Impedance Minimization by Nonlinear Tapering

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DESY XFEL
X-Ray Free-Electron Laser

Outline

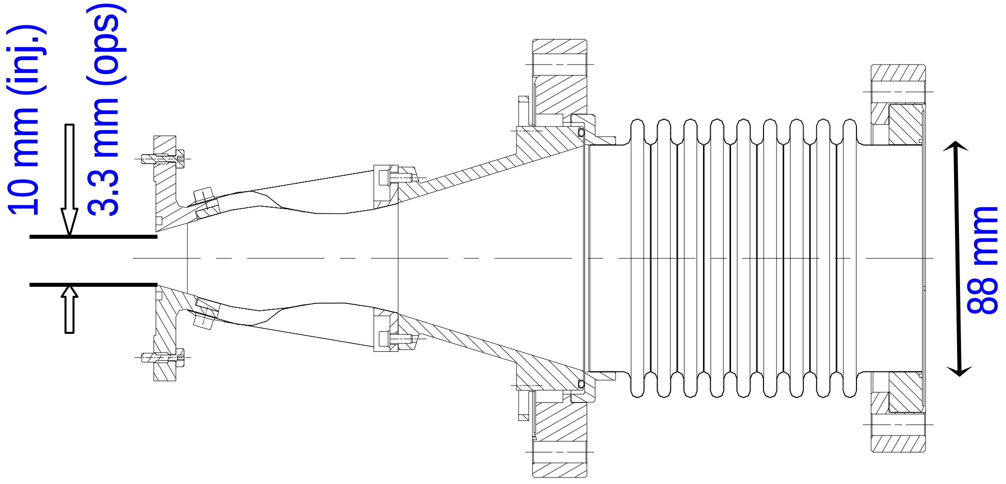
- Motivation
- Review of theoretical results
- Optimal boundary
- EM solvers used
- Results for impedance reduction
- Conclusion

Z-optimization by non-linear tapering is not that new ….

Apart from acoustics, used for gyrotron tapers, mode converters, antennae design, etc.

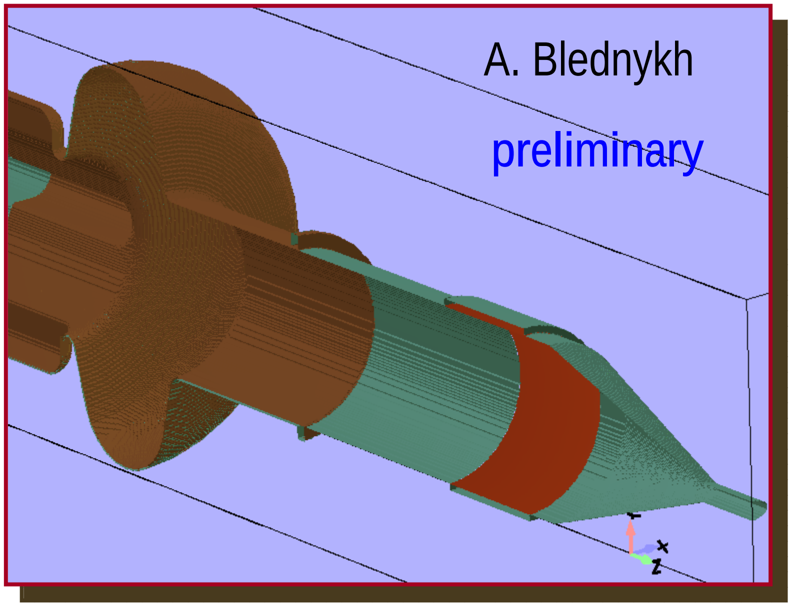
Examples of Accelerator Tapers & Motivation and Scope of This Work

NSLS MGU Taper for X13, X25, X29 & X9



h_{max}/h_{min}~ 9 (gap open) or 27 (gap closed)

NSLS-II transition to SC RF cavity



A. Blednykh

preliminary

h_{max}/h_{min}~10

Focus on large X-sectional variations and gradual tapering; study transverse, broad-band geometric impedance @ low frequency inductive regime

Goal to reduce Z to avoid instabilities (TMCI) in rings, or ε degradation in linacs ...

Theory Review

Axially symmetric taper

$$Z_\perp(k) \cong -\frac{iZ_0}{2\pi}\int_{-\infty}^{\infty} dz \frac{r'(z)^2}{r(z)^2}$$

K. Yokoya, 1990

Flat rectangular taper 2*w* × 2*h*, *w*>>*h*

$$Z_y^{rect}(k) = -\frac{iZ_0 w}{4}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^3}$$

G. Stupakov, 1995

Elliptical x-section taper 2*w* × 2*h* , *w*>>*h*

$$Z_x^{ell}(k) = -\frac{iZ_0}{4\pi}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^2}$$

$$Z_y^{ell}(k) = -\frac{iZ_0 \pi w}{16}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^3}$$

