Summary on Mini-Workshop on Space-Charge Effects in FELs

Igor Zagorodnov

Beam Dynamics Group Meeting

17.12.07

Mini-Workshop on Space-Charge Effects in FELs

12-14 December 2007

Berliner Elektronenspeicherring-Gesellschaft

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- Accommodation
- Conference-Dinner

support

High brightness electron beams with energies in GeV range have to be produced and maintained in Linac drivers for VUV and X-ray FELs. Due to the small energy spread from the RF gun, these beams behave like a cold relativistic non-neutral plasma. They are subject to collective plasma behaviour more than thermal gas-like behaviour. The propagation of such a beam takes place in a transition regime from space charge to emittance dominated dynamics, as the beam energy increases. In addition, in downstream magnetic compressor devices where the peak current increases up to kA range, the transition may occur again. Thus, the transverse and longitudinal space charge forces driving emittance dilutions and microbunching instability respectively are of great importance for design and operation of modern FELs. During the last decades several analytical methods and simulation codes (mainly based on 1d-approximation) are developed to predicate the effect of space-charge forces in driver linacs. However there are still a number of important issues related to the space charge effects which have to be addressed. In this workshop we plan to address analytical methods and numerical calculations of space-charge effects and their implementation in simulation programs.

Dates: from 12 December 2007 14:00 to 14 December 2007 14:00

Location: *Berliner Elektronenspeicherring-Gesellschaft*

Albert-Einstein-Str. 15
12489 Berlin
Germany

Wednesday, 12 December 2007

14:00	**1. Session: Welcome** (auditorium: 14:00 - 14:20)
	1. Session: Experiences with space-charge at FLASH Conveners: Dr. T. LIMBERG (DESY) (auditorium: 14:20 - 15:00)
15:00	**1. Session: Discussion** (auditorium: 15:00 - 15:30)
	Caffee break (auditorium: 15:30 - 16:00)
16:00	**2. Session: about self consistent particle & field simulations** Conveners: Dr. Martin DOHLUS (DESY) (auditorium: 16:00 - 16:45)
17:00	**2. Session: Discussion** (auditorium: 16:45 - 17:30)

Thursday, 13 December 2007

09:00	
	3. Session: Current Status of Space Charge Modeling in FEL Simulations Conveners: Dr. Sven REICHE (UCLA) (auditorium: 09:30 - 10:15)
10:00	**Caffee-break** (auditorium: 10:15 - 10:45)
	4.Session: 3D Space Charge Routines: The Software Package MOEVE and FFT Compared (auditorium: 10:45 - 11:30)
11:00	
	4.Session: Space-charge calculation using analytical Formulas (an incomplete overview) (auditorium: 11:30 - 12:30)
12:00	
	4.Session: Some Comments on HOMDYN and RETAR (auditorium: 12:30 - 13:00)
13:00	**Lunch break** (auditorium: 13:00 - 14:00)
14:00	**5.Session: Space-Charge Issues of electron bunches from Laser-P[...] Accelerators at low energy**
	Conveners: Dr. Florian GRUENER (MPQ) (auditorium: 14:00 - 14:45)

	Conveners: Dr. Florian GRUENER (MPQ) (auditorium: 14:00 - 14:45)
15:00	**5.Session: FEL driven by the LBNL laser-plasma accelerator** Conveners: Dr. Carl SCHROEDER (LBNL) (auditorium: 14:45 - 15:30)
	caffee break (auditorium: 15:30 - 16:00)
16:00	**6.Session: Discussion** (auditorium: 16:00 - 17:30)
18:00	**Meet for Dinner** (auditorium: 18:30 - 18:40)

Friday, 14 December 2007

09:00	**Space-charge: Measurements and Dignostics** (auditorium: 09:00 - 10:30)
10:00	**Caffee-break** (auditorium: 10:30 - 11:00)
11:00	**Conclusion and summery** (auditorium: 11:00 - 12:30)
12:00	

about self consistent particle & field simulations

Martin Dohlus

self consistent in general

- what is self consistent?
- about field calculations
- examples
- some critics on methods with poison solutions
- driven undulator field: SSY & GSSY publications

space charge: simple model and scaling

- examples

FEL codes

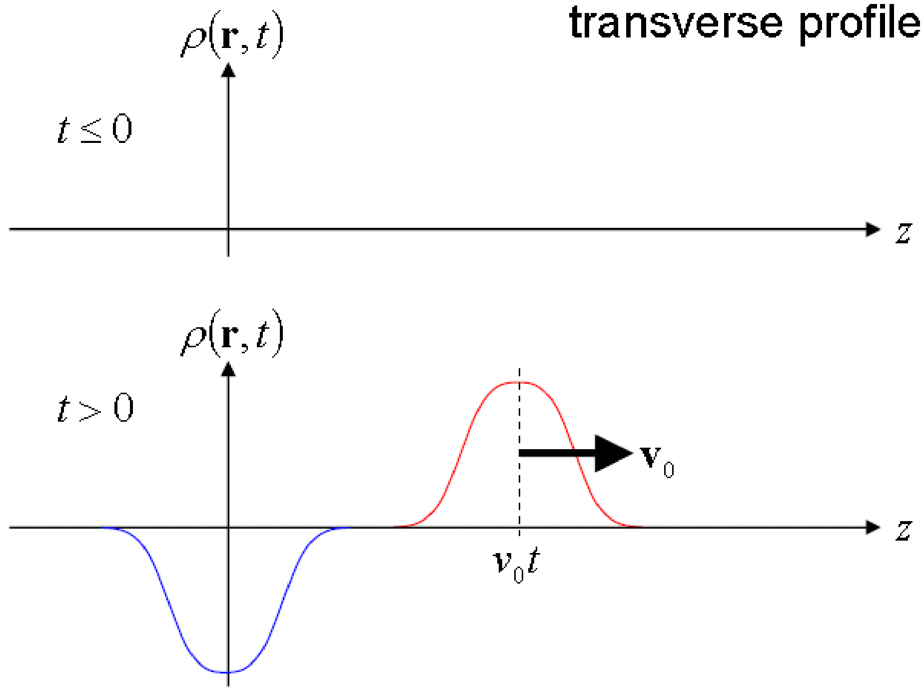
- FEL codes, in principle
- μ- and macro-effects
- program flow

example 2: driven self fields, sudden acceleration

sudden acceleration: $\rho(\mathbf{r},t) = \begin{cases} 0 & \text{if } t < 0 \\ \left(\lambda(z - v_0 t) - \lambda(z)\right)\eta(\mathbf{r}_\perp) & \text{otherwise} \end{cases}$

longitudinal charge density: $\int \lambda(z)dz = q$

transverse profile: $\int \eta(\mathbf{r}_\perp)dA_\perp = 1$



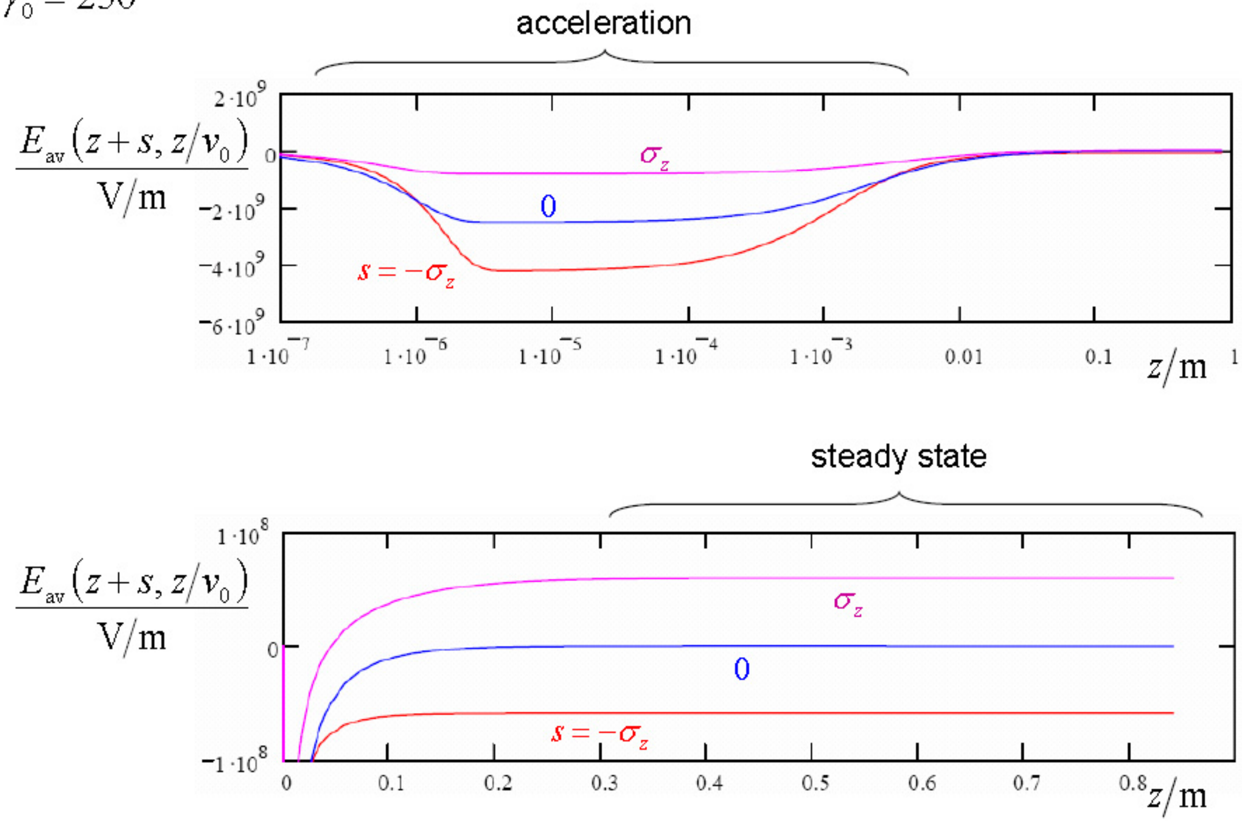
$$E_{av}(z,t) = \int E(\mathbf{r}_\perp + z, t)\eta(\mathbf{r}_\perp)dA_\perp$$

example 2: driven self field, sudden acceleration

longitudinal charge density: $\int \lambda(z)dz = q \qquad q = 0.5\,\mathrm{nC} \qquad \sigma_z = 1\,\mu\mathrm{m}$ (gaussian)

transverse profile: $\int \eta(\mathbf{r}_\perp)dA_\perp = 1 \qquad \sigma_r = 30\,\mu\mathrm{m}$ (round, gaussian)

