

ALICE is a
Lasing
Investigation
Co-
dE

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Mathematical model

1D and 3D mathematical models are described in [SSY, 1999] and coincide with those used in the code FAST of the same authors

Equations of motion correspond to effective Hamiltonian

$$H(P, \psi, z) = CP + \frac{\omega}{2c\gamma_z^2 \epsilon_0} P^2 - (Ue^{i\psi} + C.C.) + \int eE_z d\psi$$

Field Equations are used in parabolic approximation

$$c^2 \left[\Delta_{\perp} + 2i \frac{\omega}{c} \frac{\partial}{\partial z} \right] \tilde{E} = -4\pi\theta_s \omega \tilde{j}_1 \quad \tilde{E} = (E_x + iE_y) e^{-i\omega(z/c - t)}$$

with simplified **space charge model**

$$\frac{\partial}{\partial t} E_z = 4\pi(j_z - j_0)$$

[SSY, 1999] E.L.Saldin, E.A.Schneidmiller, M.Y.Yurkov, The Physics Of Free Electron Lasers, Springer, 1999

Motivation

Why write a code with the same mathematical model

- There are a lot of codes for Maxwell's equations (wakefields), why do not write **one more** for the FEL equations?
- To **study the theory** through numerical modeling
- To implement **simpler and faster** numerical methods without loss of accuracy
- To have **consistent and matched 1D, 2D and 3D models** in the same code
- To have a thoroughly **tested code** with full control and possibility of **future development**

Numerical methods

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Equations of motion

Runge-Kutta method

Leap-Frog method

Field Equation

Non-local integral representation
(two fold singular integral with
special functions)

Finite-Difference Solver with
Perfectly Matched Layer

Why other methods?

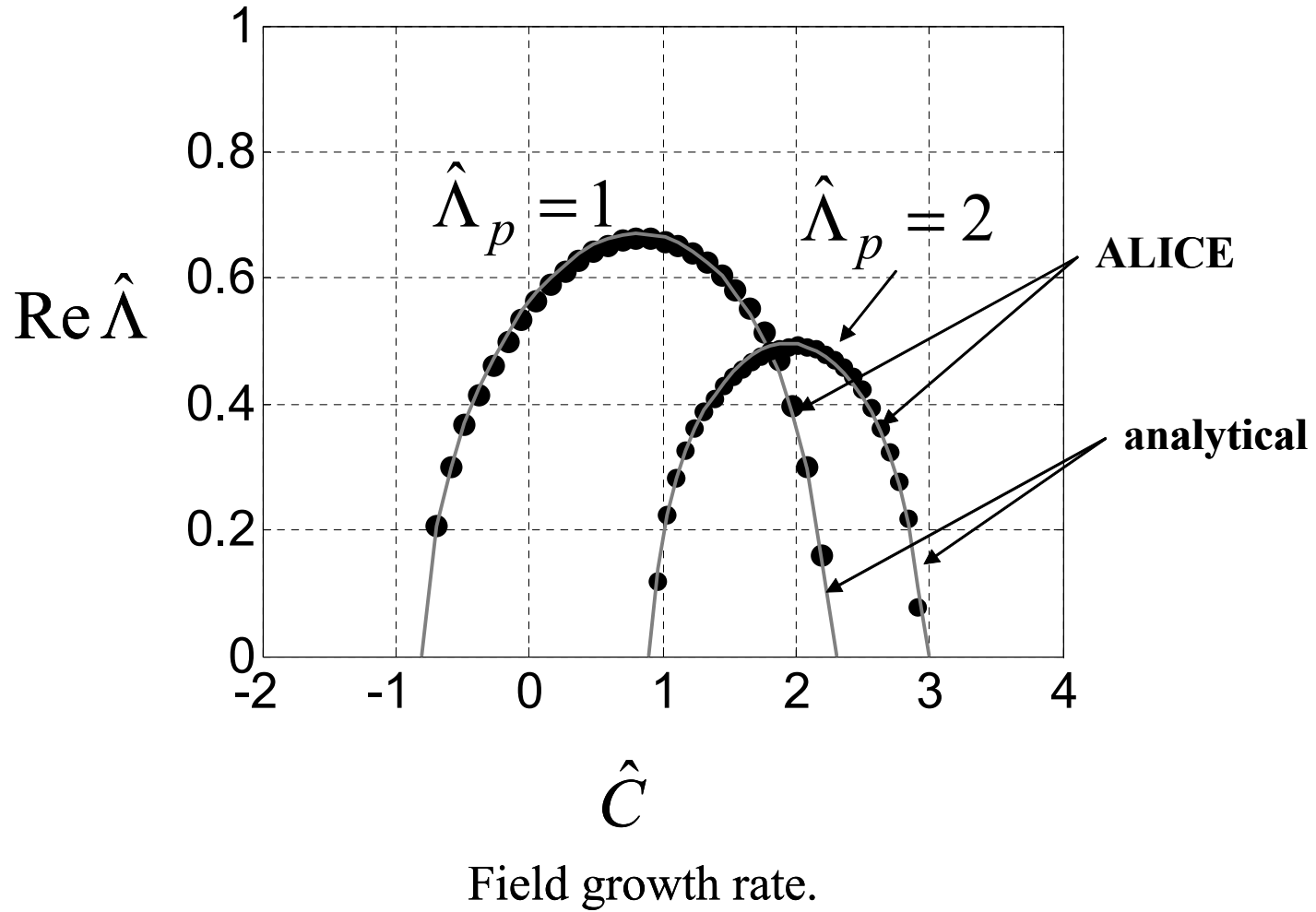
- Leap-Frog is **faster** than Runge-Kutta and „symplectic“(?)
- Finite-Difference solver is **local**: uses only information from one previous „slice“; it should be faster than non-local retarded integral representation which uses all slices in the slippage length
- Like to the integral representation the Perfectly Matched Layer approximates the „**open boundary**“ **condition accurately**

