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Numerical Studies of Resistive Wall Effects

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XFEL Beam Dynamic Meeting

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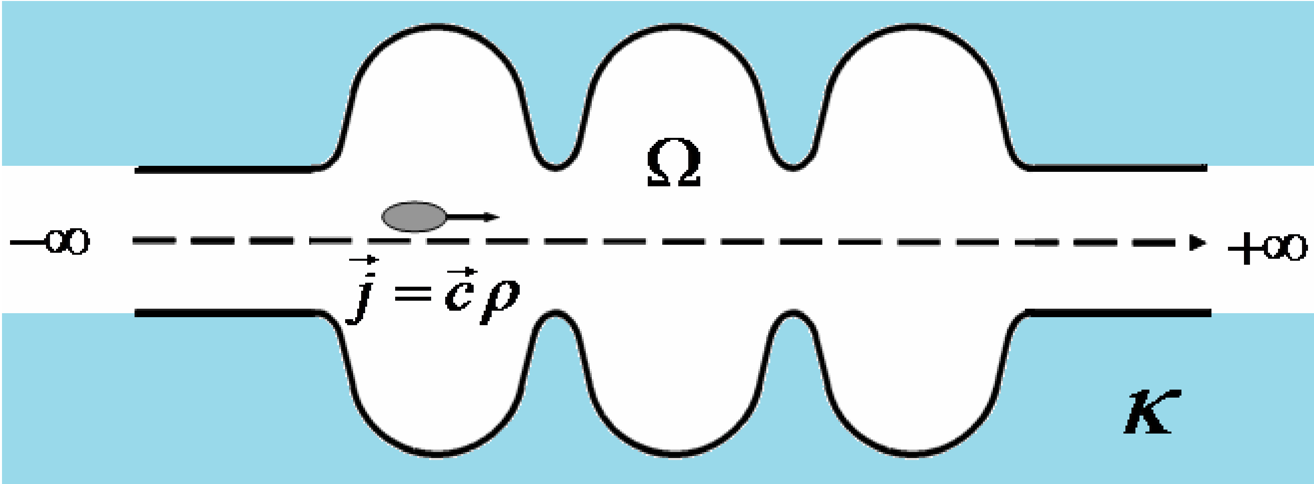
Topics

- Formulation of the problem
- Physical motivation of the model
- Algorithm description
- Numerical examples

Formulation of the Problem

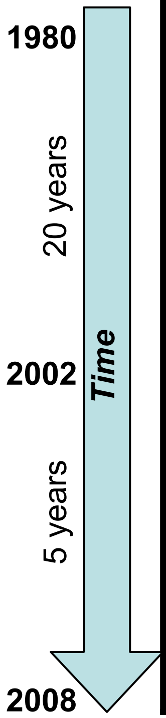


Ultra relativistic charged particle moving through an accelerating structure with finite conductive walls supplied with infinite pipes.

$$Curl\ \vec{E} = -\frac{\partial}{\partial t}\mu\vec{H} \qquad Div\ \varepsilon\vec{E} = \rho$$

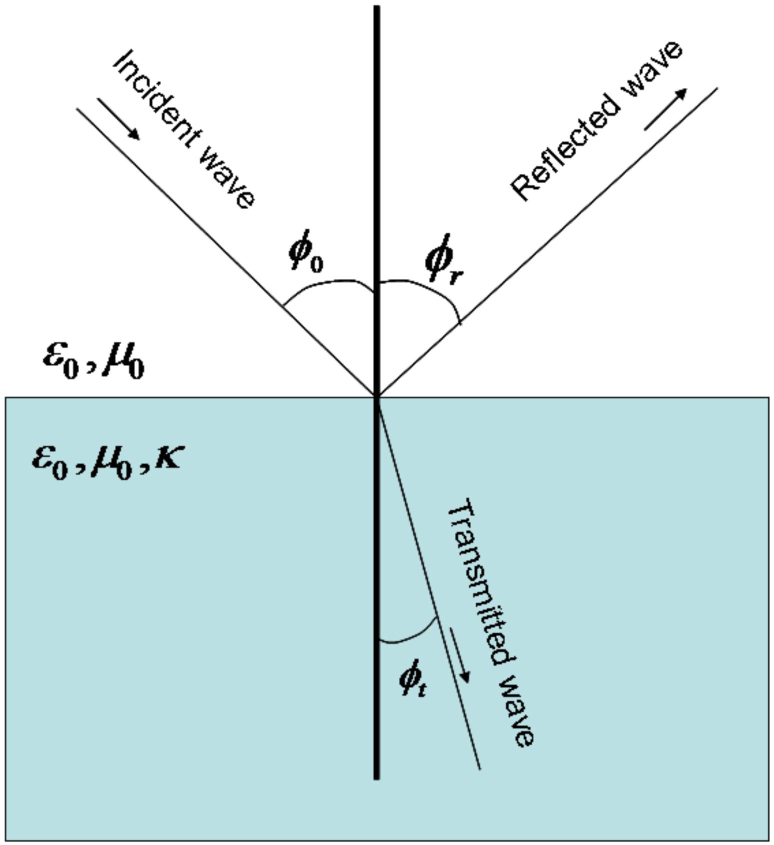
$$Curl\ \vec{H} = \vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E} \qquad Div\ \vec{H} = 0$$