Z_x: $h \ll L$, $k <\sim 1/h_{min}$

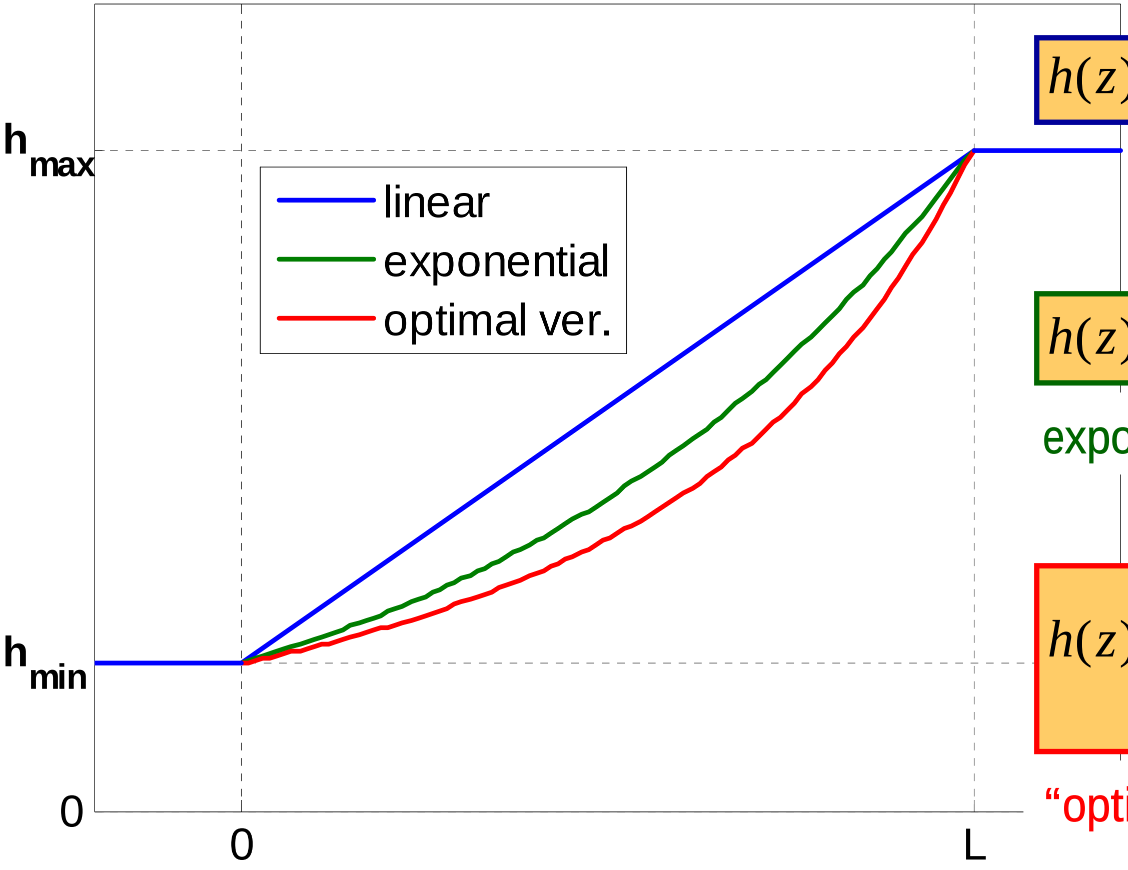
Z_y: $w \ll L$, $k <\sim 2/w_{min}$

B. Podobedov & S. Krinsky, 2006

These are inductive regime impedances. Tapers are gradual to be effective.

Functionals lend themselves to simple boundary optimization.

Optimizing boundaries



$$h(z) = h_{\min}\left(1+(h_{\max}/h_{\min}-1)\times z/L\right)$$

linear

$$h(z) = h_{\min}(h_{\max}/h_{\min})^{z/L}$$

exponential to minimize Z_x (or $Z_\perp$ *h*->*r*)