Computer realization and testing

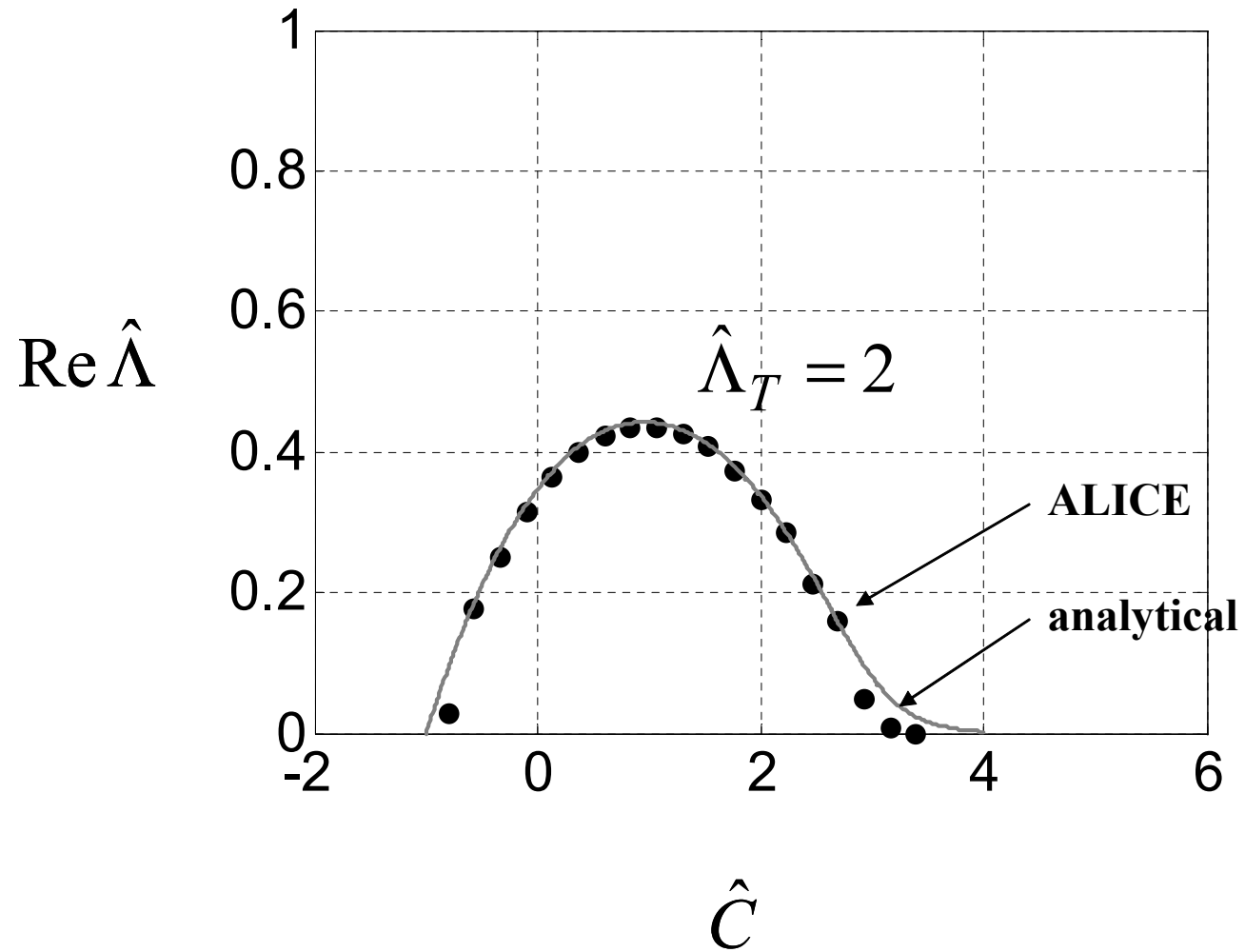
- **The code was initially developed in Matlab and then rewritten in C/C++**
- **the numerical results are compared with the analytical ones when possible (propagation of different Laguerre-Gaussian azimuthal modes, analytical results for linear regime in 1D and 3D theories)**
- **The figures from chapters 2, 3, 6 of [SSY, 1999] are reproduced with the new code**

[SSY, 1999] E.L.Saldin, E.A.Schneidmiller, M.Y.Yurkov, The Physics Of Free Electron Lasers, Springer, 1999

Tests. Space charge algorithm (1D)



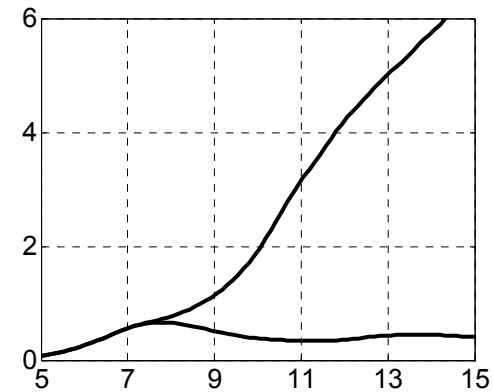
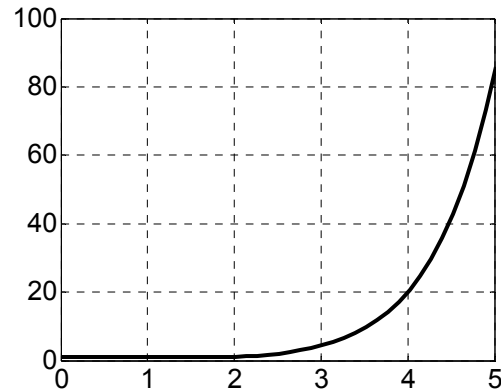
Tests. Energy spread algorithm (1D)



Field growth rate.

Tests. Tapering (3D)

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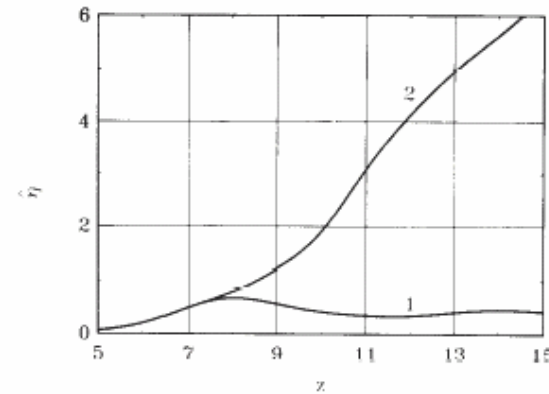
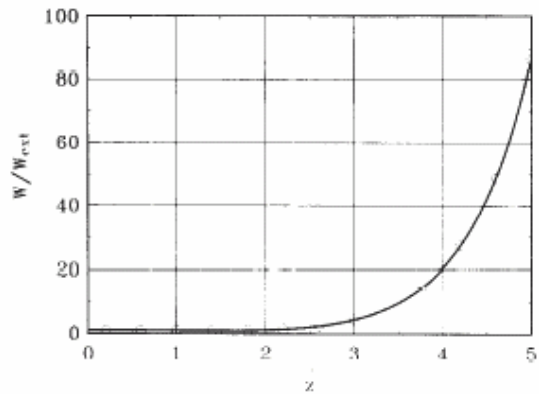