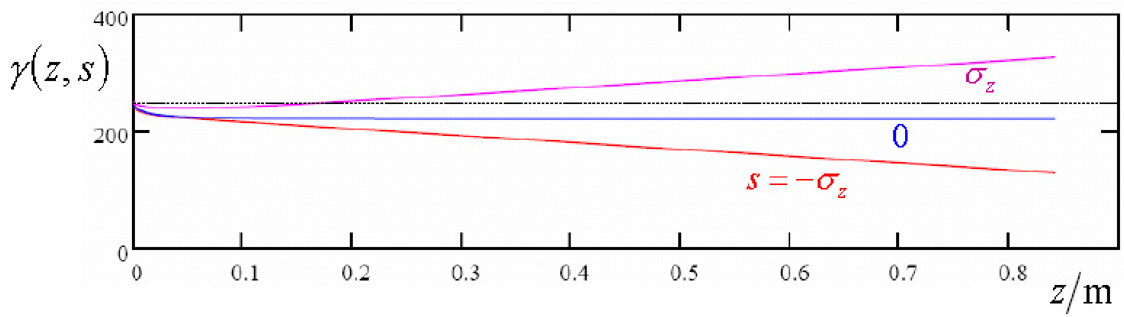
$\gamma_0 = 250$



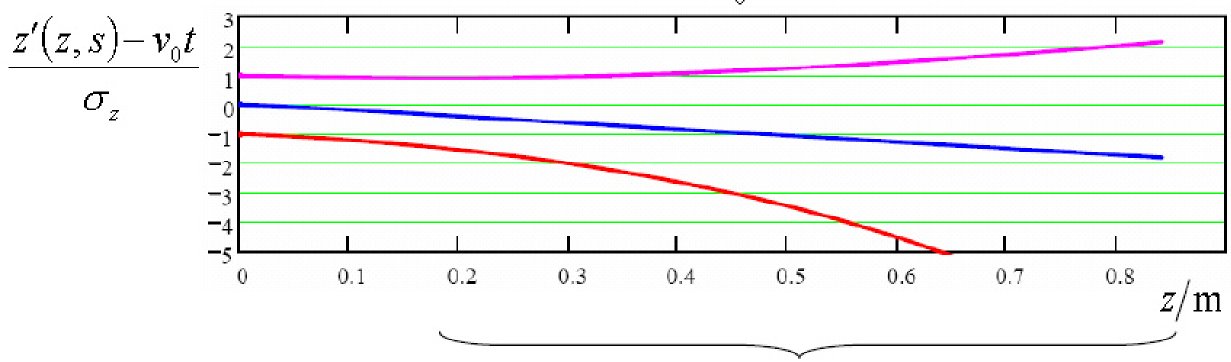
example 2: response to driven self field

sudden acceleration

$$\gamma(z,s) = \gamma_0 + \frac{q}{mc^2}\int_0^z E_{av}\left(\tilde{z}+s, \tilde{z}/v_0\right) d\tilde{z}$$



$$z'(z,s) = s + \int_0^z \frac{v(z,s)}{v_0} d\tilde{z}$$



"self consistent" calculation required

example 3: 1d, self consistent adiabatic acceleration

$\rho(\mathbf{r},t) = \lambda(z,t)\eta(\mathbf{r}_\perp)$

longitudinal charge density: $\int \lambda(z)dz = q$

$\lambda(z,t) = \sum q_\nu \delta(z - z_\nu(t))$

transverse profile: $\int \eta(\mathbf{r}_\perp)dA_\perp = 1$

macro particles: q_ν , $z_\nu(t)$, $p_\nu(t)$

approach for field calculation: $E_{\mathrm{av}}(z - vt) = \frac{1}{4\pi\varepsilon_0\sigma_r^2}\int \lambda(z - vt - \xi)F\left(\frac{\xi\gamma}{\sigma_r}\right)d\xi$

linear motion model $\lambda(z,t) = \lambda(z - vt)$(for averaged longitudinal field) extended to individual motion of disc sources:

$$E_{\mathrm{av}}(z,t) \approx \frac{1}{4\pi\varepsilon_0\sigma_r^2}\sum q_\nu F\left(\frac{(z - z_\nu(t))\gamma_\nu(t)}{\sigma_r}\right)$$

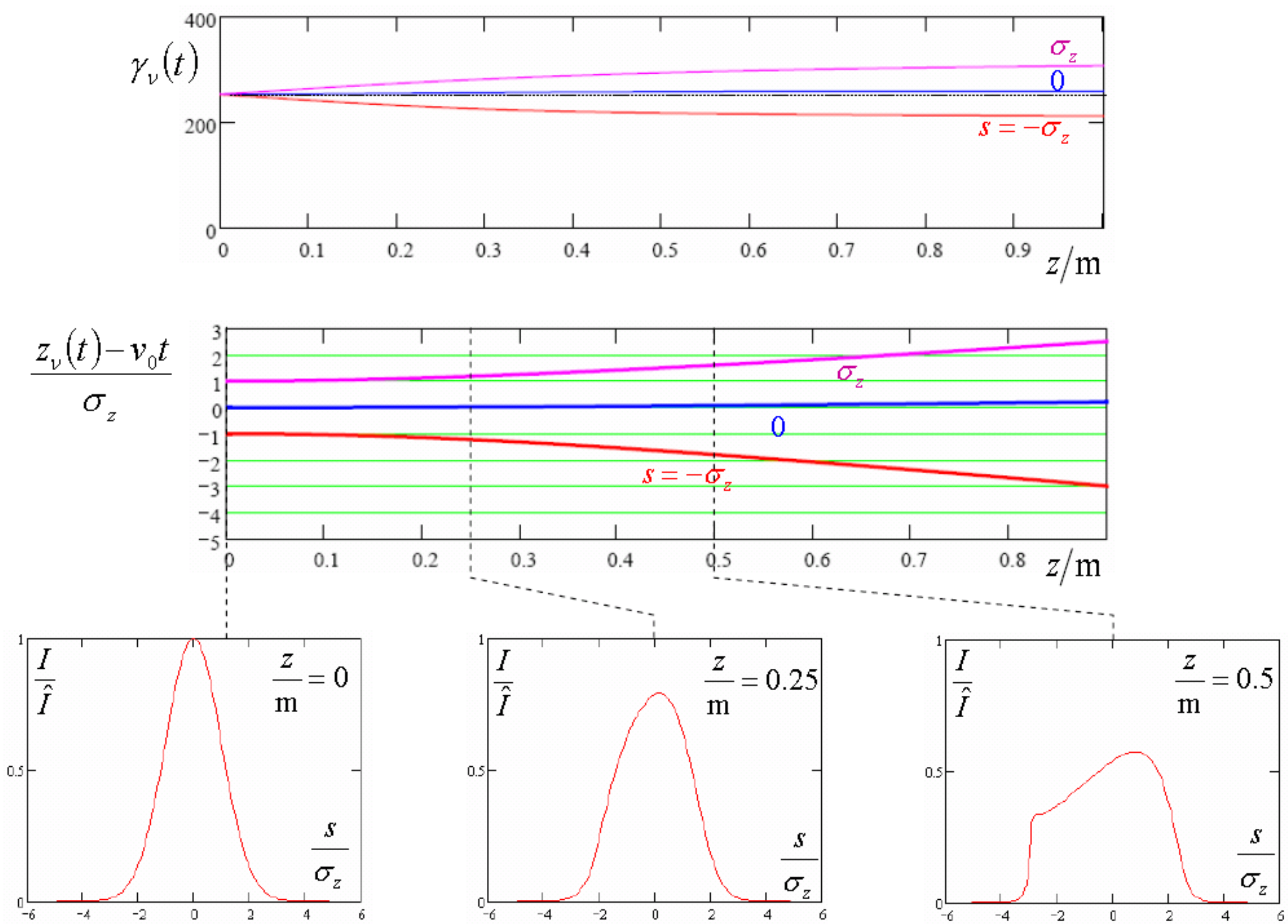
equation of motion (for rigid discs):

$$\frac{d}{dt}z_\nu(t) = v(p_\nu(t))$$

$$\frac{d}{dt}\,p_\nu(t) = qE_{\mathrm{av}}(z_\nu,t)$$

longitudinal charge density: $\int \lambda(z)dz = q$ $q = 0.5\,\mathrm{nC}$ $\sigma_z = 1\,\mu\mathrm{m}$ (gaussian)

