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# Response Matrix Measurements and Analysis at DESY


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DESY – MPY –

# Contents

# Motivation

- To achieve the maximum performance of an accelerator, the linear optics of the machine needs to be close to the design optics
- The real machine has gradient errors, alignment errors, etc. which are normally unknown and distorting the optics
- Orbit depends non-linear on the focusing of the quadrupoles. Analyze the difference orbits due to the kick of corrector magnets (Orbit-Response-Matrix) to find out the error sources
- Correct the gradient errors and restore the linear optics
- Analysis gives valuable information about the BPM system and the corrector magnets

# Definition of the Orbit Response Matrix

- Definition of the **orbit response matrix (ORM)**:

$$\mathbf{C}_{ij}^{xx} := \frac{\Delta x_i}{\Delta \theta_{x,j}} \qquad \text{for x-plane}$$

$\Delta x_i$ : change of the beam position at BPM $i$

$\Delta \theta_{x,j}$ : change of the kick angle of the corrector $j$

- Change the kick angle of all correctors one after the other and measure the orbit change with all available BPMs

- Measurement can be written as

$$\begin{pmatrix} \Delta \vec{x} \\ \Delta \vec{y} \end{pmatrix} = \begin{pmatrix} \mathbf{C}^{xx} & \mathbf{C}^{xy} \\ \mathbf{C}^{yx} & \mathbf{C}^{yy} \end{pmatrix} \cdot \begin{pmatrix} \Delta \vec{\theta}_x \\ \Delta \vec{\theta}_y \end{pmatrix}$$

- Due to coupling and/or rotated BPMs or rotated correctors the orbit changes also in the other plane. For an uncoupled machine: $\mathbf{C}^{xy} = \mathbf{C}^{yx} = \mathbf{0}$

# Form of the Orbit Response Matrix

## Beamline

- Corrector kick is changing the trajectory downstream of corrector:

$$\mathbf{C}_{ij} = \begin{cases} \sqrt{\beta_i \beta_j}\,\sin\left(2\pi|\phi_i - \phi_j|\right) & \text{if} \quad \phi_i > \phi_j \quad , \\ 0 & \text{otherwise} \quad . \end{cases}$$

- Triangle above main diagonal of $\mathbf{C}$ is zero

## Circular accelerator

- Corrector kick is changing the orbit everywhere:

$$\mathbf{C}_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin\pi Q}\cos\left(2\pi|\phi_i - \phi_j| - \pi Q\right) - \frac{D_i D_j}{\left(\alpha_c - \frac{1}{\gamma^2}\right) C}$$

$D$: dispersion function, $\alpha_c$: momentum-compaction factor,

$C$: circumference, $\gamma$: Lorentz factor

- Second term: energy shift due to kick of the corrector

# Measured Matrix vs. Computer Model Matrix

Distinguish between

- Measured matrix $\bar{\mathbf{C}}$

  Depends on scaling factors $b_i := 1 + \Delta b_i$ and $c_j := 1 + \Delta c_j$

  of BPMs and correctors. For error-less BPMs/correctors $b_i = c_j = 1$.

- Model matrix $\mathbf{C}$

  The computer model of the real machine

Assume, that model matrix depends on unknown parameters $\vec{p}$ of the lattice (e.g. quadrupole gradient errors, quadrupole roll angles, ...). Design parameters are called $\vec{p}_0$. Then $\bar{\mathbf{C}}$ can be written as:

$$\bar{\mathbf{C}}_{ij} = \frac{1}{b_i} \cdot \mathbf{C}(\vec{p}_0 + \Delta\vec{p})_{ij} \cdot c_j$$

Taylor expansion ($\Delta p_k, \Delta b_i, \Delta c_j \ll 1$):

$$\bar{\mathbf{C}}_{ij} \approx \mathbf{C}_{ij} + \sum_k \left.\frac{\partial \mathbf{C}_{ij}}{\partial p_k}\right|_{\vec{p}_0} \Delta p_k - \mathbf{C}_{ij}\Delta b_i + \mathbf{C}_{ij}\Delta c_j$$

# Fitting the Unknown Parameters

- Linear system of equations to solve:

$$\underbrace{\left(\bar{\mathbf{C}} - \mathbf{C}\right)}_{\vec{y}} = \underbrace{\left(\frac{\partial \mathbf{C}}{\partial p_k} \quad -\mathbf{C} \quad +\mathbf{C}\right)}_{\mathbf{A}} \cdot \underbrace{\begin{pmatrix} \Delta\vec{p} \\ \Delta\vec{b} \\ \Delta\vec{c} \end{pmatrix}}_{\vec{x}}$$

  with the fit-parameter vector $\vec{x}$ and the measured response matrix in $\vec{y}$.

  Matrix $\mathbf{A}$ can be computed e.g. using MAD

- Take finite resolution of BPMs $\sigma$ into account, by dividing each row of $\mathbf{A}$ by $\sigma/\theta_j$

- Solve over-determined equations by least square fit using truncated SVD:

$$\vec{x} = (\underbrace{\mathbf{A}^{\mathbf{T}}\mathbf{A}}_{\mathbf{B}})^{-1}\mathbf{A}^{\mathbf{T}}\vec{y} \qquad \text{Normal equations}$$

- Use for the next iteration optics with $\vec{p}_0 + \Delta\vec{p}$ for the model matrix. Iterate several times, until convergence achieved.

# Solving the Equations using SVD

- Matrix $\mathbf{B} := \mathbf{A}^{\mathbf{T}}\mathbf{A}$ is singular and inverse matrix $\mathbf{B}^{-1}$ is not existing due to an unknown global scaling factor $f$ between BPMs and correctors:

$$\frac{1}{f}\frac{1}{b_i} \cdot \mathbf{C}_{ij} \cdot c_j f \equiv \frac{1}{b_i} \cdot \mathbf{C}_{ij} \cdot c_j$$

  $\longrightarrow$ Two small eigenvalues of $\mathbf{B}$ ($x$ and $y$-plane)

- Fix the unknown scaling factor $f$ by measuring the dispersion function $D$

- Solution: Remove null-space from $\mathbf{B}$ using singular value decomposition:

$$\mathbf{B} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathbf{T}}$$

  with orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$ and a diagonal matrix $\mathbf{S}$ with singular values $s_i$.

- Compute pseudo-inverse of $\mathbf{B}$

$$\mathbf{B}^{+} = \mathbf{V}\mathbf{D}\mathbf{U}^{\mathbf{T}}$$

  using the truncated SVD and the cutoff-parameter $\epsilon < 1$. Set $1/s_i = 0$ for small singular values. Matrix $\mathbf{D}$ has diagonal shape with $\mathbf{D}_{ii} = 1/s_i$ if $s_i < \epsilon s_1$, otherwise $\mathbf{D}_{ii} = 0$.

# Matrix Sizes

- Assume a ring with $N$ BPMs and $M$ correctors
  - Size of response matrix: $N_{\text{MAT}} = (N_x + N_y) \cdot (M_x + M_y)$
  - Minimum number of fit parameters: $N_{\text{FIT}} = (N_x + N_y) + (M_x + M_y)$
- Examples:

| Machine | $N_x$ | $N_y$ | $M_x$ | $M_y$ | $N_{\text{MAT}}$ | $N_{\text{FIT}}$ | Memory($\mathbf{A}$) |
|---|---|---|---|---|---|---|---|
| El-Weg | 6 | 6 | 11 | 12 | 276 | 35 | 77 kB |
| PETRA-e | 113 | 113 | 118 | 111 | 51754 | 455 | 188 MB |
| HERA-p | 141 | 141 | 128 | 126 | 71628 | 536 | 307 MB |
| HERA-e | 287 | 287 | 281 | 277 | 320292 | 1132 | 2.9 GB |

- Problems for HERA-e with memory and computation time

$\longrightarrow$ Working on a subset of all corrector magnets/BPMs

$\longrightarrow$ Or: Use a different approach for optics correction!

# Scaling laws for Achievable Accuracy

Assumption: FODO lattice with $N$ BPMs and $M$ corrector magnets; same kick $\theta$ of all correctors; same $\beta$ function at all BPMs and correctors; all BPMs have the same resolution $\sigma$ (V. Ziemann, EPAC 2002).

- Fitting $N$ BPM scaling factors:

$$\sigma(b) \approx \frac{\sigma}{\beta\theta}\frac{1}{\sqrt{M}}$$

- Fitting $M$ corrector scaling factors:

$$\sigma(c) \approx \frac{\sigma}{\beta\theta}\frac{1}{\sqrt{N}}$$

- Fitting $Q$ gradient errors:

$$\sigma(\Delta kl) \approx \frac{\sigma}{\beta^2\theta}\frac{48\pi}{\sqrt{N \cdot M}}$$

➡ Small BPM resolution $\sigma$ is crucial for the sensitivity to fit errors. Use big corrector kicks $\theta$ and as much BPMs $N$ and correctors $M$ as possible.

# Beta/Phase function fit

- Matrix element of response matrix:

$$\Delta x_{ij} = \sqrt{\beta_i}\underbrace{\frac{\sqrt{\beta_j}\Delta\theta_j}{2\sin\pi Q}}_{f_j}\cos(\pm\phi_j \mp \phi_i + \pi Q) \quad \text{for} \quad \begin{cases} \phi_i > \phi_j \\ \phi_i < \phi_j \end{cases}$$

- Factorization of monitor and corrector parameters:

$$\begin{aligned}\Delta x_{ij} &= f_j\cos(\pi Q \pm \phi_j)\cdot\underbrace{\sqrt{\beta_i}\cos(\phi_i)}_{x_i} \pm f_j\sin(\pi Q \pm \phi_j)\cdot\underbrace{\sqrt{\beta_i}\sin(\phi_i)}_{y_i} \\ &= \sqrt{\beta_i}\cos(\pi Q \mp \phi_i)\cdot\underbrace{f_j\cos(\phi_j)}_{x_j} \mp \sqrt{\beta_i}\sin(\pi Q \mp \phi_i)\cdot\underbrace{f_j\sin(\phi_j)}_{y_j}\end{aligned}$$

- Alternating fit of $(\beta_i, \phi_i)$ or $(f_j, \phi_j)$ :

$$\chi^2 = \sum_{i,j}\left(\frac{\Delta x_{ij}^{\text{meas}} - \Delta x_{ij}^{\text{model}}(\beta_i, \phi_i, f_j, \beta_j)}{\sigma(\Delta x_{ij}^{\text{meas}})}\right)^2 \longrightarrow \text{min.}$$

- For optics correction phases $\phi_i$ and $\phi_j$ are used (not sensitive to scaling errors of BPMs and correctors!)

