Estimation of Emittance Growth due to Vacuum Mirror of RF Gun

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Beam Dynamics Group Meeting

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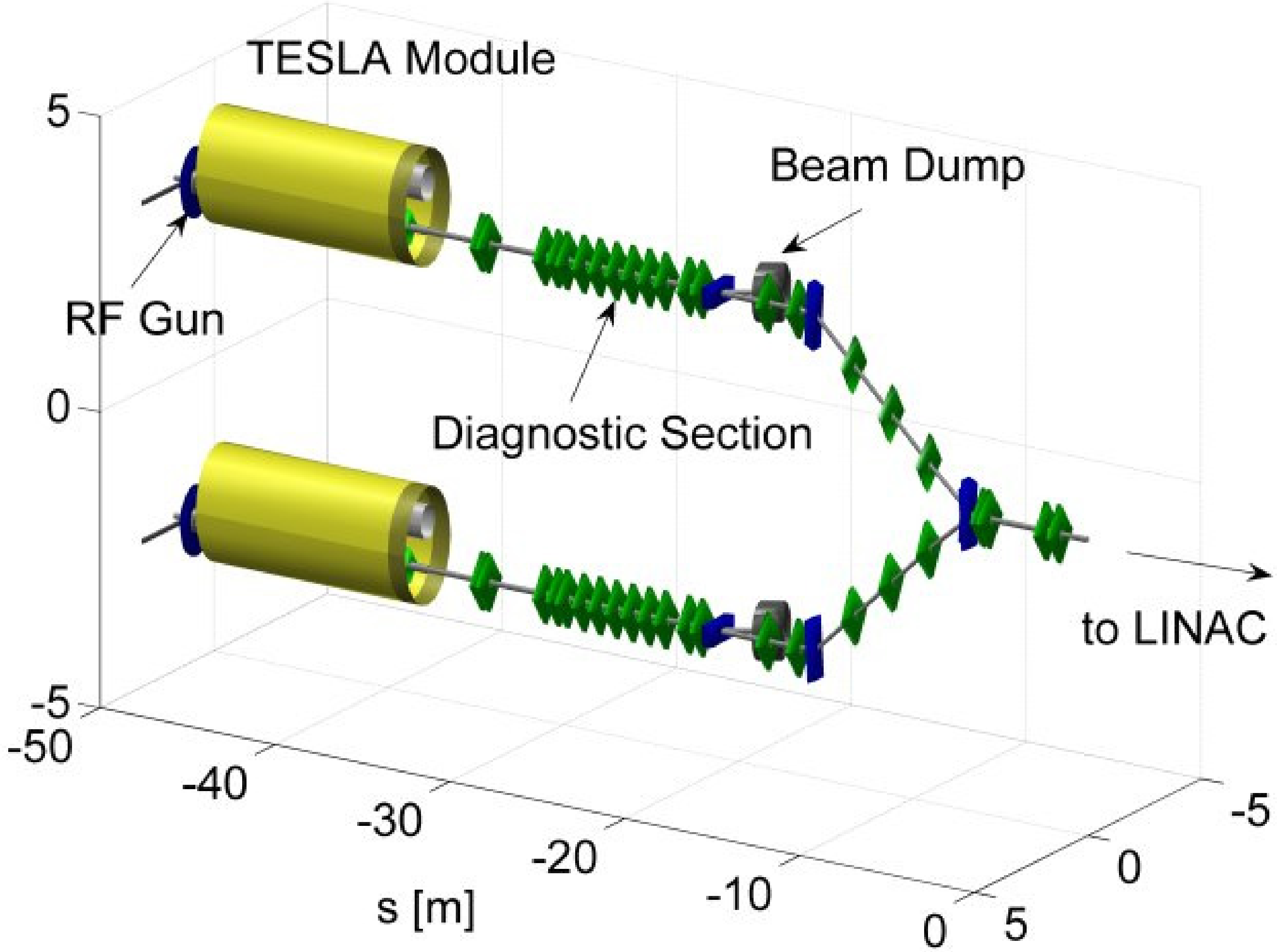
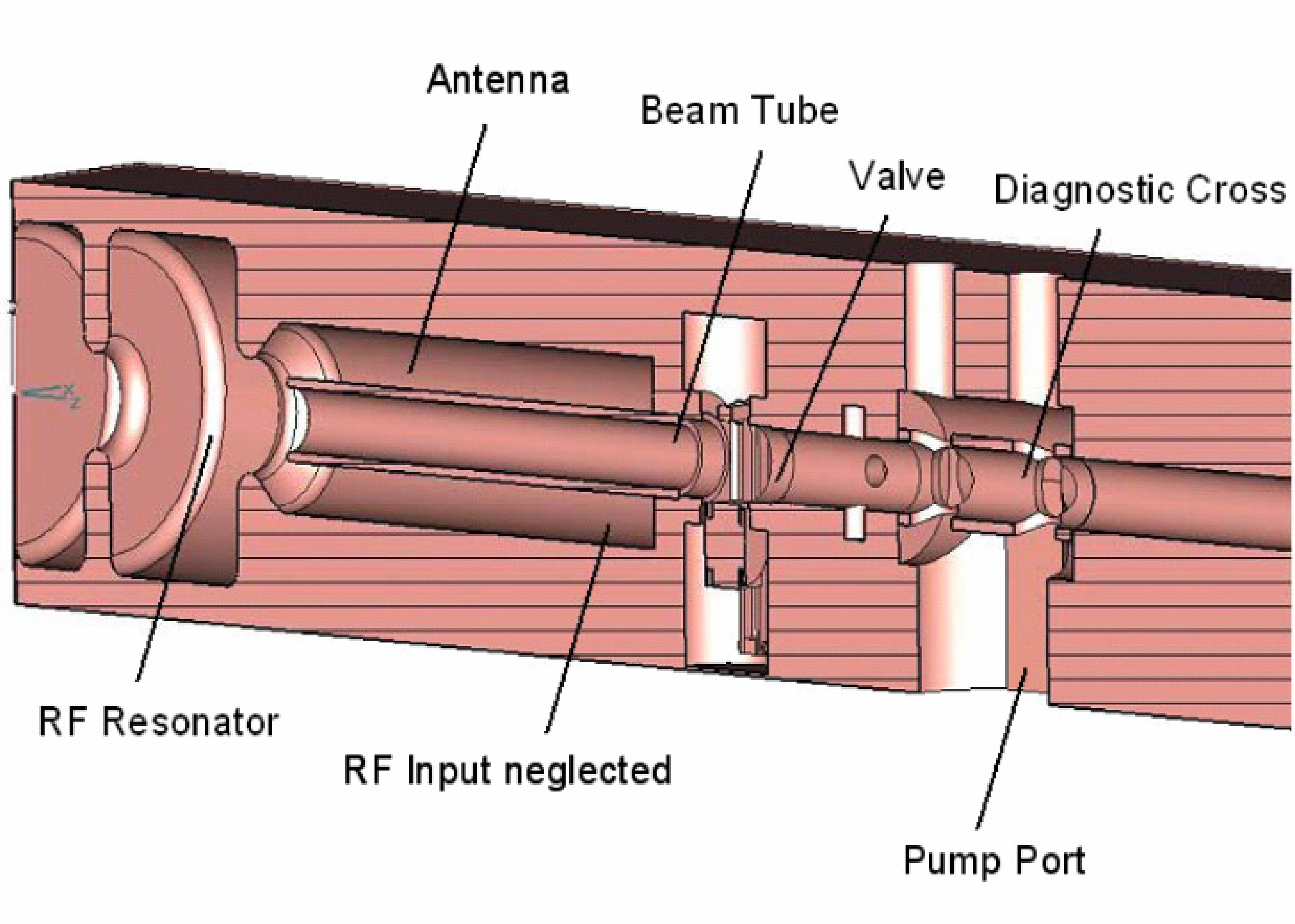
Thanks to M. Krasilnikov, K. Flöttmann, S. Schnepp, M. Dohlus