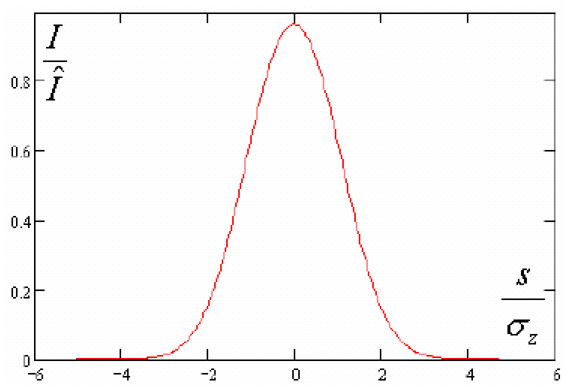
transverse profile: $\int \eta(\mathbf{r}_\perp)dA_\perp = 1$ $\sigma_r = 30\,\mu\mathrm{m}$ (round, gaussian) $\gamma_0 = 250$



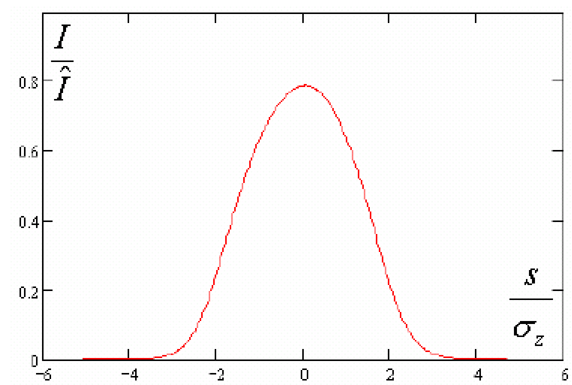
example 4: self consistent, ASTRA

3d version of example 3

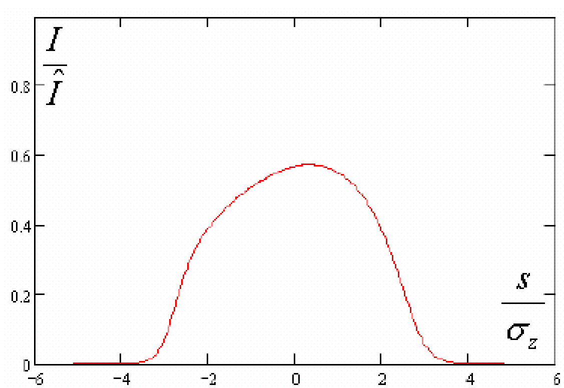
$$\frac{z}{\text{m}} = 0.09 \qquad E_{tot} = E_0 + N \cdot 0.36\,\text{MeV}$$
$$x_{\text{rms}} = 40\,\mu\text{m}$$



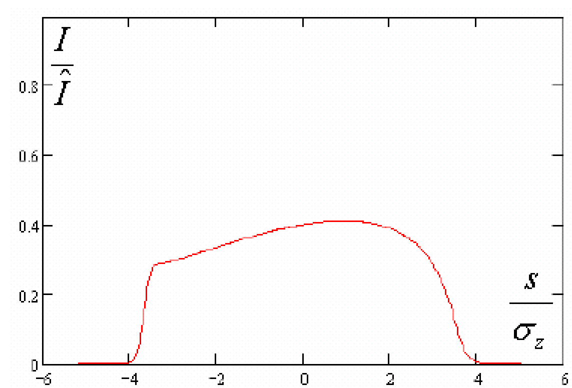
$$\frac{z}{\text{m}} = 0.29 \qquad E_{tot} = E_0 + N \cdot 1.4\,\text{MeV}$$
$$x_{\text{rms}} = 105\,\mu\text{m}$$



$$\frac{z}{\text{m}} = 0.6 \qquad E_{tot} = E_0 + N \cdot 2.1\,\text{MeV}$$
$$x_{\text{rms}} = 243\,\mu\text{m}$$



$$\frac{z}{\text{m}} = 1 \qquad E_{tot} = E_0 + N \cdot 2.4\,\text{MeV}$$
$$x_{\text{rms}} = 450\,\mu\text{m}$$



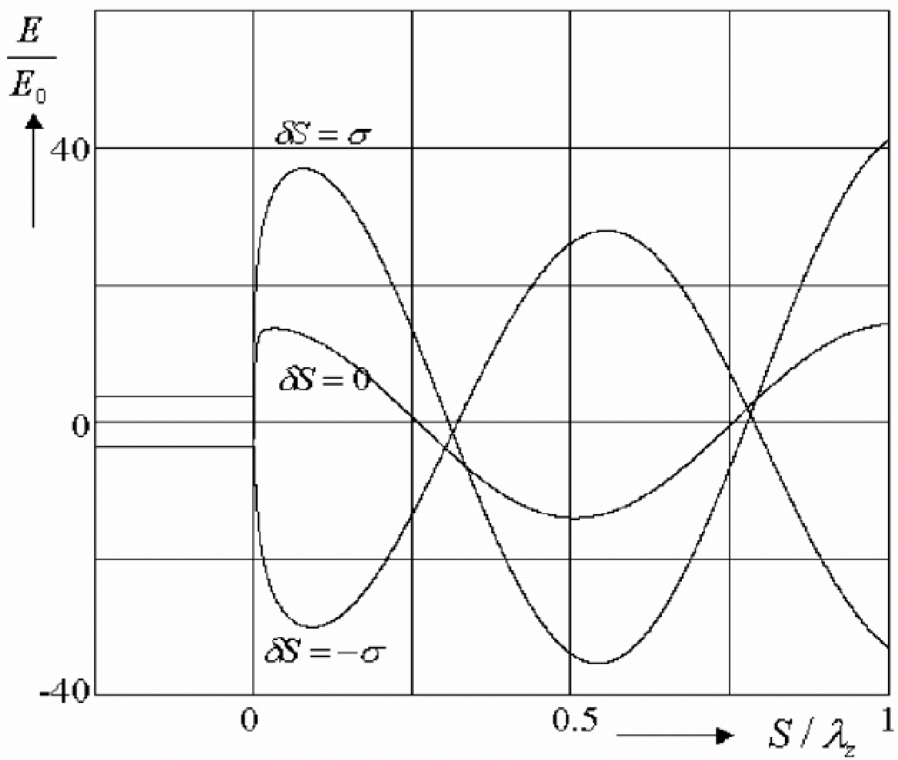
example 6: undulator motion
& retardation effects

Dohlus, Kabel, Limberg: Coherent effects of a macro-bunch in an undulator NIM, A445 (2000)

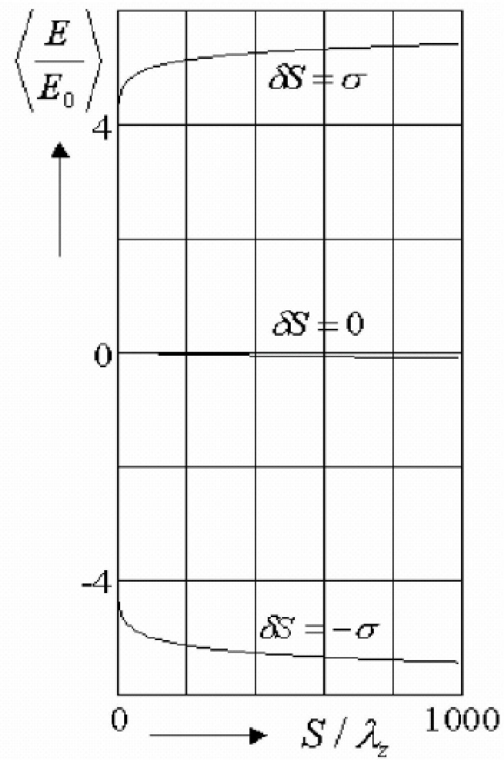
$\mathsf{E} = 1\,\mathrm{GeV},\ q = 1\mathrm{nC},\ K = 1.27,\ \lambda_u = 2.73\mathrm{cm}$

$$E_0 = \frac{q}{4\pi\varepsilon\gamma^2\sigma^2} \qquad \langle E \rangle = \frac{1}{\lambda_u}\int_{-\lambda_u/2}^{\lambda_u/2} E(z+\xi)\,d\xi$$

field in the 1st undulator period



averaged field for 1000 periods



driven undulator field: SSY & GSSY publications

Radiative interaction of electrons in a bunch moving in an undulator

Authors: Saldin E.L.; Schneidmiller E.A.; Yurkov M.V.[1]
Source: Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, Volume 417, Number 1, 1 November 1998 , pp. 158-168(11)
Publisher: Elsevier

"projected" csr model
one-dimensional (renormalized) longitudinal wake for arbitrary trajectory
analytic formula for gaussian bunch in undulator

renormalized = extraction of 1d singularity

June 2007 DESY 07-087

Longitudinal impedance and wake from XFEL undulators. Impact on current-enhanced SASE schemes

Gianluca Geloni, Evgeni Saldin, Evgeni Schneidmiller and Mikhail Yurkov
Deutsches Elektronen-Synchrotron DESY, Hamburg

3d effects included
conditions: macro bunch, steady state, free space

space charge, simple model & scaling, examples:

undulator parameter and effective gamma: $K,\ \bar{\gamma} = \dfrac{\gamma}{\sqrt{1+K^2/2}},\ \gamma_{\mathrm{rms}} \approx 1.56 \dfrac{\hat{I}}{I_A} \dfrac{L}{\sigma_z \bar{\gamma}^2} \ln\left(\dfrac{\sigma_z \bar{\gamma}}{\sigma_r}\right)$

---

parameters 1: (high s.c.)