# Correction of Beta-Beating in HERA

- BPMs and correctors have unknown scaling factors
- Scaling factors will lead to an error in the beta function but not in the phase function

  ⇨ Use phase function ($\varphi_i$, $\varphi_j$) for correction!
- Phase beating due to gradient error of a quadrupole $\Delta k_q$:

$$\Delta\varphi = \frac{\beta_q \Delta k_q l}{4\sin\pi Q}\{\sin(2\pi Q) + \sin(2\varphi_q - 2\pi Q) + \mathrm{sign}(\varphi - \varphi_q)[\sin(2\pi Q) + \sin(2|\varphi - \varphi_q| - 2\pi Q)]\}$$

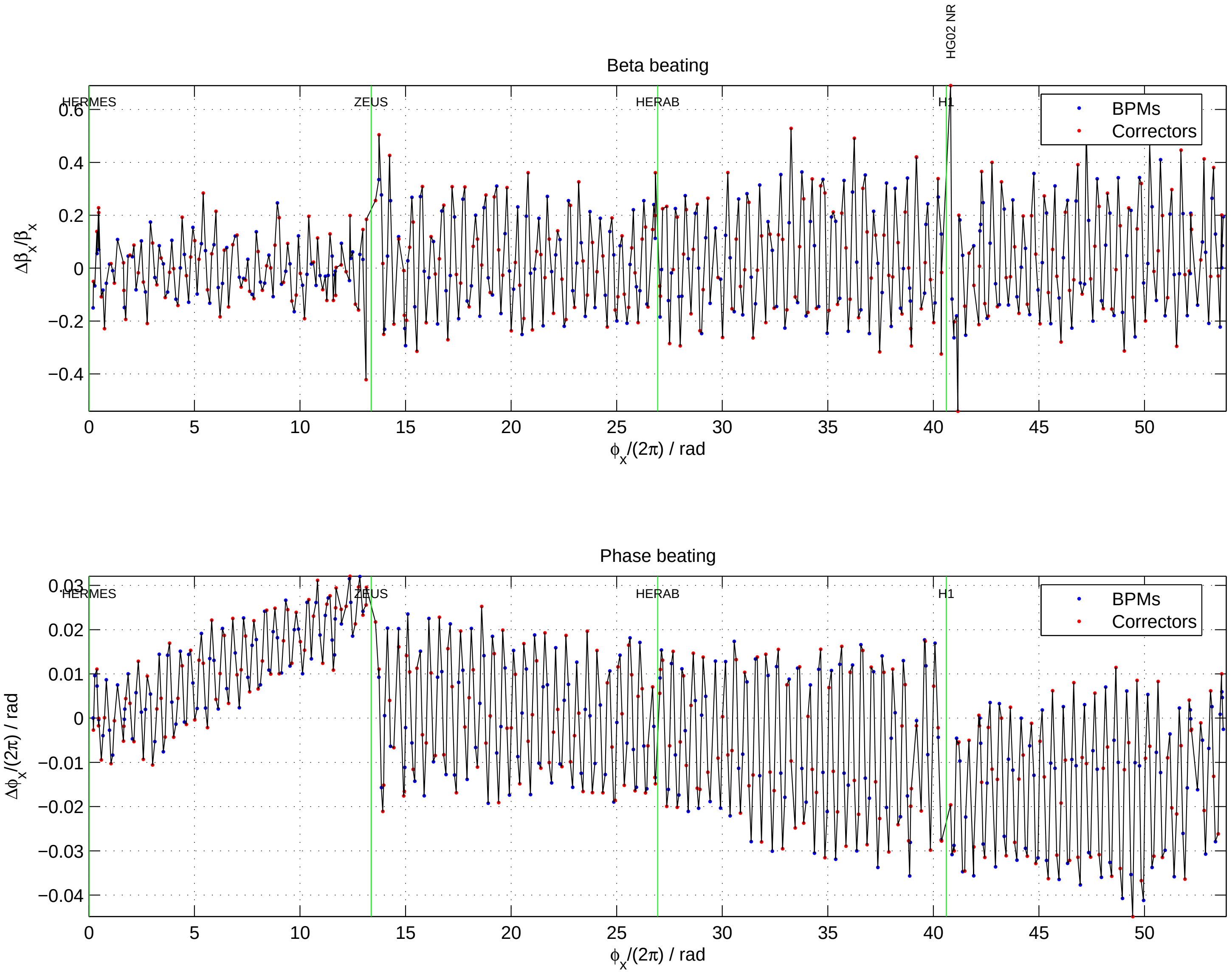
- Global correction of beta beating:

  Solve for quadrupole corrections $\Delta k_q$ using SVD or MICADO:

$$\|\varphi_{i,j} - \sum_q \frac{\partial \varphi_{i,j}}{\partial k_q} \Delta k_q\|^2 \to \text{min.}$$

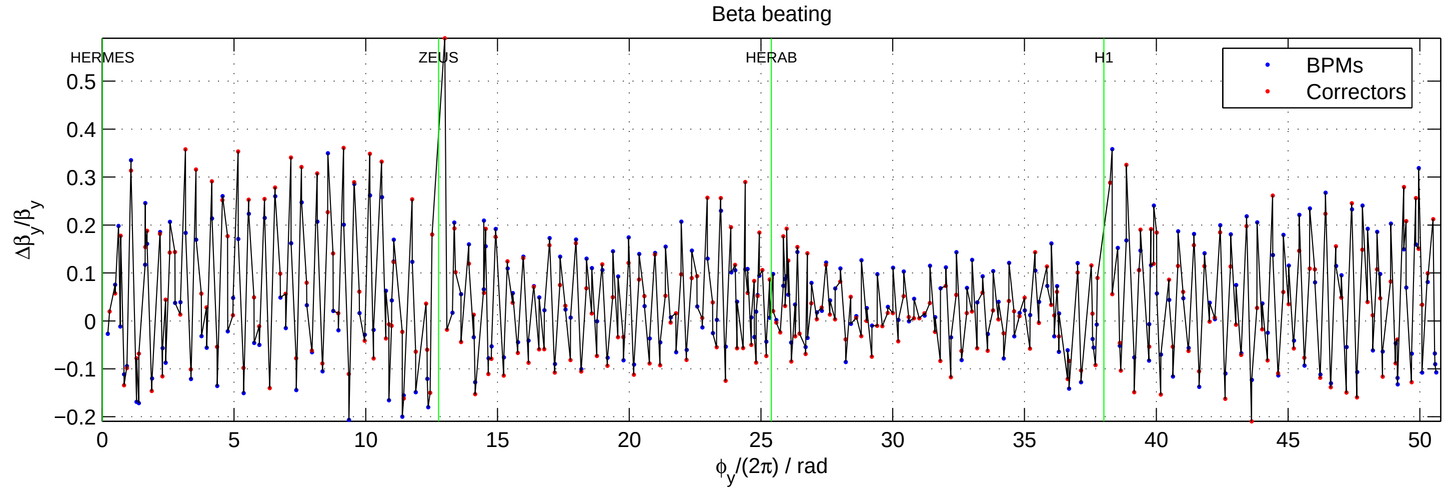
# Example: HERA-e, $x$-plane

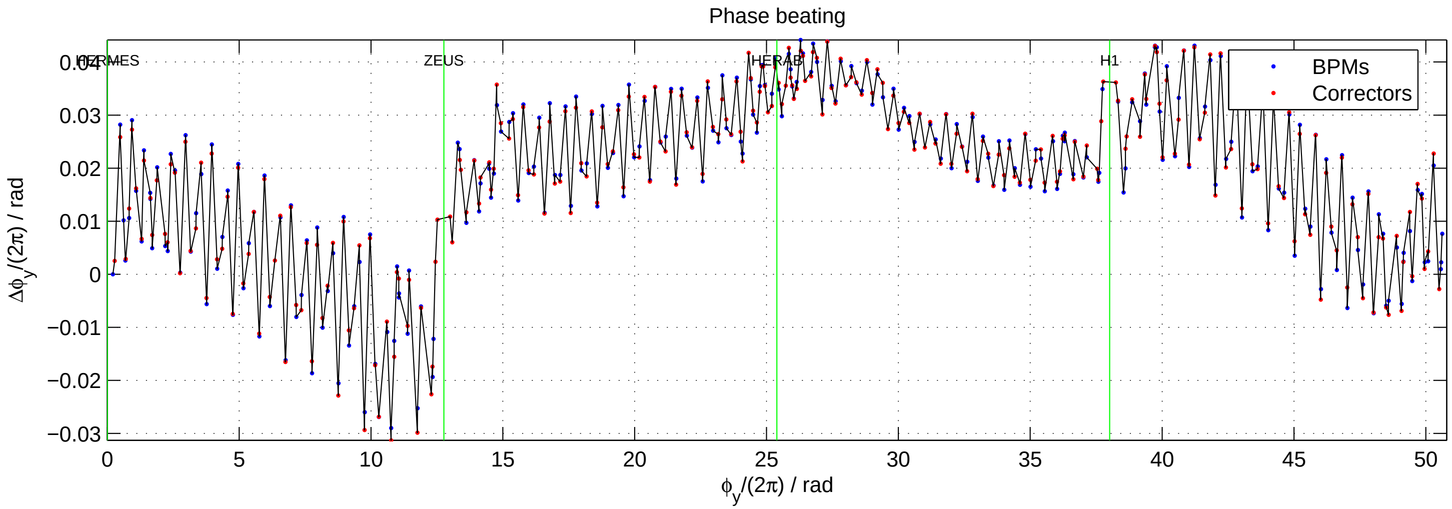
Before correction; ZEUS calorimeter closed; luminosity optics



# Example: Luminosity Optics HERA-e, $y$-plane

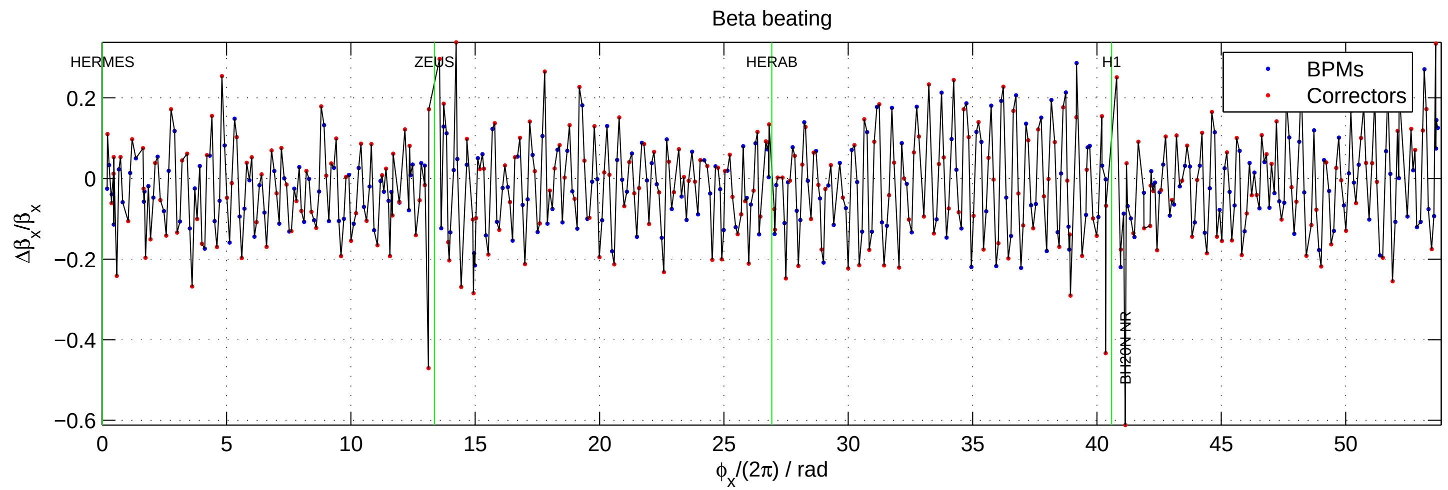
Before correction; ZEUS calorimeter closed; luminosity optics

