a numerical investigation:
emittance growth by rf coupler kicks

Philippe Piot's results (PAC 2005)

mechanisms for emittance growth

model for one coupler (= round beam + discrete kick)

ASTRA calculation for ACC1

mechanisms for emittance growth – 2nd

ASTRA calculation for ACC2

conclusions, questions

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remarks:

XFEL case
ideal operation conditions for ACC1 (on crest, steady state, no reflected power)
coupler fields from http://adweb.desy.de/~mpymax/mafia/HOM_Coupler/index.html

STEERING AND FOCUSING EFFECTS IN TESLA CAVITY DUE TO HIGH ORDER MODE AND INPUT COUPLERS

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IMPACT ON THE XFEL PHOTOINJECTOR

We present a practical example of the coupler-effects on the beam dynamics by considering the x-ray FEL photoinjector [5]. In the injector, the electron beam is photoemitted in an L-band (f = 1.3 GHz) rf-gun and directly injected into a standard TESLA-type accelerating module composed of eight cavities and accelerated to ∼190 MeV. The first two cavities of the accelerating module are operated with $\hat{E}_z$ = 23 MV/m and the remaining six cavities with $\hat{E}_z$ = 50 MV/m. Figure 7 compares the beam parameter evolutions along the injector beamline from the photocathode surface (z = 0) up to the end of the first accelerating module. The field disturbances caused by the coupler result in a 16% and 1% relative emittance growth respectively in the horizontal and vertical plane. The induced steering results in beam displacements at the accelerating module exit of $|\delta x|$ =680 μm and $|\delta y|$ =170 μm, and no significant $x - y$ coupling is observed.

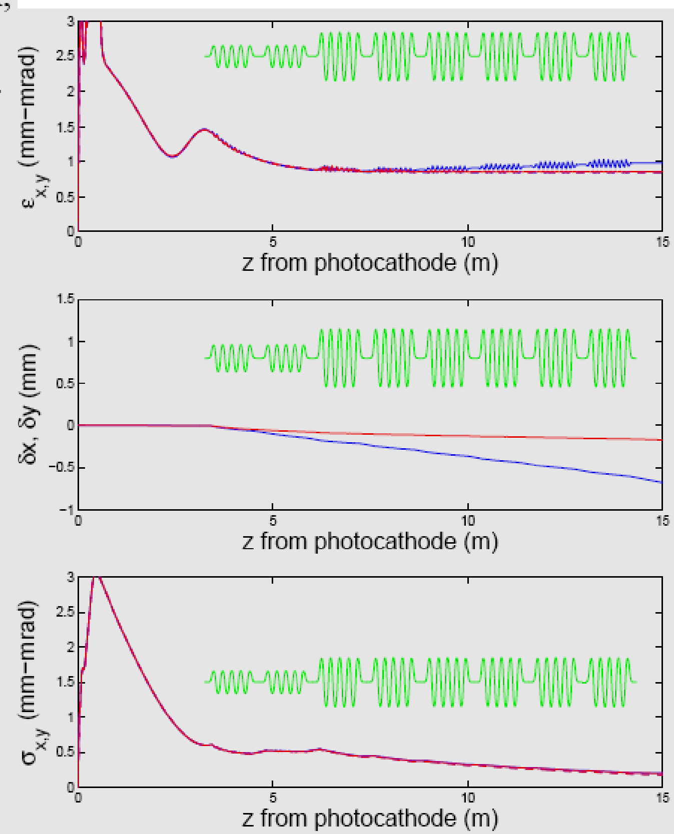

Figure 7: Simulated normalized transverse (top plot) beam position (middle plot) and beam rms sizes (bottom plot) evolution along the x-ray FEL photoinjector with (solid line) and without (dashed line) including the coupler effects. red: vertical plane; blue: horizontal plane. The linac phase is chosen for maximum energy gain. The green curves represents the axial electric field in the cavity in arbitrary units.

mechanisms for emittance growth

projected emittance:

- different centroid shift for different slices
- different optics for different slices
- mechanisms for growth of slice emittance

slice emittance:

- x-y coupling
- (non linearity)

slice emittance, x-y coupling

correlation matrix of
round non-magnetized beam:
(no angular momentum)

$$S = \varepsilon \begin{bmatrix} \beta & -\alpha & 0 & 0 \\ -\alpha & \gamma & 0 & 0 \\ 0 & 0 & \beta & -\alpha \\ 0 & 0 & -\alpha & \gamma \end{bmatrix}$$

with $\gamma = \dfrac{1+\alpha^2}{\beta}$

$$\rightarrow \det\left(\frac{1}{\varepsilon} S\right) = 1$$

$$\psi(\mathbf{x}) = \frac{1}{(2\pi\varepsilon)^2} \exp\left(-\frac{1}{2}\mathbf{x}^t \mathbf{S}^{-1}\mathbf{x}\right)$$

horizontal & vertical emittance:

$$\varepsilon_x(S) = \sqrt{\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}} \rightarrow \varepsilon_x = \varepsilon \qquad \varepsilon_y(S) = \sqrt{\begin{bmatrix} s_{33} & s_{34} \\ s_{34} & s_{44} \end{bmatrix}} \rightarrow \varepsilon_y = \varepsilon$$

symplectic kick matrix:
(discrete kick)

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ v & 1 & u & 0 \\ 0 & 0 & 1 & 0 \\ u & 0 & w & 1 \end{bmatrix}$$

emittance after kick:

$$\varepsilon_x\left(TST^t\right) = \varepsilon_y\left(TST^t\right) = \varepsilon\sqrt{1+(u\beta)^2}$$

(discrete) model for one coupler

coupler fields from http://adweb.desy.de/~mpymax/mafia/HOM_Coupler/index.html

$$\to V_x \mathbf{e}_x + V_y \mathbf{e}_y + V_z \mathbf{e}_z = \int \left(\mathbf{E}^{(\mathrm{mode})} + \mathbf{v} \times \mathbf{B}^{(\mathrm{mode})} \right) \exp\left(i \frac{\omega}{c} z \right) dz$$

kick matrix (on crest): $T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ v & 1 & u & 0 \\ 0 & 0 & 1 & 0 \\ u & 0 & w & 1 \end{bmatrix}$

$$u = \frac{V^{(\mathrm{cavity})}}{V_{\parallel}^{(\mathrm{beam})}} \mathrm{Re}\left\{ \frac{\partial V_x / \partial y}{V_z} \right\} = \frac{V^{(\mathrm{cavity})}}{V_{\parallel}^{(\mathrm{beam})}} \mathrm{Re}\left\{ \frac{\partial V_y / \partial x}{V_z} \right\}$$

$$v = \frac{V^{(\mathrm{cavity})}}{V_{\parallel}^{(\mathrm{beam})}} \mathrm{Re}\left\{ \frac{\partial V_x / \partial x}{V_z} \right\} \qquad w = \frac{V^{(\mathrm{cavity})}}{V_{\parallel}^{(\mathrm{beam})}} \mathrm{Re}\left\{ \frac{\partial V_y / \partial y}{V_z} \right\}$$

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typical numbers for 1st coupler (=upstream, cavity 1)

$$\left. \begin{matrix} V_{\parallel}^{(\mathrm{beam})} \approx 6.8\,\mathrm{MV} \\ V_{\parallel}^{(\mathrm{cavity})} \approx 12\,\mathrm{MV} \end{matrix} \right\} \to \mathrm{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1.89 \times 10^{-3} & 1 & 5.928 \times 10^{-3} & 0 \\ 0 & 0 & 1 & 0 \\ 5.928 \times 10^{-3} & 0 & -1.741 \times 10^{-3} & 1 \end{pmatrix}$$

unit of off-diagonal elements: 1/m

$$\beta \approx 3\,\mathrm{m}$$

relative emittance growth:

$$\sqrt{1 + (u\beta)^2} \approx 1.00017$$

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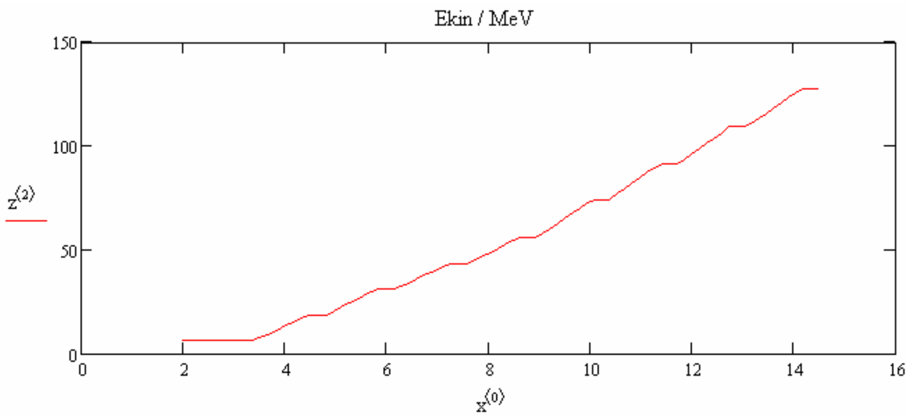
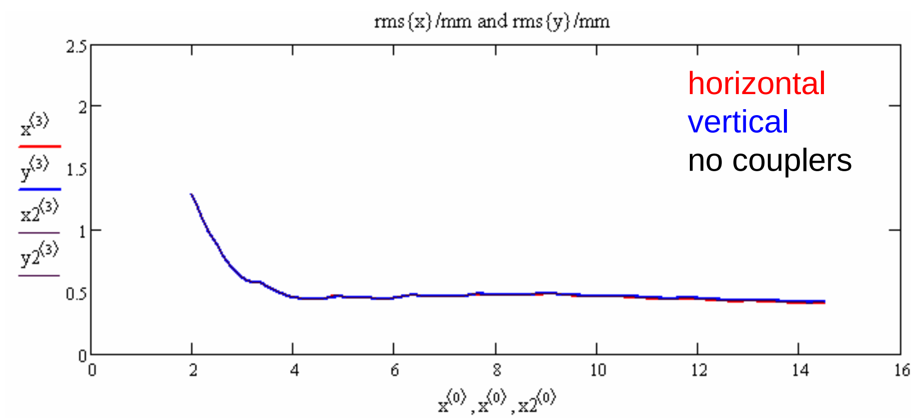
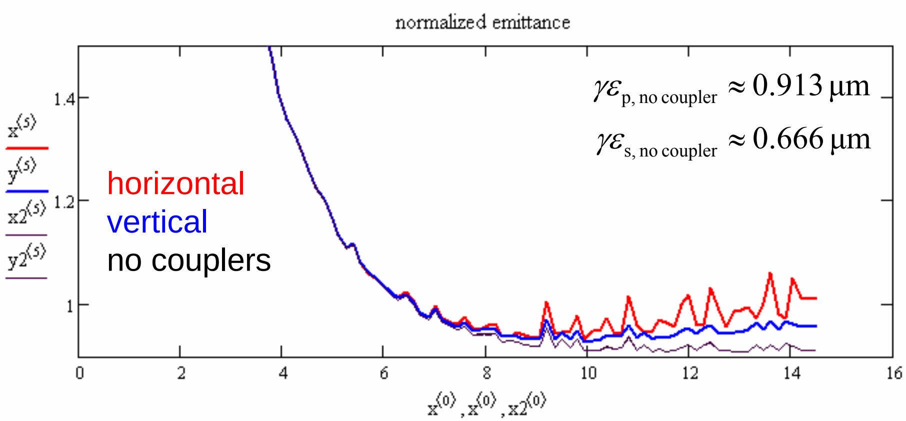
upstream coupler, cavity 5:

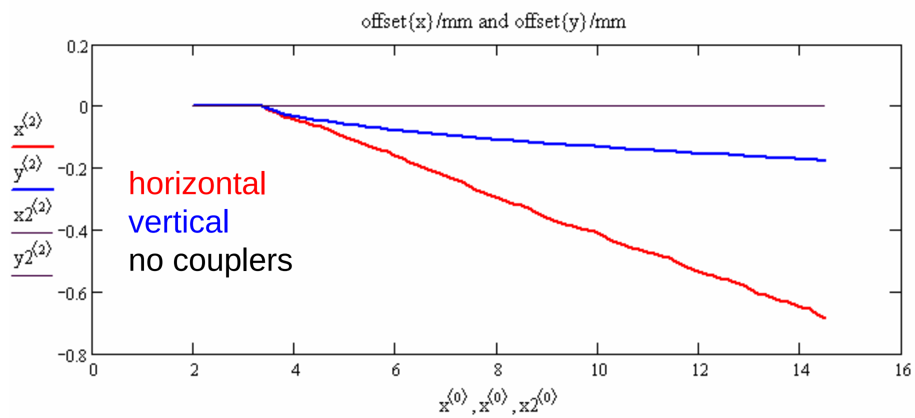
$$\left. \begin{matrix} V_{\parallel}^{(\mathrm{beam})} \approx 62\,\mathrm{MV} \\ V_{\parallel}^{(\mathrm{cavity})} \approx 18\,\mathrm{MV} \end{matrix} \right\} \to u \approx 0.001/\mathrm{m} \qquad \beta \approx 30\,\mathrm{m} \qquad \sqrt{1 + (u\beta)^2} \approx 1.00045$$

ASTRA calculation for ACC1









projected emittance

$$\frac{\varepsilon_{\text{p,x}}}{\varepsilon_{\text{p, no coupler}}} \approx 1.11 \qquad \frac{\varepsilon_{\text{p,y}}}{\varepsilon_{\text{p, no coupler}}} \approx 1.051$$

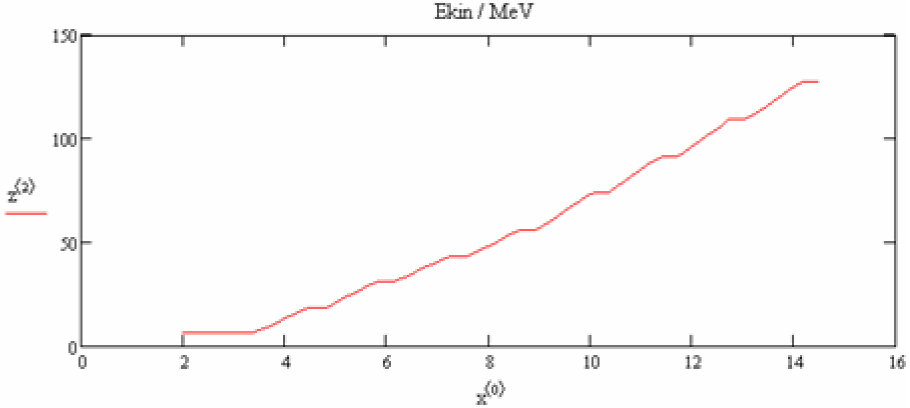
slice emittance
(center 10% charge)

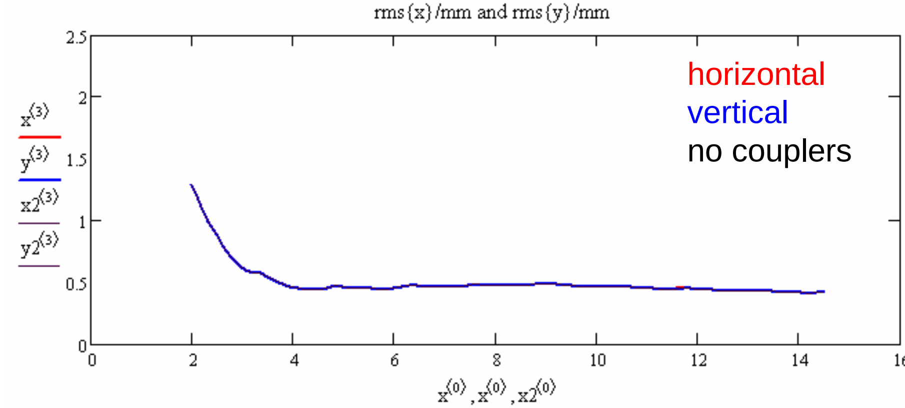
$$\frac{\varepsilon_{\text{s,x}}}{\varepsilon_{\text{s, no coupler}}} \approx 1.028 \qquad \frac{\varepsilon_{\text{s,y}}}{\varepsilon_{\text{s, no coupler}}} \approx 1.021$$