$\hat{I} = 50\,\mathrm{kA}$ $\quad \gamma = 300$ $\quad \sigma_z = 1\,\mu\mathrm{m}$ $\quad L = 0.1\,\mathrm{m}$ $\quad \rightarrow \gamma_{\mathrm{eff}}/\gamma = 5.2\cdot 10^{-2}$

$\bar{\gamma} = 250$ $\quad \sigma_r = 30\,\mu\mathrm{m}$

self consistent integration required

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parameters 2: (~ FLASH)

$\hat{I} = 1.5\,\mathrm{kA}$ $\quad \gamma = 1000$ $\quad \sigma_z = 1\,\mu\mathrm{m}$ $\quad L = 30\,\mathrm{m}$ $\quad \rightarrow \gamma_{\mathrm{eff}}/\gamma = 2.1\cdot 10^{-3}$

$\bar{\gamma} = 820$ $\quad \sigma_r = 280\,\mu\mathrm{m}$ $\left(\gamma\varepsilon = 4\,\mu\mathrm{m},\ \beta = 20\,\mathrm{m}\right)$

perturbation method

---

parameters 3: (~ XFEL)

$\hat{I} = 5\,\mathrm{kA}$ $\quad \gamma = 35000$ $\quad \sigma_z = 25\,\mu\mathrm{m}$ $\quad L = 300\,\mathrm{m}$ $\quad \rightarrow \gamma_{\mathrm{eff}}/\gamma = 1.1\cdot 10^{-5}$

$\bar{\gamma} = 12000$ $\quad \sigma_r = 24\,\mu\mathrm{m}$ $\left(\gamma\varepsilon = 1\,\mu\mathrm{m},\ \beta = 20\,\mathrm{m}\right)$

perturbation method

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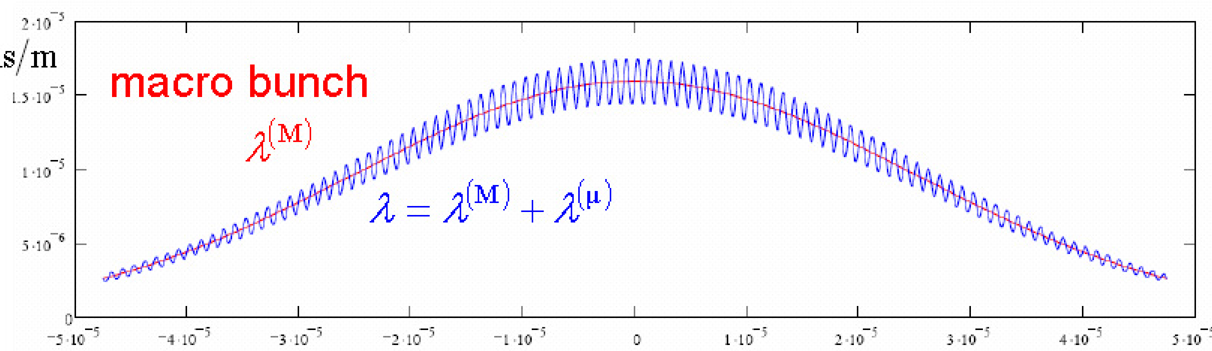
parameters 4: (“ESASE”)

$\hat{I} = 18\,\mathrm{kA}$ $\quad \gamma = 28000$ $\quad \sigma_z = 50\,\mathrm{nm}$ $\quad L = 50\,\mathrm{m}$ $\quad \rightarrow \gamma_{\mathrm{eff}}/\gamma = 1.8\cdot 10^{-3}$

$\bar{\gamma} = 10000$ $\quad \sigma_r = 25\,\mu\mathrm{m}$

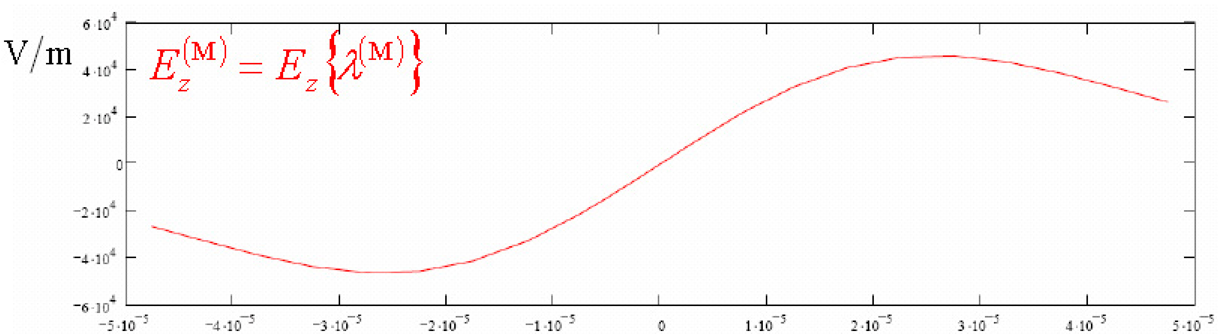
perturbation method

μ- and macro-effects

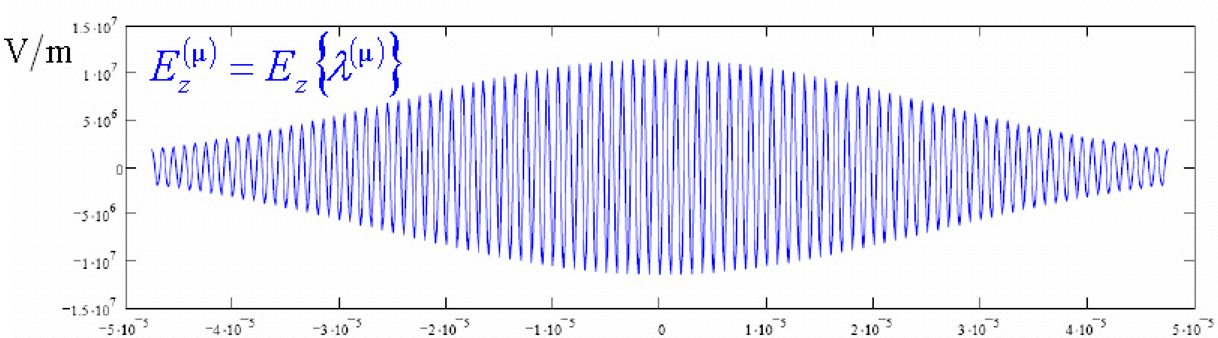


$$\rho(\mathbf{r}, t) = \lambda(z - vt)\eta(r_\perp)$$

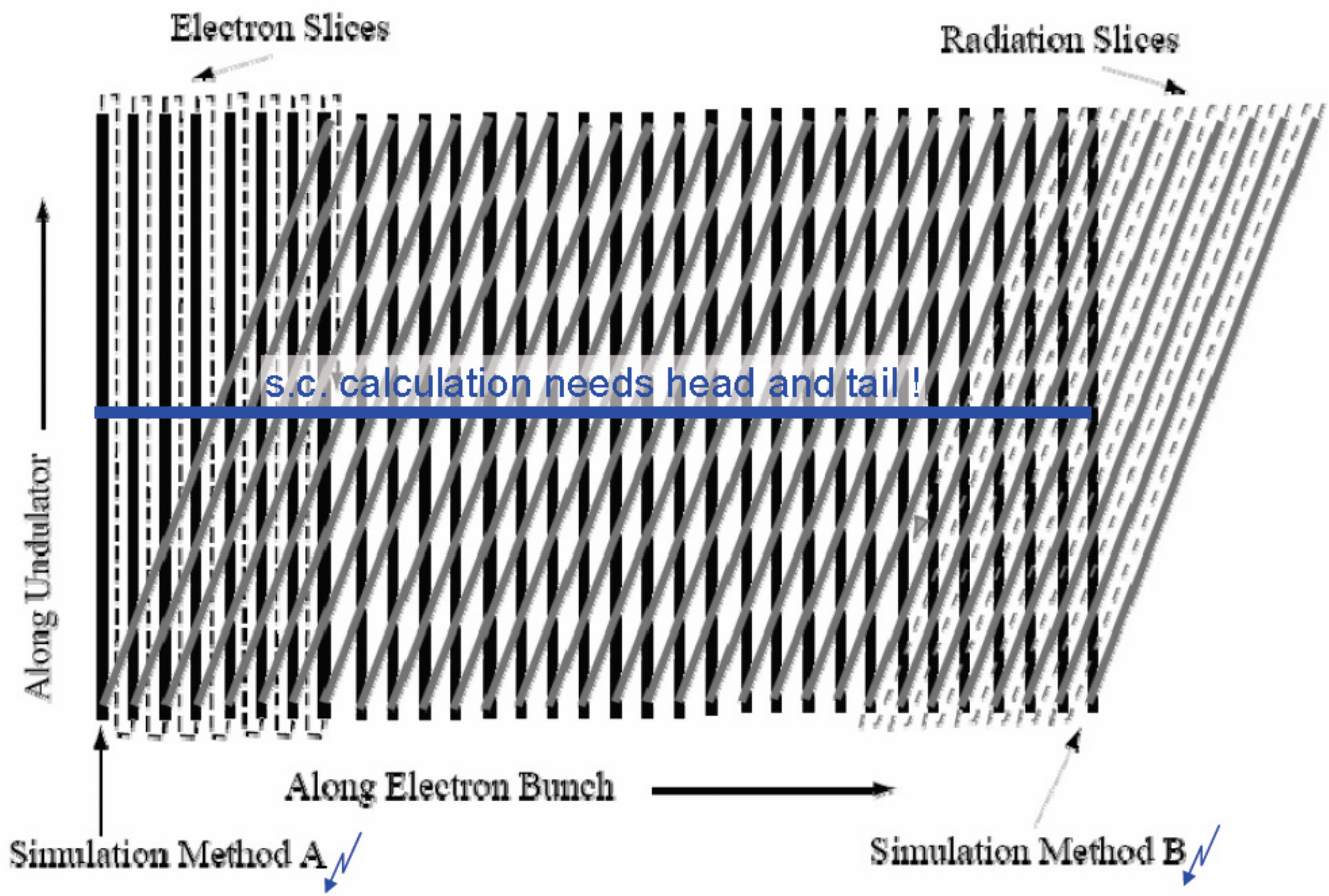
macroscopic effect



μ effect, treated by FEL code



program flow



from a famous PhD thesis: ... *Et Facta Est Lux*, Genesis 1.3
Sven Reiche 1999

Current Status of Space Charge Modeling in FEL Simulations

Sven Reiche

BESSY - 12/12/07

Limitation of Sequential Approach

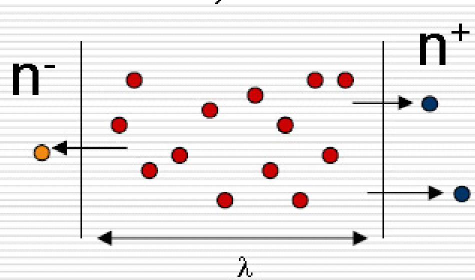
- The simulation cannot be self-consistent because the current electron slice is unaware of the other electron slices (except for the slippage field) and their history.
- The codes prohibits a re-arrangement of the electron slices and a change in the current profile within the undulator.
- The macro particle charge depends on the local current and is not constant for the whole bunch
- The codes does not allow to transfer particles from one electron slice to another due without affecting the bunching factor (macro particles are arrange to yield an initial small bunching factor)