$$h(z) = \frac{h_{\min}}{\left(1+((h_{\min}/h_{\max})^{1/2}-1)\,z/L\right)^2}$$

“optimal vertical” to minimize Z_y

Reduced slope @ small *h*(z); big difference when $h_{\max}/h_{\min}$>>1

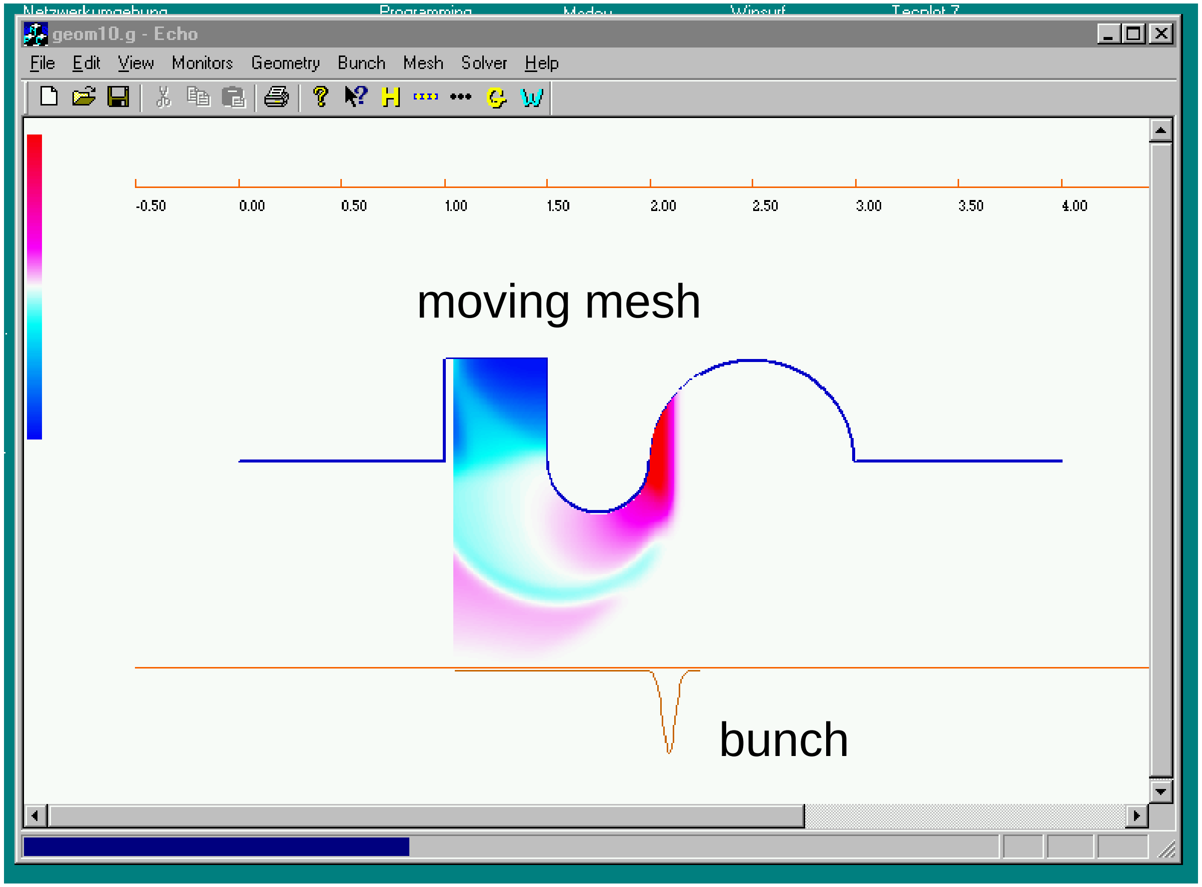
At $h_{\max}/h_{\min}$=20 predict factor of 2 reduction for $Z_\perp$ or Z_x, factor of ~3 for Z_y

What Was Done

We attempted to check the accuracy of theoretical predictions for impedance reduction by non-linear tapers in axially-symmetric, elliptical, and rectangular geometry using EM field solvers

- ABCI (axially symmetric)
- ECHO (axially symmetric & 3D)
- GDFIDL (3D)

Wakefield code ECHO
(TU Darmstadt / DESY)



Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

Zagorodnov I, Weiland T., *TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation*// Physical Review – STAB,8, **2005**.

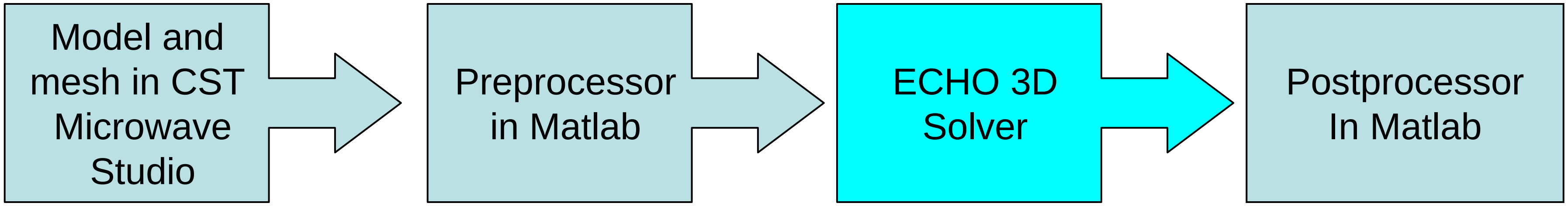
Wakefield code ECHO

(TU Darmstadt / DESY)

- **zero dispersion** in z-direction
- **staircase free** (second order convergent)

