# Long Range Wake Potential of BPM in Undulator Section

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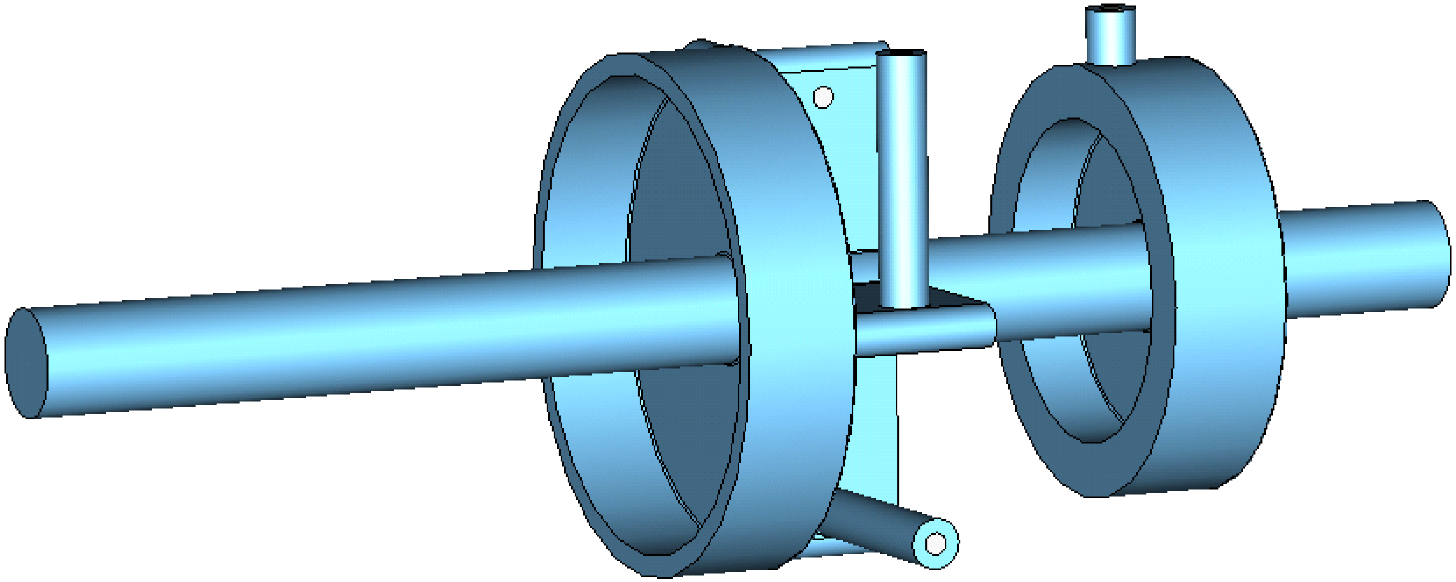
Beam Dynamics Group Meeting

29.01.08

# BPM Geometry (Dirk Lipka)



# Longitudinal wake

$$w_{||}(s) = \sum_i 2k_i \cos\left(\omega_i \frac{s}{c}\right) e^{-\alpha_i \frac{s}{c}}$$

$$\lambda(s) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{s^2}{2\sigma^2}}$$

$$W_{||}(s) = \int_{-\infty}^{s} w_{||}(s-s')\lambda(s')ds' \approx \sum_i 2k_i e^{0.5\frac{\sigma^2}{c^2}\left(\alpha_i^2-\omega_i^2\right)} \cos\left(\omega_i \frac{s}{c} - \omega_i \alpha_i \frac{\sigma^2}{c^2}\right) e^{-\alpha_i \frac{s}{c}}, \quad s \gg \sigma$$

$$W_{||}(s) \approx \sum_i 2k_i^{(\sigma)} \cos\left(\omega_i \frac{s}{c}\right) e^{-\alpha_i \frac{s}{c}}, \quad s \gg \sigma$$

$$\alpha_i = \frac{\omega_i}{2Q_i}$$

$$k_i^{(\sigma)} = k_i e^{-0.5\frac{\sigma^2}{c^2}\omega_i^2\left(1-\frac{1}{4Q_i^2}\right)} \approx k_i e^{-0.5\frac{\sigma^2}{c^2}\omega_i^2}$$

$$k_i\left(\vec{r}_\perp^{(1)}, \vec{r}_\perp^{(2)}\right) = \frac{V^*\left(\vec{r}_\perp^{(1)}\right) V\left(\vec{r}_\perp^{(2)}\right)}{4U_i}$$

