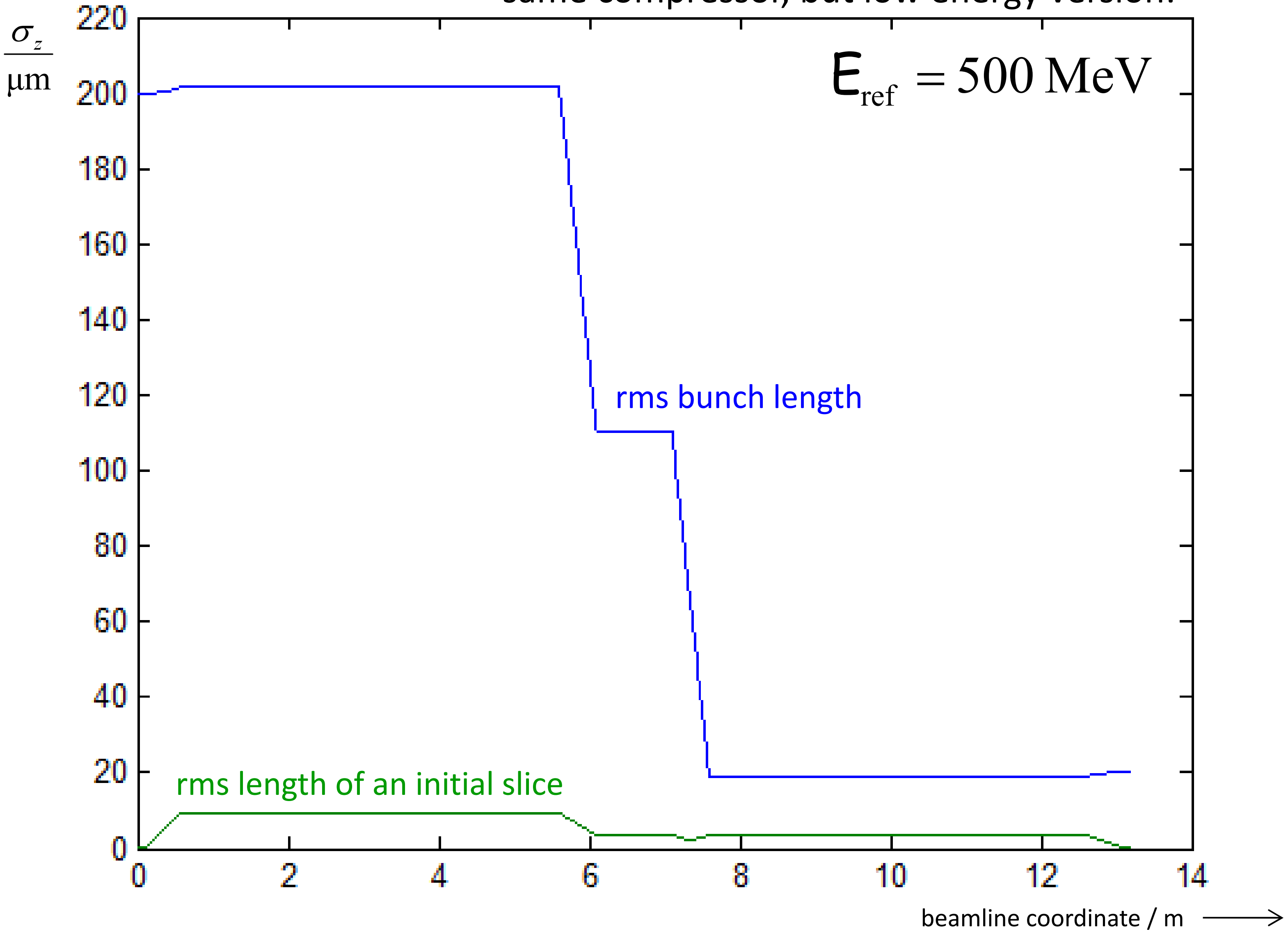
in the chicane these particles may be in different slices and may observe different longitudinal fields (, even with the projected CSR model)

growth of slice emittance, the “typical” bunch compressor



growth of slice emittance, the “typical” bunch compressor

same compressor, but low energy version:



other forces / effects

rough estimation / scaling

for Gaussian bunch with peak current $I = \frac{cq}{\sqrt{2\pi}\sigma_z}$

space charge $E_\parallel \propto \frac{q}{4\pi\varepsilon_0}\frac{1}{(\gamma\sigma_z)^2} \propto \frac{Z_0 I}{3\gamma^2\sigma_z}$ for $\gamma\sigma_z >> \sigma_r$