} accurate results with coarse mesh

- **moving mesh** without interpolation
- in **2.5D** stand alone application



Model and mesh in CST Microwave Studio

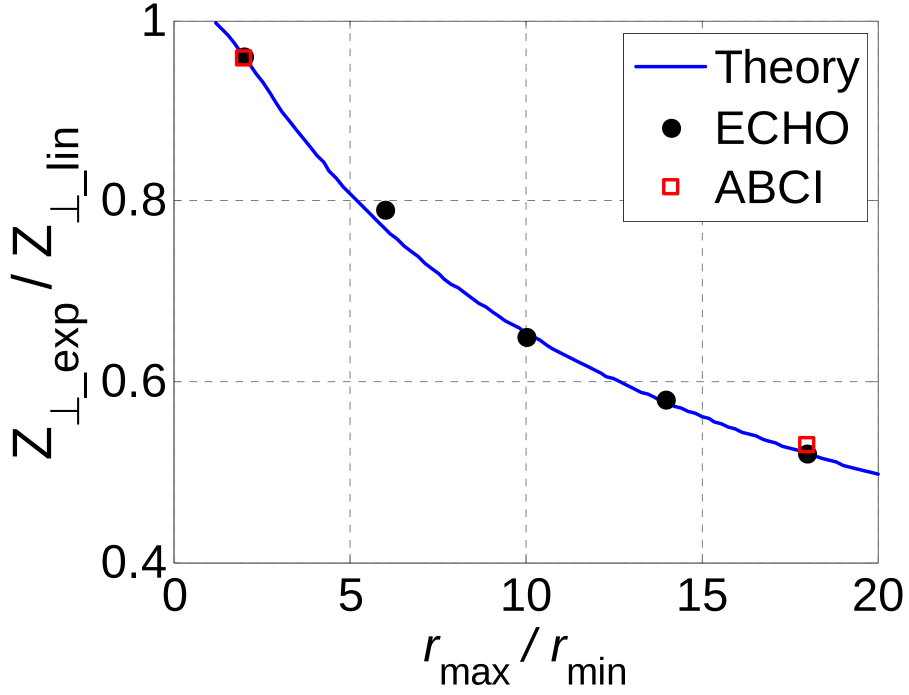
Preprocessor in Matlab

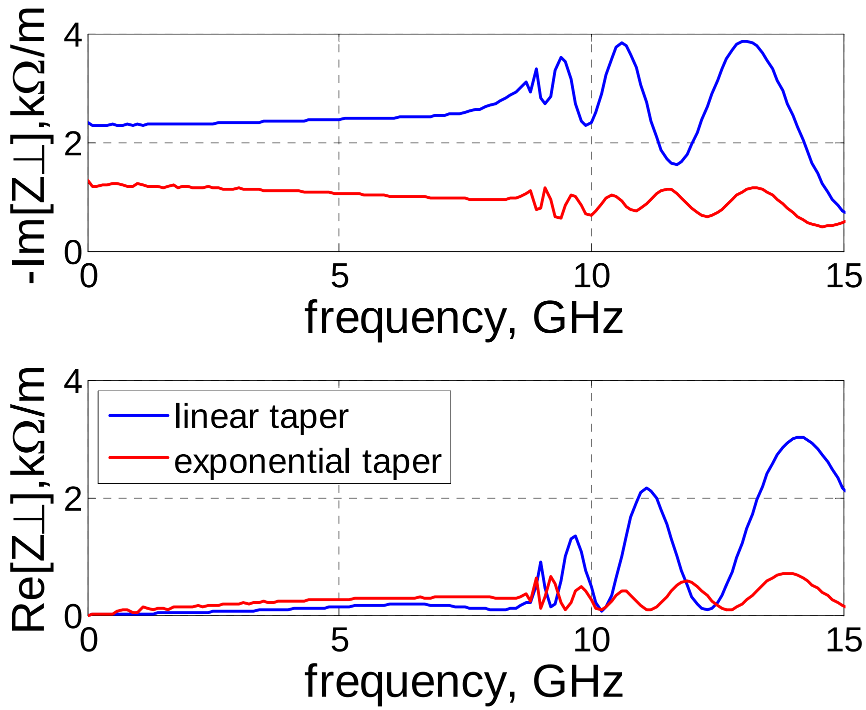
ECHO 3D Solver

Postprocessor In Matlab

- in **3D** only solver, modelling and meshing in CST Microwave Studio
- allows for accurate calculations on conventional single-processor PC
- To be parallelized ...