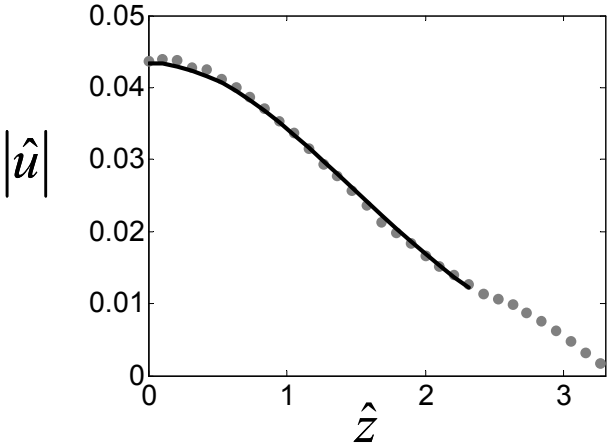
Fig. 11.1. The power gain at the linear stage of amplification. The curve is calculated with the initial problem solution code and the circles are calculated with the nonlinear simulation code. Here $B = 1$, $\hat{C} = 0$, $\hat{A}_p^2 = 0$, $\hat{A}_T^2 = 0$, $N = 5$, $M = 100$, $\hat{\omega} = 1.2$.

Fig. 11.2. The reduced efficiency $\hat{\eta}$ versus the interaction length. Here $B = 1$, $\hat{C} = 0$, $\hat{A}_p^2 = 0$, $\hat{A}_T^2 = 0$, $N = 5$, $M = 100$, $\hat{\omega} = 1.2$ and $\hat{W}_{ext} = 10^{-3}$. Curve (1): without tapering and curve (2): the tapering according to formula (11.6).

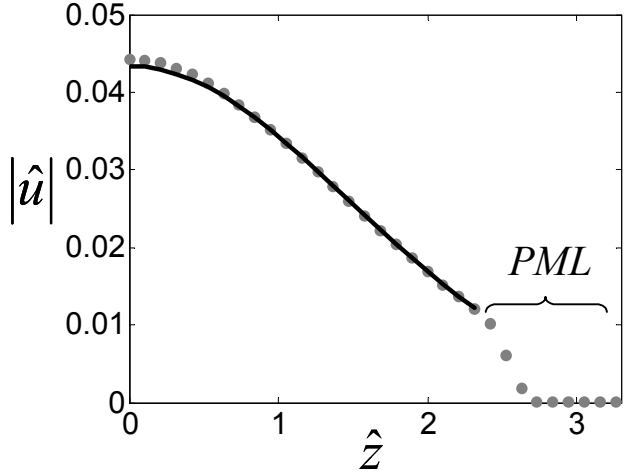
*[SSY, 1995] E.L.Saldin, E.A.Schneidmiller, M.Y.Yurkov, The Physics Of Free Electron Lasers, An Introduction, Physics Reports 260 (1995) 187-327

Tests. Field Solver, Perfectly Matched Layer (3D)

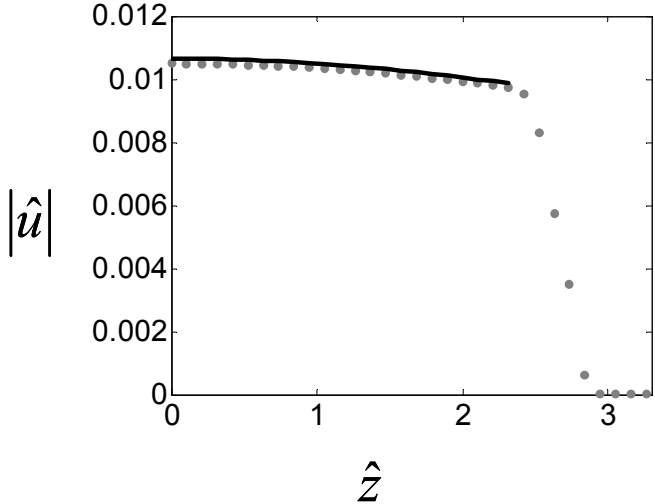
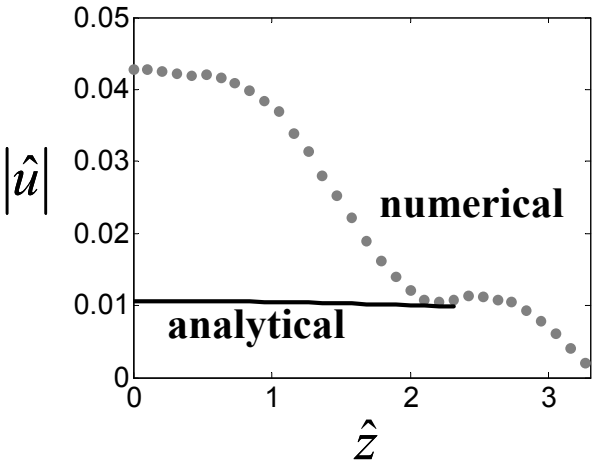
Dirichlet BC