- Numerical estimation of the kick
- Gun layout and vacuum mirror geometry
- Analytical estimation of the kick
- ASTRA simulations of emittance growth
- Analytical estimation of emittance growth

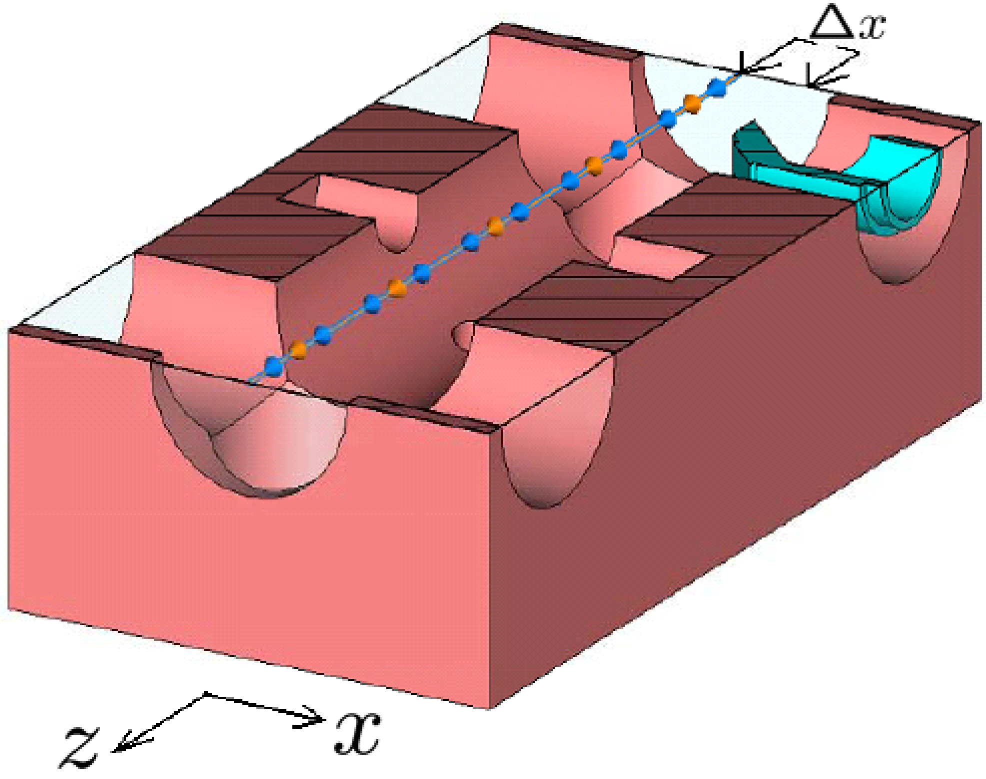


TESLA Module
Beam Dump
RF Gun
Diagnostic Section
to LINAC
5
0
-5
-50
-40
-30
-20
-10
0
s [m]
-5
0
5

Sasha Schnepp (Darmstadt)



Sasha Schnepp (Darmstadt)