Impedance Reduction for Axially Symmetric Tapers

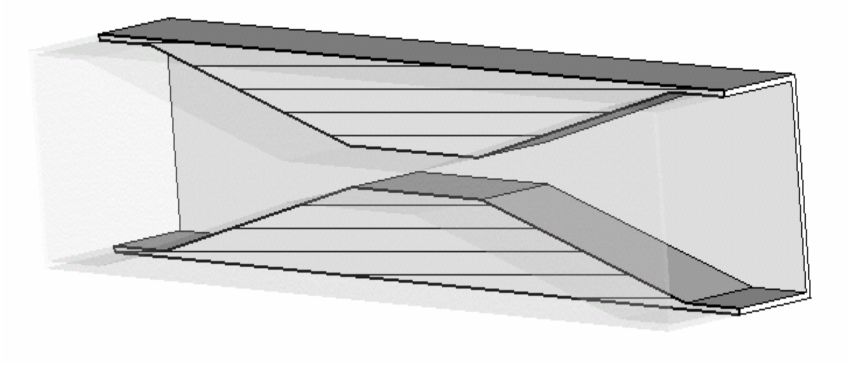

$Z_\perp$ reduction for exponential tapering


$Z_\perp(f)$ for r_{max}/r_{min}= 18, r_{min} =1 cm

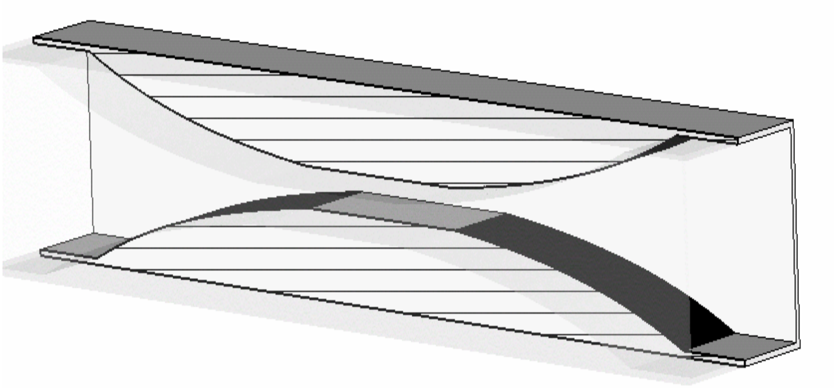
$Z_\perp$[kΩ/m] and reduction due to exponential taper agree well with theory

Impedance reduction extends through inductive regime (k~$1/r_{min}$) & beyond

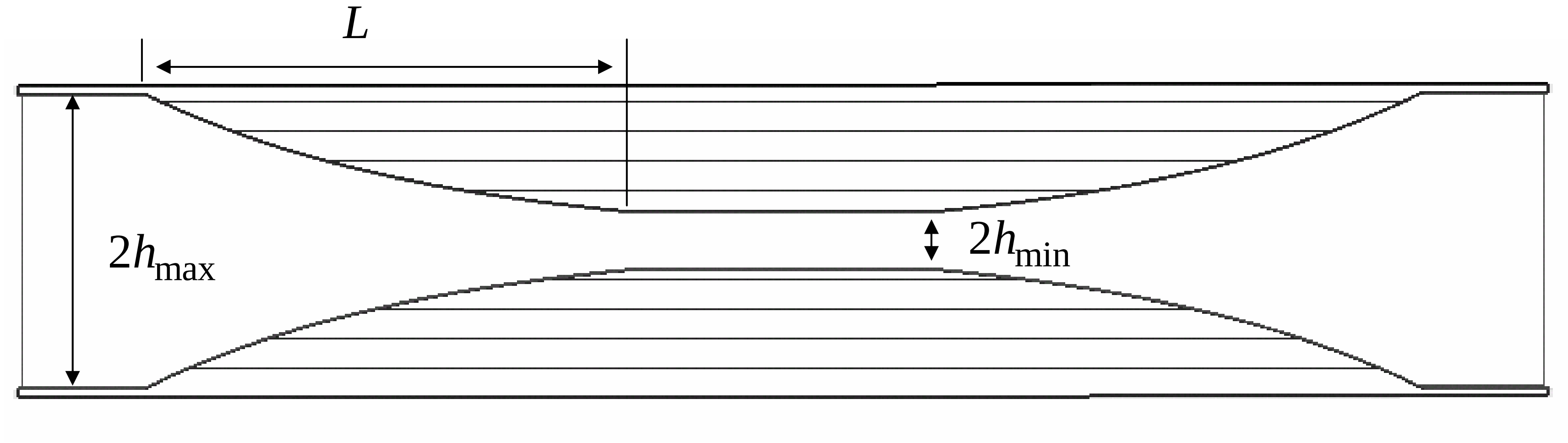
Geometry for Rectangular Taper Calculations