Straight-Forward Improvements

- Transverse Space-Charge Fields:
  - The particle tracking is replaced by solving the Lorenz-force equation for the transverse motion. The corresponding differential equations are included in the ODE solver (typically Runge-Kutta).
  - Slight/moderate increase in the CPU time.
- Generalize the import of externally calculated wakefields, CSR potentials and space charge fields.
  - Based on the implementation of wakefields, a look-up table of the field is used to determine the long-range electric fields.
  - Very simple implementation with no significant increase in CPU time
  - Restrictions remain the same. No change in the current profile is allowed.

Implementation to Allow Changes in Current Profile

- Dynamic Update of Current Profile
  - Each integration step the flux out of the slice is calculated and compared to the influx from adjacent slices. Local current is updated to the net change of particles (though the particles are not exchanged).
  - Requires to store full electron beam and radiation field on disk (GBytes of data) and enforces the direction of integration by looping over the bunch per integration step instead of tracking on slice over entire undulator length (change of loop order).
  - Wakefield/Space Charge can be updated with each step.
  - Models for wakefields and space charge fields have to be hard-coded into the FEL code.
  - Moderate increase in CPU time (file access, space-charge/wakefield model calculations)

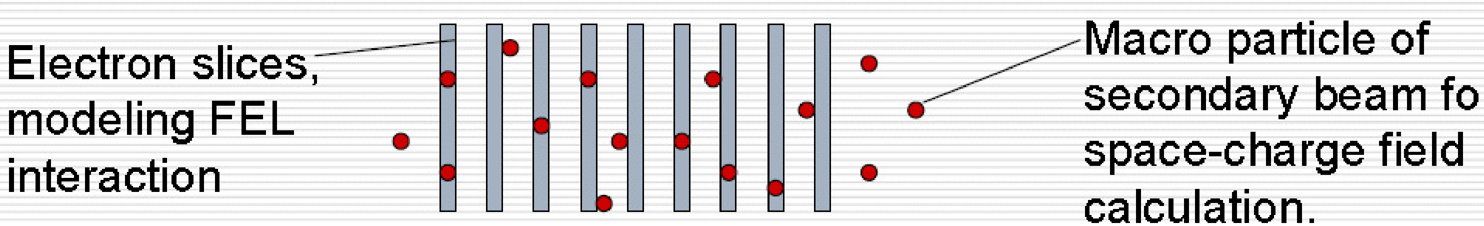


$$\Delta n_j = n^+_{j-1} + n^-_{j+1} - n^+_j - n^-_j$$

$$I_j \rightarrow \left(1 + \frac{\Delta n_j}{n_j}\right) I_j$$

Self-Consistent Approach - 2

- Tracking of Secondary Beam
  - Secondary beam has low number of particles, describing the distribution of the electron beam.
  - Particles are tracked through the undulator field with a self-consistent solution to space charge field and wakefields but excluding FEL interaction.
  - The derived long-range field and local current is than included in the FEL equation of the main particle distribution.
  - It allows to simulate even for subsections of the electron beam as long as the secondary beam is fully representing the bunch.
  - Moderate increase in the CPU time with little change in the core algorithm



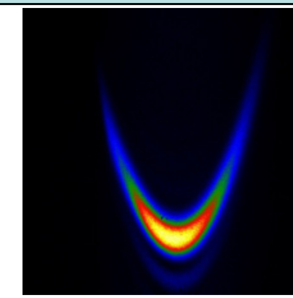
Outlook for Genesis 1.3 (To-Do List / Wishlist)

- Support of higher order wakefields, CSR fields and space charge fields similar to current implementation of longitudinal wakefields (look-up table).
- Stand-alone support of wakefields, separating it from the beam envelope (BEAMFILE) file.
- Integration of Genwake into Genesis to derive wakefields directly from external particle distribution or beam envelope file.
- Option for solving transverse Lorenz force equation to allow for transverse electric fields.

Space-Charge Issues of Electron Bunches from Laser-Plasma-Accelerators at Low Energy

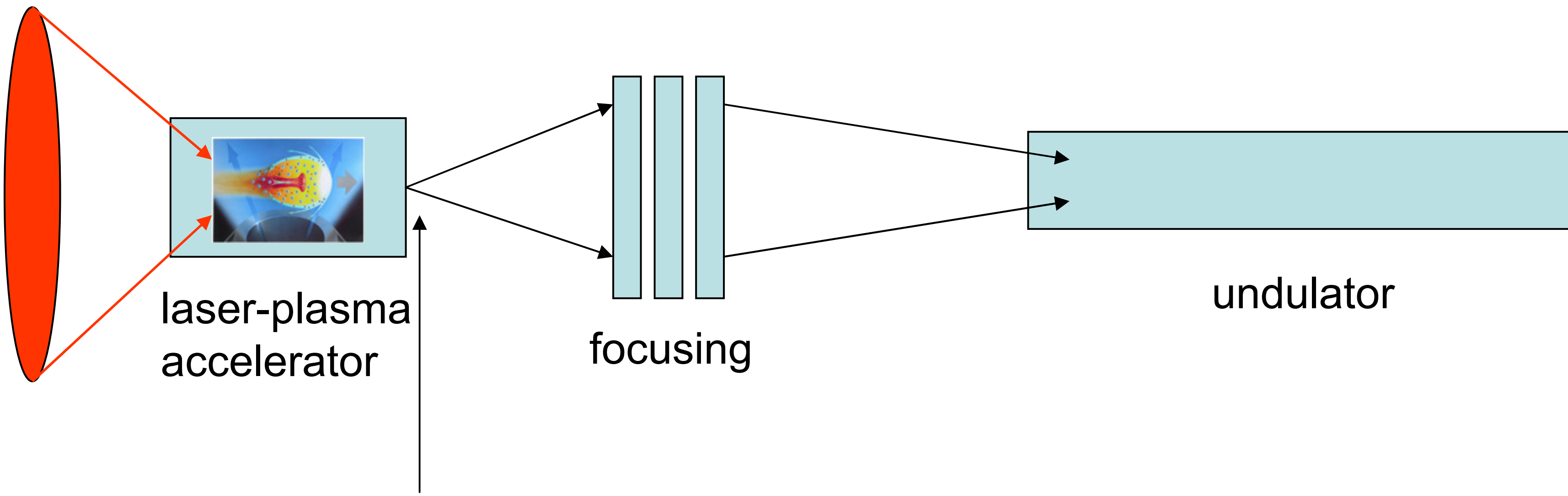
Max-Planck-Institut für Quantenoptik
MPQ

F. Grüner, MPQ
Space-charge workshop
BESSY, Dec 12-14, 2007



- basics of laser-plasma accelerators
- GSSY paper
- Bosch paper
- GPT for GSSY/Bosch case
- GPT for complete setup
- analytical approach

Laser plasma accelerators: setup for table-top FEL



beam size: ~ **µm**
charge: ~ **nC**
divergence: ~ mrad
energy: ~ 150 MeV

GSSY paper

Nuclear Instruments and Methods in Physics Research A 578 (2007) 34–46

equations according to arXiv version

their ansatz

the Helmholtz equation

$$c^2\nabla^2\vec{\bar{E}} + \omega^2\vec{\bar{E}} = 4\pi c^2\vec{\nabla}\bar{\rho} - 4\pi i\omega\vec{\bar{j}}\,, \qquad (3)$$

Eq. (3) can be solved with the help of an appropriate Green's function $G(z_o - z', \vec{r}_{\perp o} - \vec{r'}_\perp)$ yielding

$$\bar{E}_z(z_o, \vec{r}_{\perp o}, \omega) = -4\pi \int_{-\infty}^{\infty} dz' \int d\vec{r'}_\perp \left(\frac{i\omega}{c^2}\bar{j}_z - \frac{\partial\bar{\rho}}{\partial z'}\right) G(z_o - z', \vec{r}_{\perp o} - \vec{r'}_\perp)\,, \qquad (4)$$

Bosch

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **10**, 050701 (2007)

Longitudinal wake of a bunch of suddenly accelerated electrons within the radiation formation zone