CSR, circular motion $E_\parallel \propto \frac{1}{\pi}\frac{Z_0 I}{L_o}$

$L_o = \sqrt[3]{24R_0^2\sigma_z}$

$\sigma_z >> R_0/\gamma^3$

CSR, after magnet
(distance S) $E_\parallel \propto \frac{1}{2\pi}\frac{Z_0 I}{(0.5L_o + S)}$

$S << 2\gamma^2\sigma_z$

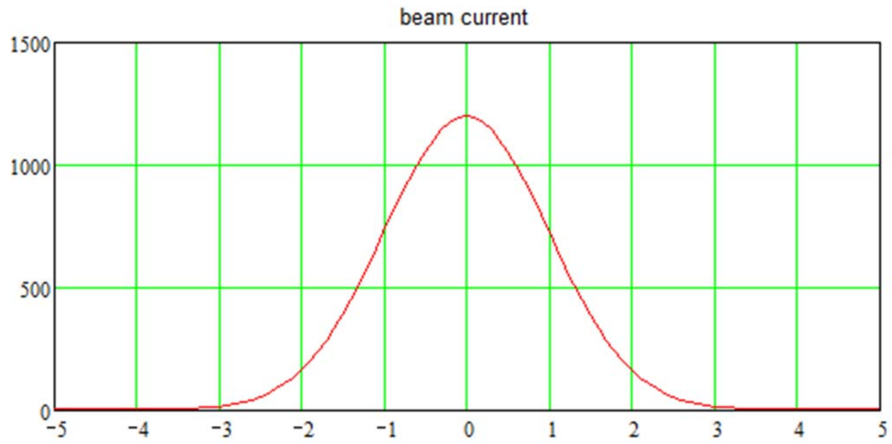
resistive wall wake
(round pipe, radius a) $E_\parallel \propto \frac{Z_0 I}{8a\sqrt{\sigma_z \kappa Z_0}}$

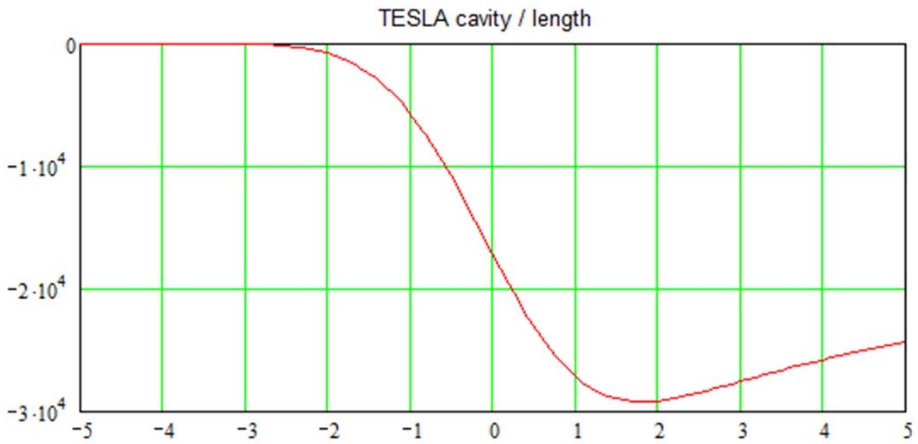
$\sigma_z >> S_{\text{ch}} = \sqrt[3]{\frac{a^2}{2\kappa Z_0}}$