Linear taper

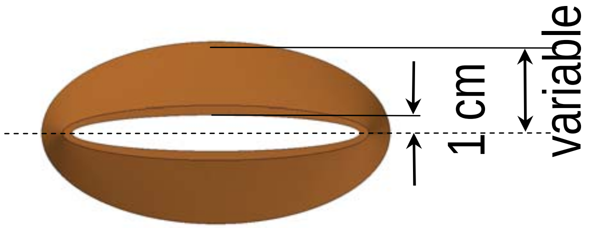


“Optimal” taper

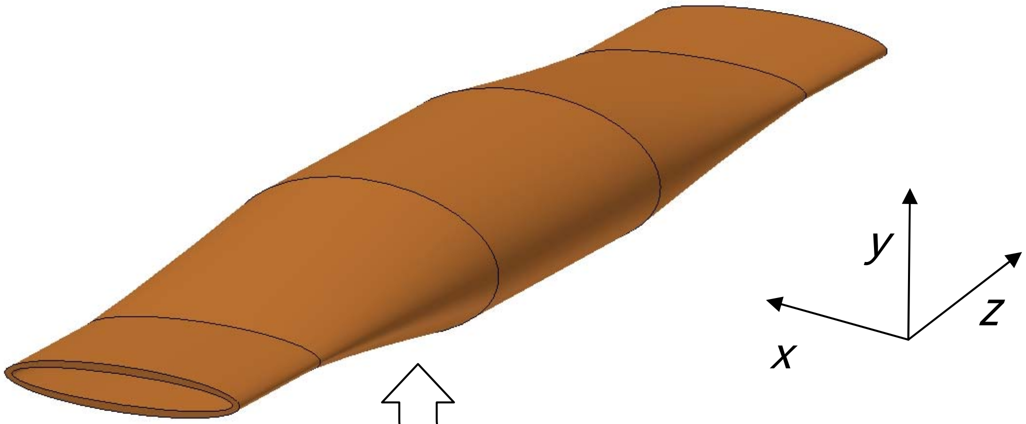


Geometry for Elliptical Taper Calculations

confocal geometry $w(z)^2 - h(z)^2 = \text{const.}$



4:1 or 8:1 aspect ratio
@ min X-section

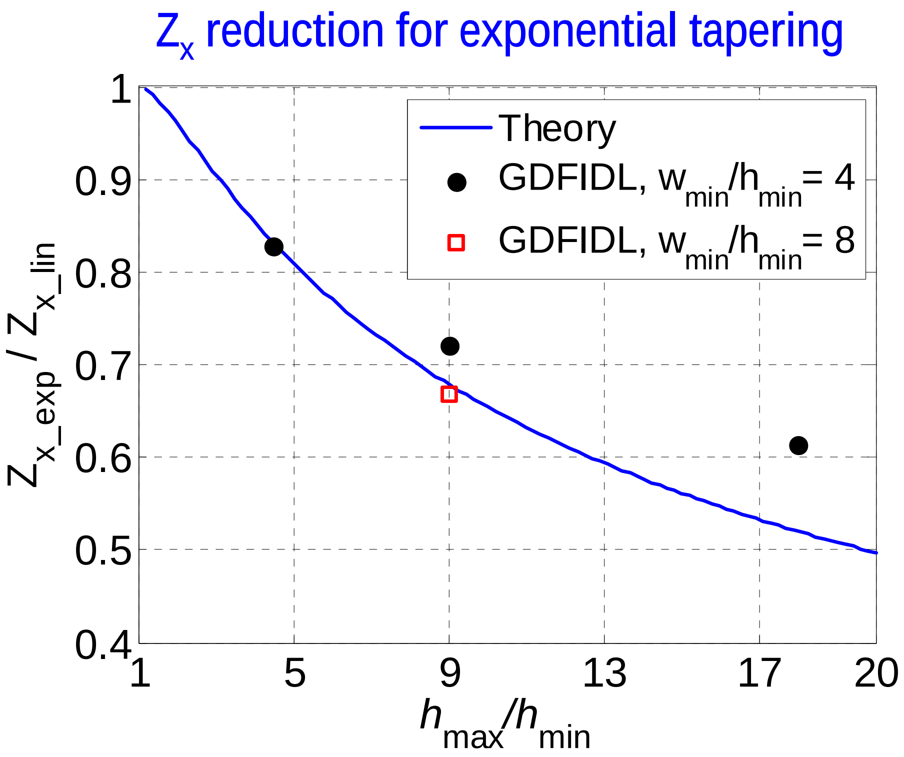


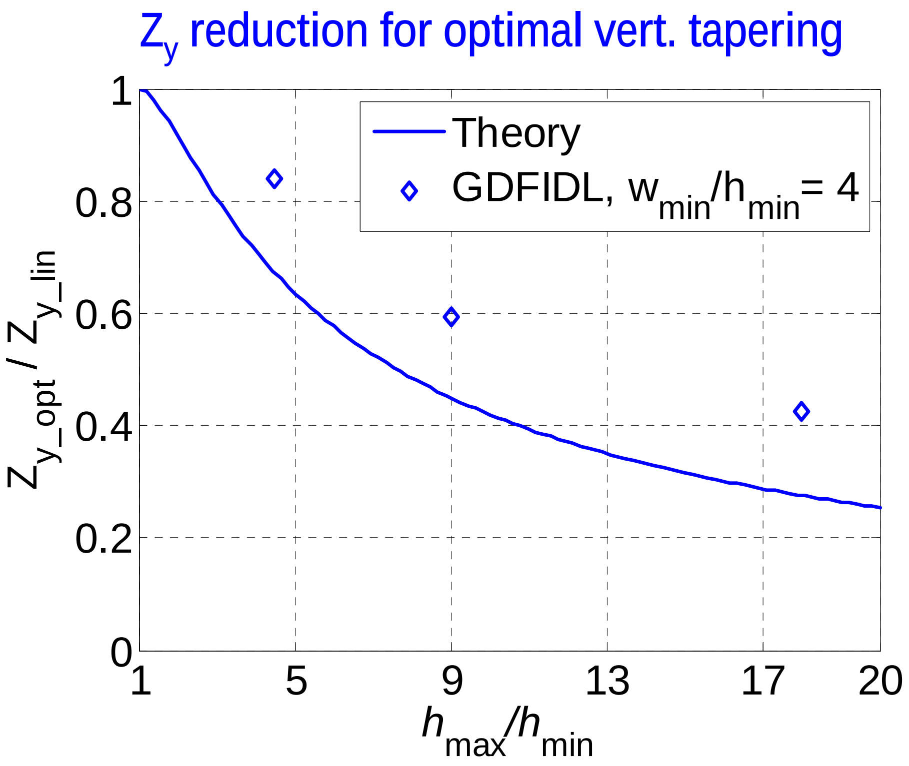
Each taper subdivided into 4 linearly tapered pieces to approx. nonlinear boundary.