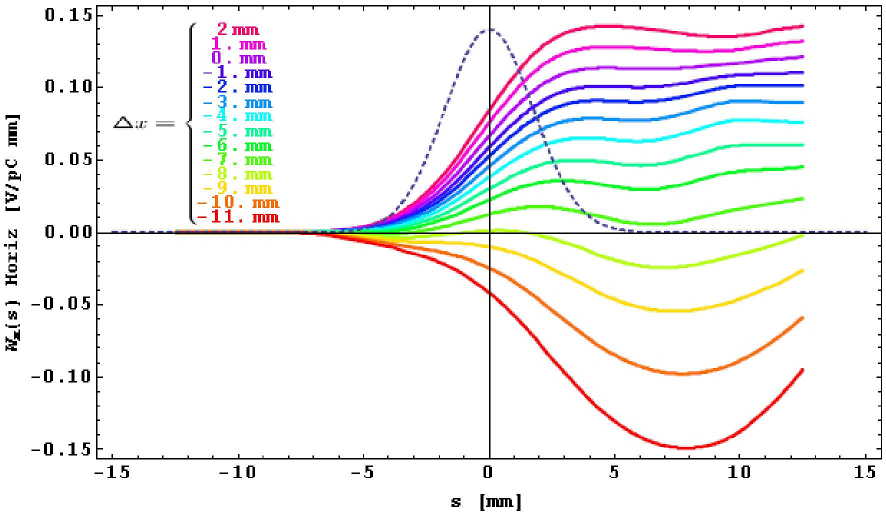


xz **plane**

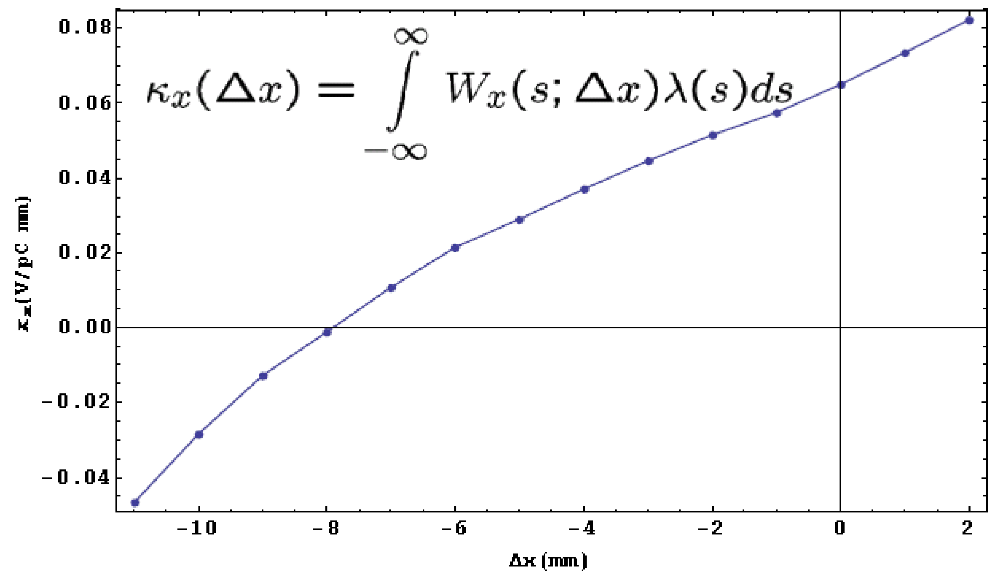
PBCI (Darmstadt), Sasha Schnepp

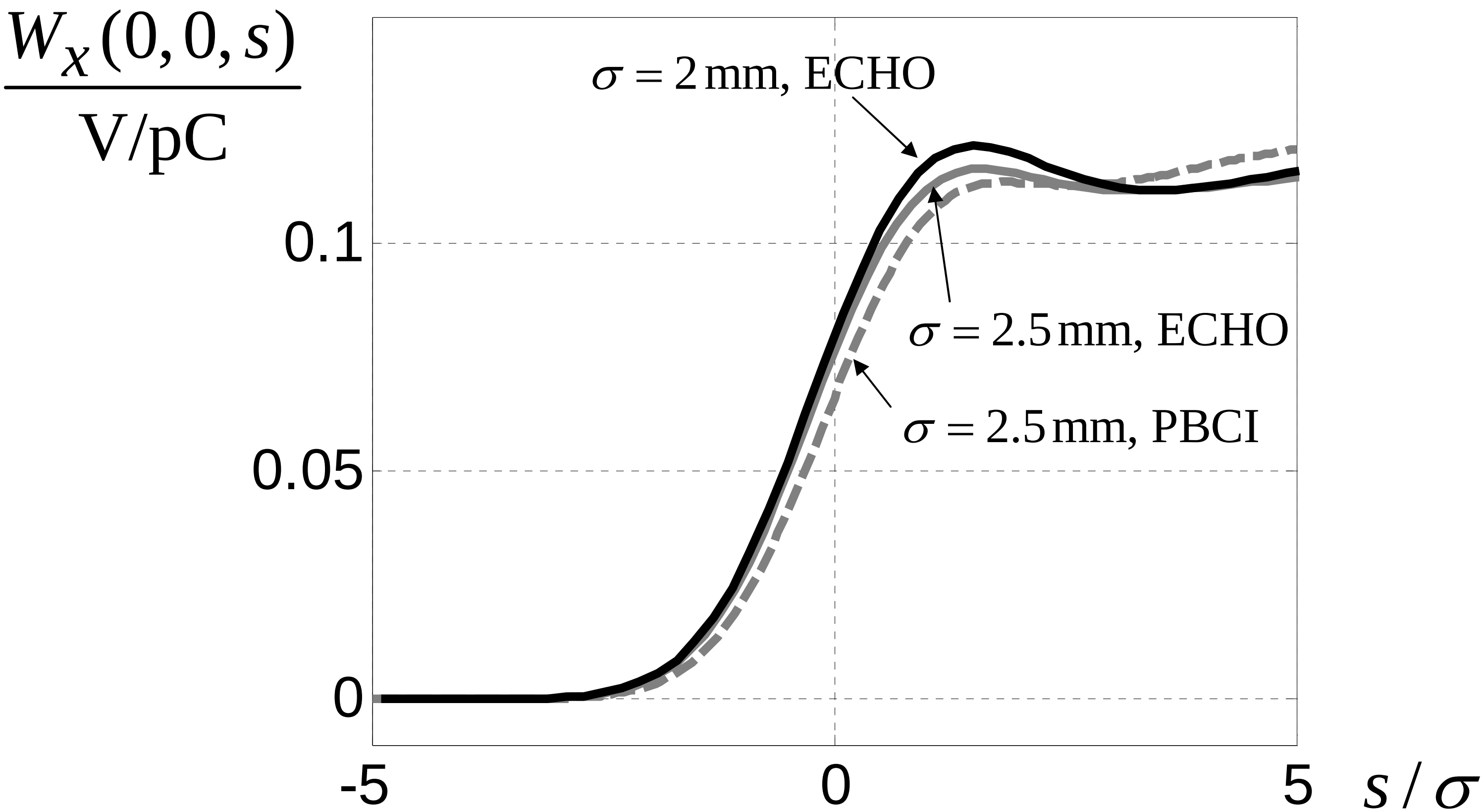
$$\sigma = 2.5\,\mathrm{mm}$$

$$K_x(0,0) = 0.064\,\frac{\mathrm{kV}}{\mathrm{nC}}$$



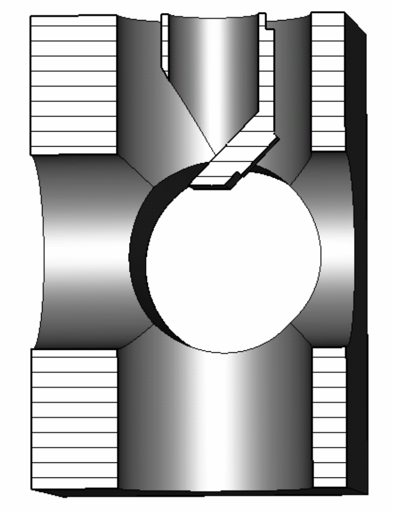
Plot of the horizontal wake potential for different shifts Δx of the particle path with respect to the longitudinal axis.