For an electron traveling on the z-axis, which passes through the origin at time $t = 0$, the longitudinal Coulomb field expressed in coordinates $r = R\theta$ and z is [4]

$$\tilde{E}_{\mathrm{Coul}}(r, z, \omega) = e^{ikz} \exp(ikz/2\gamma^2)\left(\frac{-ie\omega}{2\pi\varepsilon_0 c^2 \gamma^2}\right) K_0\left(\frac{|\omega| r}{c\gamma}\right).$$

The wake is the sum of a Coulomb (space charge) wake and a coherent-radiation wake, where the Coulomb wake is

$$W_{\mathrm{Coul}}(0, z, \Delta t) = N \int_{-\infty}^{\infty} F(\omega) e^{-i\omega\Delta t} d\omega \int_0^{\infty} r' dr' \rho(r') \times \left[\left(\frac{-ie\omega}{2\pi\varepsilon_0 c^2\gamma^2}\right) K_0\left(\frac{|\omega| r'}{c\gamma}\right)\right] \qquad (10)$$

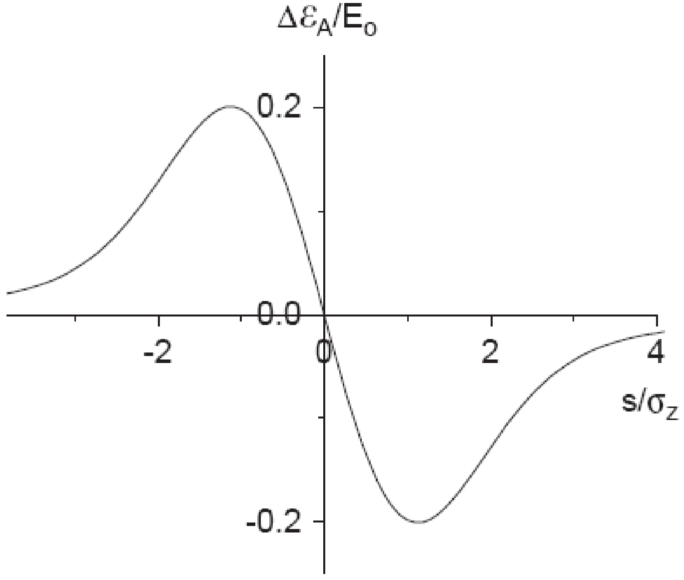
For a uniform radial density distribution within a bunch radius of r_b [$\rho(r) = 1/\pi r_b^2$ for $r < r_b$], the wakes are

$$W_{\mathrm{Coul}}(0, z, \Delta t) = \frac{-Ne}{2\pi^2 \varepsilon_0 r_b^2} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega\Delta t}\left(\frac{i}{\omega}\right) \times \left[1 - \frac{|\omega| r_b}{c\gamma} K_1\left(\frac{|\omega| r_b}{c\gamma}\right)\right] d\omega \qquad (12)$$

GSSY-chirp

$$\frac{\Delta\mathscr{E}_A}{\mathscr{E}_0}\left(\frac{s}{\sigma_z};\eta\right) = \frac{I_{\max}\hat{z}}{\gamma I_A}F\left(\frac{s}{\sigma_z};\eta\right)$$

$\gamma = 300,\ \eta = 10\ \text{and}\ I = 50\,\text{kA}$

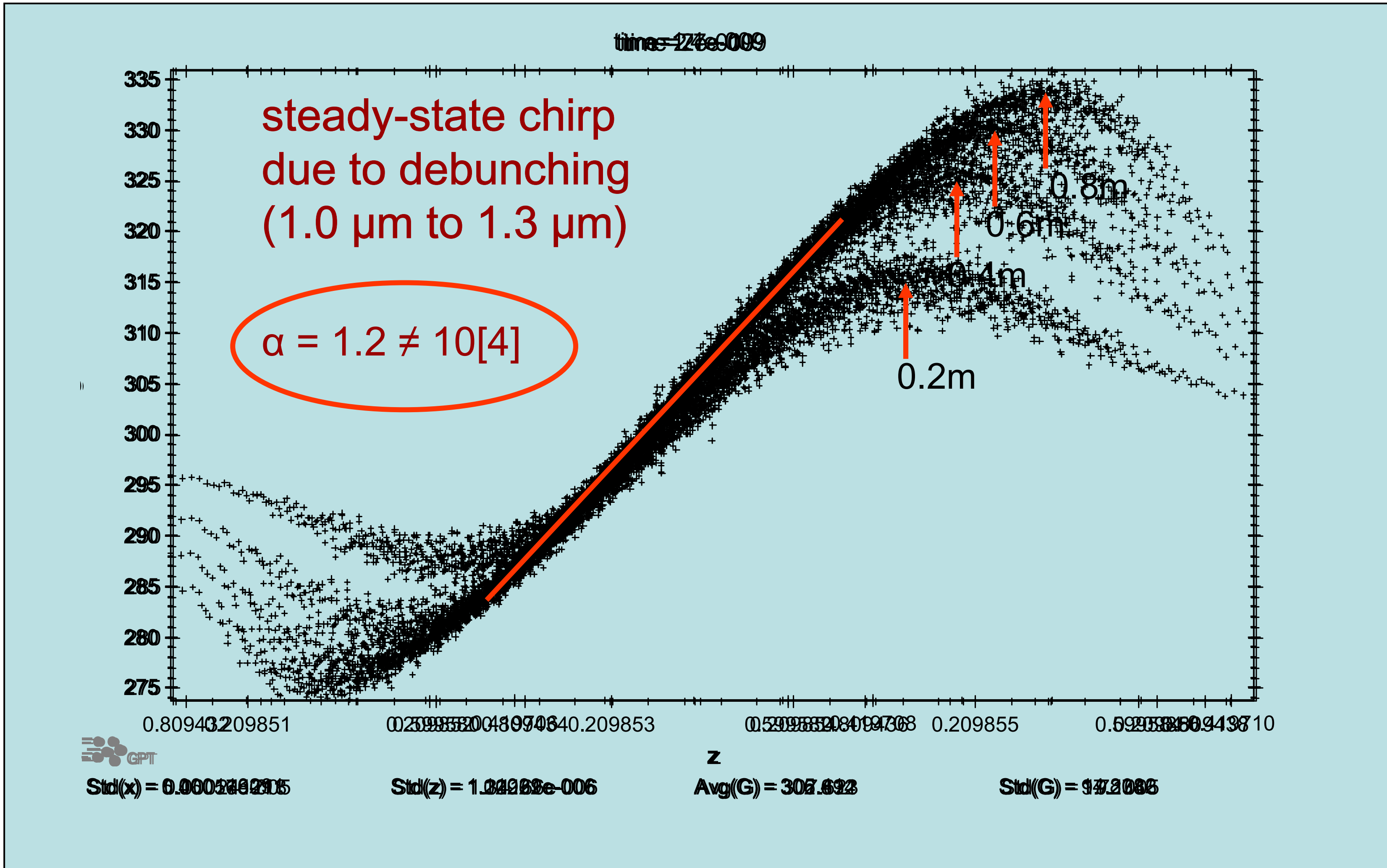


$$\hat{\alpha} = -(\gamma\omega\rho_{1D}^2)^{-1}\cdot \mathrm{d}\gamma/\mathrm{d}t \longrightarrow \hat{\alpha} \simeq 10$$

(normalized) chirp over one (ideal) cooperation length → (actually ~4)

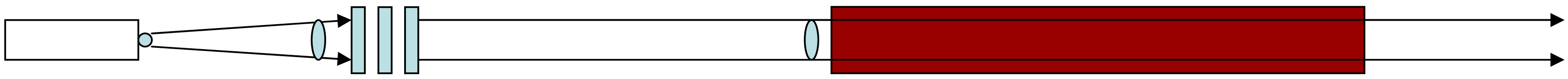
GPT for GSSY/Bosch

parameter: $\gamma = 300$, $\sigma_z = 1\ \mu m$, $\sigma_\perp = 30\ \mu m$ and zero emittance
stop at z = 0.8 m



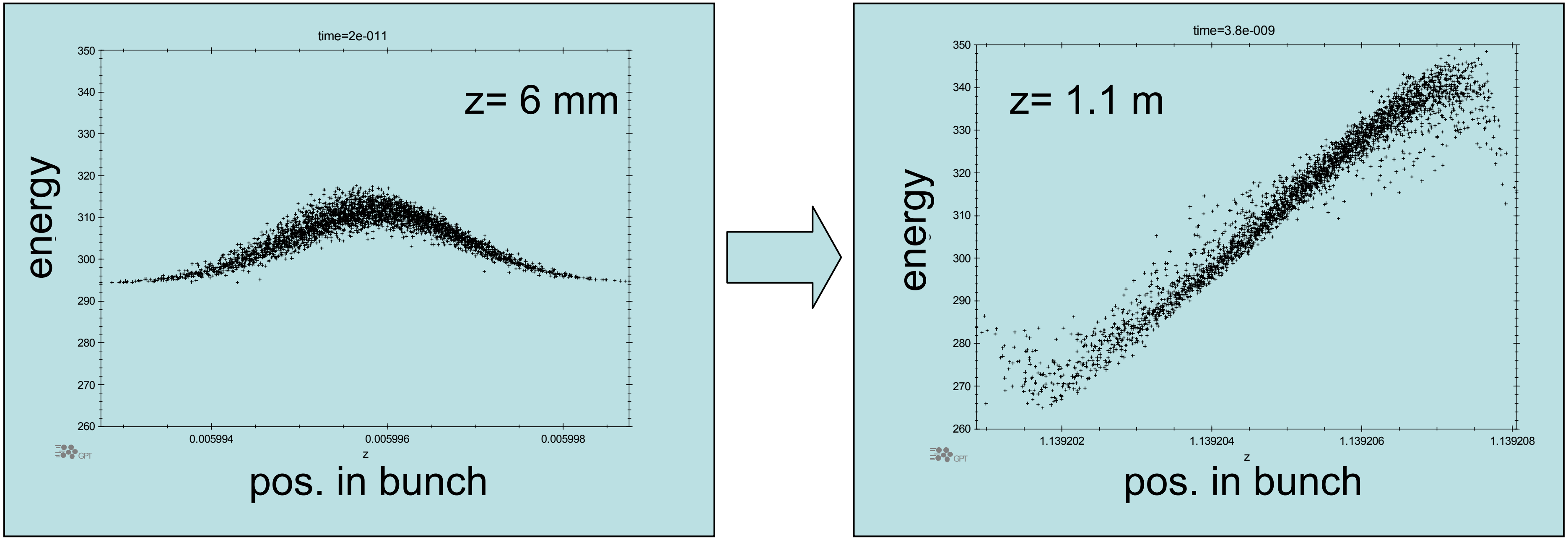
GPT for complete setup

bunch evolves



transverse expansion

longitudinal expansion



Conclusion

- GSSY/Bosch correct for „frozen bunch“
  - ever-increasing chirp
- GPT reaches soon steady-state chirp
  - in agreement with expansion time scales
  - in agreement with finite self-energy