Gradual tapers in convex geometry

Long straight pipes to avoid “interaction” between two tapers

Impedance Reduction for Elliptical X-Section Tapers

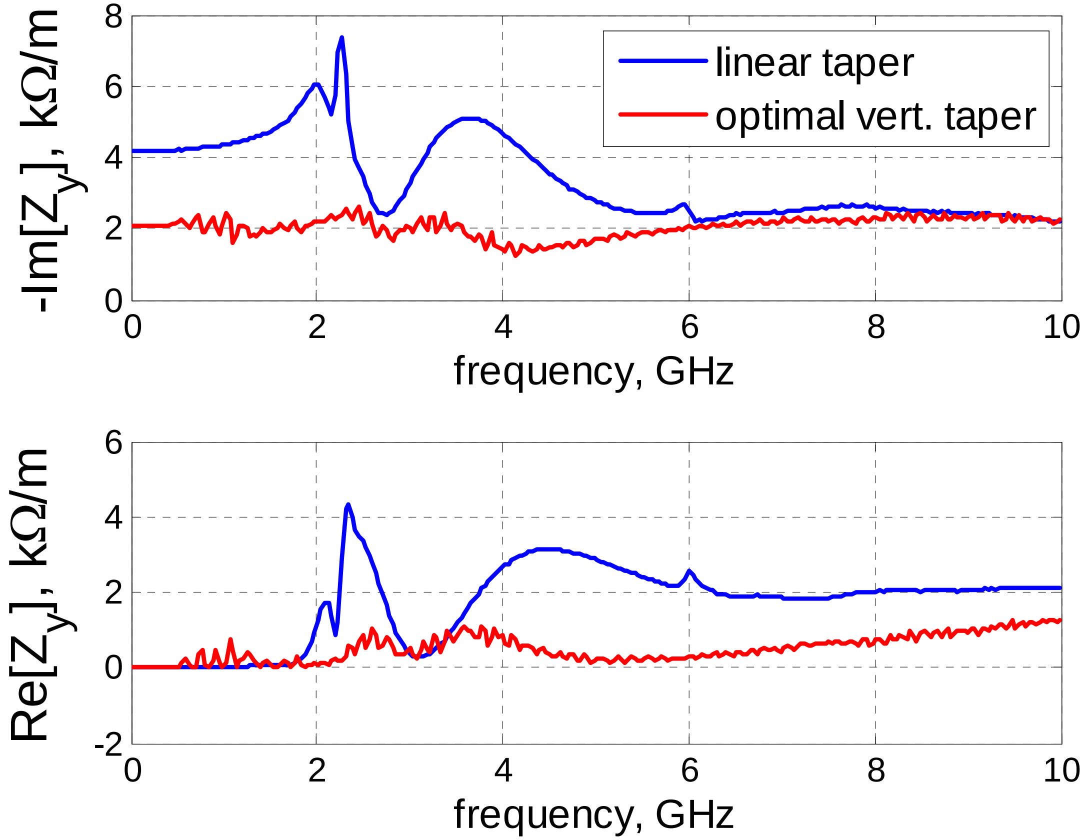

Z_x reduction for exponential tapering


Z_y reduction for optimal vert. tapering

Z_x[kΩ/m] and reduction due to exponential taper agree well with theory

Z_y[kΩ/m] is less than theory; Z_y gets reduced due to optimal taper less than predicted