PML

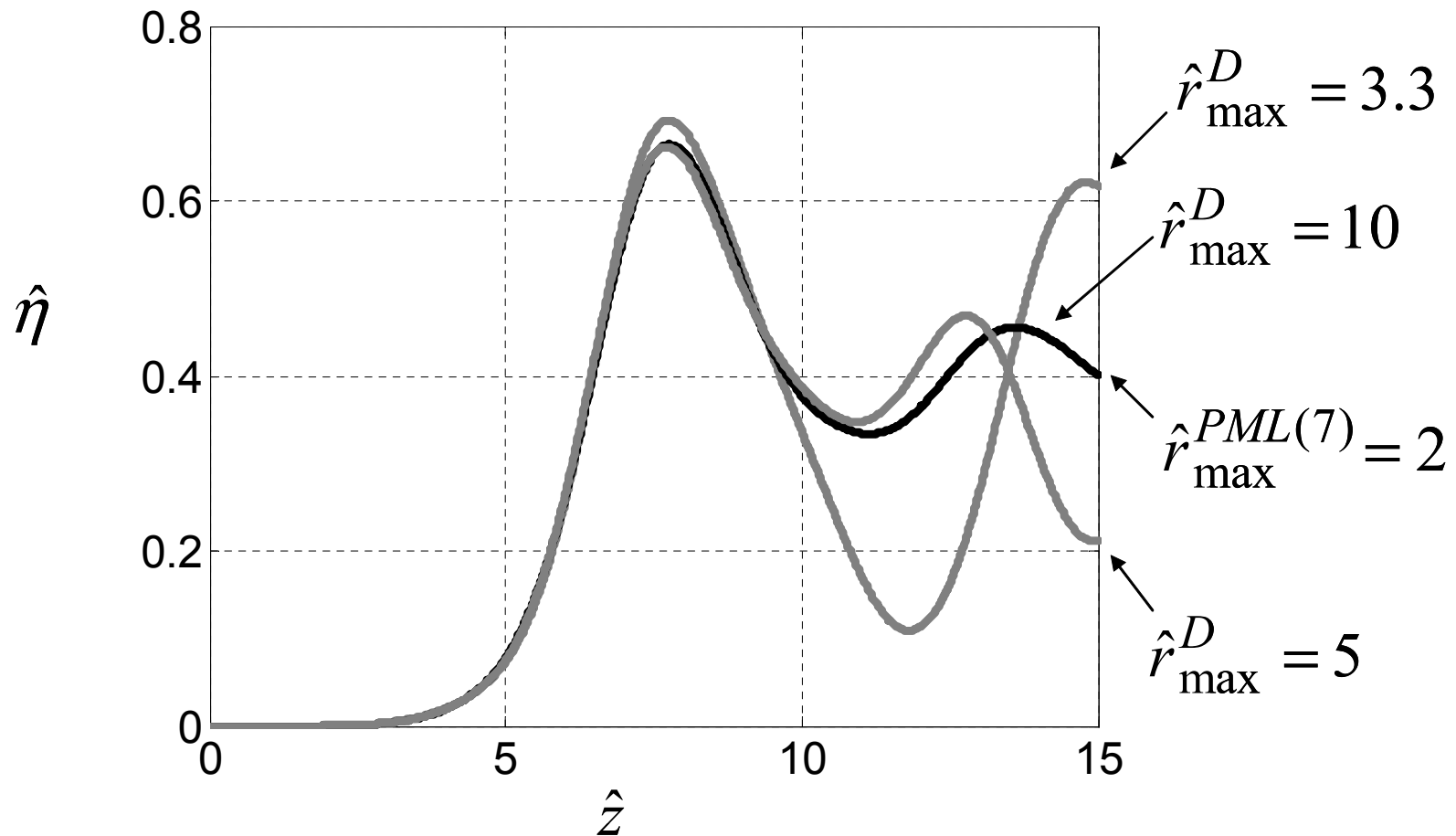


$\hat{z} \approx 1$



$\hat{z} \approx 5$

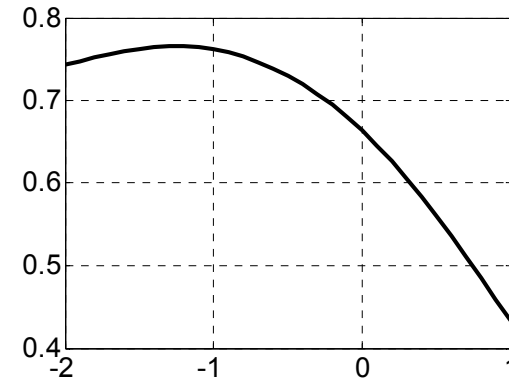
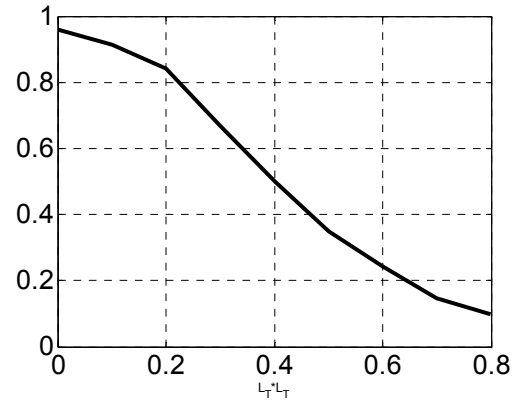
Tests. Field Solver, Perfectly Matched Layer (3D)



Dirichlet BC vs. PML

Tests. Energy spread and diffraction (3D)

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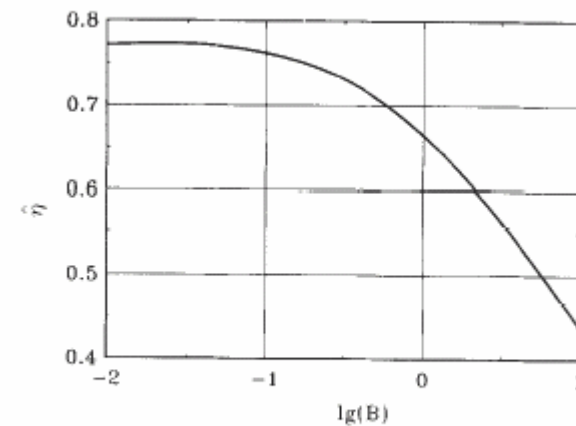
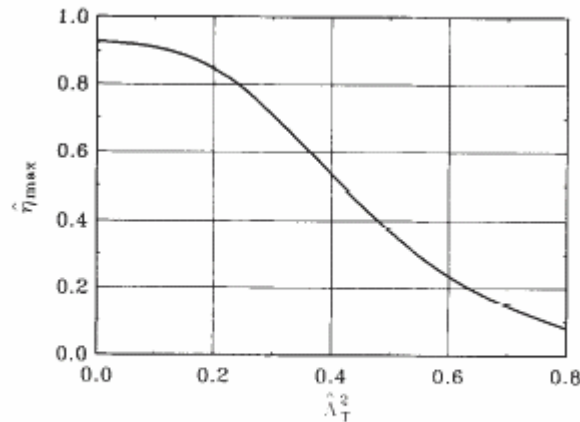


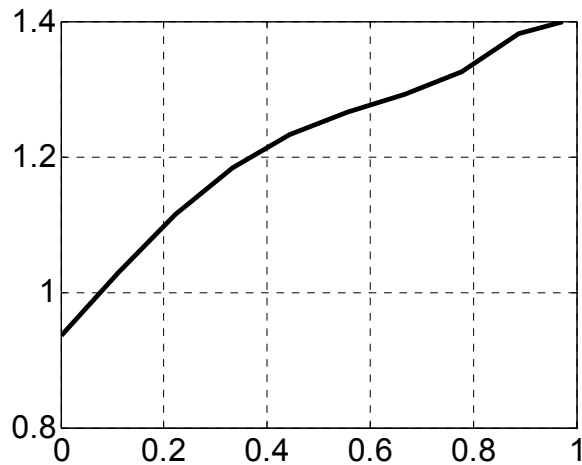
Fig. 11.5. The reduced efficiency $\hat{\eta}$ at the saturation versus the energy spread parameter $\hat{\Delta}_T^2$. Here $B = 1$ and $\hat{\Delta}_p^2 = 0$. (The detuning parameter corresponds to the maximum gain at the linear high-gain limit).