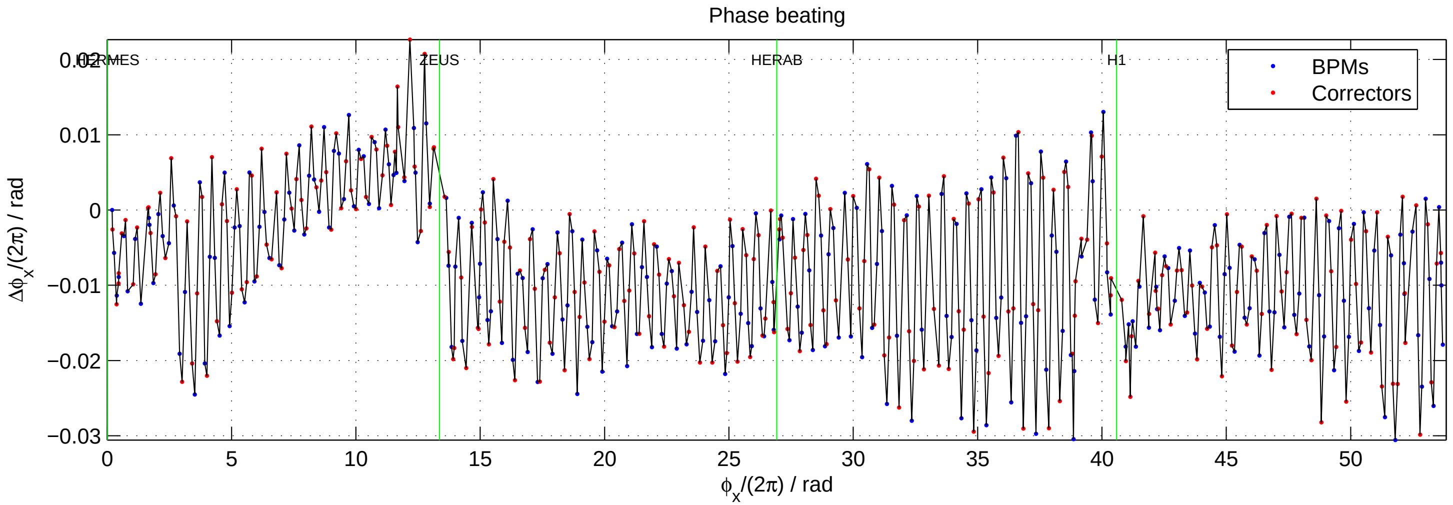


# Example: HERA-e, $x$-plane, corrected

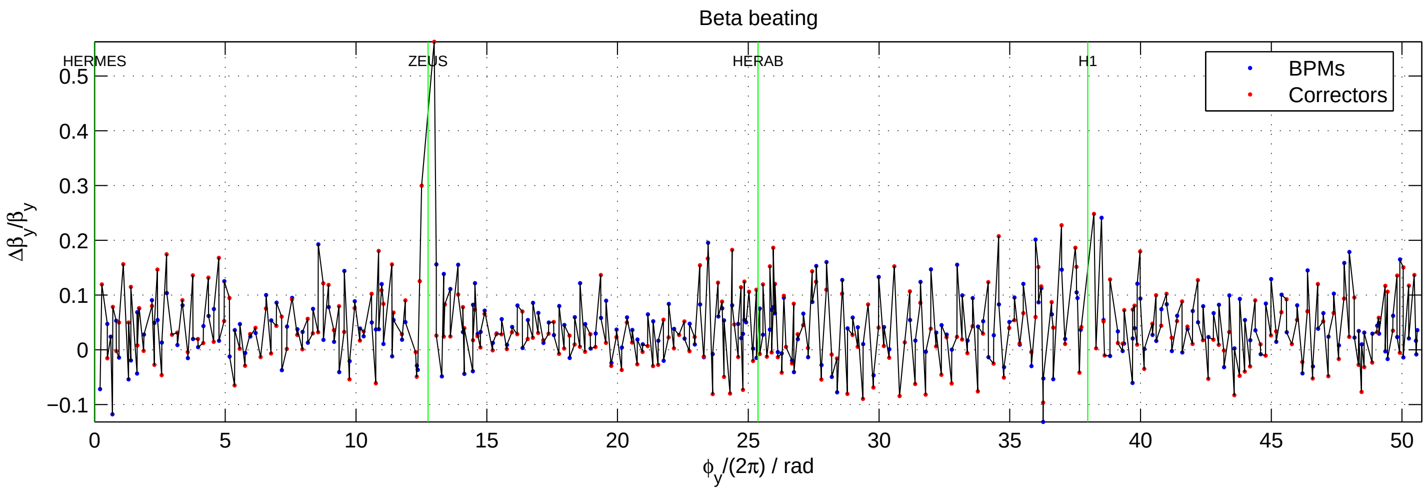
After correction with 10 quadrupoles ($\Delta k/k$ up to 4 %)

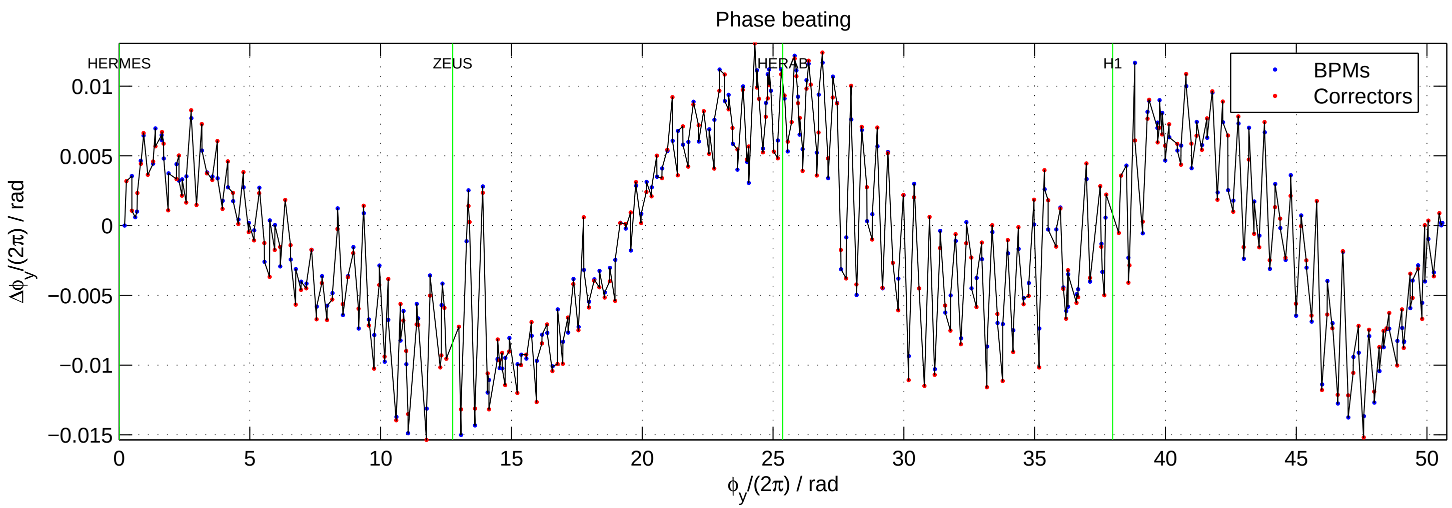


# Example: HERA-e $y$-plane, corrected

After correction with 10 quadrupoles ($\Delta k/k$ up to 4 %)