# Transverse wake

$$\vec{w}_\perp\left(s,\vec{r}_\perp^{(1)},\vec{r}_\perp^{(2)}\right)=\vec{w}_\perp(0,0)+\vec{\vec{W}}_\perp^{D}(s)\vec{r}_\perp^{(1)}+\vec{\vec{W}}_\perp^{Q}(s)\vec{r}_\perp^{(2)}+O(2)$$

For structure with symmetry group of rotations $C_4$

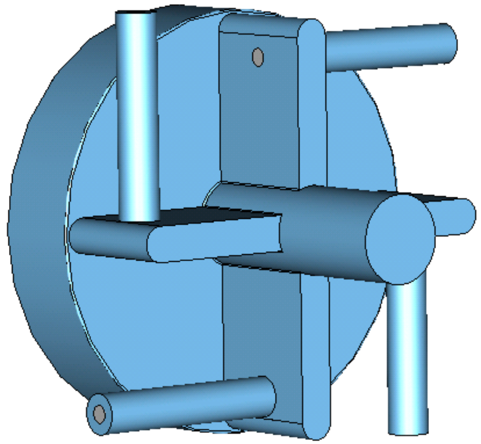
$$\vec{w}_\perp(s,\vec{r}_\perp)\approx\vec{r}_\perp w_\perp(s)$$

$$w_\perp(s)\approx\sum_i c\left(\frac{R}{Q}\right)_i \sin\left(\omega_i\frac{s}{c}\right)e^{-\alpha_i\frac{s}{c}},\qquad s \gg \sigma$$

$$\left(\frac{R}{Q}\right)_i=\frac{2k_i}{\omega}\qquad\qquad k_i=\frac{V^*(\vec{r}_\perp)V(\vec{r}_\perp)}{4U_i\left|\vec{r}_\perp\right|^2}$$

$$W_\perp(s)\approx\sum_i c\left(\frac{R}{Q}\right)_i^{(\sigma)}\sin\left(\omega_i\frac{s}{c}\right)e^{-\alpha_i\frac{s}{c}}$$

$$\left(\frac{R}{Q}\right)_i^{(\sigma)}=\left(\frac{R}{Q}\right)_i e^{-0.5\frac{\sigma^2}{c^2}\omega_i^2}$$