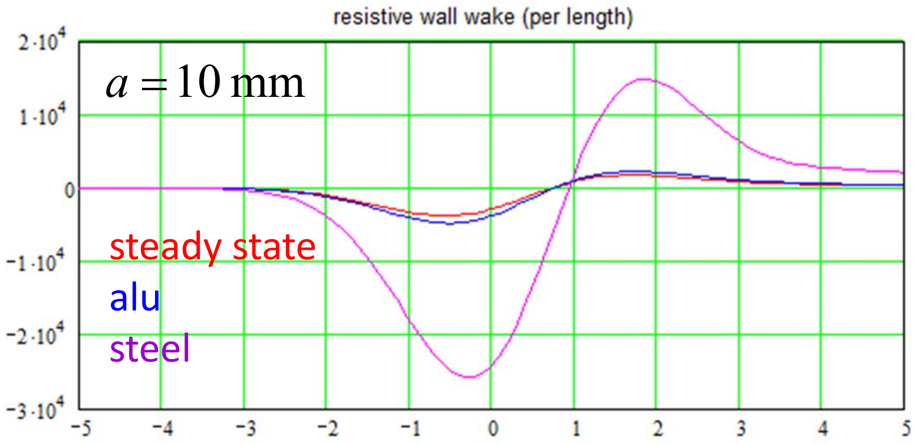
other forces / effects

$$q = 1\,\mathrm{nC}$$

$$\sigma_z = 100\,\mu\mathrm{m}$$

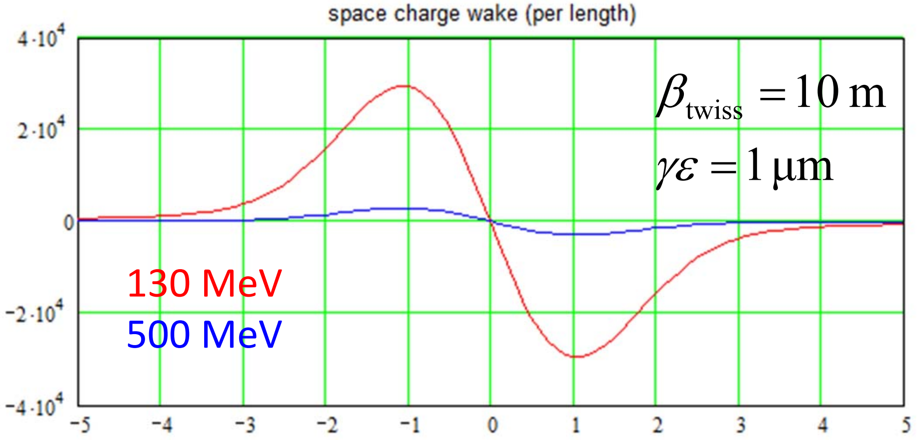

beam current


TESLA cavity / length


resistive wall wake (per length)

$\kappa := 1.5\cdot10^6 \qquad a := 0.01$

$$\frac{Z0\cdot1200}{8\cdot a\cdot\sqrt{\sigma\cdot\kappa\cdot Z0}} = 2.378\times10^4$$

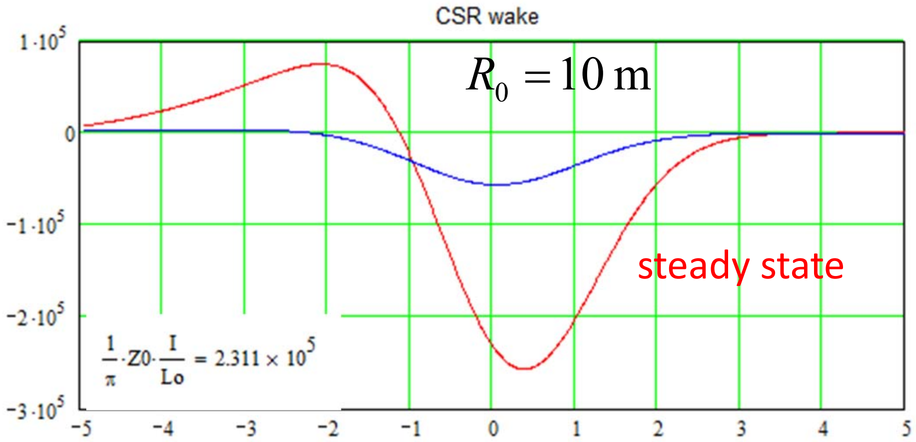

space charge wake (per length)

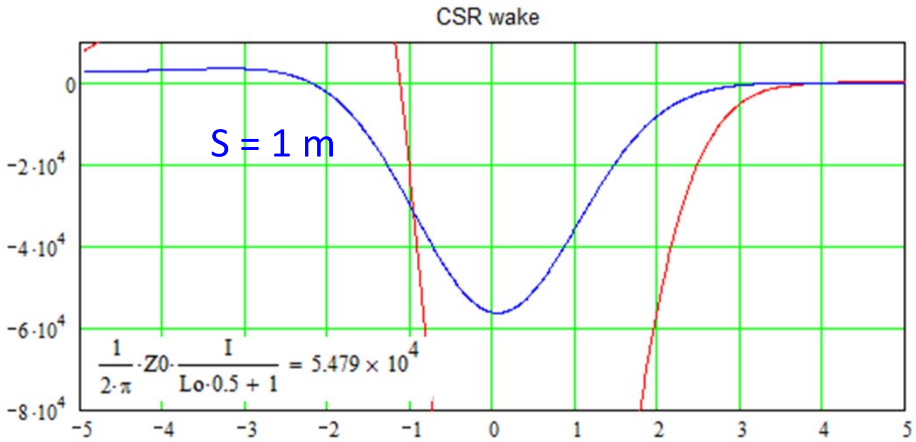
$$\frac{Z0\cdot I}{3\cdot\left(\frac{130}{0.511}\right)^2\cdot\sigma} = 2.324\times10^4$$