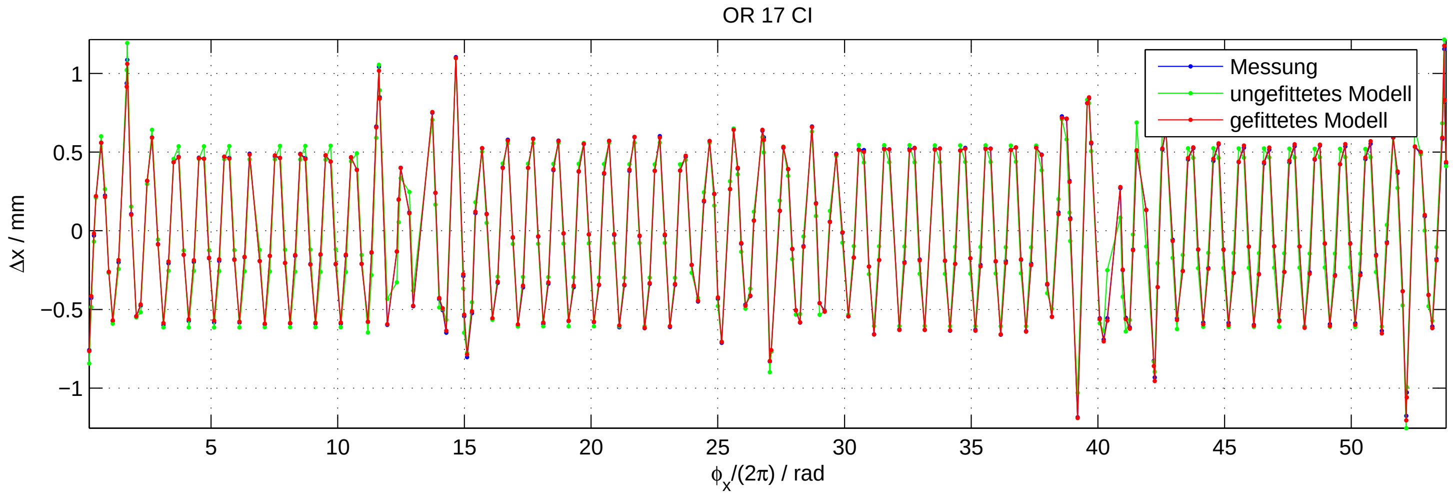


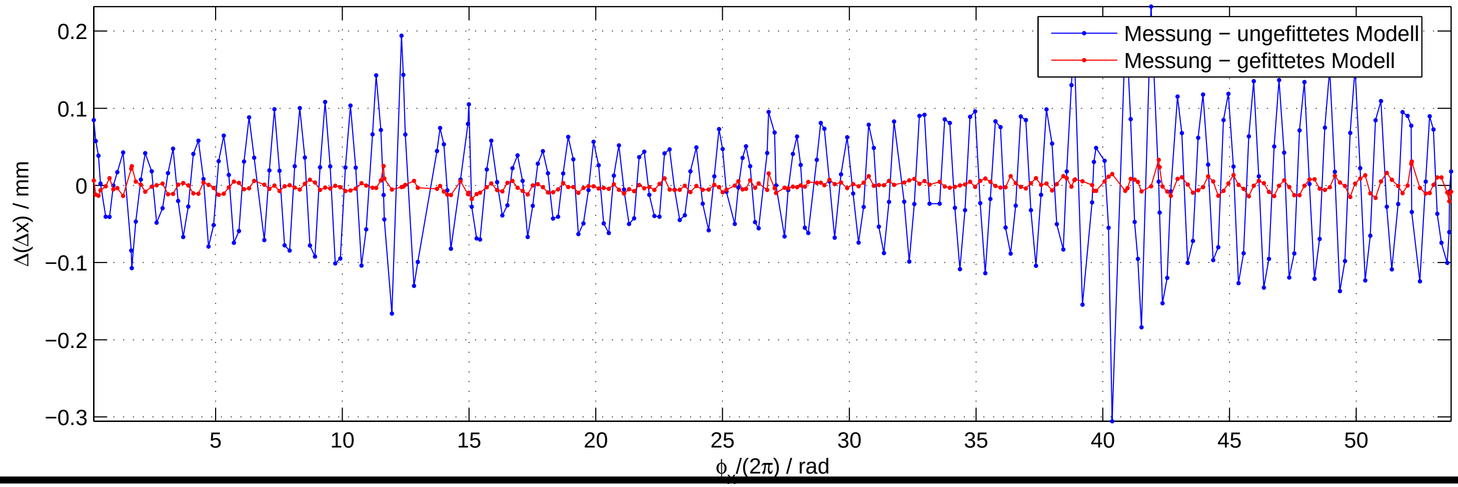



# Response-Matrix Analysis: Accuracy

**Top**: Difference orbits (Measurement, unfitted and fitted model) for corrector OR 17 CI

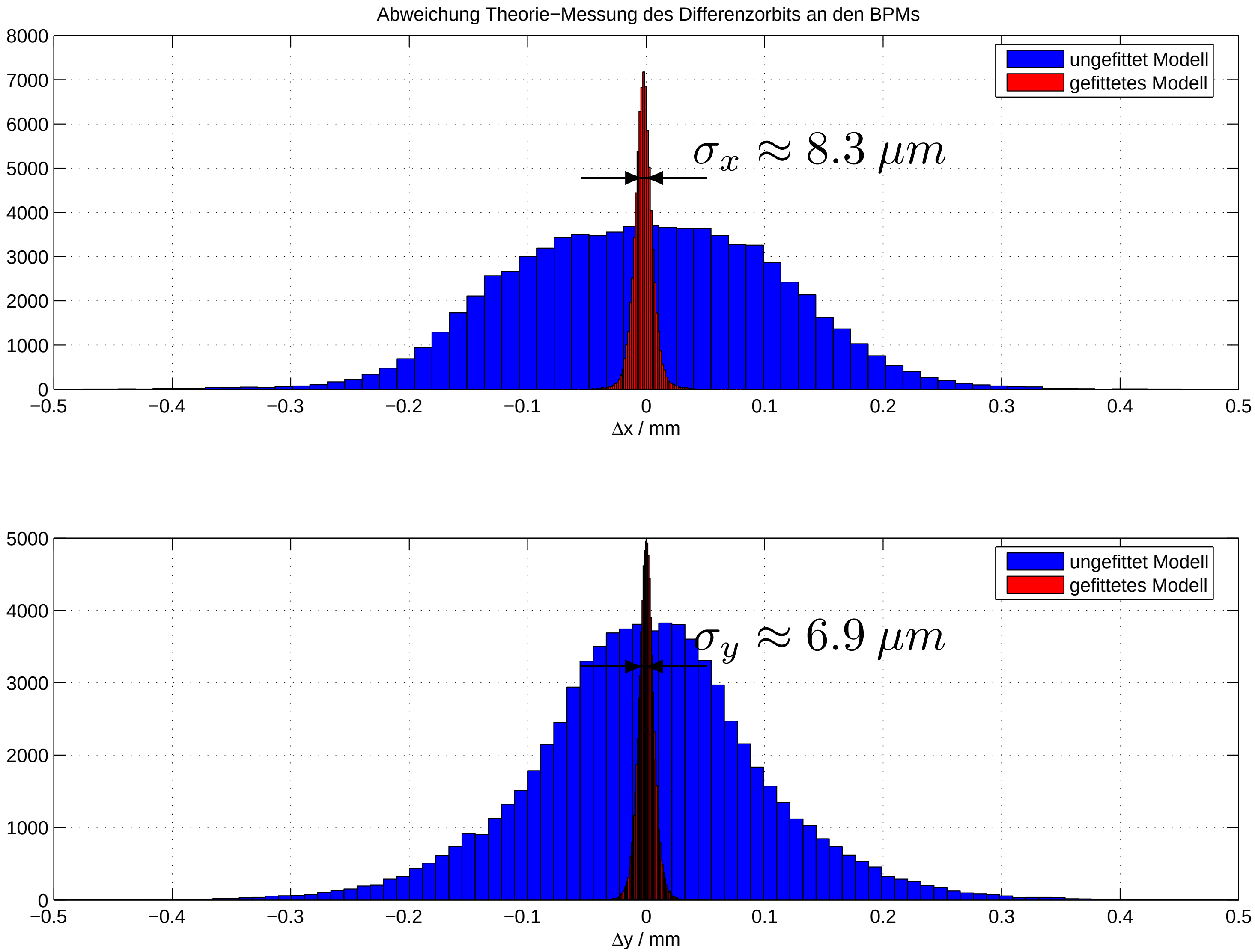
**Bottom**: Difference between measurement and model before and after fit



# Response-Matrix Analysis: Accuracy

Orbit difference at all BPMs before (blue) and after (red) fit (BPM resolution $\sigma \approx 7/4\ \mu$m):



# HERA: Bugs found with ORM

HERA-e:

- Wrong longitudinal position of corrector magnets (VO, VG) in lattice file
- 20 % magnetic field reduction for CV 27 corrector magnets
- Wrong longitudinal position of 8 BPMs in rotator section N & S
- Global scaling factor of BPM system (software bug)
- Many BPMs with wrong cabling or bad buttons signals
- Longitudinal permutation of three BPMs in HERA-e

HERA-p:

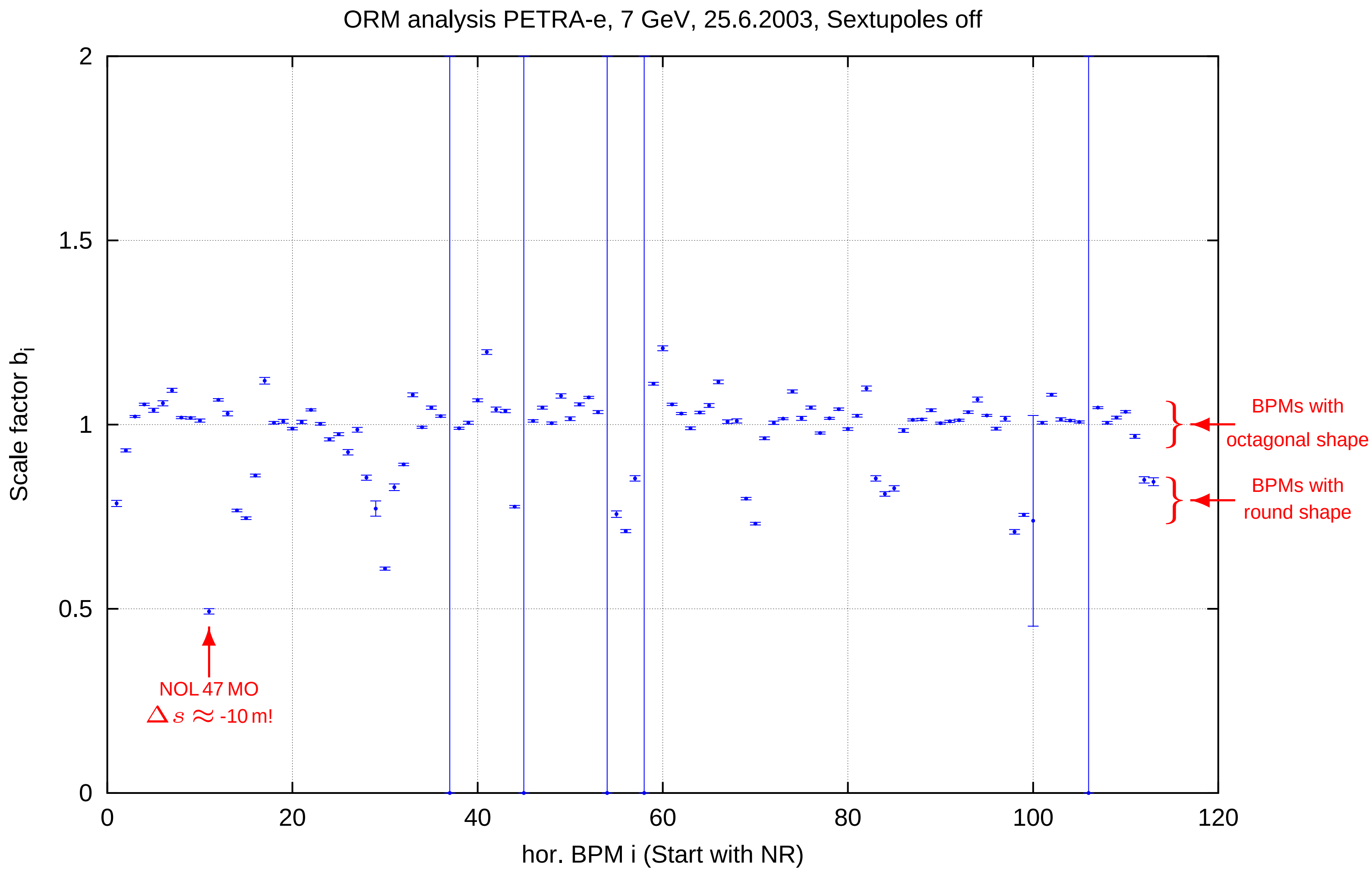
- Interchanged cables of s.c. quadrupoles QP33/35 NL
- Wrong length entry in magnet database for QP33/35 NL & QP33/35 SL
- Wrong calibration curve of IR quadrupole family GA/GB
- Wrong calibration curve of corrector magnet CZ 27
- Many bad BPMs

# Example: PETRA-e

- **BPMs:**
  - x & y-plane: 113 BPMs
  - In control system (2003): 3 different BPM types (octagonal shape, round chamber $\varnothing = 100$ mm and $\varnothing = 120$ mm)
  - But: 8 different BPM types installed in PETRA!
  - POISSON: $K_x$ of BPMs with octagonal shape 20 % too small (50 % of all BPMs)!
- **Corrector magnets:**
  - x-plane: 118 correctors
    - 23 CH (separate)
    - 83 CB, 6 C4, 6 C5 (backleg winding)
  - y-plane: 111 CV (separate)
- **Quadrupoles:**
  - 23 independent quadrupole families