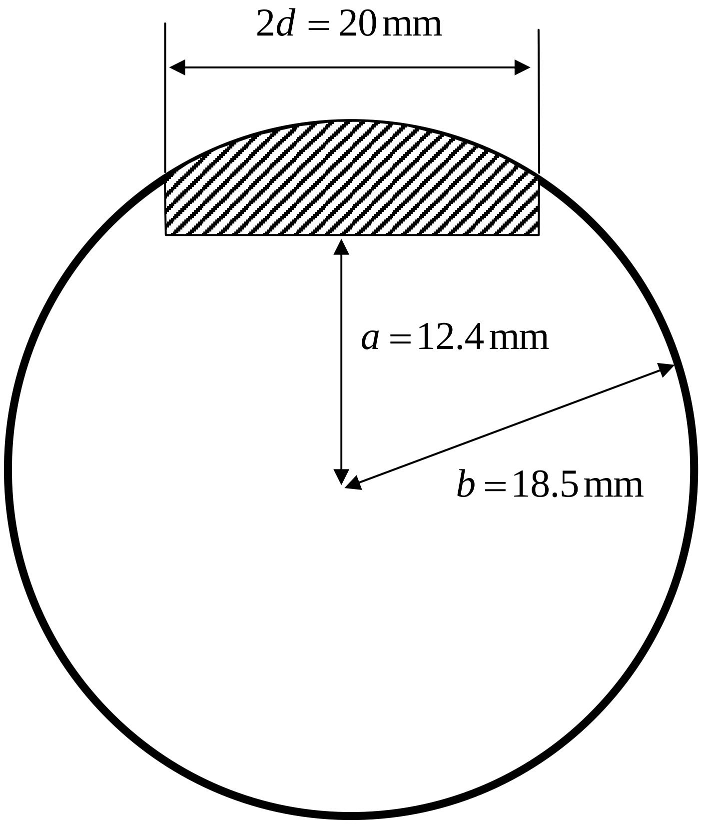




	$k_x(0,0)$ kV/nC
PBCI, σ=2.5 mm	**0.064**
ECHO, σ=2.5 mm	0.071
ECHO, σ=2 mm	0.075

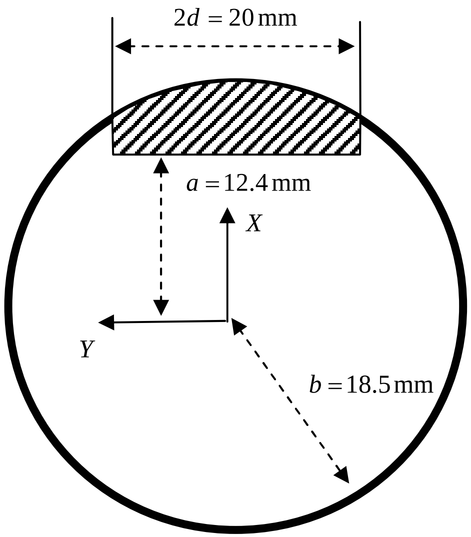
Sasha Schnepp (Darmstadt)





G.Stupakov, K.Bane, I.Zagorodnov, Optical Approximation …, PR-STAB, 2007

$$Z_{\|}(\vec{r}_1, \vec{r}_2) = \frac{1}{2\pi c}\left[\int_{S_B} \nabla\varphi_B(\vec{r}_1, \vec{r})\nabla\varphi_B(\vec{r}_2, \vec{r})ds - \int_{S_{ap}} \nabla\varphi_A(\vec{r}_1, \vec{r})\nabla\varphi_B(\vec{r}_2, \vec{r})ds\right]$$

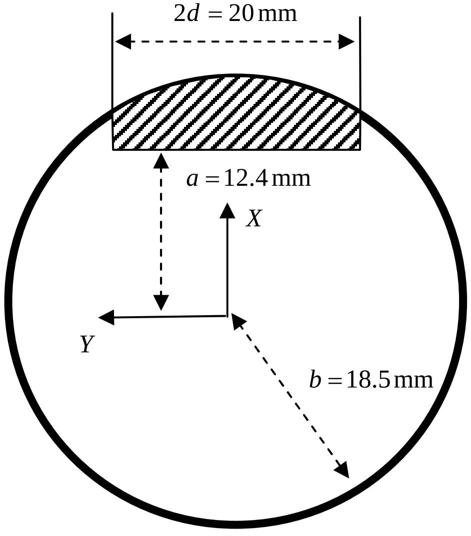


$$k_x(x_1, x_2) = k_x(0,0) + k_x^D x_1 + k_x^Q x_2$$

$$k_x(0,0) = \frac{c\,Z0\left(\left(-2a^2 + b^2\right)\mathrm{ArcTan}\left[\frac{d}{a}\right] + a\,d\left(1 + 2\,\mathrm{Log}[b] + \mathrm{Log}\left[\frac{1}{a^2+d^2}\right]\right)\right)}{4\,a\,b^2\,\pi^2}$$

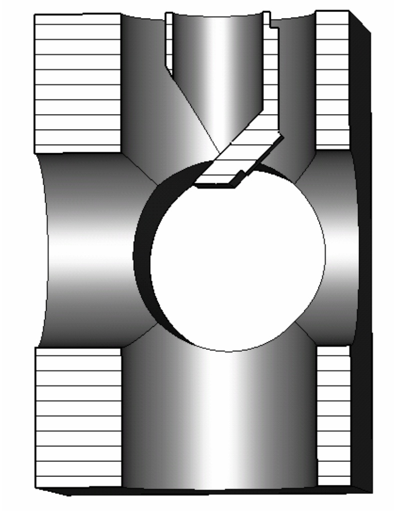
$$k_x^Q = \frac{1}{8\,a^2\,b^4\,d^2\left(a^2+d^2\right)\pi^2}\,c\,Z0\left(a\,d\left(-a^2 b^4 + b^4 d^2 + a^3\sqrt{b^2-d^2}\left(b^2+6d^2\right) + a\,d^2\sqrt{b^2-d^2}\left(b^2+6d^2\right)\right) + \left(a^2+d^2\right)\left(a^2\left(b^4 - 8d^4\right)\left(\mathrm{ArcCot}\left[\frac{d}{\sqrt{b^2-d^2}}\right] - \mathrm{ArcTan}\left[\frac{a}{d}\right]\right) + \left(-8a^4 + b^4\right)d^2\,\mathrm{ArcTan}\left[\frac{d}{a}\right]\right)\right)$$

$$k_x^D = \frac{1}{8\,a^2\,b^4\,d^2\,\pi^2}\,c\,Z0\left(a\,d\left(b^4 - a\,b^2\sqrt{b^2-d^2} - 2\,a\,d^2\left(2a + \sqrt{b^2-d^2}\right)\right) + b^2\left(-a^2\left(b^2 - 4d^2\right)\left(\mathrm{ArcCot}\left[\frac{d}{\sqrt{b^2-d^2}}\right] - \mathrm{ArcTan}\left[\frac{a}{d}\right]\right) + \left(4a^2 + b^2\right)d^2\,\mathrm{ArcTan}\left[\frac{d}{a}\right]\right)\right)$$