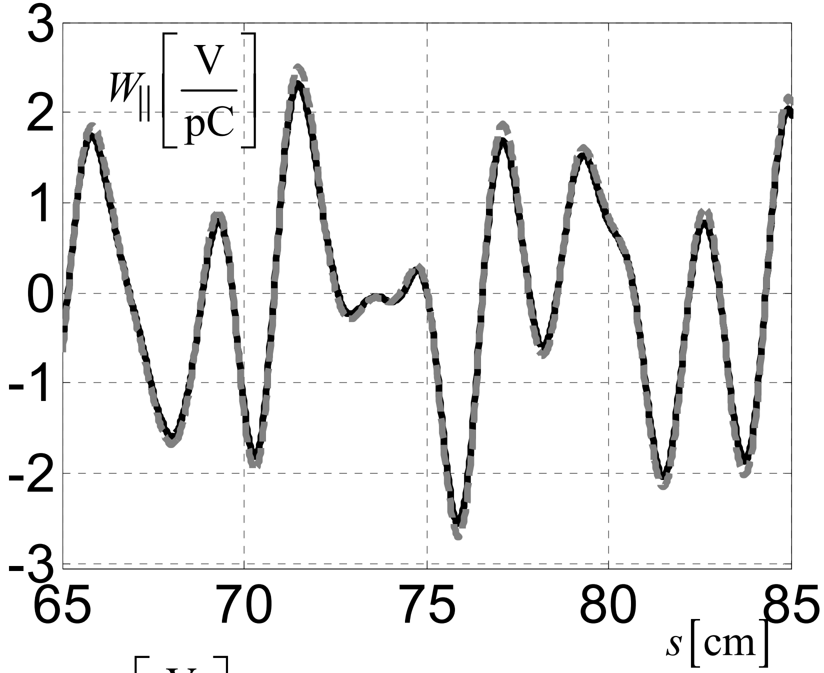


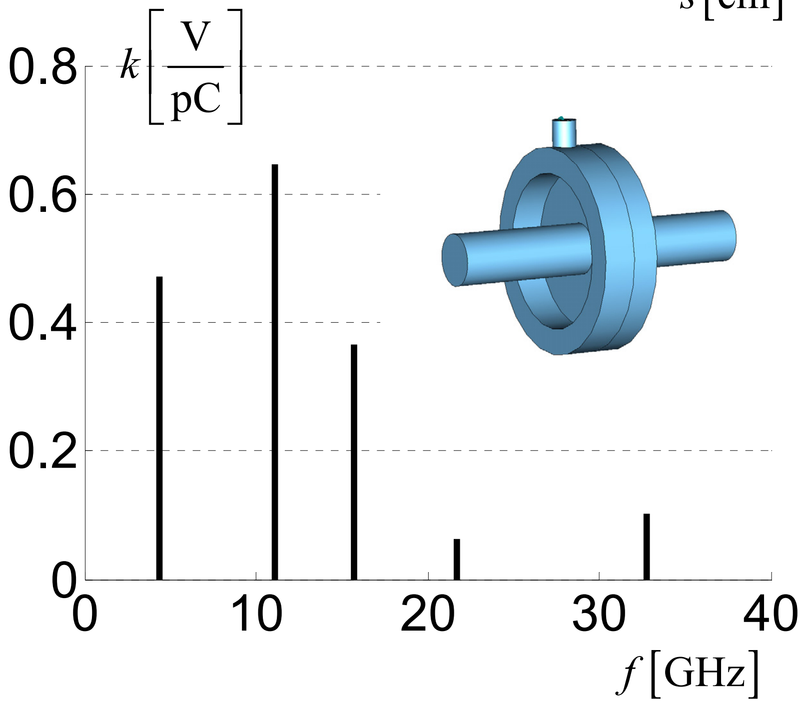
FD - Frequency Domain
TD - Time Domain

MWS - CST Microwave Studio
PS - CST Particle Studio

ECHO – time-domain wakefield code
Prony – Prony-Pisarenko method

# Reference Cavity (PEC)




$$f_{cut} = 2.4048\frac{c}{2\pi a} \sim 23\mathrm{GHz}$$

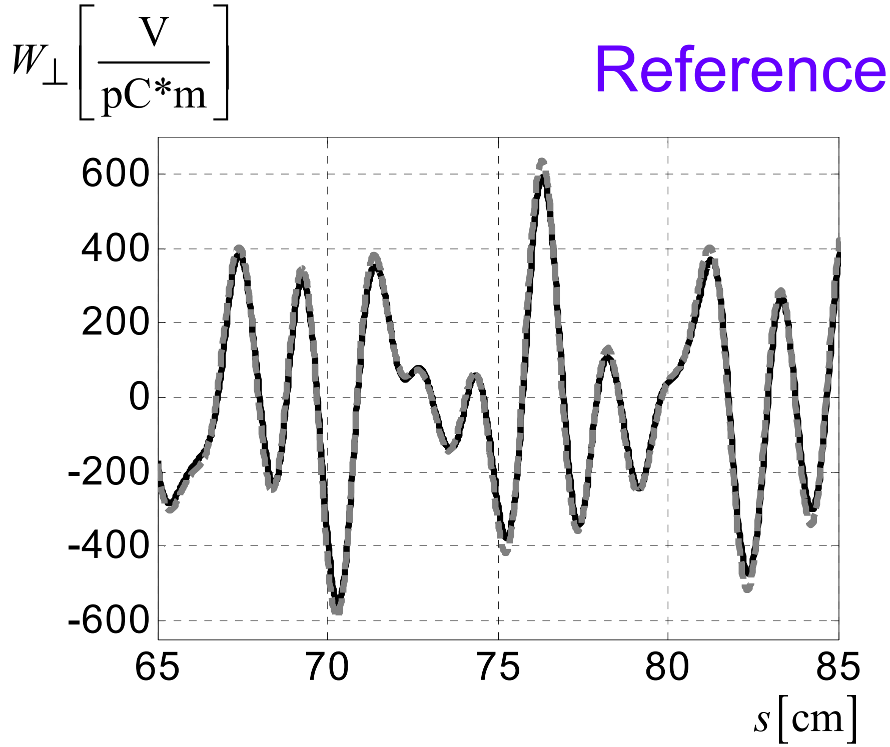
ECHO + Prony

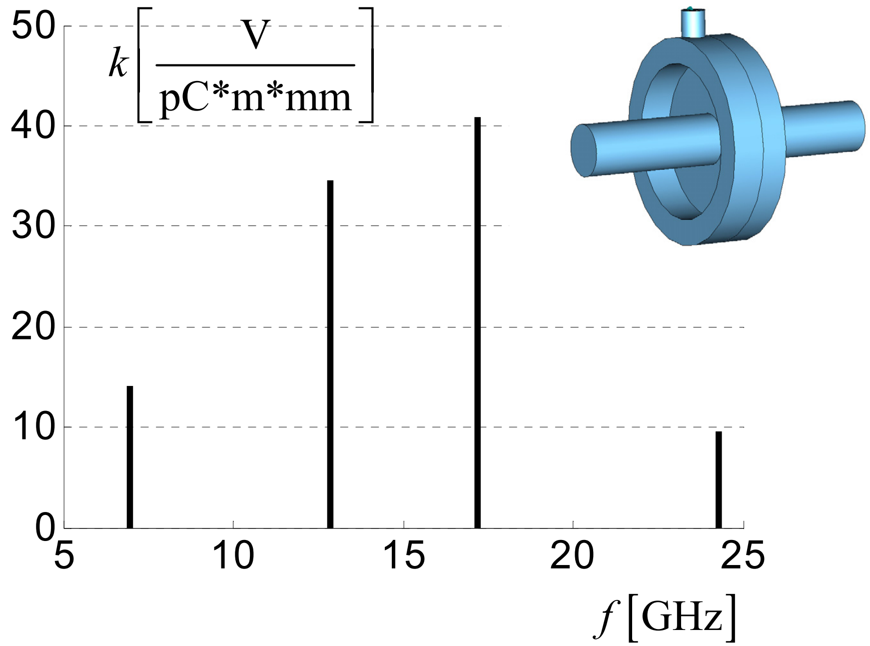
| f, GHz | k, V/pC |
|---|---|
| **4.3744** | **0.4696** |
| **11.0772** | **0.6294** |
| **15.7603** | **0.3466** |
| **21.7338** | **0.0577** |

CST PS + Prony

| f, GHz | k, V/pC |
|---|---|
| **4.3713** | **0.4721** |
| **11.066** | **0.6505** |
| **15.7501** | **0.36574** |
| **21.677** | **0.0566** |

# Reference Cavity (PEC)

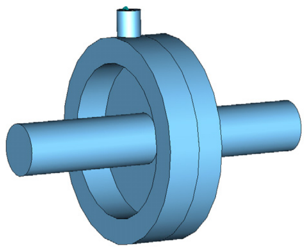



ECHO + Prony

| f, GHz | k, V/(pC*m*m) |
|---|---|
| **6.9594** | **14.1565e3** |
| **12.8695** | **34.5164e3** |
| **17.1830** | **40.8118e3** |

$$f_{cut} = 1.841 \frac{c}{2\pi a} \sim 17.6\text{GHz}$$



# Reference Cavity (Lossy)

| Stainless Steel 304 | conductivity 1,40E+06 |
|---|---|

## longitudinal (monopole)

| f, GHz (MWS, FD) | k, V/pC (MWS, FD) | k, V/pC (PS, TD) | k, V/pC (ECHO, TD) | Q (MWS, FD) | Q (PS, TD) | $\alpha$, $sec^{-1}$ |
|---|---|---|---|---|---|---|
| **4.37** | **0.46** | 0.47 | 0.47 | 636 | 602 | **2.2e7** |
| **11.06** | **0.60** | 0.65 | 0.63 | 995 | 890 | **3.5e7** |
| **15.75** | **0.33** | 0.36 | 0.35 | 1113 | 1075 | **4.4e7** |
| **21.72** | **0.16** | 0.07 | 0.06 | 1712 | 1126 | **4.0e7** |