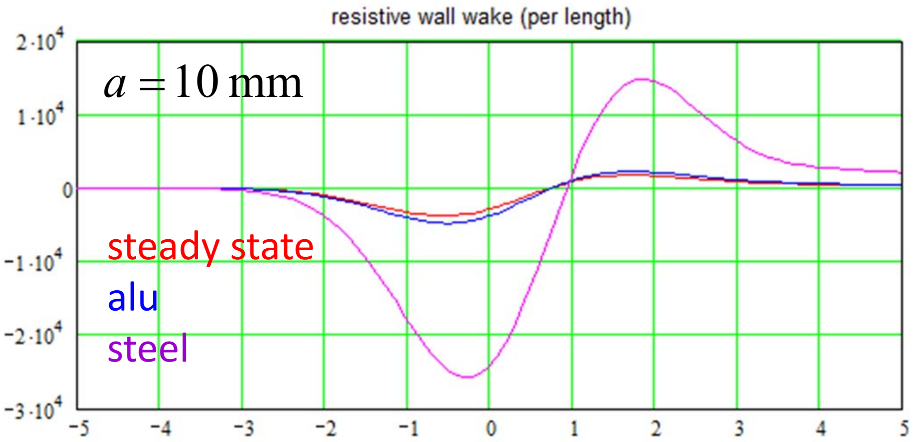
other forces / effects

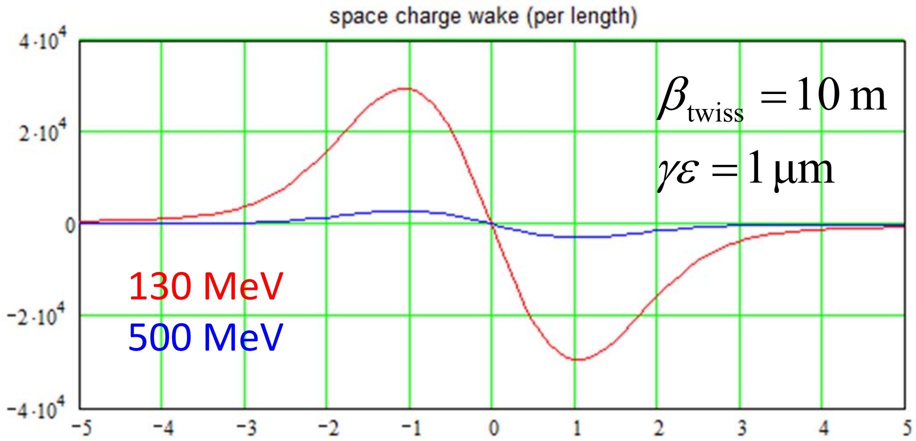
$q = 1\,\mathrm{nC}$

$\sigma_z = 100\,\mu\mathrm{m}$









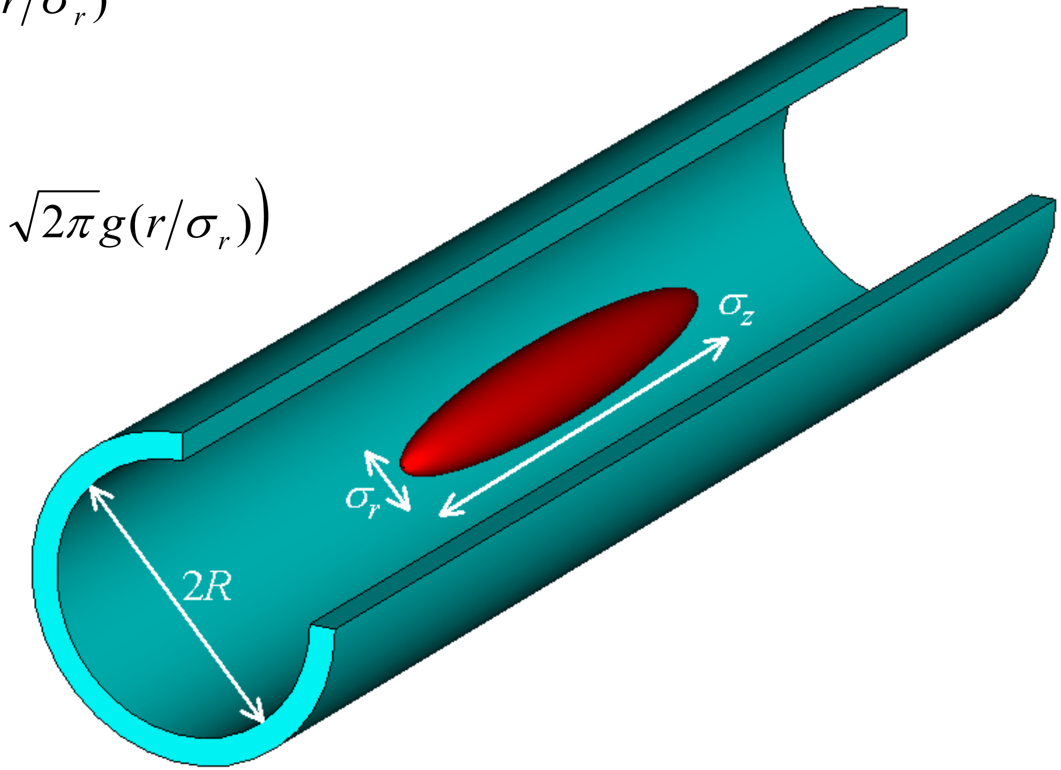
compression work

$$\rho(r,z) = \frac{q}{\sqrt{2\pi}\sigma_z\sigma_r^2} g(z/\sigma_z) g(r/\sigma_r)$$

$$\gamma \to \infty$$