ECHO vs. analytical results for the model case

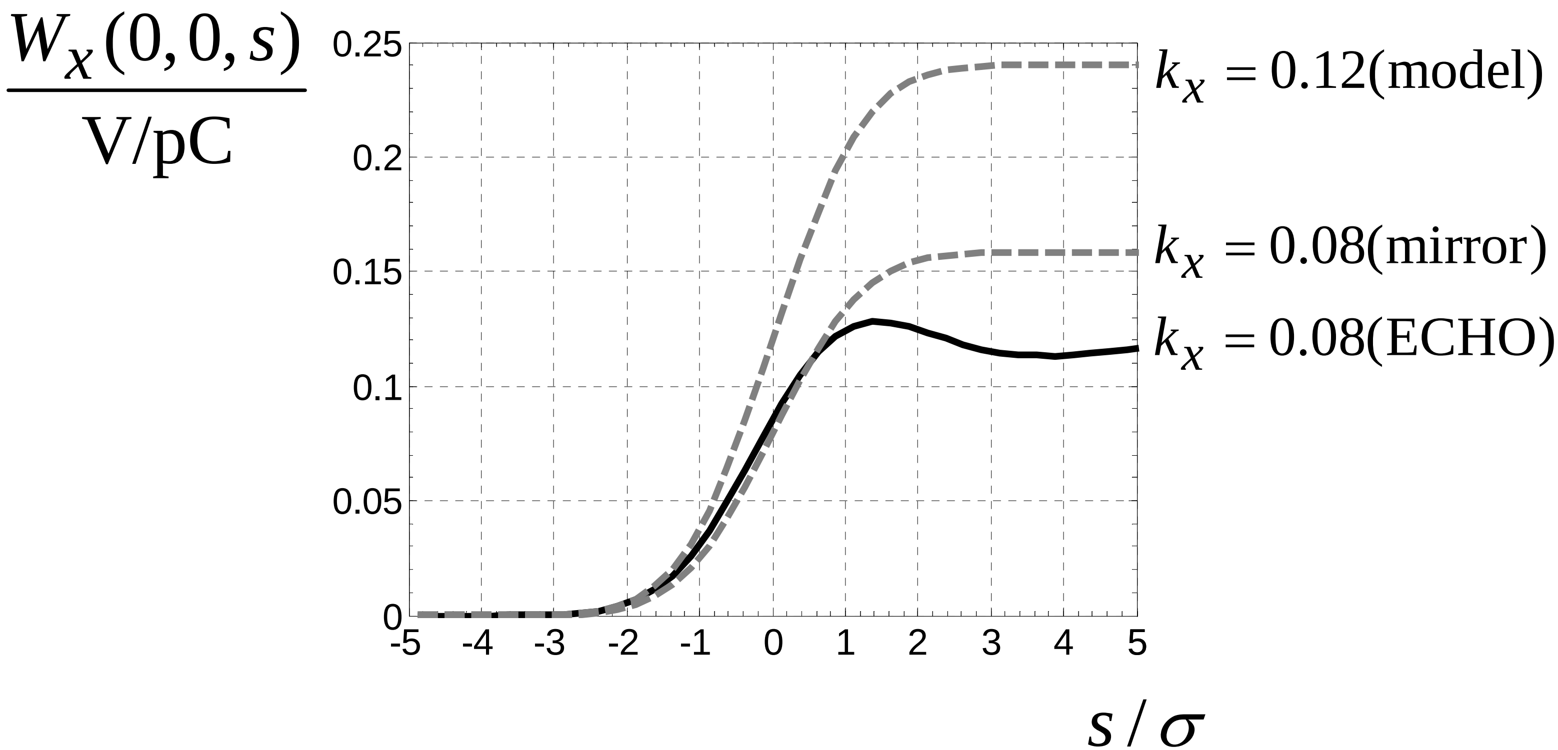
	$k_x(0,0)$ kV/nC	k_x^D kV/nC/m	k_x^Q kV/nC/m
Analytical	**0.124**	13.09	12.1
Numerical (σ=0.5 mm)	0.120	13.08	11.6
Numerical (σ=2 mm)	0.103		



ECHO for mirror, σ=2 mm

$k_x(0,0)$ kV/nC	k_x^D kV/nC/m	k_x^Q kV/nC/m
0.075	24.3	7.5

$$\sigma = 2\,\text{mm}$$



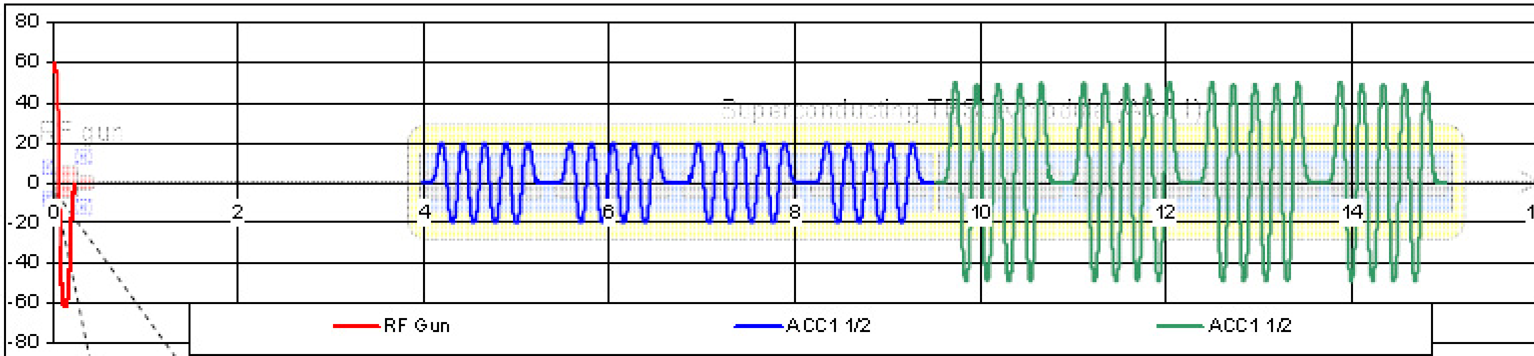
$$w_x(x_1, x_2, s) = 2\left[k_x(0,0) + k_x^D x_1 + k_x^Q x_2\right]H(s)$$

$$\lambda(s) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{s^2}{2\sigma^2}}$$