## transverse (dipole)

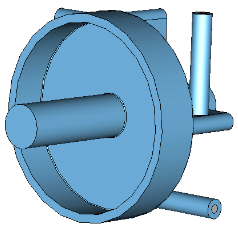
| f, GHz (MWS, FD) | k, V/(pC*m*m) (MWS, FD) | k, V/(pC*m*m) (PS, TD) | k, V/(pC*m*m) (ECHO, TD) | $2\pi R/Q$, $\Omega/mm^2$ | Q (MWS, FD) | Q (PS, TD) | $\alpha$, $sec^{-1}$ |
|---|---|---|---|---|---|---|---|
| **6.96** | 13e3 | 15e3 | 14e3 | **3.74** | **797** | 1989 | **2.7e7** |
| **12.86** | 31e3 | 32e3 | 34e3 | **4.82** | **1000** | 1129 | **4.0e7** |
| **17.22** | 39e3 | 48e3 | 41e3 | **4.53** | **1117** | 608 | **4.8e7** |
| **24.43** | 25e3 | 32e3 | 10e3 | **2.05** | **1586** | 2353 | **4.8e7** |

# Resonator Cavity (Lossy)

| Stainless Steel 304 | conductivity 1,40E+06 |
|---|---|

longitudinal

| f, GHz (MWS, FD) | K, 1e12 (MWS, FD) | K, 1e12 (PS, TD) | Q (MWS, FD) | α, $sec^{-1}$ |
|---|---|---|---|---|
| **3.51** | **0.12** | 0.11 | 288 | **3.8e7** |
| **8.56** | **0.23** | 0.19 | 428 | **6.3e7** |
| **13.44** | **0.22** | 0.19 | 543 | **7.8e7** |
| **18.31** | **0.16** | 0.17 | 683 | **8.4e7** |



transverse

| f, GHz (MWS, FD) | k, V/(pC*m*m (MWS, FD) | k, V/(pC*m*m (PS, TD) | Q (MWS, FD) | 2πR/Q, Ω/$mm^2$ | α, $sec^{-1}$ |
|---|---|---|---|---|---|
| **4.46** | 5.1e3 | 6.5e3 | 367 | **2.3** | **3.8e7** |
| **7.67** | 2.0e3 | 2.7e3 | 469 | **0.5** | **5.1e7** |
| **9.3** | 2.3e3 | 2.7e3 | 470 | **0.5** | **6.2e7** |
| **10.46** | 6.1e3 | 6.1e3 | 549 | **1.2** | **6.0e7** |
| **14.07** | 11.1e3 | 12.5e3 | 607 | **1.6** | **7.3e7** |
| **15.16** | 2.4e3 | 2.5e3 | 719 | **0.3** | **6.6e7** |
| **19.17** | 13.5e3 | 17e3 | 715 | **1.4** | **8.4e7** |
| **20.4** | 17.7e3 | 4.9e3 | 902 | **1.7** | **7.1e7** |

# Asymptotic multi-bunch loss

$$W_{||}^{(\infty)}(s)=\sum_{j=1}^{\infty}W_{||}(j\Delta+s)=\sum_{j=1}^{\infty}\sum_{i}2k_i^{(\sigma)}\cos\left(\omega_i\frac{j\Delta+s}{c}\right)e^{-\alpha_i\frac{j\Delta+s}{c}}\leq$$

$$\leq\frac{e^{-\alpha\frac{\Delta+s}{c}}}{1-e^{-\alpha\frac{\Delta}{c}}}\sum_i 2k_i^{(\sigma)}\approx\frac{e^{-\alpha\frac{\Delta}{c}}}{1-e^{-\alpha\frac{\Delta}{c}}}\sum_i 2k_i$$