$$E_r(r,z) = \frac{q}{2\pi\varepsilon_0 r\sigma_z} g(z/\sigma_z)\left(1-\sqrt{2\pi}\, g(r/\sigma_r)\right)$$

$$B_\varphi(r,z) = c_0 E_r(r,z)$$

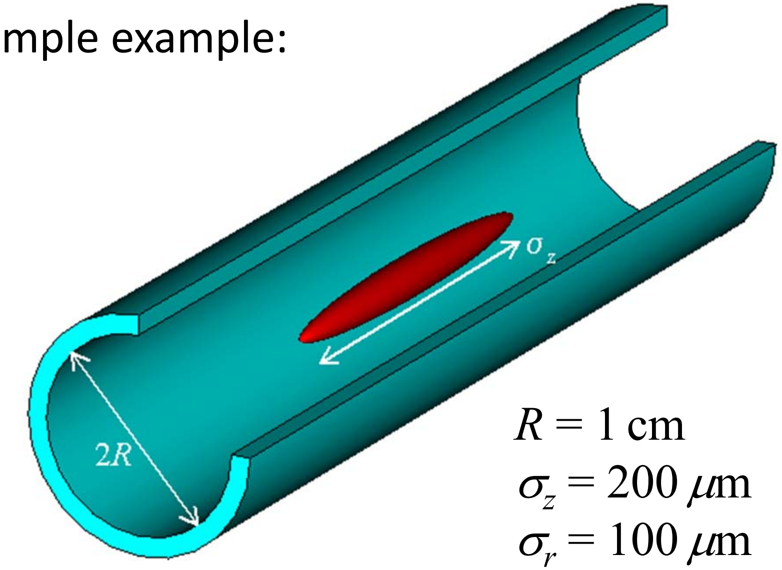


$$\gamma >> R/\sigma_z$$

$$W_{tot} = W_e + W_m = \frac{q^2}{4\pi^{3/2}\varepsilon_0\sigma_z} \ln\left(\frac{R}{1.5\sigma_r}\right)$$