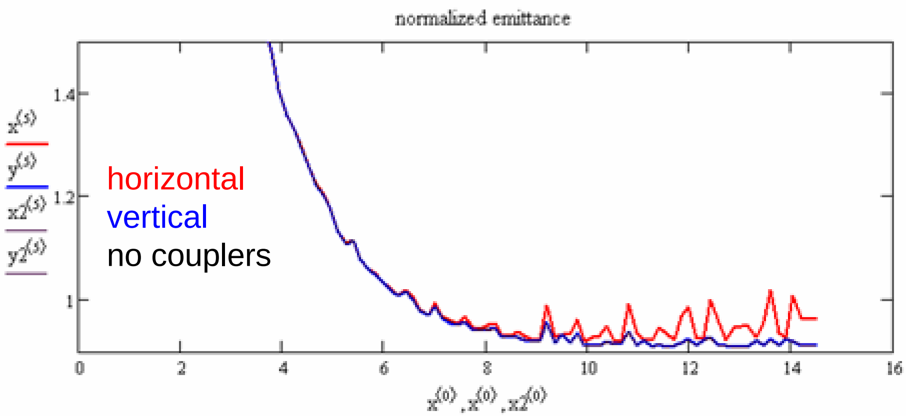
mechanisms for emittance growth – 2nd

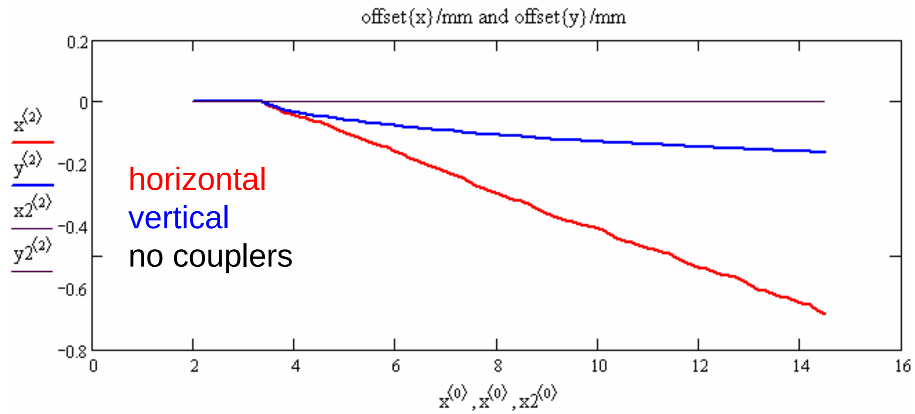
offset independent fields

$$\mathbf{E}(x,y,z) := \mathbf{E}(0,0,z) \qquad \mathbf{B}(x,y,z) := \mathbf{B}(0,0,z)$$









projected emittance

$$\frac{\varepsilon_{\mathrm{p,x}}}{\varepsilon_{\mathrm{p,\,no\,coupler}}} \approx 1.055 \qquad \frac{\varepsilon_{\mathrm{p,y}}}{\varepsilon_{\mathrm{p,\,no\,coupler}}} \approx 1.00$$

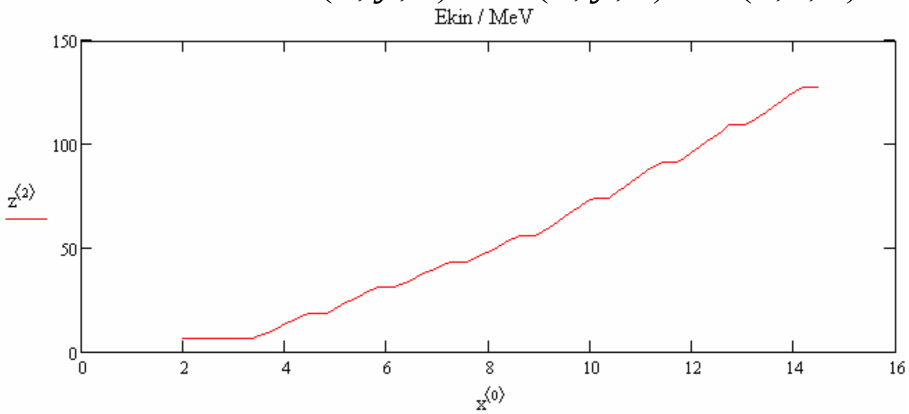
slice emittance
(center 10% charge)

$$\frac{\varepsilon_{\mathrm{s,x}}}{\varepsilon_{\mathrm{s,\,no\,coupler}}} \approx 1.00 \qquad \frac{\varepsilon_{\mathrm{s,y}}}{\varepsilon_{\mathrm{s,\,no\,coupler}}} \approx 1.00$$