- main question: does FEL cope with (correlated!) debunching: global debunching vs. local micro-bunching

FEL driven by the LBNL laser-plasma accelerator

Carl B. Schroeder

Lawrence Berkeley National Laboratory

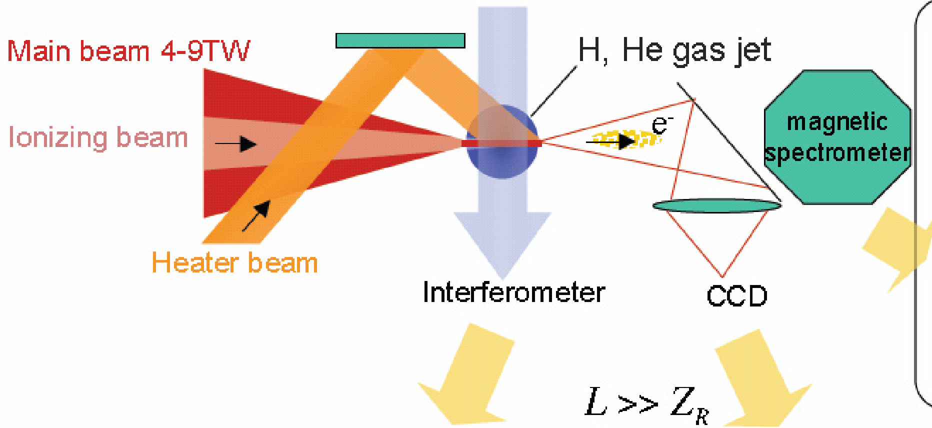
Mini-Workshop on Space Charge Effects in FELs
BESSY: 12-14 December 2007

Work supported by Office of Science, US DOE, Contract DE-AC02-05CH11231.

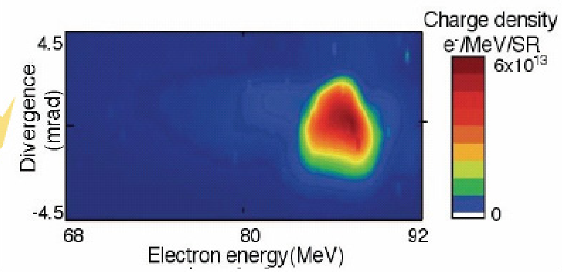
Outline

- Quality and stability of GeV-class laser-plasma beams at LBNL
- FEL application and electron beam requirements
- Space-charge effects
- Coherent transition radiation generation and measurements

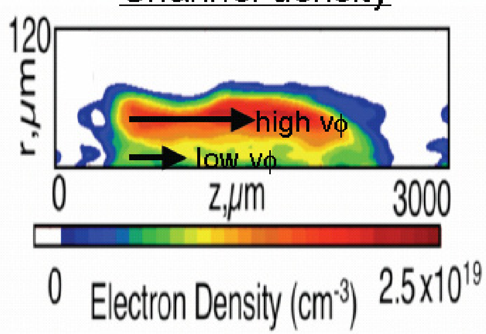
2004 LBNL Expts: Plasma channels produce high quality e-beams by guiding relativistic intensities



9TW: high-quality e-bunches
hundreds of pC, 2% ΔE/E, 3 mrad

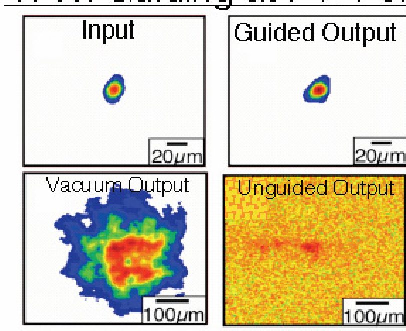


Channel density



variation of n_e guides laser

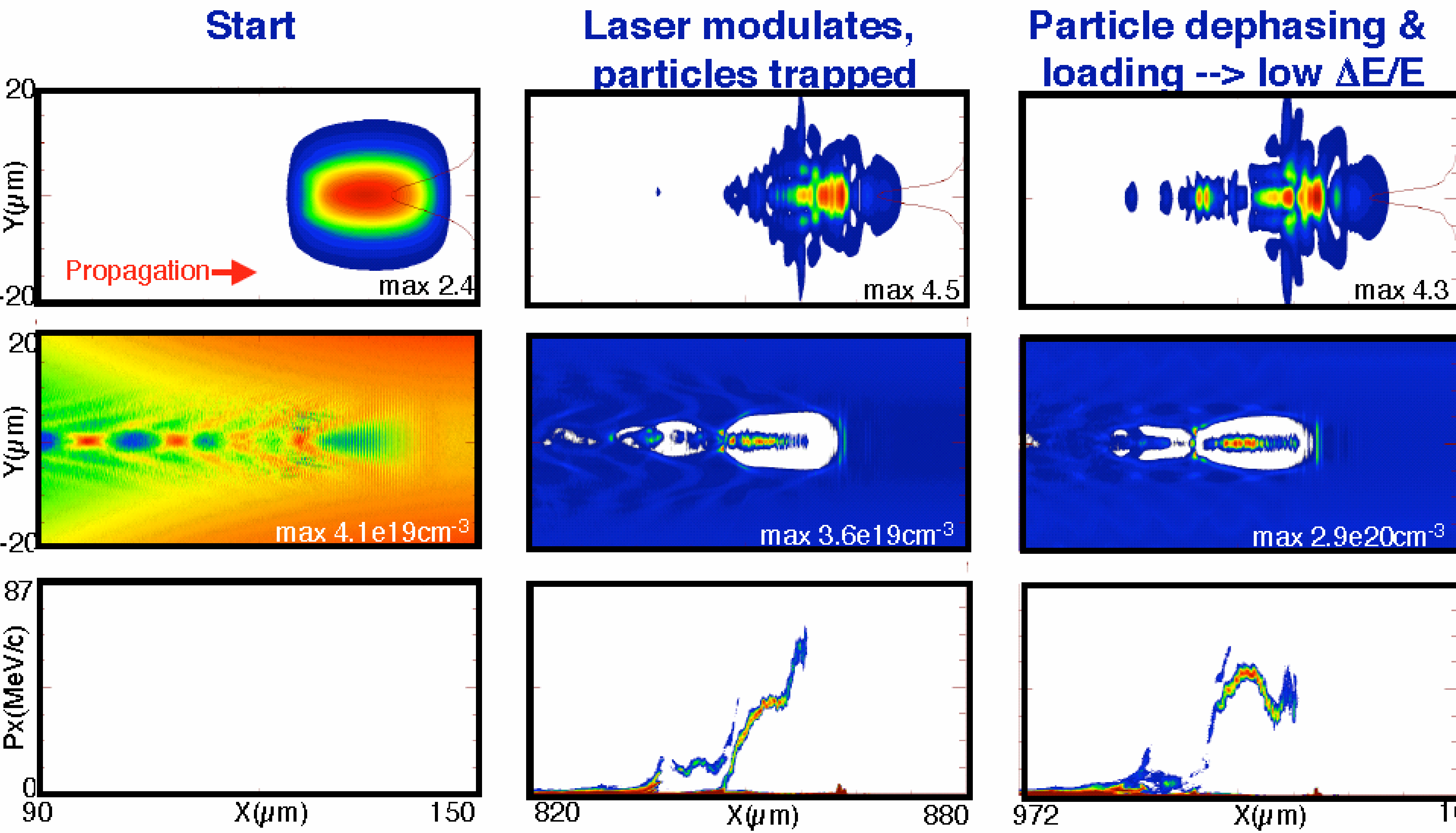
4TW: Guiding at P > Pcrit



Density profile tuned to compensate for self guiding

Geddes *et al*, PRL 2004, Nature 2004

extension of interaction to dephasing



PIC Simulation [resolve laser period (0.8 micron) over mm distance in 3D]:

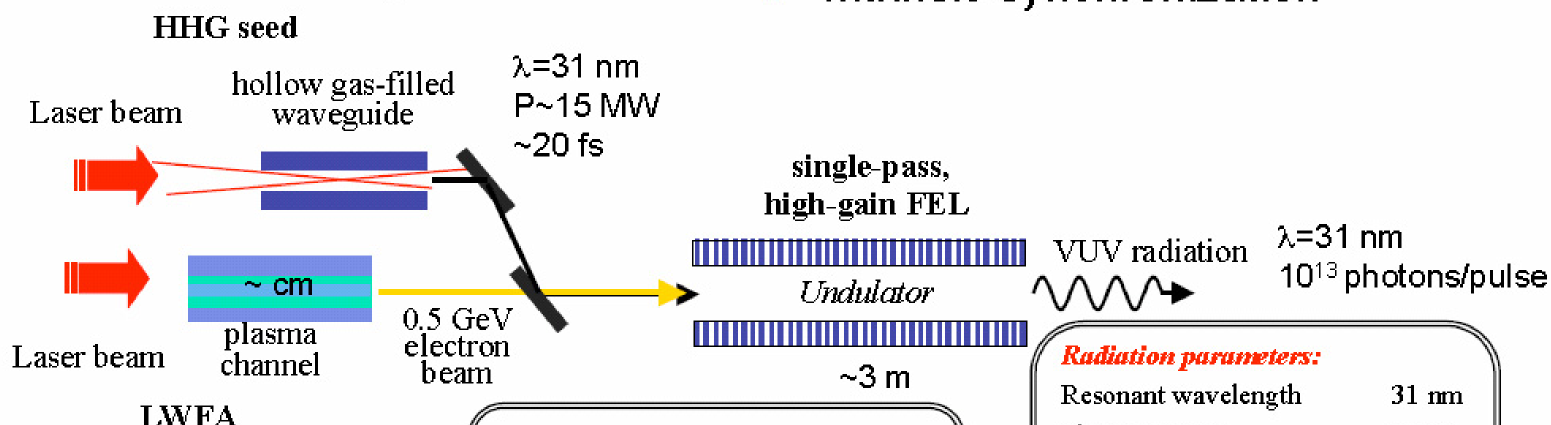
Charge 400 pC, E= 100 MeV, ΔE/E 5-10% (approaching experiment)

High harmonic generation (HHG) seeding

- Reduced undulator length
- Reduced power fluctuations
- Improved coherence
- Intrinsic synchronization

HHG* seed:

Harmonic wavelength	31 nm
Power	15 MW
Duration, FWHM	20 fs



LWFA Electron Beam:

Beam Energy	0.5 GeV
Peak current	10 kA
Charge	0.2 nC
Bunch duration, FWHM	20 fs
Energy spread (slice)	0.25 %
Norm. Emittance	1 mm-mrad

Undulator Parameters:

Undulator type	planar
Undulator period	2.18 cm
Number of periods	220
Peak Field	1.02 T
Undulator parameter, K	1.85
Beta function	3.6 m

Radiation parameters:

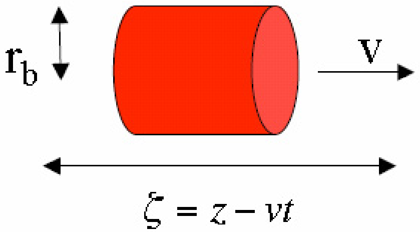
Resonant wavelength	31 nm
Photon energy	40 eV
Pierce parameter	$5x10^{-3}$
1D Gain length	0.19 m
3D Gain length	0.31 m
Steady-state sat. power	12 GW
Spontaneous rad. Power	4 kW
Slippage length	7 µm

* E. Takahashi et al., Phys. Rev. E 66, 021802 (2002).

Longitudinal space-charge wakefield

- Consider flat-top transverse profle: R. Bosch, Phys. Rev. ST-Accel. Beams (2007);

$$g(r) = \begin{cases} 1/(\pi r_b^2), & \text{if } r < r_b \\ 0, & \text{if } r > r_b \end{cases}$$



Total longitudinal force along axis:

$$F(r=0,\zeta) = \frac{4ie^2N}{r_b^2}\int \frac{dk}{2\pi} e^{-ik\zeta}\frac{F(k)}{k}\left[1 - \left(\frac{|k|r_b}{\gamma}\right)K_1\left(\frac{|k|r_b}{\gamma}\right)\right]$$

Total differential force:

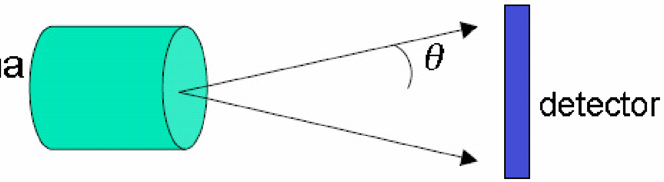
$$\frac{dF}{d\zeta}(r=0,\zeta) = \frac{2e^2N}{\pi r_b^2}\int dk e^{-ik\zeta} F(k)\left[1 - \left(\frac{|k|r_b}{\gamma}\right)K_1\left(\frac{|k|r_b}{\gamma}\right)\right]$$

$\Rightarrow$ valid after formation length: $L_f \approx 2\sigma_z\gamma^2$

Diverging beam

- Diverging beam:

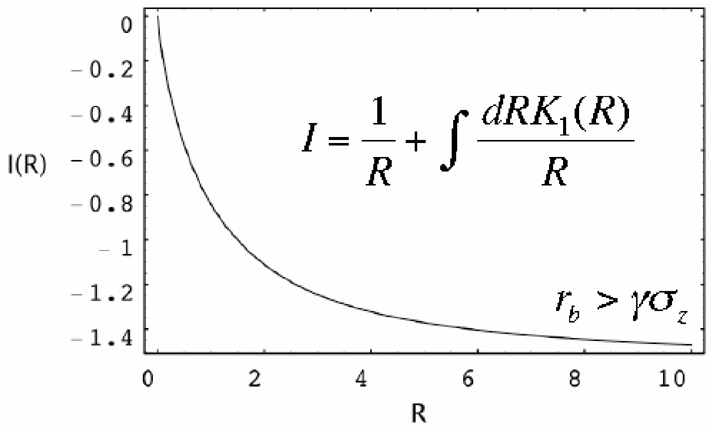
  $r_b(z) = \theta z$



- Generalization of energy change for diverging beam:

$$\Delta W(r=0,\Delta z,\zeta) = \frac{2ie^2N}{\pi\gamma\theta}\int dk e^{-ik\zeta}F(k)\int_{R_0}^{R_1} dR R^{-2}\left[1 - RK_1(R)\right]$$

$$= \frac{2ie^2N}{\pi\gamma^2}\Delta z\int dk e^{-ik\zeta}kF(k)\frac{[I(R_0)-I(R_1)]}{R_1-R_0}$$

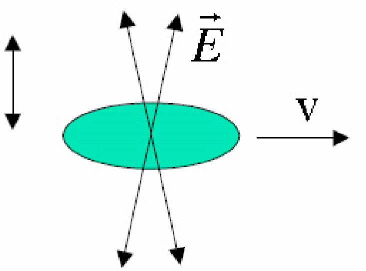


$$I = \frac{1}{R} + \int \frac{dRK_1(R)}{R}$$

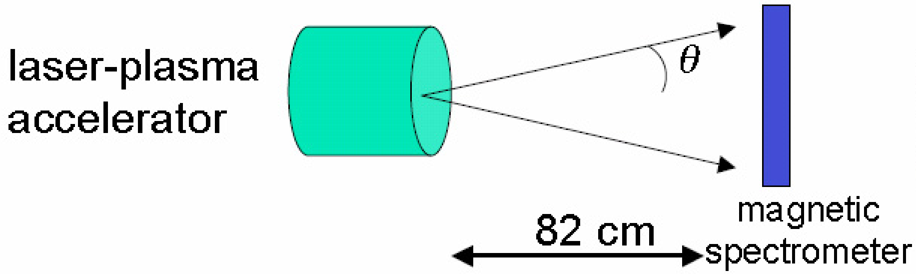
$r_b > \gamma\sigma_z$

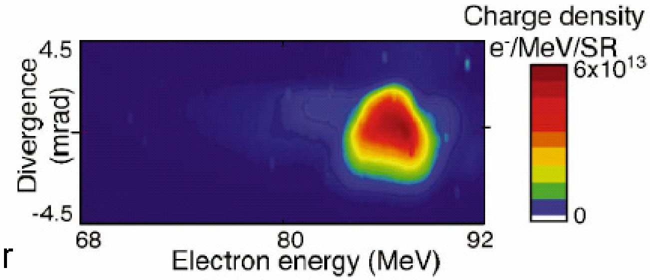
$R = |k| r_b / \gamma$

$\sim \gamma\lambda \sim \gamma\sigma_z$



Space-charge: 2004 LBNL LWFA expt.



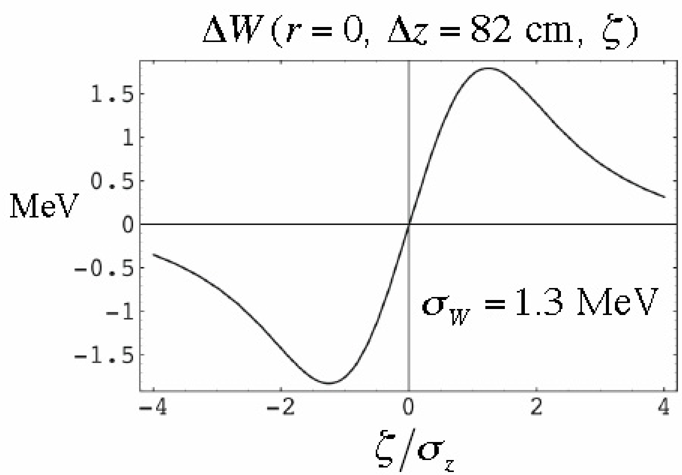


Formation length ⇒ L_f ~ 8 cm

86 ±1.8 MeV,

3 mrad divergence,

10 kA



$\sigma_\gamma / \gamma = 1.5\%$

⇒ Constant radius beam (24 micron) predicts 12% energy spread