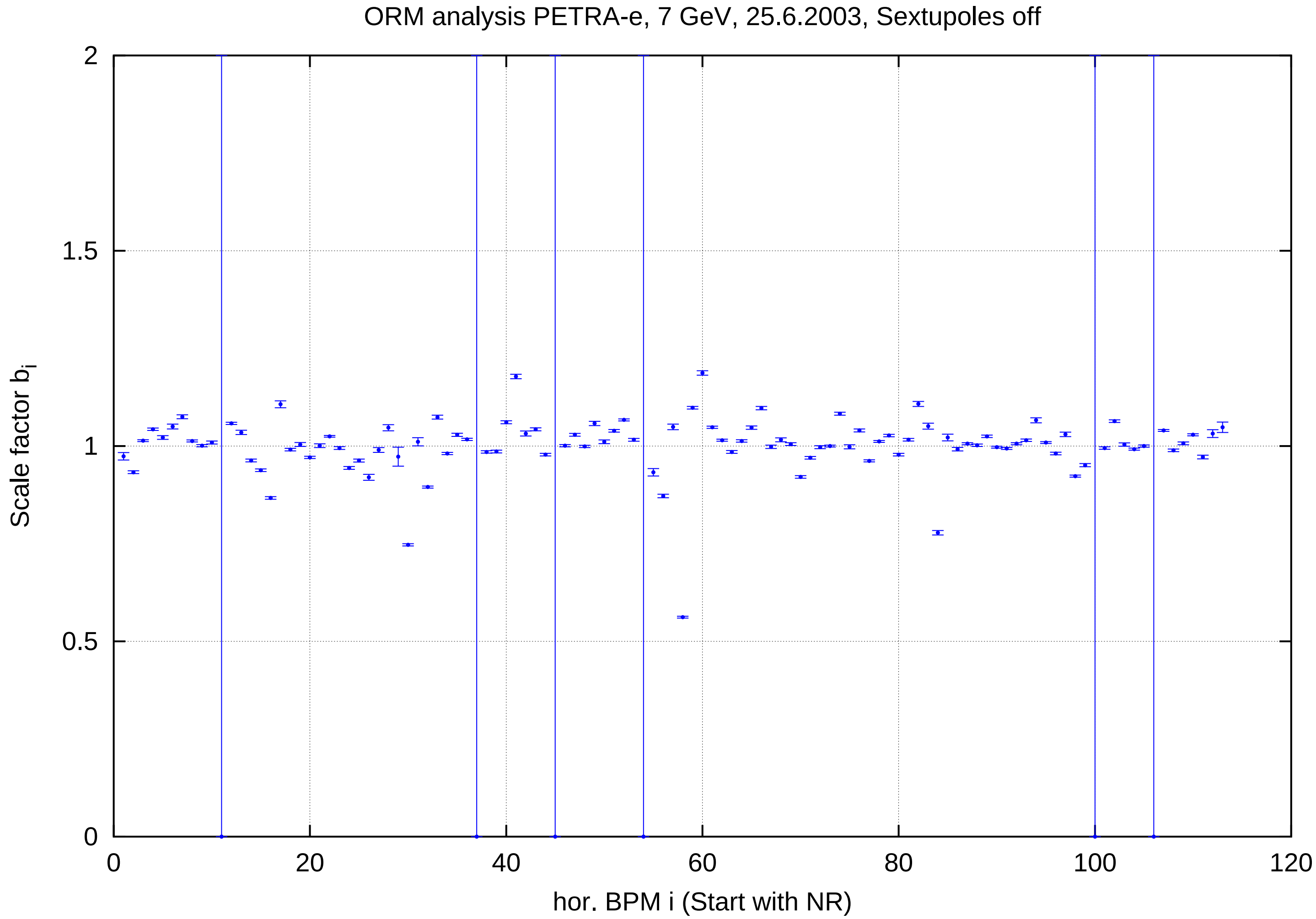
# PETRA: BPM Scaling Factors

Before correction of monitor constants of BPMs with octagonal shape



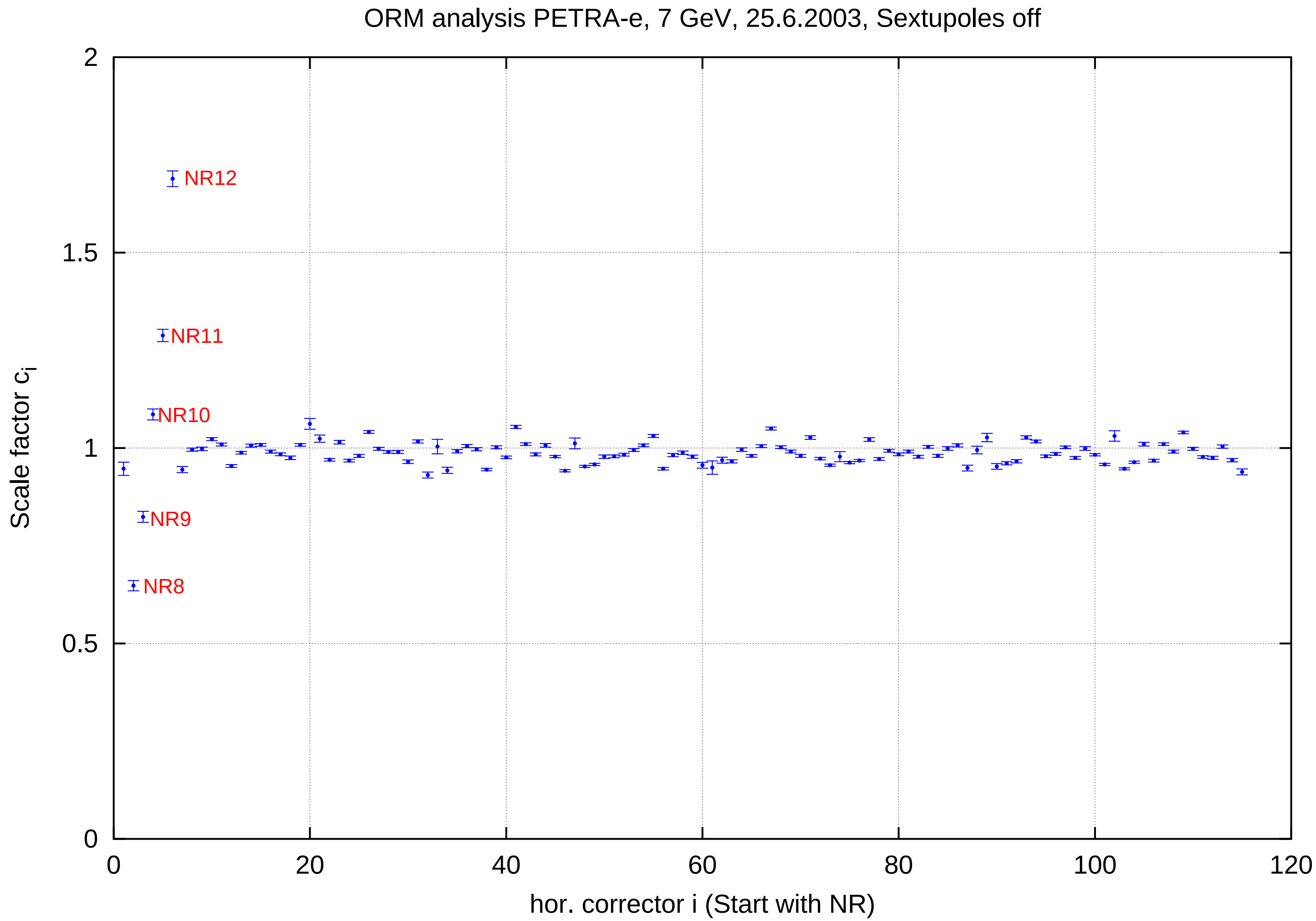
# PETRA: BPM Scaling Factors

After correction of monitor constants of BPMs with octagonal shape



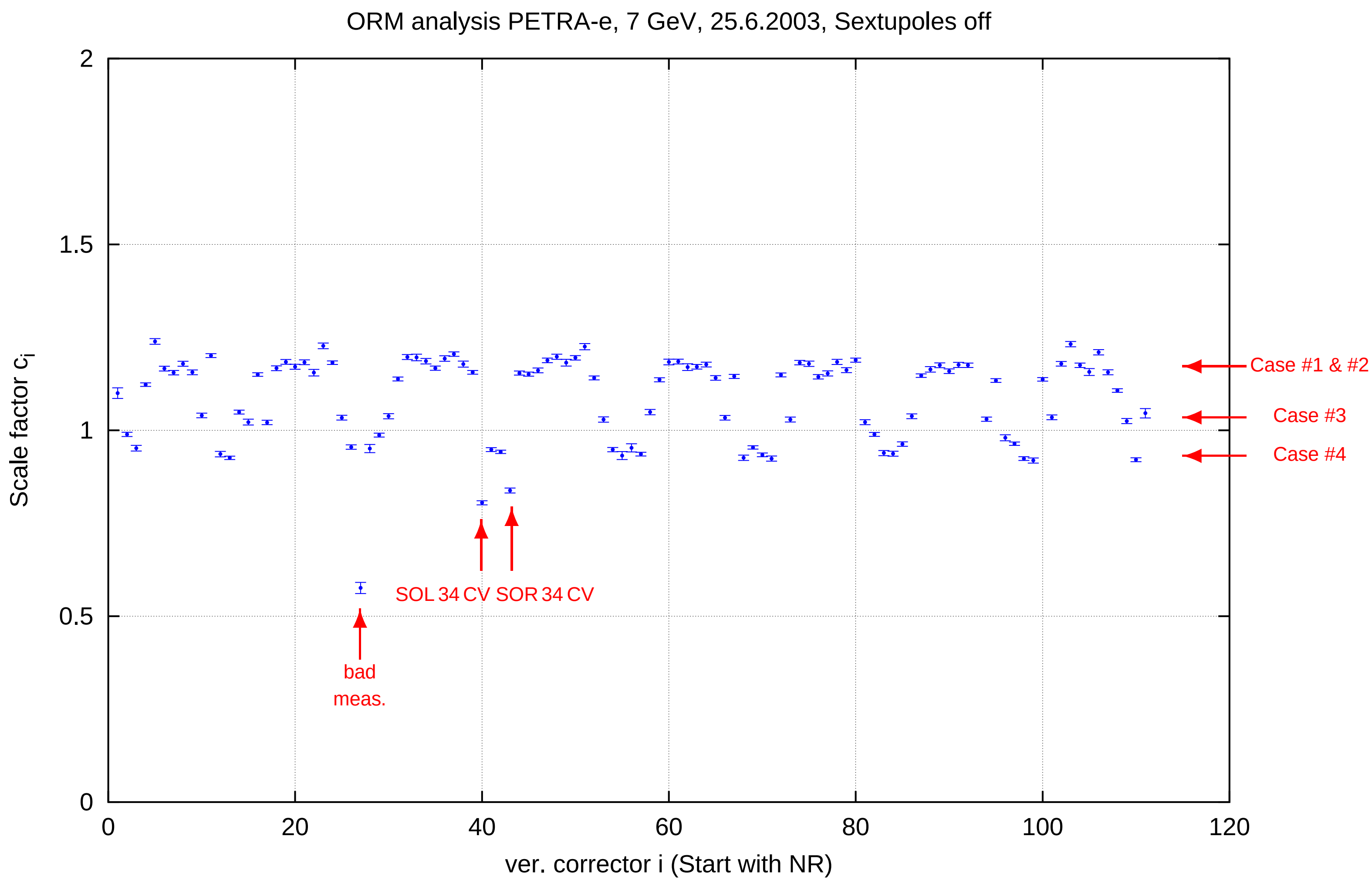
# PETRA: Horizontal Correctors

Four correctors longitudinally permutated: NR 8 CH $\leftrightarrow$ NR 12 CH, NR 9 CH $\leftrightarrow$ NR 11 CH



# PETRA: Vertical Correctors

Found four groups of corrector scaling factors. In addition two correctors with increased field near DESY II and DESY III beam line (SOL/SOR34 CV) were found.