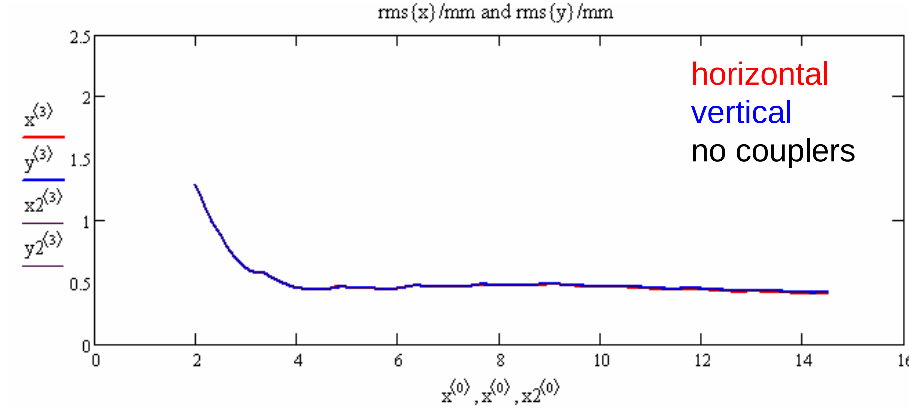
mechanisms for emittance growth – 2nd

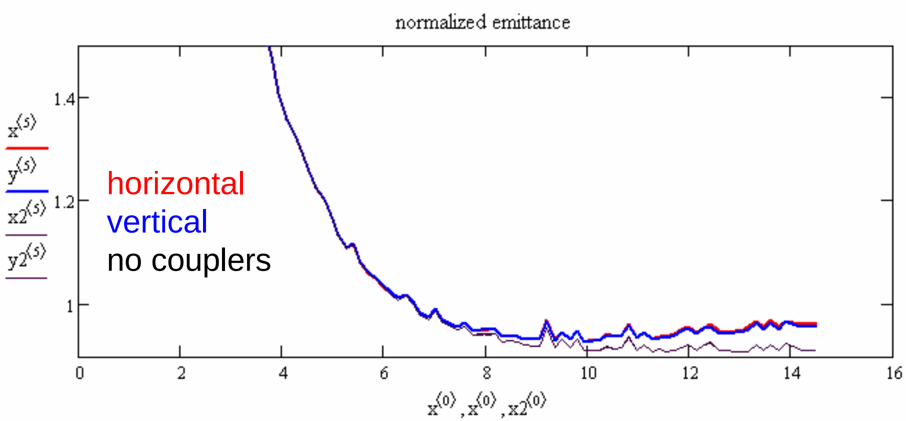
zero field at center

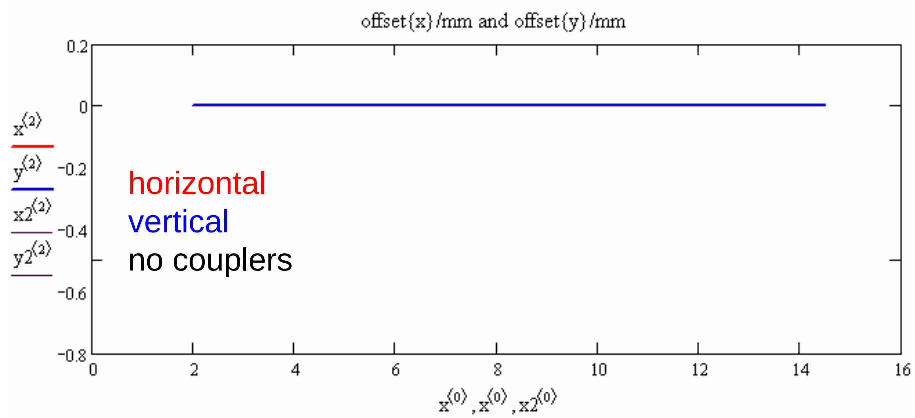
$$\mathbf{E}(x,y,z) := \mathbf{E}(x,y,z) - \mathbf{E}(0,0,z)$$



$$\mathbf{B}(x,y,z) := \mathbf{B}(x,y,z) - \mathbf{B}(0,0,z)$$







projected emittance

$$\frac{\varepsilon_{\mathrm{p,x}}}{\varepsilon_{\mathrm{p,\,no\,coupler}}} \approx 1.057 \qquad \frac{\varepsilon_{\mathrm{p,y}}}{\varepsilon_{\mathrm{p,\,no\,coupler}}} \approx 1.051$$

slice emittance
(center 10% charge)