An (incomplete) survey of available codes



	Non-dispersive in longitudinal direction	Second order convergence	Conductivity
BCI/TBCI	No	No	No
NOVO	Yes	No	No
ABCI	No	No	No
MAFIA	No	No	No
XWAKE	No	Yes	No
Gdfidl	No	No	No
Tau3P	No	Yes	No
ECHO	Yes	Yes	No
CST	No	Yes	Yes
PBCI	Yes	No	No
NEKCEM	No	Yes	No

Transmission of EM wave on vacuum-conductor boundary surface.



$$\sin \phi_t = \frac{1}{n(\phi_0, \omega, \kappa)} \sin \phi_0$$

$$\kappa >> \varepsilon_0 \omega \Rightarrow \phi_t \sim 0$$

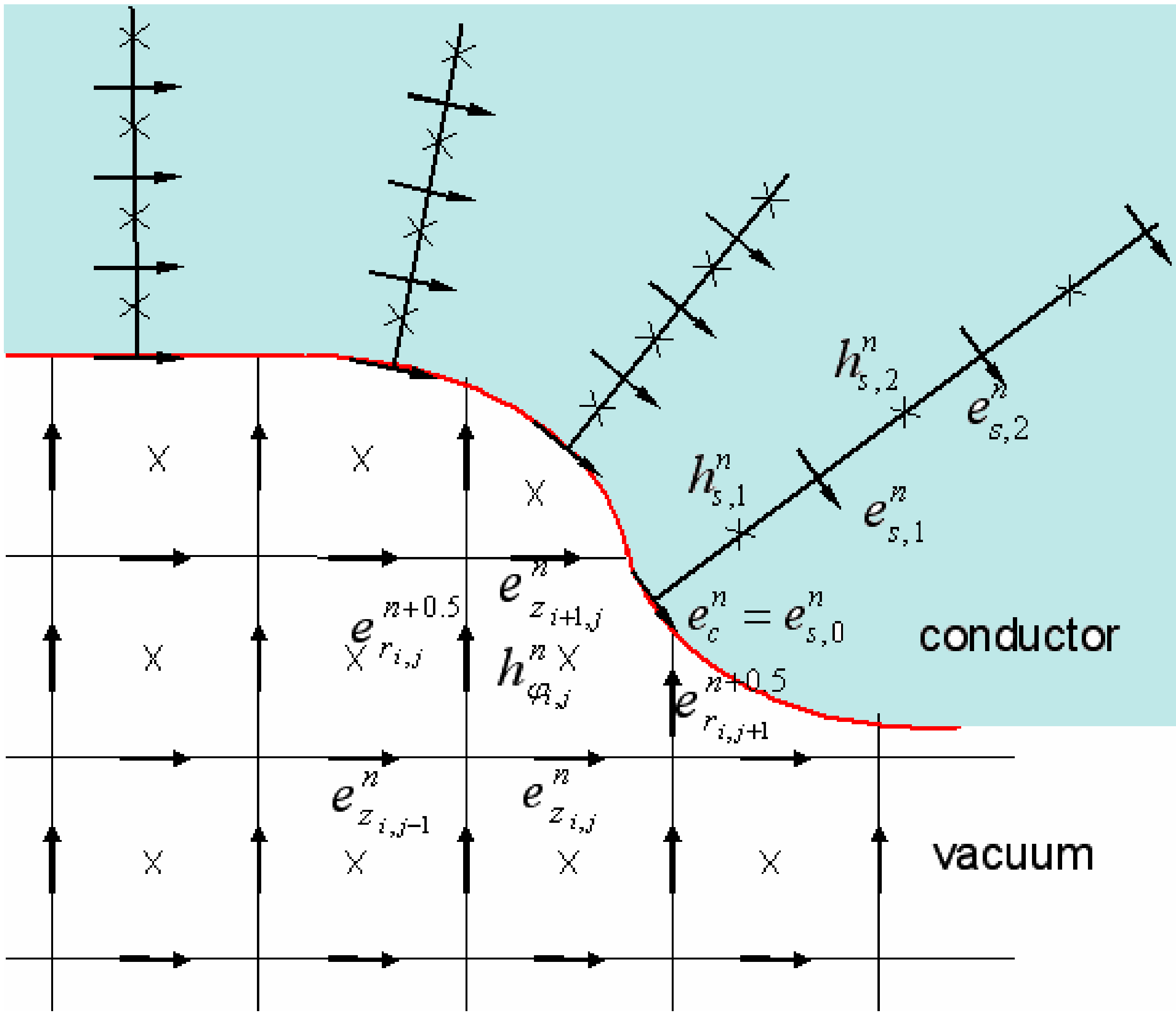
Example

Stainless Steel - $\kappa = 1.4 \cdot 10^6\ \Omega^{-1} m^{-1}$

r.m.s bunch lenght - 25 μm

$$\kappa / \varepsilon_0 \omega \sim 10^4$$

Vacuum grid with 1D conducting lines at the boundary



$$\lambda_{metal} = \frac{2\pi}{\sqrt{\omega\mu\sigma}}$$

$$\lambda_{vacuum} = \frac{2\pi c}{\omega}$$

Stainless Steel

$$\kappa = 1.4 \cdot 10^{6}\ \Omega^{-1} m^{-1}$$

$$\sigma = 25\,\mu m \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 80$$

$$\sigma = 1 mm \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 500$$

Copper

$$\kappa = 58 \cdot 10^{6}\ \Omega^{-1} m^{-1}$$