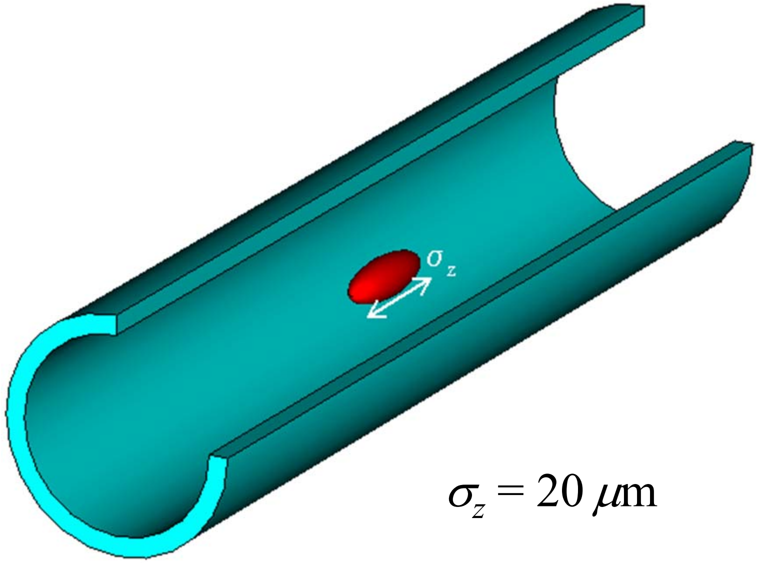
compression work

simple example:



$R = 1\ \mathrm{cm}$
$\sigma_z = 200\ \mu\mathrm{m}$
$\sigma_r = 100\ \mu\mathrm{m}$
$q = 1\ \mathrm{nC}$

$\Rightarrow$



$\sigma_z = 20\ \mu\mathrm{m}$

$W_{\mathrm{tot}} = 0.107\ \mathrm{mJ}$ $\Delta W_{\mathrm{tot}} = 0.958\ \mathrm{mJ}$ $W_{\mathrm{tot}} = 1.065\ \mathrm{mJ}$

for comparison: steady state CSR energy loss in magnet

$R_0 = 10\ \mathrm{m}$, $L = 0.5\ \mathrm{m}$, $\sigma_z = 20\ \mu\mathrm{m} \rightarrow P = 375\ \mathrm{kW}$, $P\ L/c_0 = 0.625\ \mathrm{mJ}$

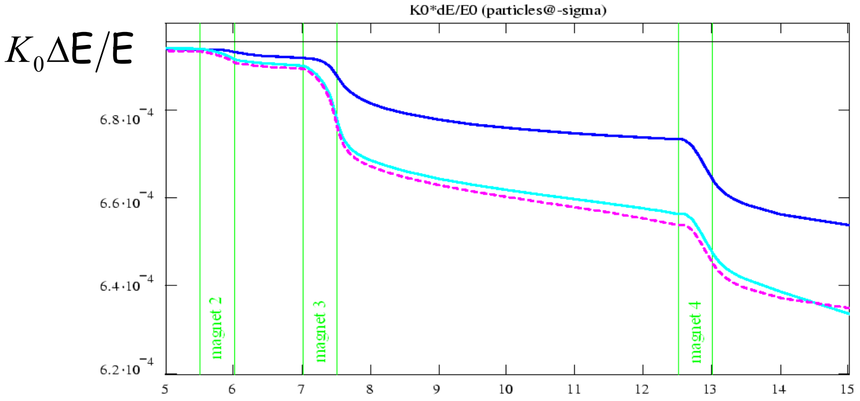
transverse effects

see: Transverse Effects of Microbunch Radiative Interaction
Y. Derbenev, V. Shiltsev, SLAC-PUB-7181

example: the "typical" bunch compressor (5 GeV case)

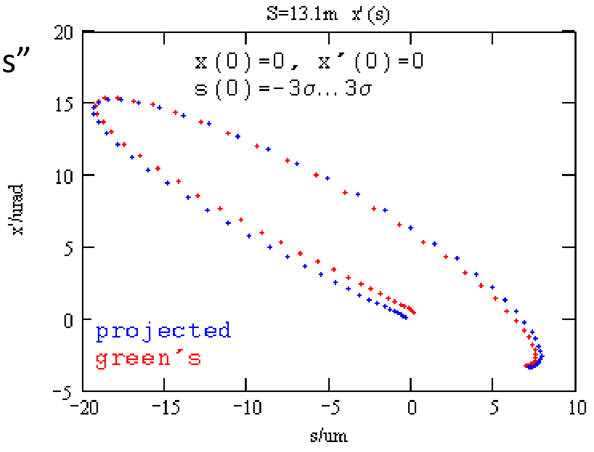
methods for tracking: projected model
full field, approach (1)
full field, approach (2)
} "greens"