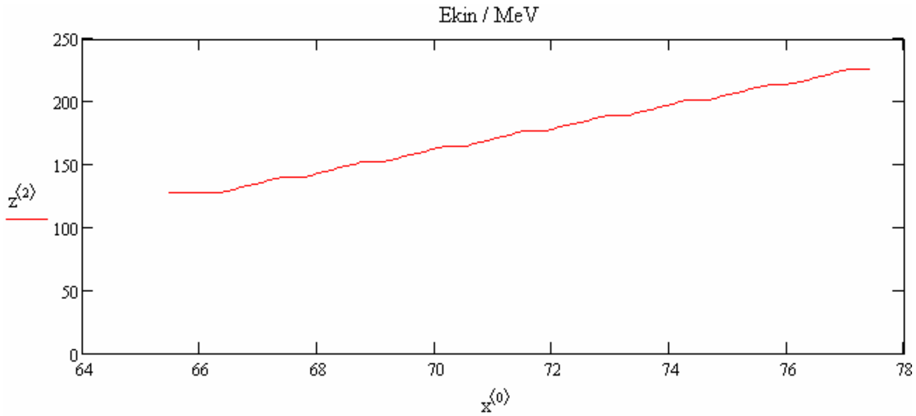
$$\frac{\varepsilon_{\mathrm{s,x}}}{\varepsilon_{\mathrm{s,\,no\,coupler}}} \approx 1.028 \qquad \frac{\varepsilon_{\mathrm{s,y}}}{\varepsilon_{\mathrm{s,\,no\,coupler}}} \approx 1.021$$