$$\sigma = 25\,\mu m \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 500$$

$$\sigma = 1 mm \Rightarrow \frac{\lambda_{vacuum}}{\lambda_{metal}} \approx 3000$$

Longitudinal dispersion free TE/TM numerical scheme

Field update in vacuum

$$\hat{\mathbf{e}}_r^{n+0.5} = \hat{\mathbf{e}}_r^{n-0.5} - \Delta\tau\,\mathbf{M}_{\varepsilon_r^{-1}}\mathbf{P}_z^*\hat{\mathbf{h}}_\varphi^n$$

$$\hat{\mathbf{h}}_\varphi^{n+0.5} = \hat{\mathbf{h}}_\varphi^n + \frac{\Delta\tau}{2}\mathbf{M}_{\mu^{-1}}\left[\mathbf{P}_z\hat{e}_r^{n+0.5} - \mathbf{P}_r\hat{e}_z^n + \hat{\mathbf{e}}_c^n\right]$$

$$\mathbf{W}\frac{\hat{\mathbf{e}}_z^{n+1} - \hat{\mathbf{e}}_z^n}{\Delta\tau} = \mathbf{M}_{\varepsilon_z^{-1}}\left[\mathbf{P}_r^*\hat{\mathbf{h}}_\varphi^{n+0.5} + \hat{\mathbf{j}}_z^{n+0.5}\right]$$

$$\hat{\mathbf{h}}_\varphi^{n+1} = \hat{\mathbf{h}}_\varphi^{n+0.5} + \frac{\Delta\tau}{2}M_{\mu^{-1}}\left[\mathbf{P}_z\hat{e}_r^{n+0.5} - \mathbf{P}_r\hat{e}_z^{n+1} + \hat{\mathbf{e}}_c^{n+1}\right]$$

where $\mathbf{W} = \mathbf{I} + \frac{\Delta\tau^2}{4}\mathbf{M}_{\mu_\varphi^{-1}}\mathbf{P}_r\mathbf{M}_{\varepsilon_r^{-1}}\mathbf{P}_r^*$