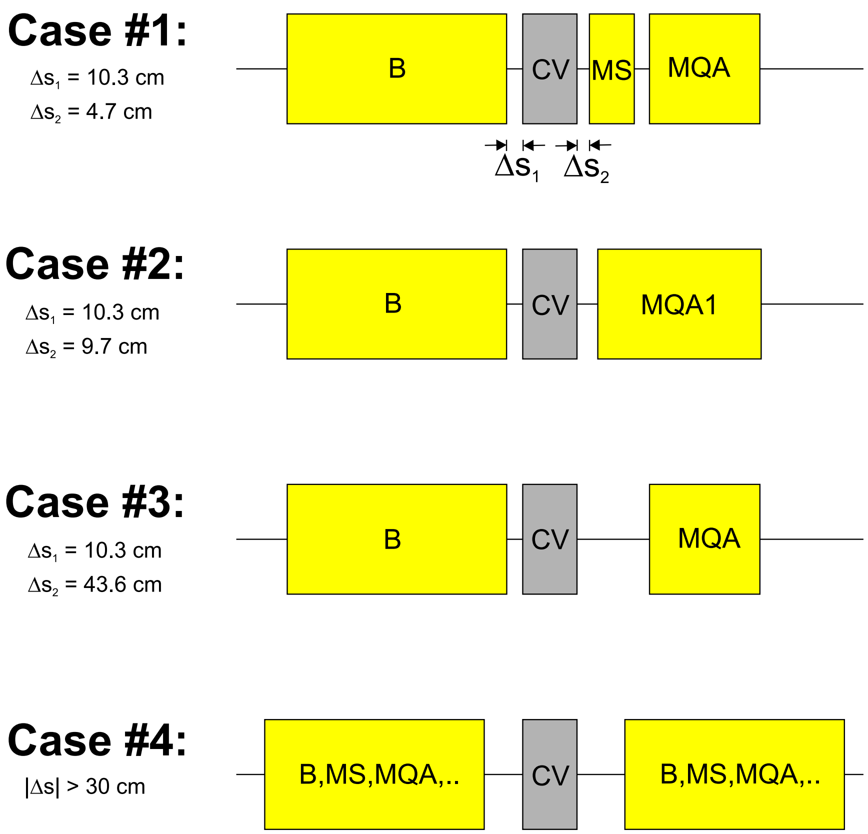
# PETRA: Vertical Correctors

- Vertical correctors (CV) near quadrupoles, sextupoles and dipoles

  $\longrightarrow$ Magnetic short-circuit between CV and adjacent magnet?
- Classification by distance between CV and nearby magnets
- B = bending magnet

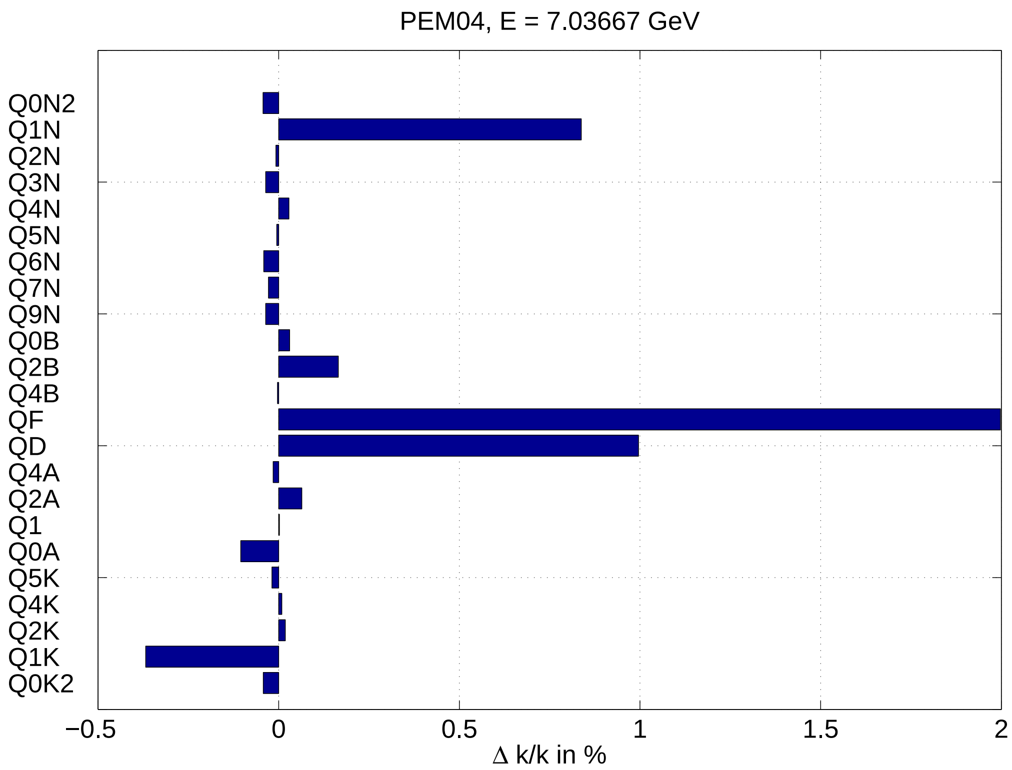
  MQA, MQA1 = quadrupoles

  MS = sextupole



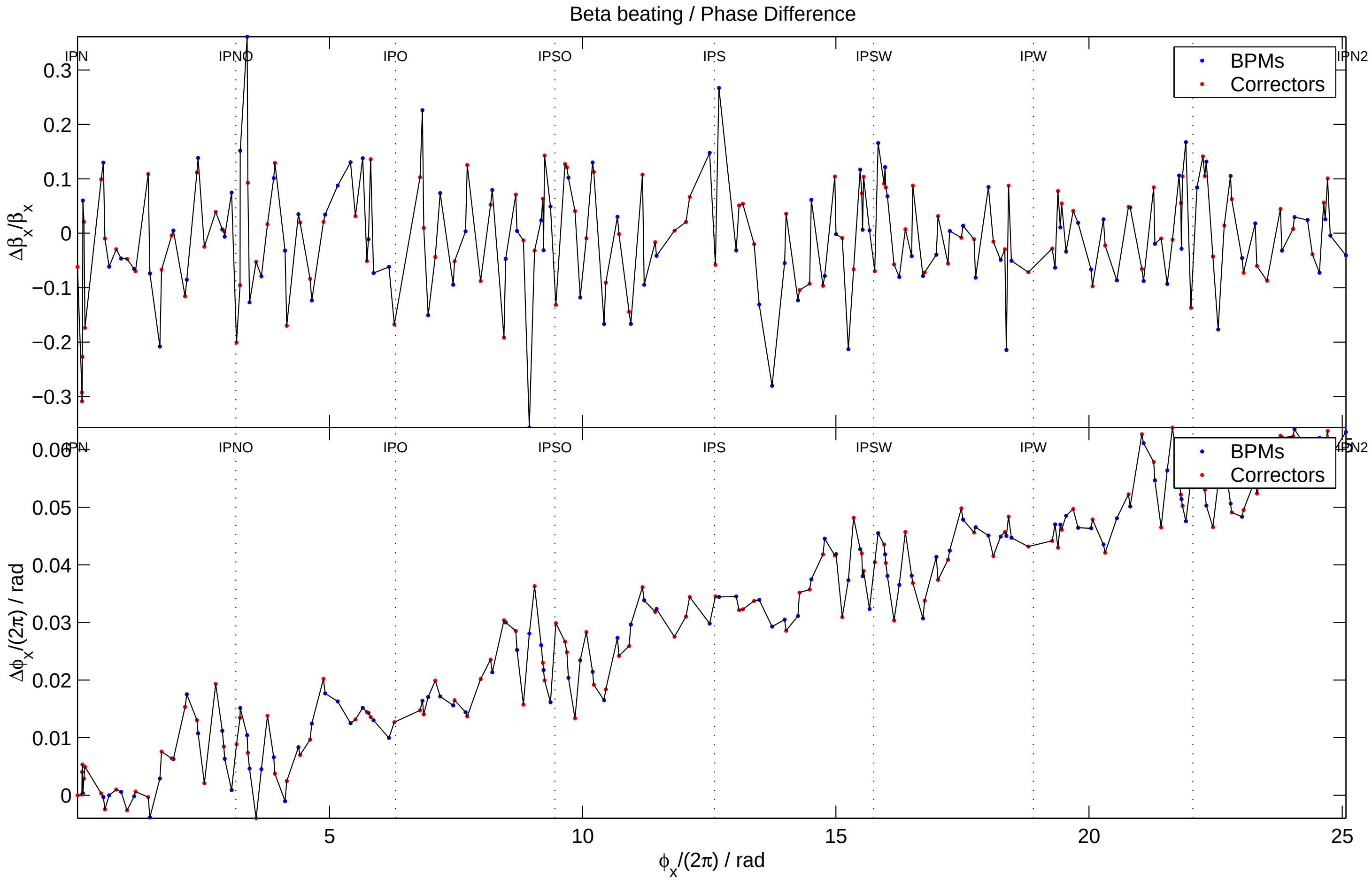
# PETRA-e: Gradient Errors

- Result of fitting the gradients of quadrupole families with CALIF (PEM04 optics, 7 GeV)
- Relative deviations of the $k$-values from theory (Q1 = doublet, Q4A = triplet A, Q4B = triplet B)
- Currents of quadrupole families had to be changed to achieve nominal tunes
- Explanation: calibration curves of quadrupoles are based on a different magnet cycling procedure; empirical corrections did correct this effect