$$\alpha=\min\left(\alpha_i\right)$$

$$\frac{\alpha\sigma}{c}\ll 1$$

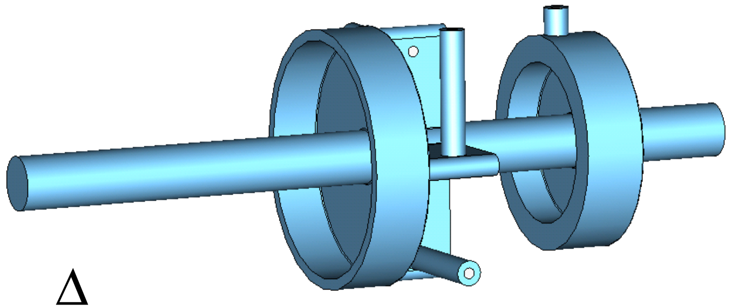
# Asymptotic multi-bunch kick

$$W_{\perp}^{(\infty)}(s)\leq\frac{e^{-\alpha\frac{\Delta}{c}}}{1-e^{-\alpha\frac{\Delta}{c}}}c\sum_i\left(\frac{R}{Q}\right)_i$$

# Multi-bunch effect

$$\frac{\Delta}{c} = 2e-7\ [\mathrm{sec}]$$ - bunch spacing

$$\alpha_{\|} = 2.2e7\ [\mathrm{sec}^{-1}] \qquad \frac{e^{-\alpha_{\|}\frac{\Delta}{c}}}{1-e^{-\alpha_{\|}\frac{\Delta}{c}}} = 0.01$$



$$W_{\|}^{(\infty)}(s) \le 0.01 \sum_i 2k_i \approx 0.05 \frac{\mathrm{kV}}{\mathrm{nC}}$$

$$\alpha_{\perp} = 2.7e7\ [\mathrm{sec}^{-1}] \qquad \frac{e^{-\alpha_{\perp}\frac{\Delta}{c}}}{1-e^{-\alpha_{\perp}\frac{\Delta}{c}}} = 0.005$$

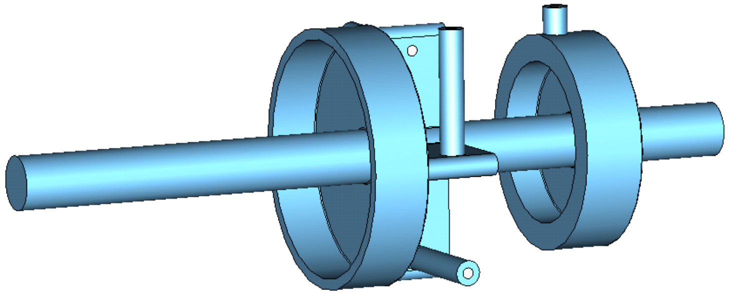
$$W_{\perp}^{(\infty)}(s) \le 0.005c \sum_i \left(\frac{R}{Q}\right)_i \approx 0.0066 \frac{\mathrm{kV}}{\mathrm{nC}\times\mathrm{mm}}$$

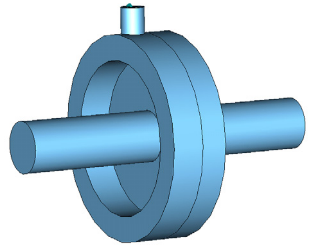
# Long Range vs. Short Range

## Long Range Wakes

$$W_{\|}^{(\infty)}(s) \leq 0.01 \sum_i 2k_i \quad \approx 0.05 \frac{\text{kV}}{\text{nC}}$$

$$W_{\perp}^{(\infty)}(s) \leq 0.005c \sum_i \left(\frac{R}{Q}\right)_i \quad \approx 0.0066 \frac{\text{kV}}{\text{nC}\times\text{mm}}$$





## Short Range Wakes

$$\text{Loss} = \frac{Z_0 c}{4a\pi^{2.5}} \Gamma\left(\frac{1}{4}\right)\sqrt{\frac{g}{\sigma}} = O(\sigma^{-0.5})$$

$$k_{\|}^{25\mu\text{m}} \approx 16.6 \frac{\text{kV}}{\text{nC}} \approx \sum_i 2k_i \, e^{-\frac{\sigma^2}{c^2}\omega_i^2}$$

$$\text{Kick} = \frac{2}{a^3} \frac{Z_0 c}{\pi^{2.5}} \Gamma\left(\frac{3}{4}\right)\sqrt{g\sigma} = O(\sigma^{0.5})$$

$$k_{\perp}^{25\mu\text{m}} \approx 0.045 \frac{\text{kV}}{\text{nC}\times\text{mm}} \approx c \sum_i \left(\frac{R}{Q}\right)_i D(\omega_i \sigma)$$