Fig. 11.6. The reduced efficiency $\hat{\eta}$ at the saturation versus the diffraction parameter B . Here $\hat{C} = 0$, $\hat{\Delta}_p^2 = 0$ and $\hat{\Delta}_T^2 = 0$.

*[SSY, 1995] E.L.Saldin, E.A.Schneidmiller, M.Y.Yurkov, The Physics Of Free Electron Lasers, An Introduction, Physics Reports 260 (1995) 187-327

Tests. Space charge (3D)

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FAST*

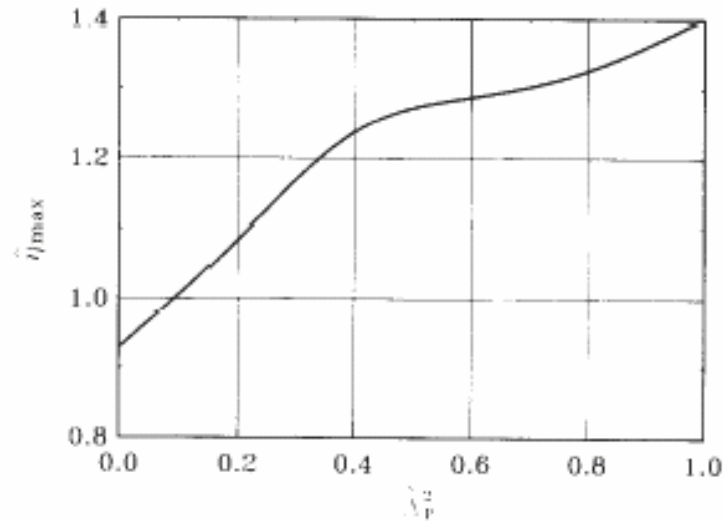
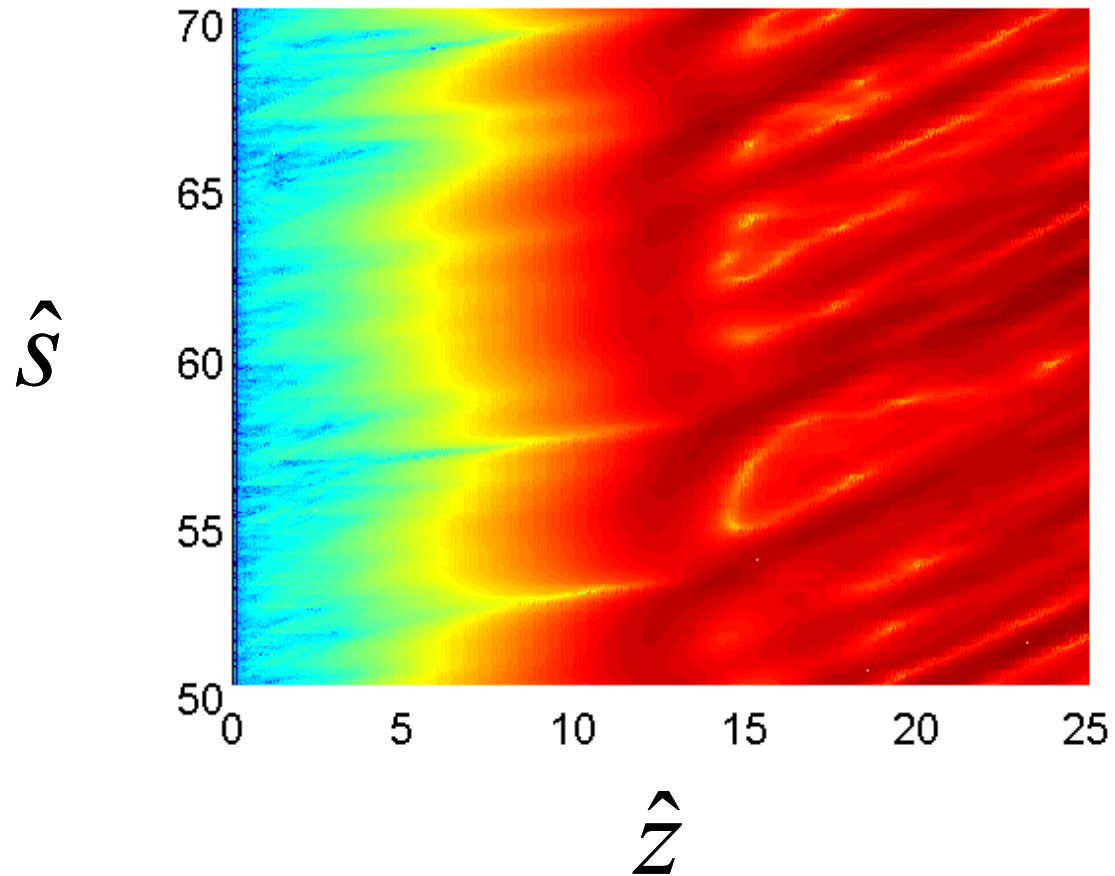


Fig. 11.4. The reduced efficiency $\hat{\eta}$ at the saturation versus the space charge parameter $\hat{\lambda}_p^2$. Here $B = 1$ and $\hat{\lambda}_p^2 = 0$. (The detuning parameter corresponds to the maximal increment at the linear high-gain limit).