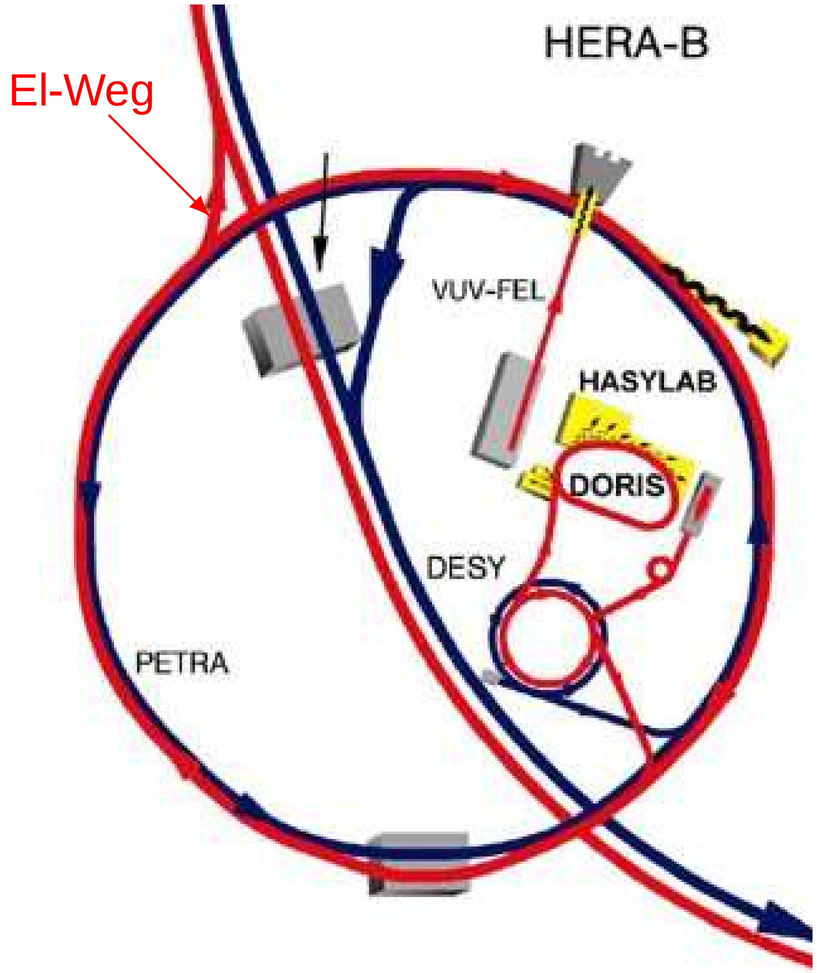
# PETRA-e: Beta & phase function

Beta function and phase function fit, PEM04 optic, 7 GeV

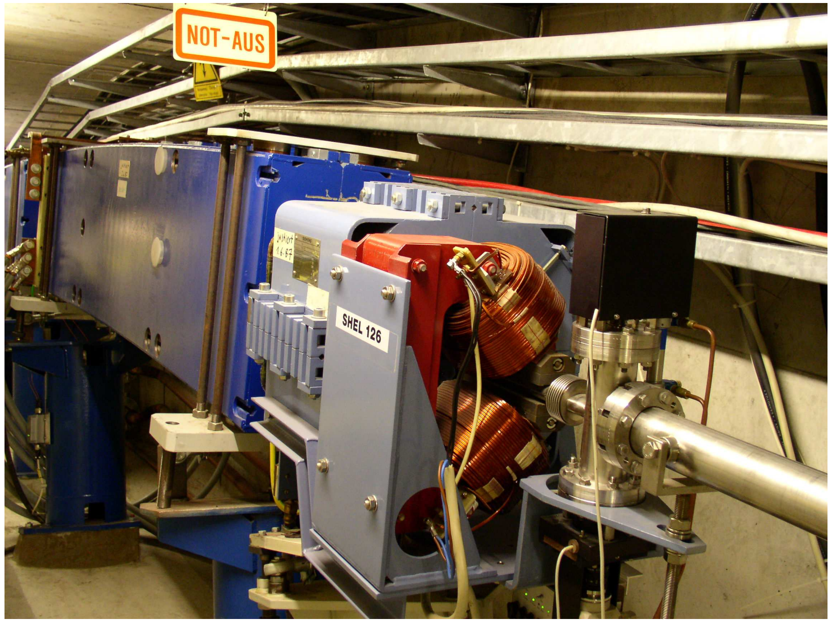


# Example: El-Weg

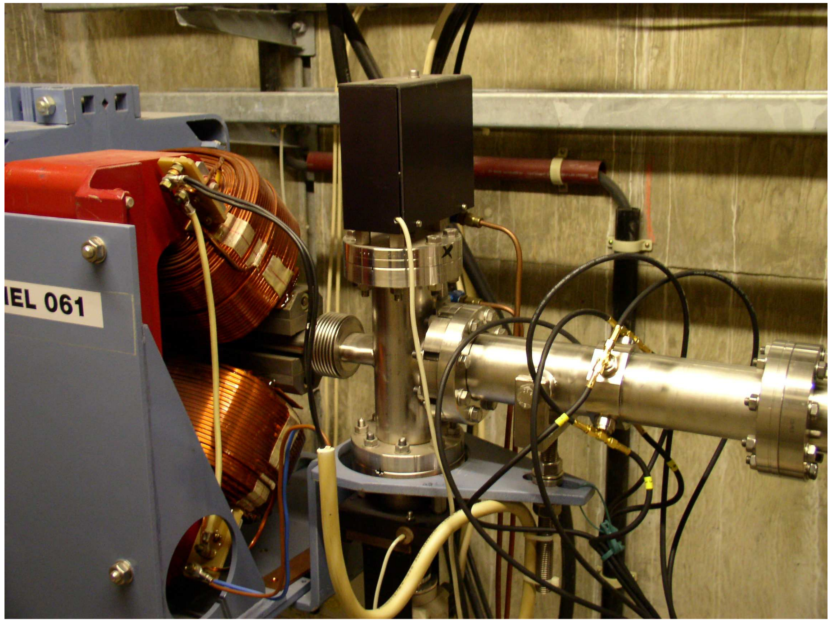
- El-Weg is e$^{\pm}$-transport line between PETRA and HERA-e
- PETRA and HERA are located on different levels and have different slopes
  - ➪ Coupled beamline
  - ➪ $x$- and $y$-bending
- Transfer efficiency in HERA II was sometimes $\ll 100\,\%$ and non-reproducible
- Summer 2004: six BPMs were installed
- Dec. 2004: Optic was checked by measuring a response matrix