# Comparison with beam parameters in the undulator

$$E_0 = 17.5\,\text{GeV}$$

$$\sigma_{x'} = \sqrt{\frac{\varepsilon_n}{\gamma\beta}} = \sqrt{\frac{1.4\,\text{mm*mrad}}{34247 * 45\text{m}}} \approx 1\,\mu\text{rad}$$

$$W_{\|}^{(\infty)}(s) \le 0.05 \frac{\text{kV}}{\text{nC}}$$

$$W_{\perp}^{(\infty)}(s) \le 0.0066 \frac{\text{kV}}{\text{nC}\times\text{mm}}$$

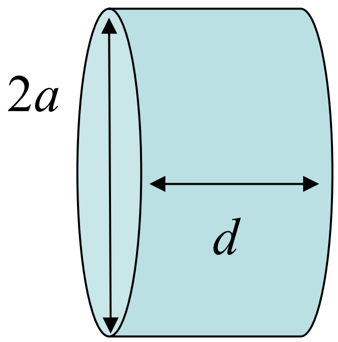
$$\left\langle \Delta E \right\rangle = eQ \left\langle W_{\|}^{(\infty)} \right\rangle \le 0.05\text{keV}$$

$$\frac{\left\langle \Delta r' \right\rangle}{r} = \frac{eQ}{E} \left\langle W_{\perp}^{(\infty)} \right\rangle \le 0.0004 \frac{\mu\text{rad}}{\text{mm}}$$

$$\frac{\left\langle \Delta E \right\rangle}{E_0} \le \frac{50\,\text{eV}}{17.5\,\text{GeV}} = 3e-9$$

$$\frac{\left\langle \Delta r' \right\rangle}{r\sigma_{x'}} \le \frac{4e-4}{\text{mm}}$$

# Geometry scaling




$$Q_i \frac{\delta_s}{\lambda_i} = F\left(i, \frac{a}{d}\right) \qquad \delta_s = \sqrt{\frac{2}{\omega_i \mu \kappa}}$$

(see R.E.Collin „Foundations for microwave engineering“)

If we scale the cavity by factor λ as

$$a, d \rightarrow \lambda a, \lambda d$$

then

$$\omega_i \rightarrow \frac{\omega_i}{\lambda}$$

$$Q_i \rightarrow Q_i \sqrt{\lambda}$$

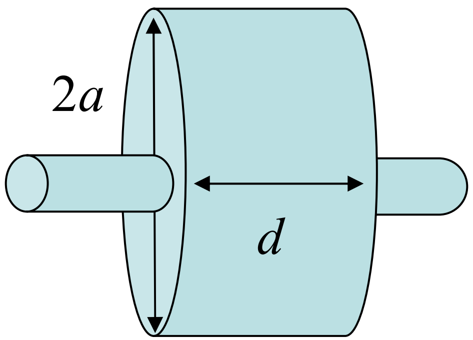
$$\alpha_i \rightarrow \frac{\alpha_i}{\lambda^{1.5}}$$

$$k_{i\|} \rightarrow k_{i\|} \frac{1}{\lambda}$$

$$k_{i\perp} \rightarrow k_{i\perp} \frac{1}{\lambda^3}$$

$$\left(\frac{R}{Q}\right)_i \rightarrow \left(\frac{R}{Q}\right)_i \frac{1}{\lambda^2}$$

# Geometry scaling




If we ***scale only the cavity*** by factor $\lambda$ as

$$a, d \to \lambda a, \lambda d$$

then

$$\omega_i \to \frac{\omega_i}{\lambda} \qquad \alpha_i \to \frac{\alpha_i}{\lambda^{1.5}} \qquad k_i \to k_i \frac{1}{\lambda}$$

$$W_{||}^{(\infty)}(s,\lambda) \le \frac{(0.01)^{\lambda^{-1.5}}}{\lambda} \sum_i 2k_i$$

$$W_{\perp}^{(\infty)}(s,\lambda) \le \frac{(0.005)^{\lambda^{-1.5}}}{\lambda^2} c \sum_i \left(\frac{R}{Q}\right)_i$$