*[SSY, 1995] E.L.Saldin, E.A.Schneidmiller, M.Y.Yurkov, The Physics Of Free Electron Lasers, An Introduction, Physics Reports 260 (1995) 187-327

Tests. SASE (1D)



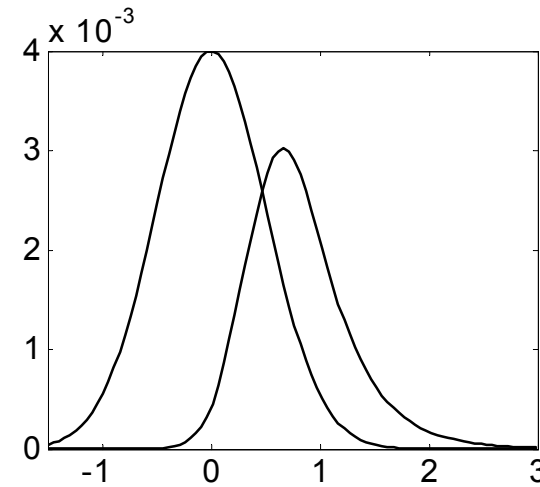
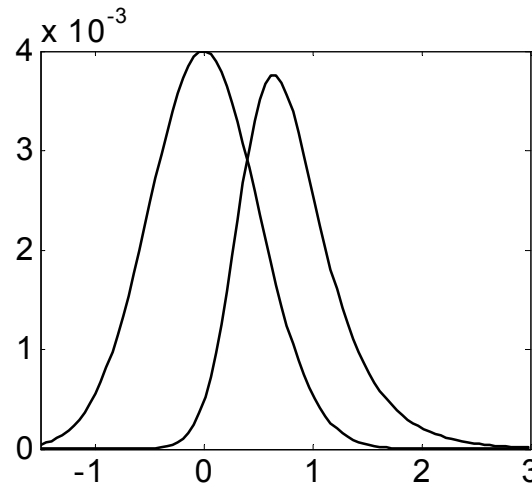
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Normalized power in the radiation pulse

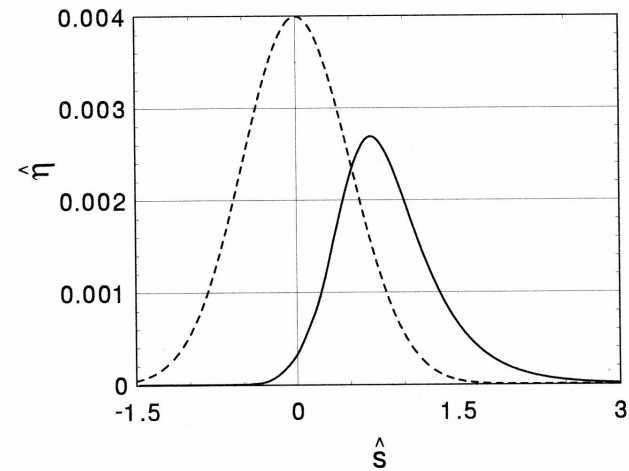
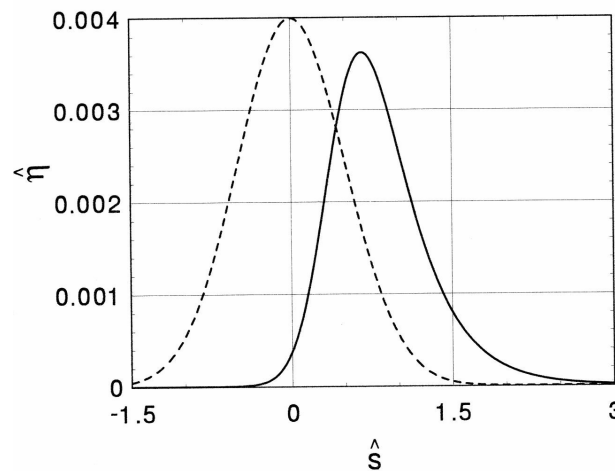
Tests. SASE, Gaussian axial bunch profile (1D)

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$$\hat{\sigma}_b = 0.5$$



FAST*

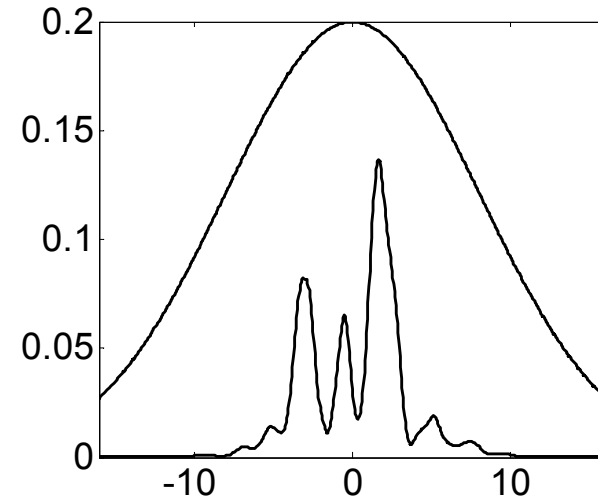
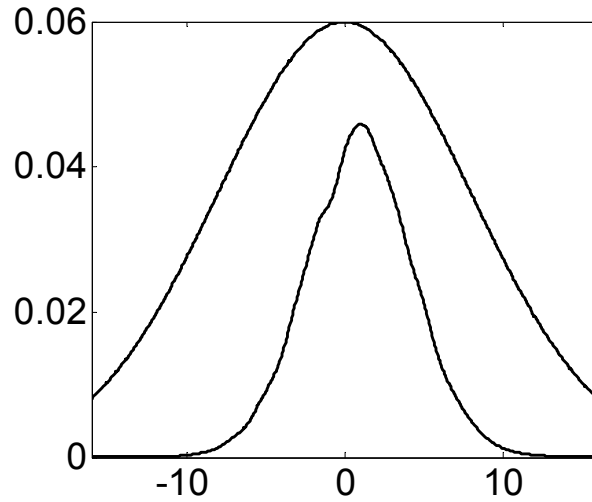


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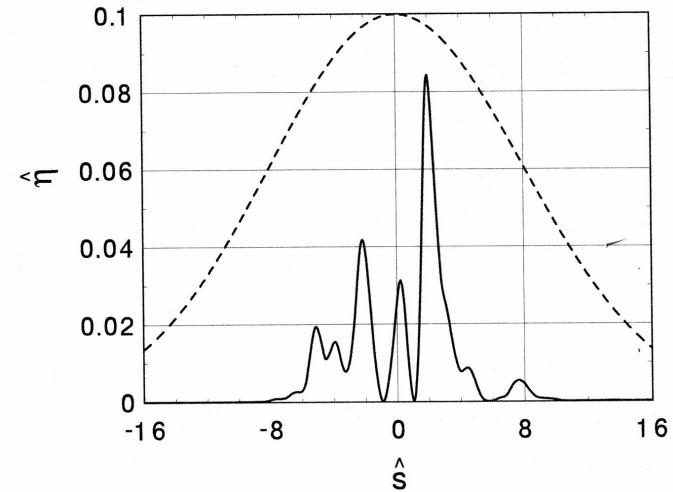
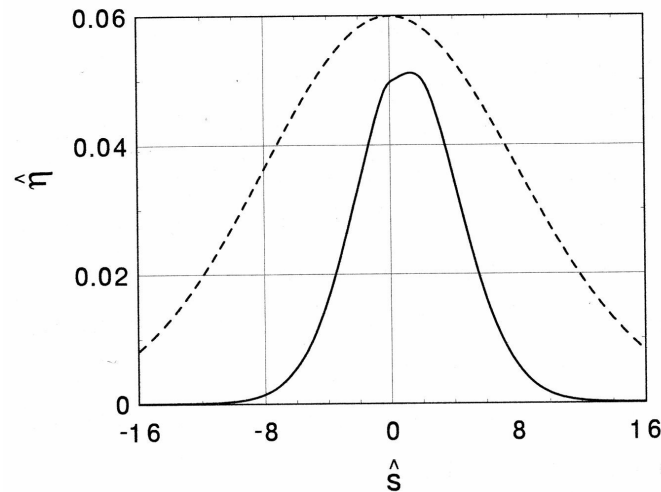
Tests. SASE, Gaussian axial bunch profile (1D)