Field update in conductor

$$\hat{\mathbf{e}}_s^{\,n+1} = \mathbf{A}\mathbf{e}_s^{\,n} + \mathbf{B}\mathbf{P}_s\frac{\hat{\mathbf{h}}_s^{\,n+1} + \hat{\mathbf{h}}_s^{\,n}}{2}$$

$$\hat{\mathbf{h}}_s^{\,n+1} = \hat{\mathbf{h}}_s^{\,n} + \Delta\tau\mathbf{P}_s^*\frac{\hat{\mathbf{e}}_s^{\,n+1} + \hat{\mathbf{e}}_s^{\,n}}{2}$$

$$a_{ii} = e^{-\kappa Z_0\Delta\tau}$$

$$a_{00} = \mathrm{e}^{-0.5\kappa Z_0\Delta\tau}$$

$$b_{ii} = \frac{(1-a_{ii})}{\kappa Z_0}$$

Algorithm Stability Condition

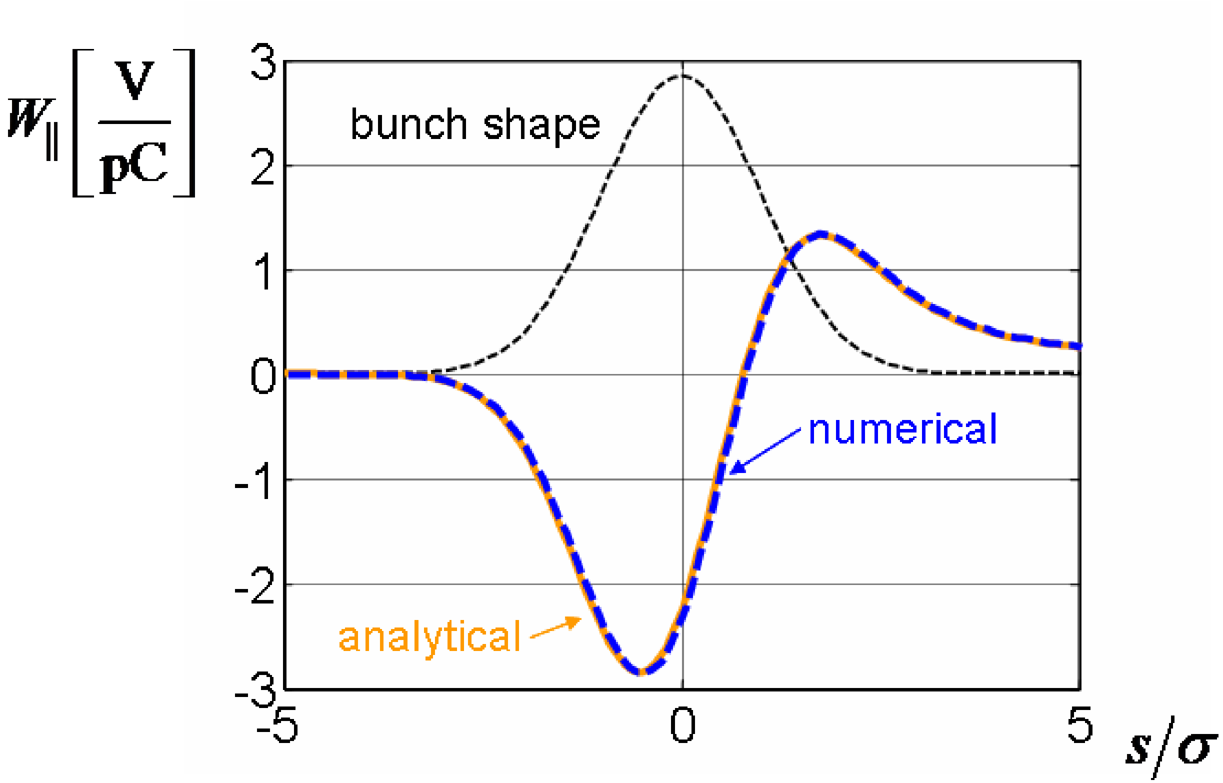
$$\Delta\tau \le \Delta z$$

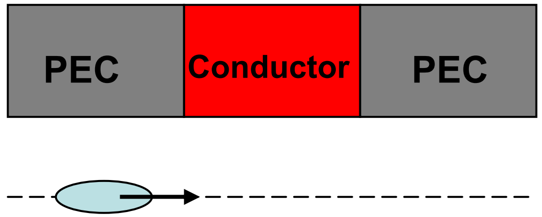
Longitudinal Dispersion Free Condition

$$\Delta\tau = \Delta z$$

Numerical Examples

Comparison of numerical and analytical steady state wakes of the Gaussian bunch with rms length σ=1mm in round pipe of radius a=1 cm and of the conductivity κ=1e5 S/m and the.