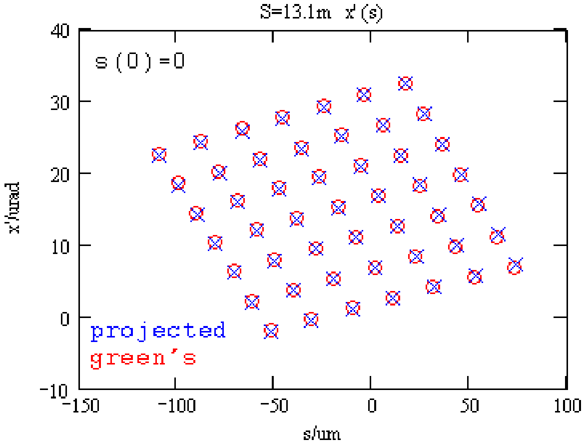
energy loss of one "typical" particle



transverse phase space after chicane

"on axis particles"





middle slice

equation of horizontal motion

$$x'' + \left(K^2 - n\right)x + x'\frac{E'}{E} = \frac{K\Delta E + F_x}{E}$$

external fields: $K(z) = 1/R$ inverse curvature

$n(z)$ external focusing quadrupole field index

energy: $E = E_{\mathrm{ref}} + \boxed{\Delta E_{\mathrm{ch}}} + \boxed{\Delta E_{\mathrm{CSR}}}$

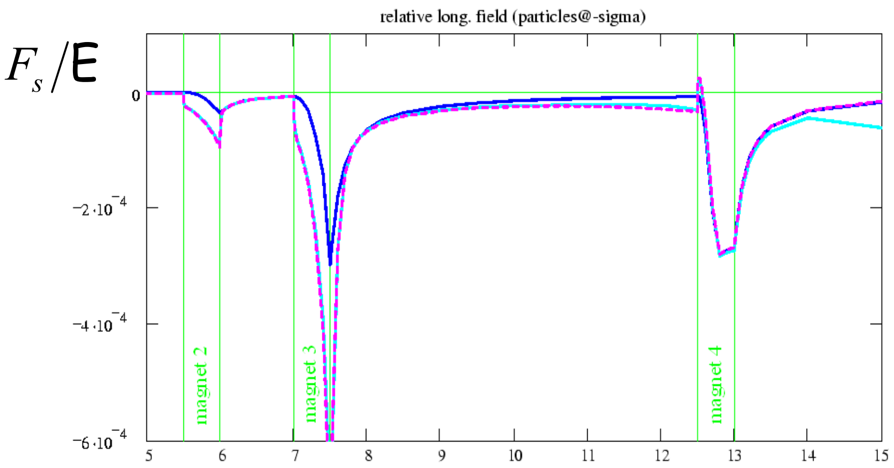
chirp self effects

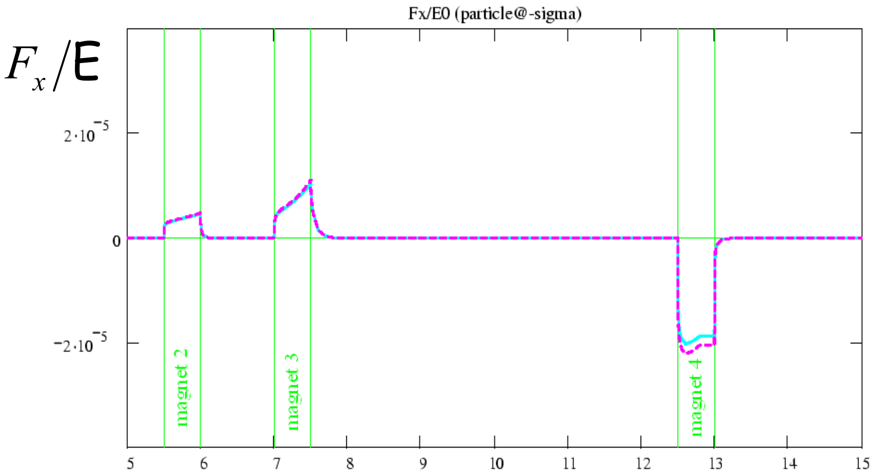
horizontal CSR force: $\boxed{F_x} = q_0\left(E_x^{(\mathrm{CSR})} - vB_y^{(\mathrm{CSR})}\right)$

to first order: $$x'' + \left(K^2 - n\right)x = \frac{K\Delta E_{\mathrm{ch}} + \boxed{K\Delta E_{\mathrm{CSR}} + F_x}}{E_{\mathrm{ref}} + \Delta E_{\mathrm{ch}}}$$