ALICE

$$\hat{\sigma}_b = 8$$

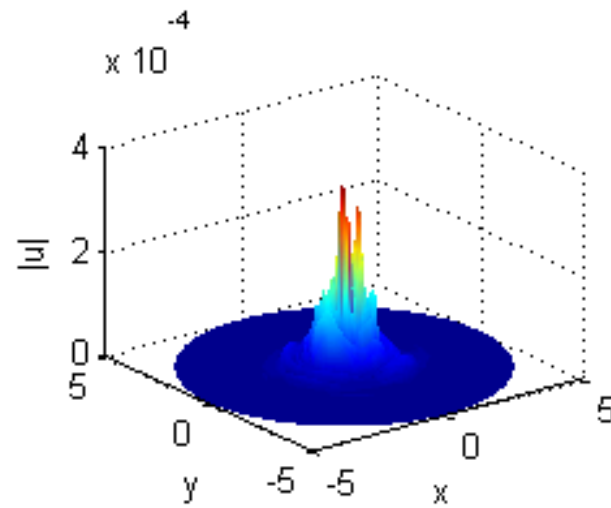


FAST*

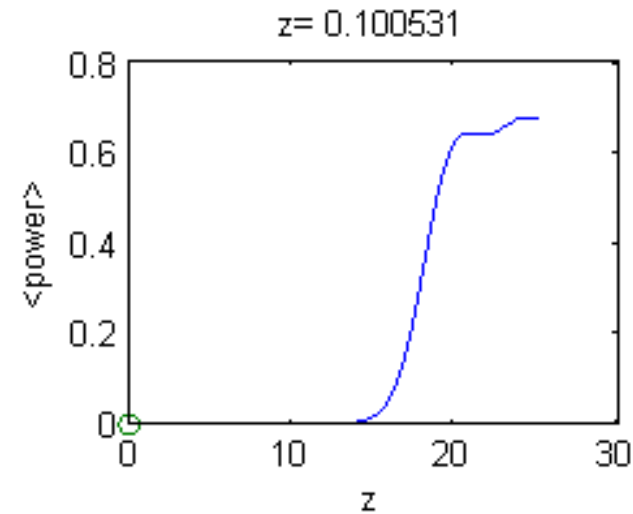
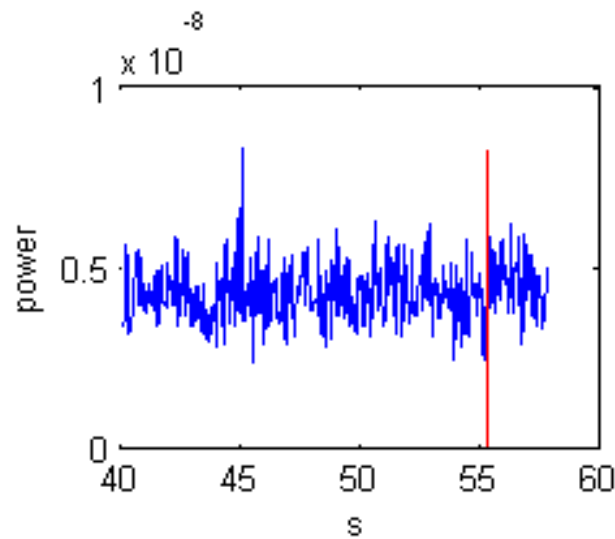
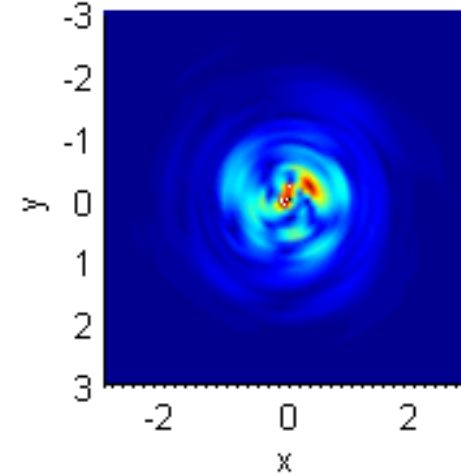