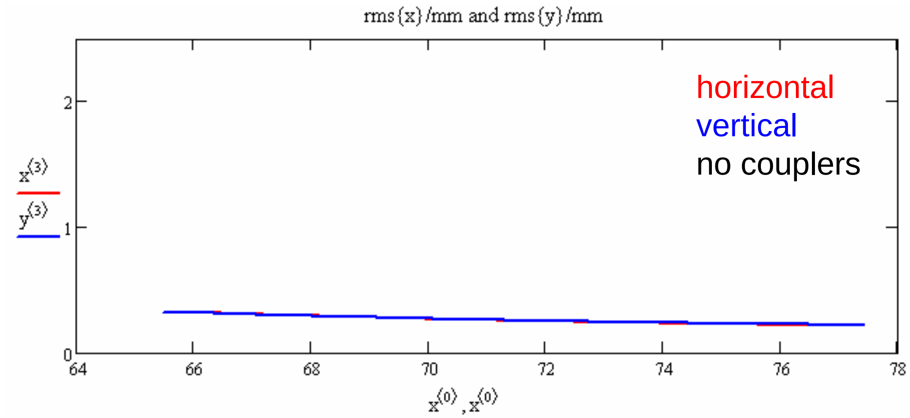
mechanisms for emittance growth – 2nd

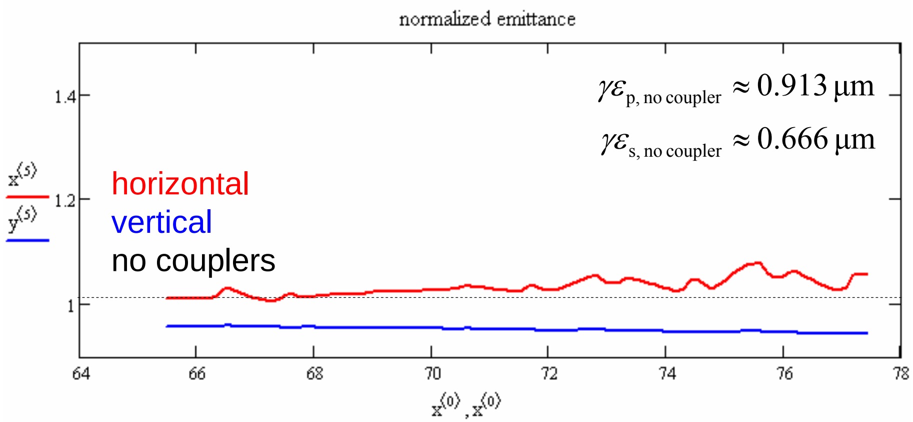
	projected emittance	projected emittance (extracted centroid shift)	slice emittance (center, 10% charge)
no coupler fields	$\varepsilon_{\text{p, no coupler}} \approx 3.64 \cdot 10^{-9}\,\text{m}$ $\gamma\varepsilon_{\text{p, no coupler}} \approx 0.913\,\mu\text{m}$		$\varepsilon_{\text{s, no coupler}} \approx 2.66 \cdot 10^{-9}\,\text{m}$ $\gamma\varepsilon_{\text{s, no coupler}} \approx 0.666\,\mu\text{m}$
full calculation	$\frac{\varepsilon_{\text{p}}}{\varepsilon_{\text{p, no coupler}}} \approx$ 1.11 for x 1.051 for y	1.057 for x 1.051 for y	$\frac{\varepsilon_{\text{s}}}{\varepsilon_{\text{p, no coupler}}} \approx$ 1.028 for x 1.021 for y
offset independent fields	1.055 for x 1.00 for y	1 for x 1 for y	1 for x 1 for y
zero field at center	1.057 for x 1.051 for y	1.057 for x 1.051 for y	1.028 for x 1.021 for y

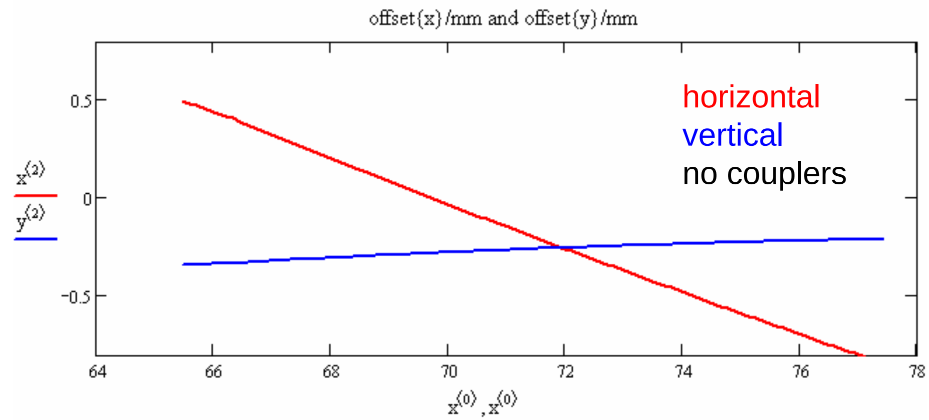
ASTRA calculation for ACC2

(ACC2 on crest)









projected emittance

$$\frac{\varepsilon_{\text{p,x}}}{\varepsilon_{\text{p, no coupler}}} \approx 1.162 \qquad \frac{\varepsilon_{\text{p,y}}}{\varepsilon_{\text{p, no coupler}}} \approx 1.036$$

slice emittance (center 10% charge)

$$\frac{\varepsilon_{\text{s,x}}}{\varepsilon_{\text{s, no coupler}}} \approx 1.026 \qquad \frac{\varepsilon_{\text{s,y}}}{\varepsilon_{\text{s, no coupler}}} \approx 1.018$$

conclusions, questions

similar results as Philippe

growth of horizontal emittance in ACC1: projected 11%, slice 3%

1st order effect = offset independent field 5.5%
(centroid shift)

2nd order effect = offset dependent field 5.7%, 2.8%

horiz. emittance growth to end of ACC2: projected 16%, slice 3%

influence of working point (reflected waves, off crest)

is that all?

is it acceptable?

1st order compensation of kicks (could be) possible with compensation elements

higher order compensation is difficult (→ keep symmetries of geometry)

required: investigation for 3rd harmonic coupler