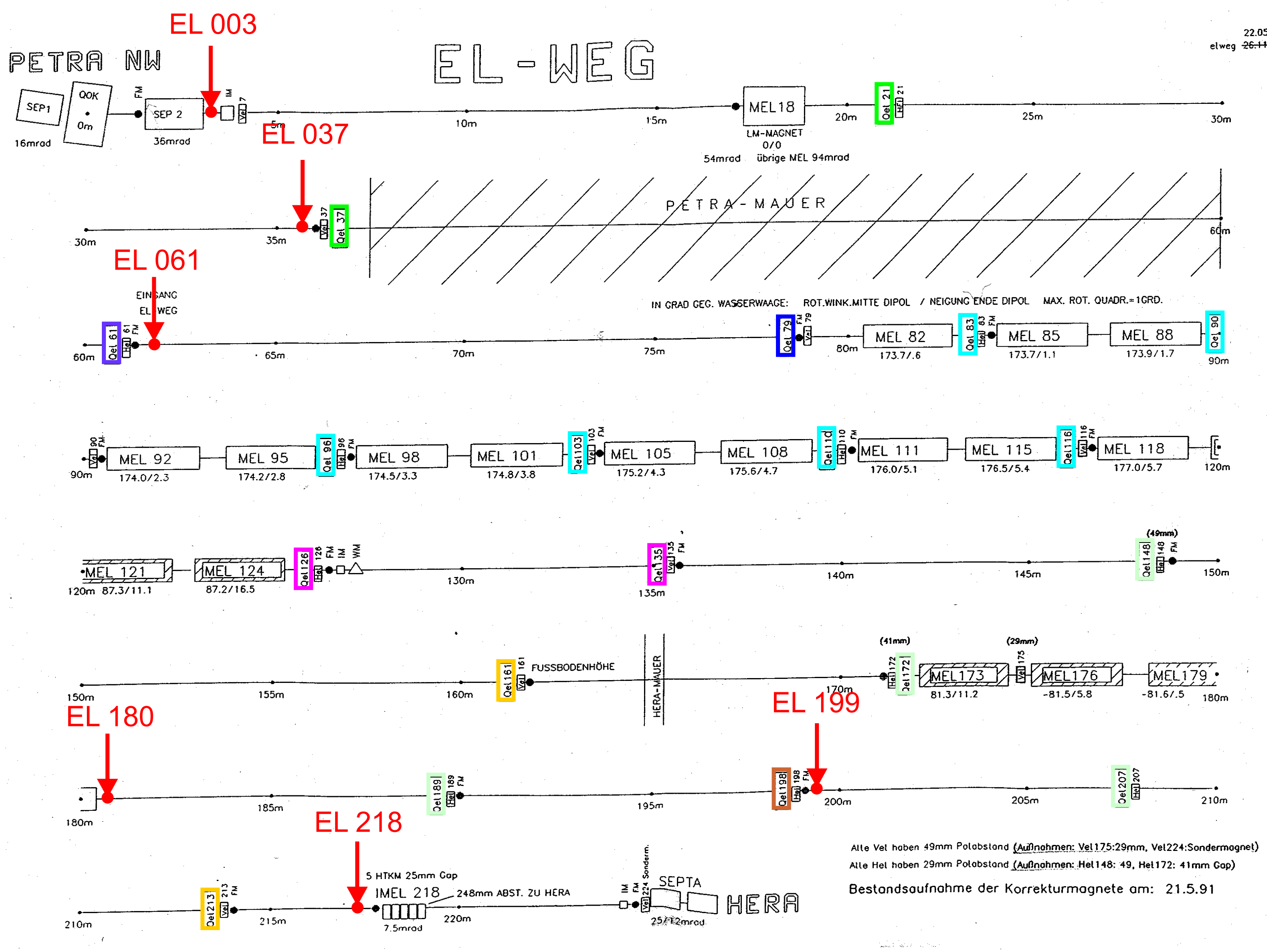
# El-Weg: BPMs, Correctors, Quadrupoles







# Layout of El-Weg

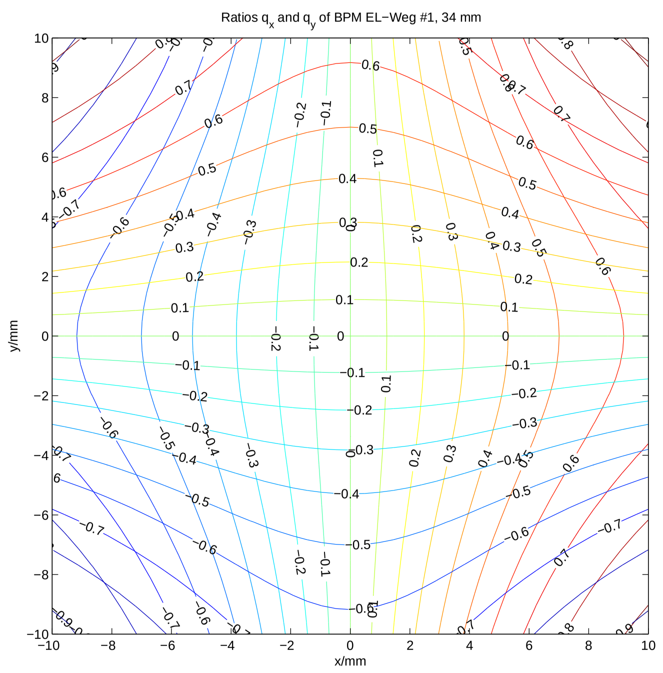


# El-Weg ORM Analysis

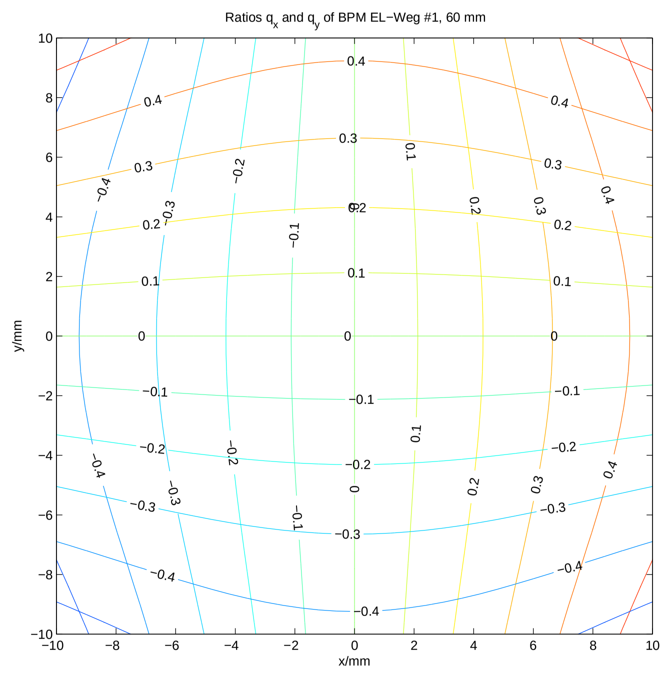
- Hardware components of the El-Weg:
  - BPMs: $N_x = 6$, $N_y = 6$
  - Correctors: $M_x = 11$, $M_y = 11$
  - Quadrupoles: $Q = 19$ in $8$ families
    - ➪ Matrix elements: $N_{x,\text{MAT}} = 36$, $N_{y,\text{MAT}} = 33$
    - ➪ BPM/corrector fit parameters: $N_{\text{FIT}} = 6 + 11 = 17$
- Bad : Number of fit parameters $\approx$ number of matrix elements
- Strategy: Use a precise BPM-model for position reconstruction, rely on correct calibration of correctors, fit only quadrupole families
- Result of ORM analysis: Quadrupole families are 2-4 % too strong

# El-Weg-BPM Calibration

Isolines of BPM EL 003 ($r = 17$ mm)




Isolines of BPM EL 037 ($r = 30$ mm)



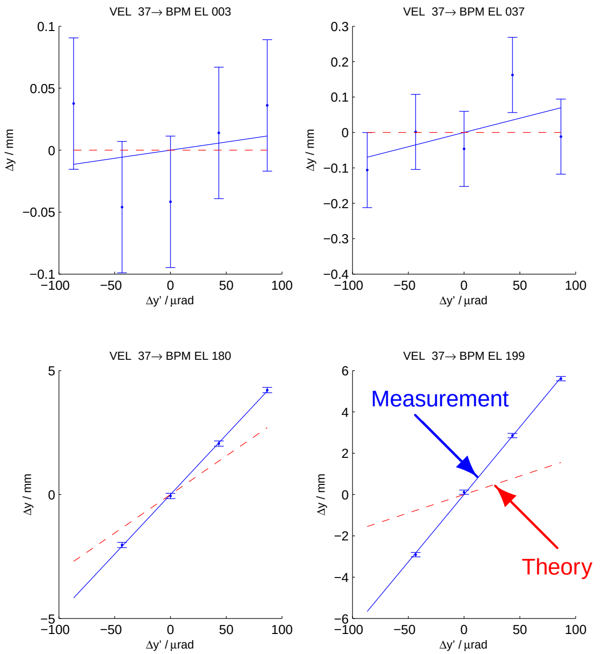

- $q = \Delta/\Sigma$ of the four BPM buttons were calculated as function of $(x,y)$ on grid
- Positions are the solution of the non-linear equations using interpolated $q$:

$$\begin{cases} q_x^{\text{theo}}(x,y) = q_x^{\text{meas}} \\ q_y^{\text{theo}}(x,y) = q_y^{\text{meas}} \end{cases}$$

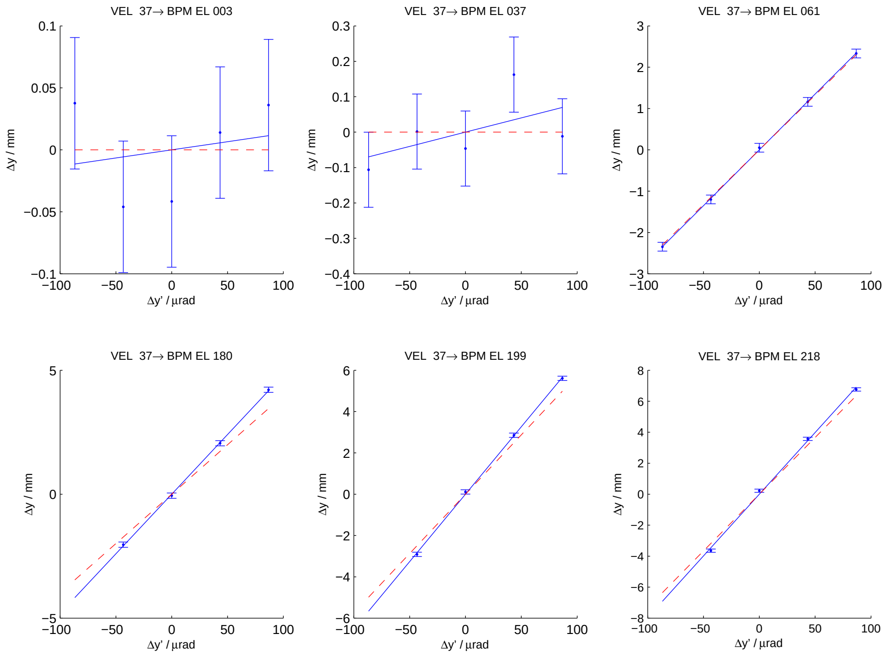
# El-Weg: $R_{12}$-Measurement

Example: Trajectory change due to the kick angle of vertical corrector VEL 37 and theoretical prediction with old and new optics model

Old optics model



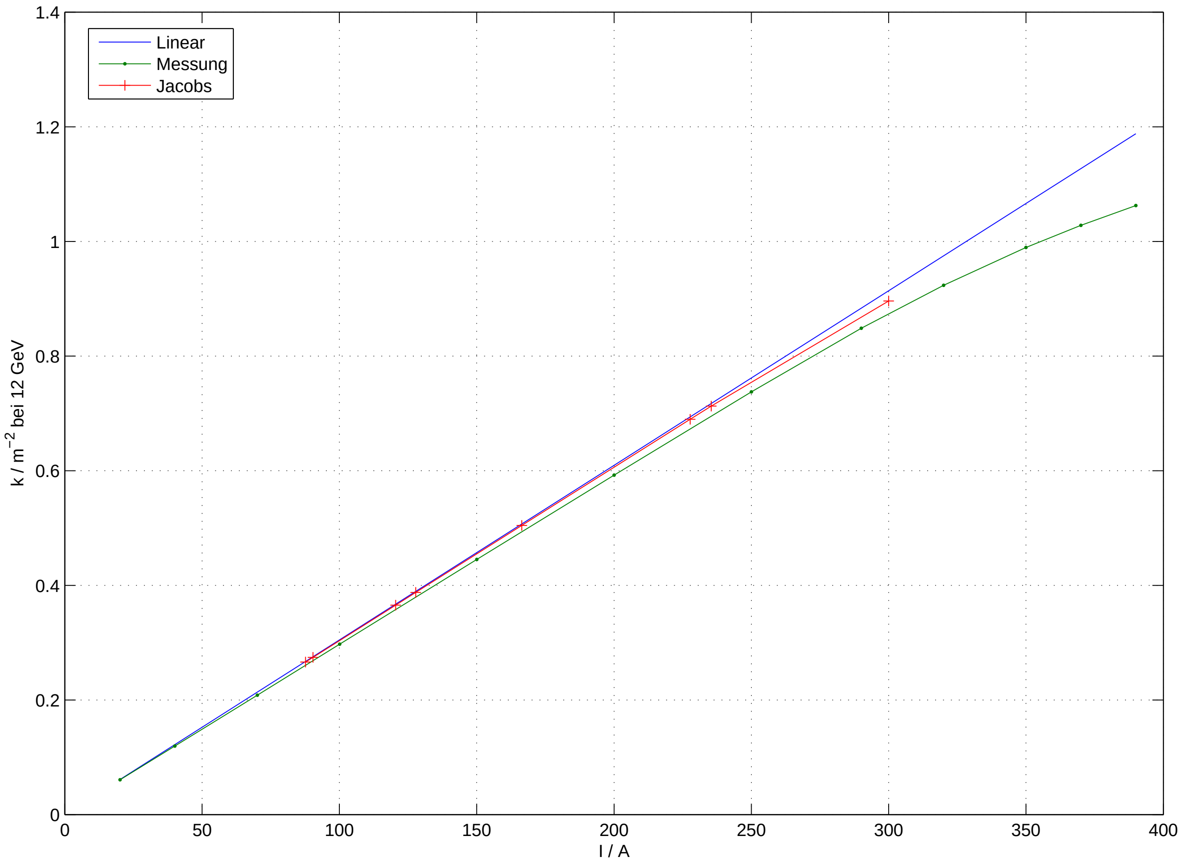

New optics model



# El-Weg: Result of Analysis

Measured QEL-quadrupole field was different from calibration curve assumed!

From optics calculations: $\Delta\beta/\beta \approx 100$ % in second part of beamline



# Summary

- Response matrix analysis is a valuable tool to understand and debug the accelerator
- Fit of the response matrix allows to find out gradient errors, calibration errors of BPMs and calibration errors of corrector magnets
- But: not everything can be fitted; it depends on the number of BPMs/correctors, the kick amplitude and the resolution of the BPMs
- Comprehensive analysis of data can give information about faulty hardware
- Many errors found in HERA-e, HERA-p, PETRA and El-Weg
- Method also useful for VUV-FEL and XFEL?