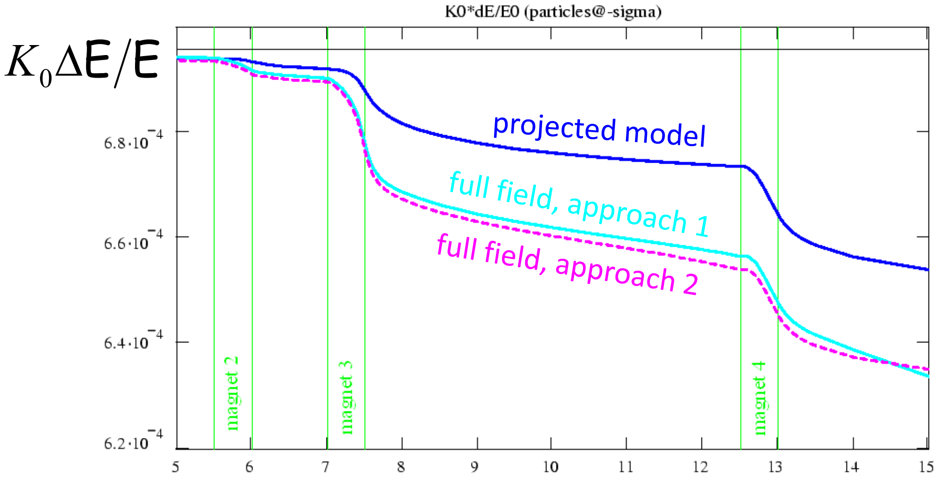
example: the “typical” bunch compressor (5 GeV case)

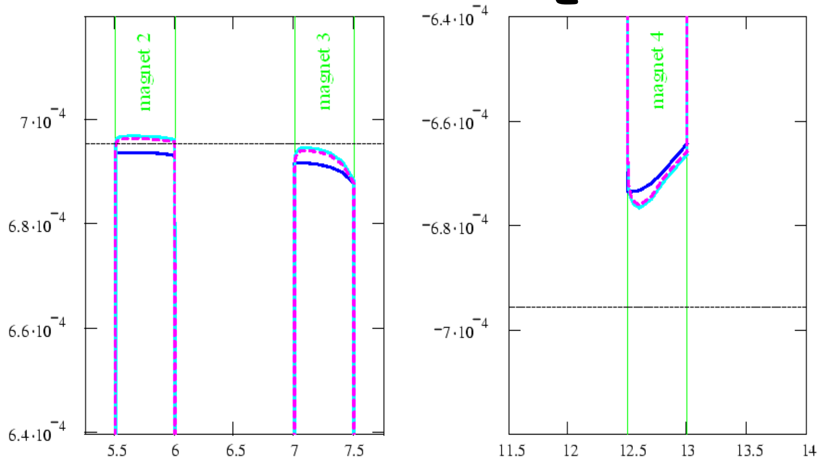
test particle particle with $(x \quad x' \quad y \quad y' \quad s)_0 = (0 \quad 0 \quad 0 \quad 0 \quad -\sigma)$ and $E_0 = E_{\mathrm{ref}} + E_{\mathrm{ch}}(s)$





$$x'' + (K^2 - n)x = \frac{K\Delta E + F_x}{E}$$





compensation

$$F_x = q_0 \vec{e}_x \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$F_x = q_0 \vec{e}_x \cdot \left(-\nabla\Phi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \nabla \times \vec{A} \right)$$

$$= q_0 \vec{e}_x \cdot \left(-\nabla\left(\Phi - \vec{A}\cdot\vec{v}\right) - \frac{d\vec{A}}{dt} \right)$$

$$= -q_0 \vec{e}_x \cdot \nabla\left(\Phi - \vec{A}\cdot\vec{v}\right) - q_0 \frac{d}{dt}\left(\vec{e}_x \cdot \vec{A}\right) + q_0 \underbrace{\frac{d\vec{e}_x}{dt}}_{\frac{v}{R}\vec{e}_{\parallel}} \cdot \vec{A}$$

weak

$$\frac{d}{dt}\left(\mathsf{E} + q_0 \Phi\right) = q_0 \frac{\partial}{\partial t}\left(\Phi - \vec{A}\cdot\vec{v}\right)$$

$$K\Delta\mathsf{E} = -q_0 K \Phi + \cdots$$

$$\boxed{\Phi \approx v\vec{A}\cdot\vec{e}_{\parallel}}$$

≈ compensation

some literature

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