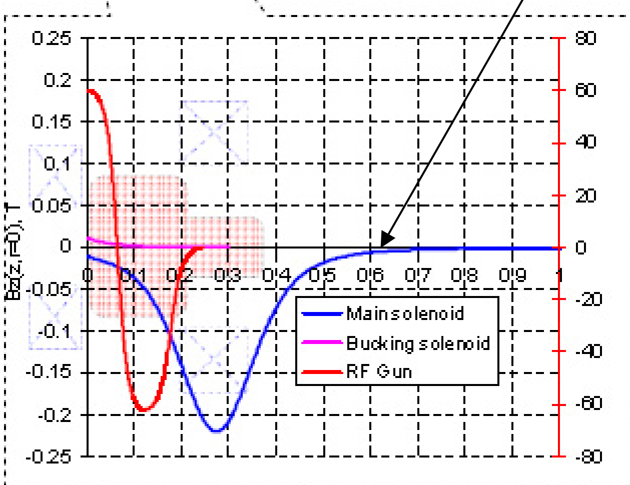
$$W_x(s) = \int_{-\infty}^{s} w_x(s-s')\lambda(s')ds' = 2k_x \int_{-\infty}^{s} \lambda(s')ds' = k_x\left(1 + Erf\left(\frac{s}{\sqrt{2}\sigma}\right)\right)$$

ASTRA input desk from M.Krasilnikov

XFEL Photo Injector Layout



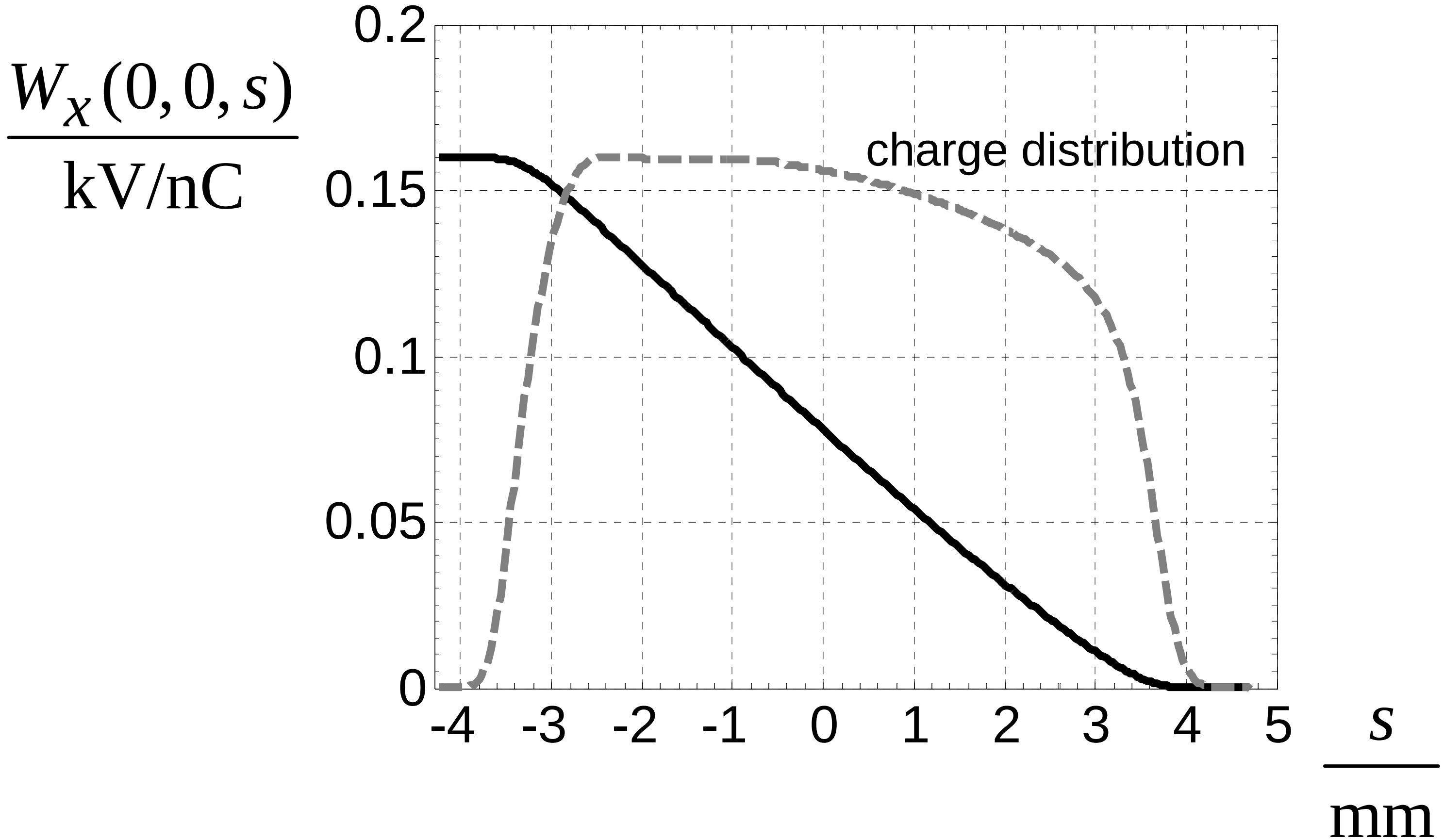
$z = 0.62\,\mathrm{m}$, vacuum mirror



Injector parameters to be optimized:

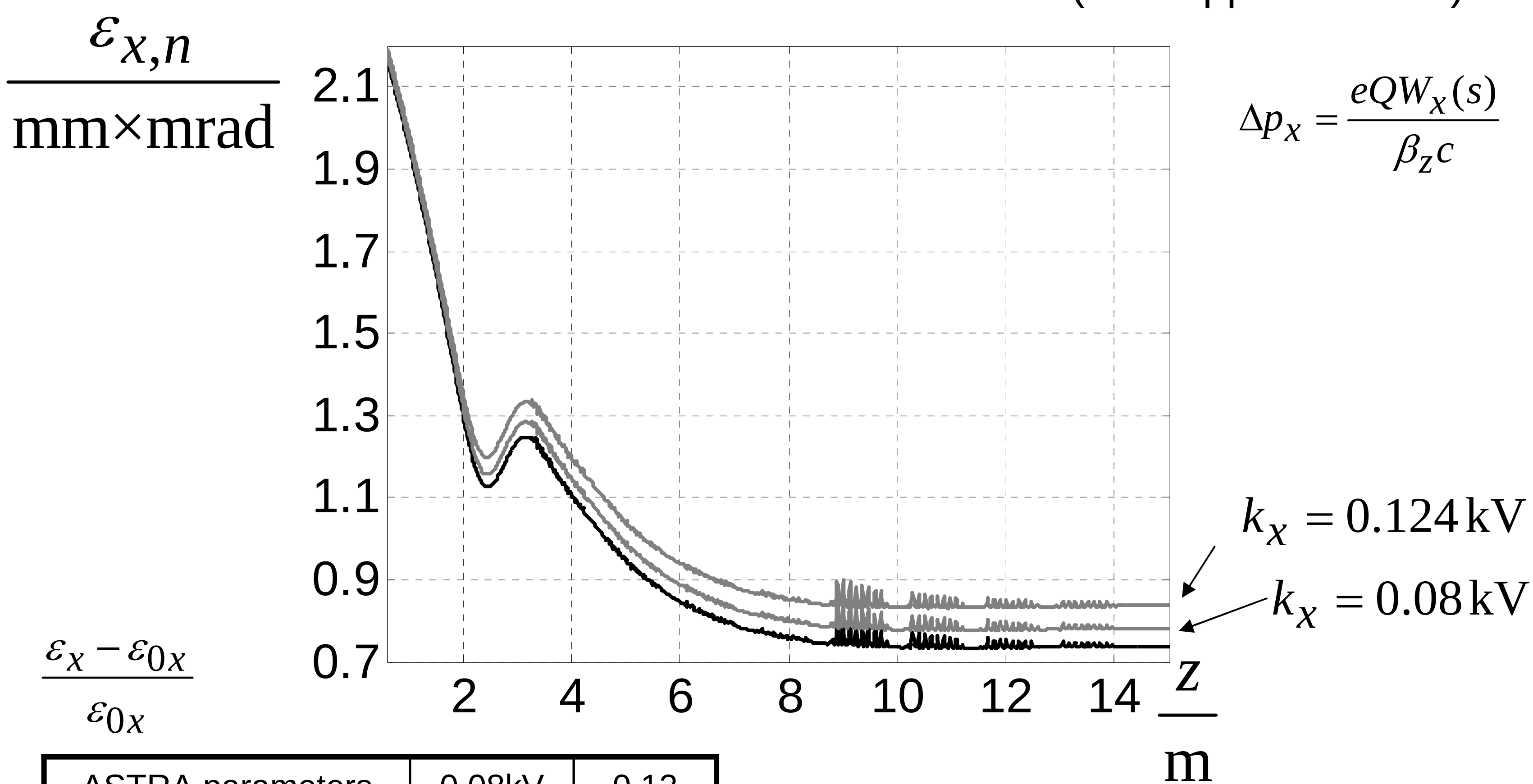
- Cathode laser: XYrms, (Trms)
- RF-Gun: launch RF phase
- Solenoid: position, peak field (current)
- Booster: position, gradient, RF phase

$$z = 0.62\,\mathrm{m}$$



$$\sigma_{\mathrm{rms}} = 1.96\,\mathrm{mm}$$

ASTRA+GlueTrack simulation (Kick approximation)



$$\Delta p_x = \frac{eQW_x(s)}{\beta_z c}$$

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}}$$

ASTRA parameters	0.08kV	0.12
20 000 particles, mesh: 15*25	4.5%	10.3%
100 000 particles, mesh: 30*40	5.9%	13.5%

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = O({k_x}^2)$$

ASTRA input desk from M.Krasilnikov

$$w_x(s) = 2k_x H(s)$$

$$\lambda(s) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{s^2}{2\sigma^2}} \qquad W_x(s) = \int_{-\infty}^{s} w_x(s-s')\lambda(s')ds' = 2k_x \int_{-\infty}^{s} \lambda(s')ds' = k_x\left(1 + Erf\left(\frac{s}{\sqrt{2\sigma}}\right)\right)$$

$$\Delta x'(s) = \frac{\Delta p_x}{p_z} = \frac{eQW_x(s)}{\beta_z^2 E_{kin}} = S\left(1 + Erf\left(\frac{s}{\sqrt{2\sigma}}\right)\right) \qquad S = \frac{eQk_x}{\beta_z^2 E}$$

$$\rho_0(x, x', s) = \frac{1}{2\pi\varepsilon_{0x}} \exp\left(-\frac{\gamma x^2 + 2\alpha xx' + \beta x'^2}{2\varepsilon_{0x}}\right)\lambda(s)$$

$$\rho(x, x', s) = \frac{1}{2\pi\varepsilon_{0x}} \exp\left(-\frac{\gamma x^2 + 2\alpha x(x' + \Delta x'(s)) + \beta(x' + \Delta x'(s))^2 +}{2\varepsilon_{0x}}\right)\lambda(s)$$

$$\varepsilon_x = \sqrt{\varepsilon_{0x}{}^2 + S^2 \frac{\varepsilon_{0x}\beta}{3}} \approx \varepsilon_{0x} + S^2 \frac{\beta}{6}$$

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = \sqrt{1 + S^2 \frac{\beta}{3\varepsilon_{0x}}} - 1 \approx S^2 \frac{\beta}{6\varepsilon_{0x}}$$

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = \sqrt{1 + S^2 \frac{\beta}{3\varepsilon_{0x}}} - 1 \approx S^2 \frac{\beta}{6\varepsilon_{0x}} \qquad S = \frac{eQk_x}{\beta_z^2 E}$$