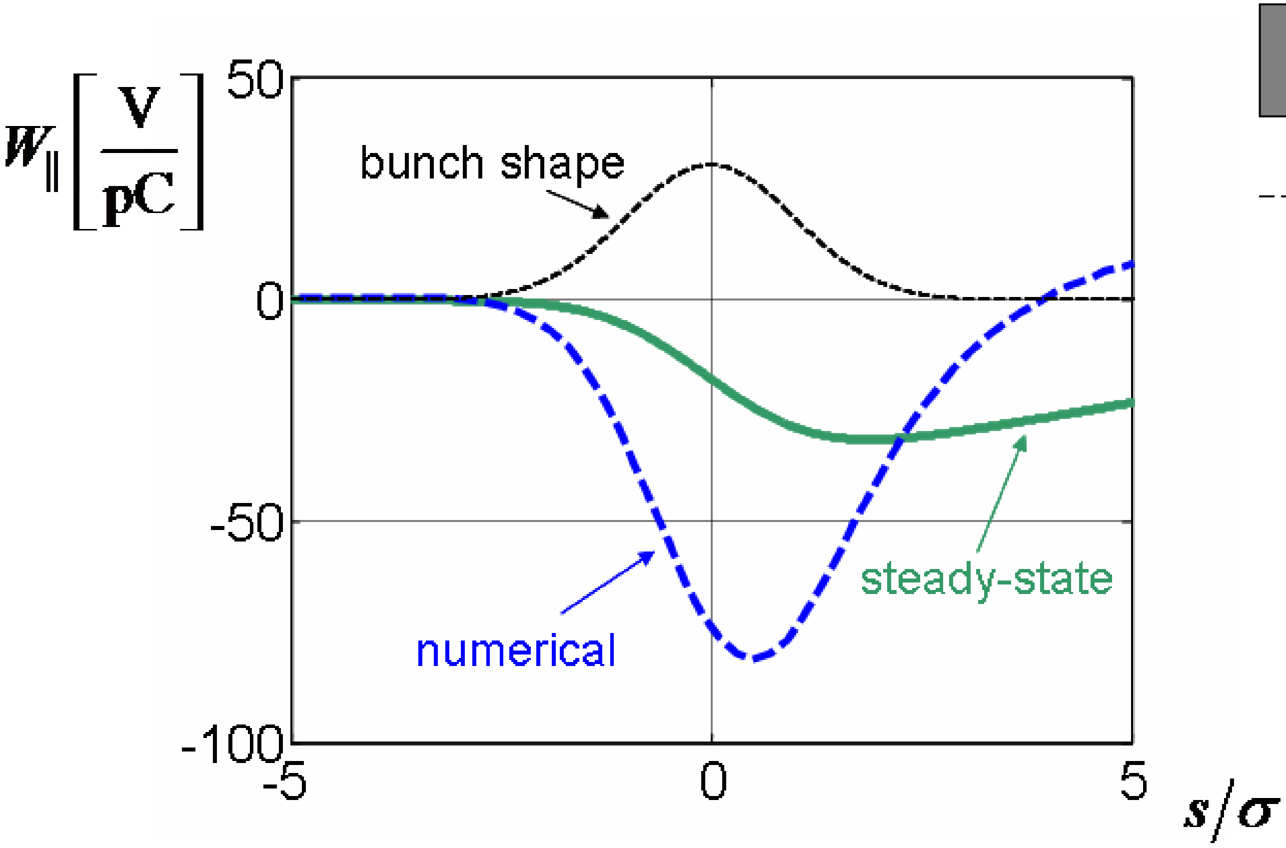


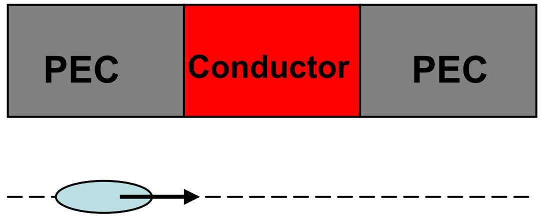


For mesh resolution of 10 points on σ
error in loss factor is 3%

Numerical Examples

The wake potential of finite length resistive cylinder with radius a=1cm, length b=10cm and conductivity κ=1e4 S/m. The Gaussian bunch r.m.s. length is σ=25 μm.