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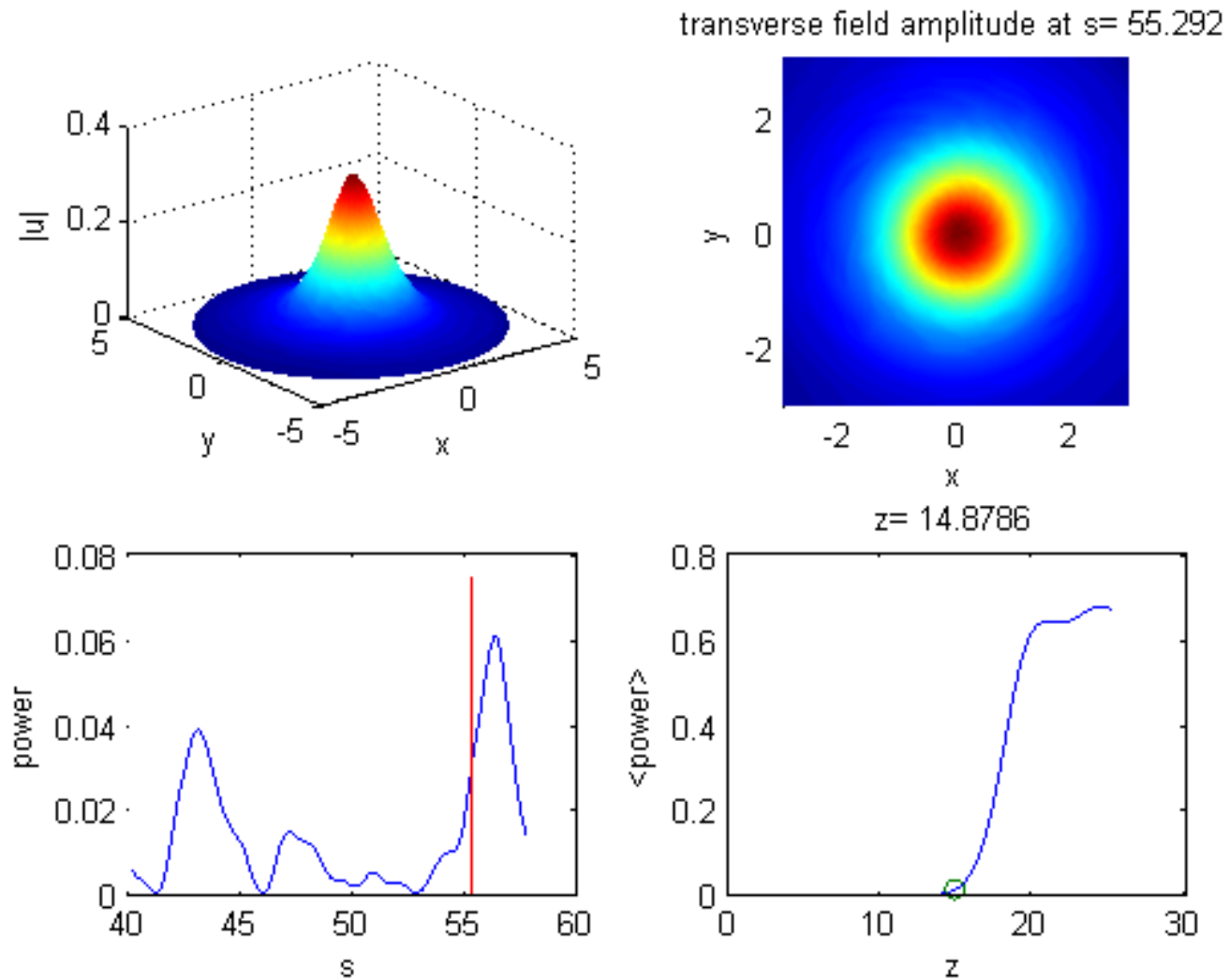
Longitudinal and transverse coherence (SASE, 3D)



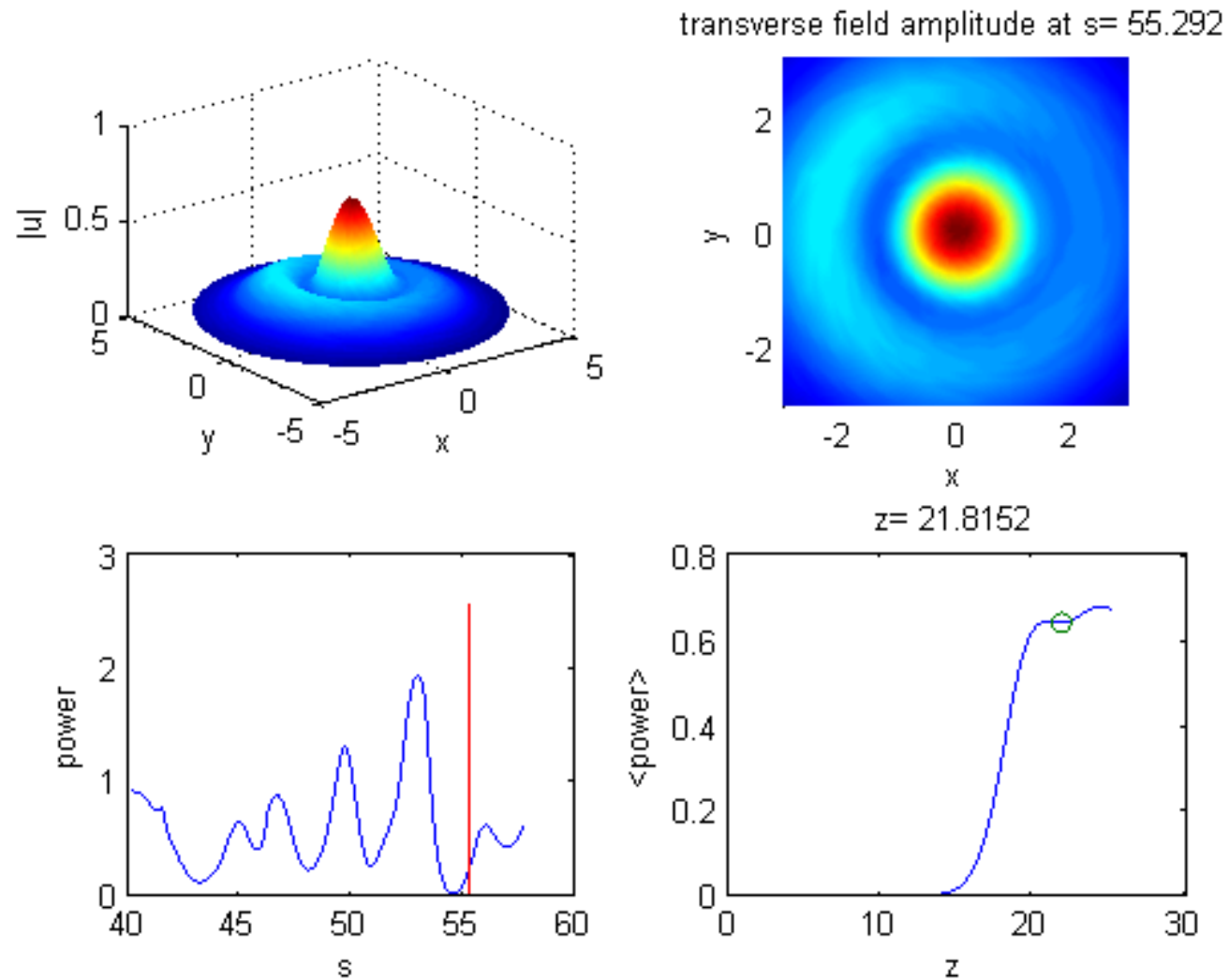
transverse field amplitude at $s = 55.292$



Longitudinal and transverse coherence (SASE, 3D)



Longitudinal and transverse coherence (SASE, 3D)



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Martin Dohlus,
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