$\beta = 8.4\,\mathrm{m}$ $\quad Q = 1\,\mathrm{nC}$

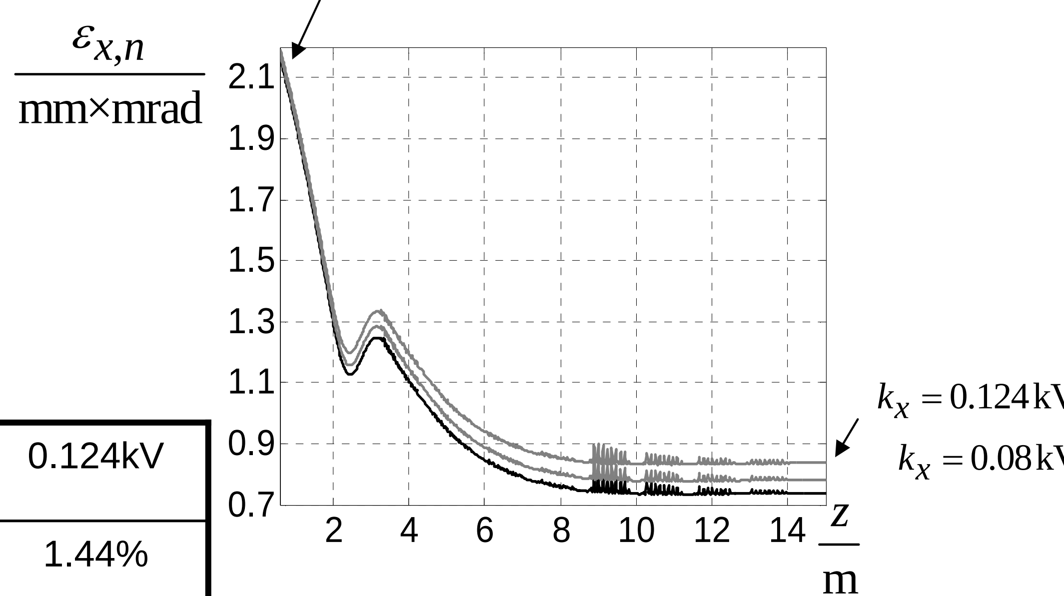
$\beta_z \approx 1$ $\quad E = 6.6\,\mathrm{MeV}$

$k_x = 0.124\,\mathrm{kV/nC}$

$\varepsilon_{n0,x} = 2.156\,\mathrm{mm \times mrad}$

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = 0.3\%$$

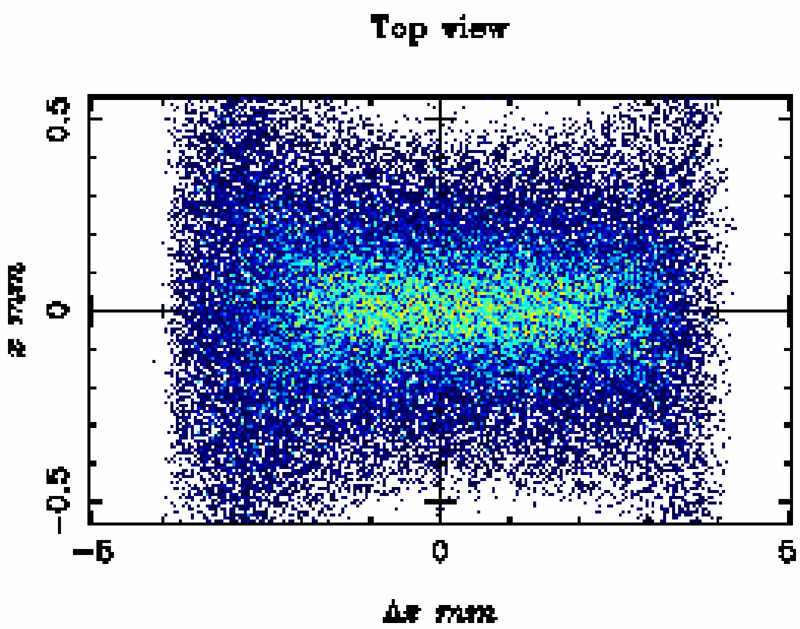
$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = 1.44\%$$



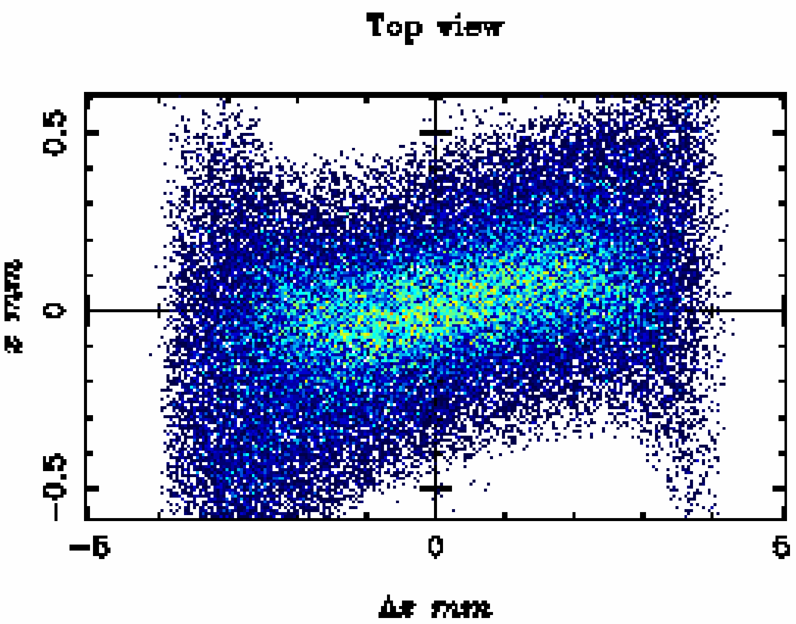
$k_x(0,0)$	0.08kV	0.124kV
Emittance growth at z=0.615 m	0.6%	1.44%
Emittance growth at z=15 m	5.9%	13.5%

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = O(k_x^2)$$

$k_x = 0$



$k_x = 0.124\,\mathrm{kV}$



$$w_x(x_1, x_2, s) = 2\left[k_x(0,0) + k_x^D x_1 + k_x^Q x_2\right]H(s)$$

ECHO for mirror, σ=2 mm

$k_x(0,0)$ kV/nC	k_x^D kV/nC/m	k_x^Q kV/nC/m
0.075	**24.3**	**7.5**

$$\frac{\varepsilon_x - \varepsilon_{0x}}{\varepsilon_{0x}} = O(k_x^2)$$