Loss Factor

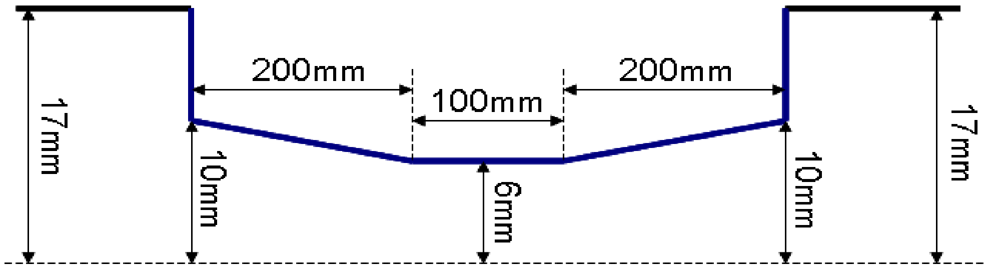
Numerical = 58 V/pC

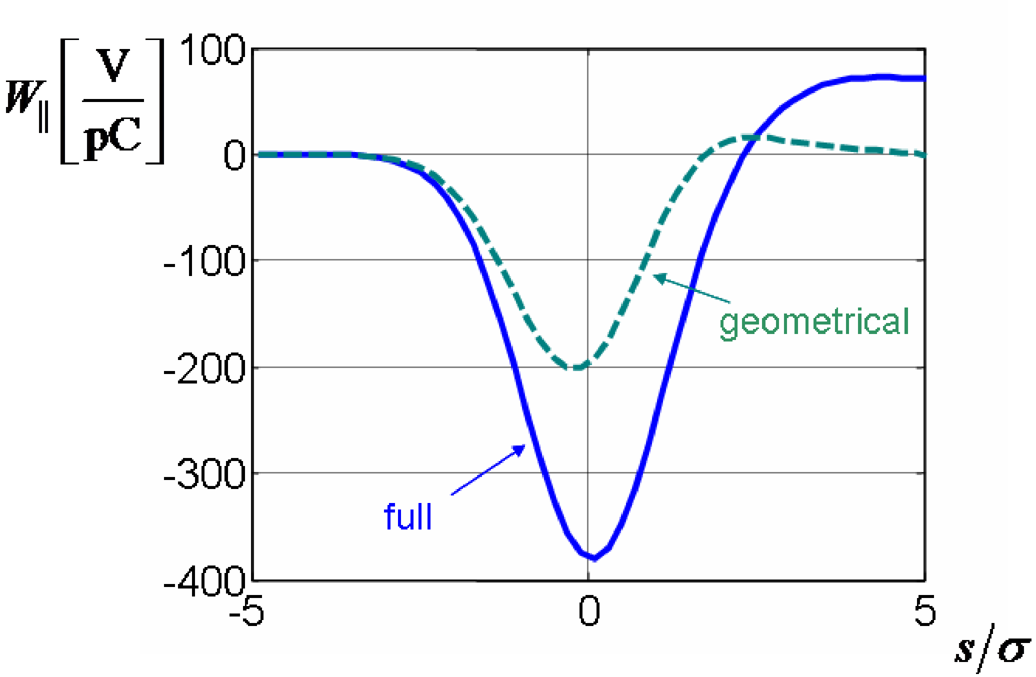
Analytic = 57 V/pC

S.Krinsky and B. Podobedov, PR-STAB, 7, 114401 (2004)

Numerical Examples

Comparison of wake potentials of tapered collimator "with" and "without" resistivity for Gaussian bunch σ = 50 μm .





Loss factor for finite conductive walls cannot be obtain as direct sum of the geometrical and the steady-state solution.