Impedance Reduction vs. Frequency for Elliptical X-Section

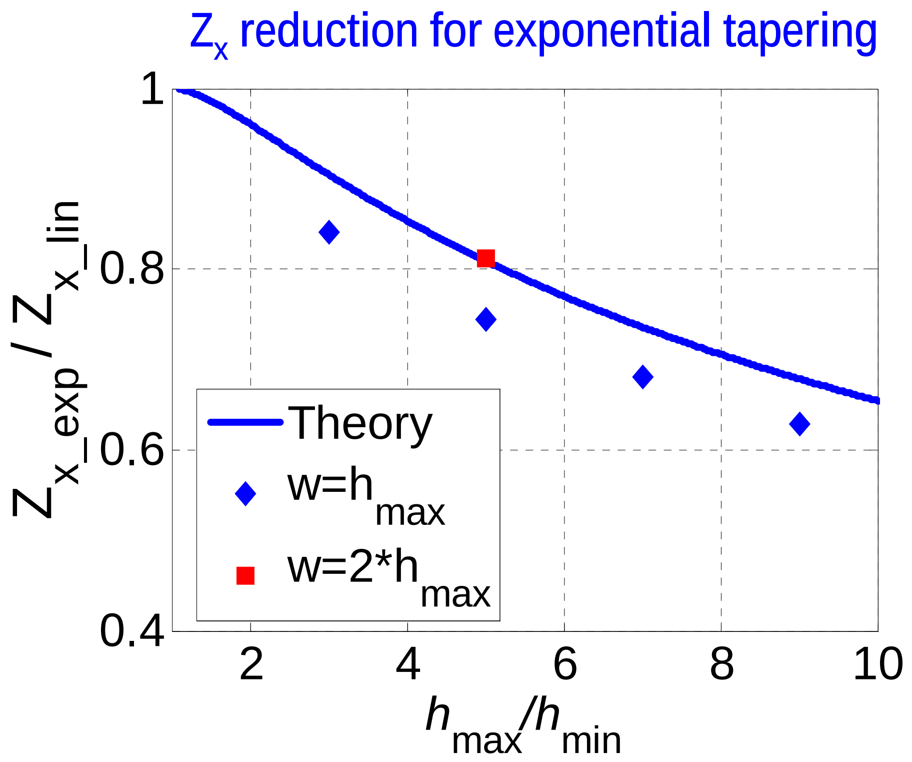


$h_{max}/h_{min} = 18$

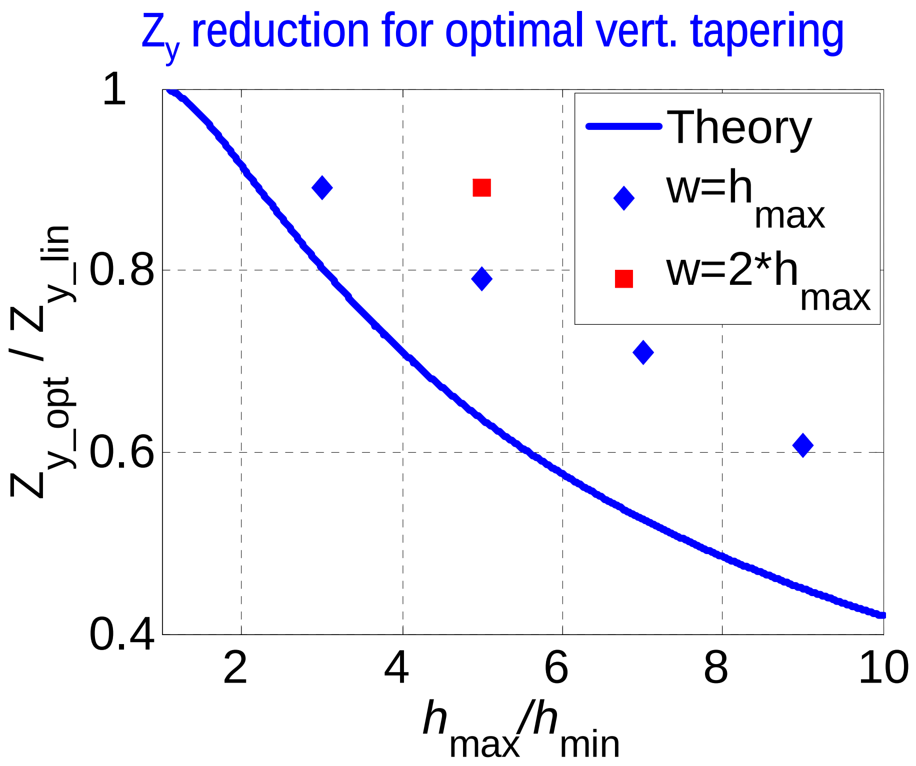
Z_y reduction extends through inductive regime ($k \sim 1/w_{min}$) & beyond

Z_x reduction extends through inductive regime ($k \sim 1/h_{min}$) & beyond

Impedance Reduction for Rectangular X-Section Tapers



Z_x reduction for exponential tapering



Z_y reduction for optimal vert. tapering

Z_x[kΩ/m] and reduction due to exponential taper agree well with theory

Z_y[kΩ/m] is less than theory; Z_y gets reduced due to optimal taper less than predicted

Results are very similar to elliptical structure

Conclusion

- For gradual tapers with large cross-sectional changes substantial reduction in geometric impedance is achieved by nonlinear taper.
- Theoretical predictions for impedance reduction are confirmed by EM solvers for axially symmetric structures and for Z_x of flat 3D structures. The vertical impedance gets reduced less than predicted, but the linear taper Z_y is lower as well.
- Optimal tapering for Z_x reduces Z_y as well and vice versa. Impedance reduction holds with frequency through the entire inductive impedance range and beyond.
- For fixed transition length, the *h*(z) tapering we consider appears to be the only “knob” to reduce transverse broadband geometric impedance of tapered structures.

Acknowledgements

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