monopole modes (single cavity)

(more: s2e March 2007 & TESLA FEL 2003-01)

Appendix A: Monopole Modes

The results from a MAFIA calculation with electric boundary conditions are summarized in the following table.

mode #	f/GHz	(R/Q)/ Ohm	G1/Ohm
1	3.7465	0.007	265.3
2	3.7589	0.060	266.2
3	3.7796	0.086	267.5
4	3.8061	0.166	269.3
5	3.8325	0.278	281.2
6	3.8585	0.194	283.1
7	3.8799	0.317	284.7
8	3.8940	0.134	285.8
9	3.8991	374.748	261.8
10	5.7689	19.568	388.9
11	5.7689	1.267	388.9
12	6.0129	3.373	408.0
13	6.0129	12.026	408.0
14	6.4409	18.580	442.8
15	6.4409	3.243	442.8
16	6.9339	16.700	467.7
17	6.9339	4.694	467.8
18	7.0410	0.225	435.2
19	7.0790	1.586	436.9
20	7.1349	0.123	439.2
21	7.2036	4.334	443.0
22	7.2799	1.204	449.5
23	7.3585	3.594	459.5
24	7.4344	15.939	471.1
25	7.5064	23.219	476.5
26	7.5765	42.874	438.8
27	7.6398	2.134	646.1
28	7.7180	18.580	603.4
29	7.7984	0.621	607.9
30	7.8737	0.876	622.8
31	7.9477	0.576	644.3
32	8.0163	0.019	670.7
33	8.0768	0.021	600.8
34	8.1265	0.040	631.9
35	8.1578	0.006	657.8
36	8.3334	1.943	690.7
37	8.3334	3.886	690.7
38	9.1175	7.103	816.8
39	9.1175	0.914	816.8
40	9.7897	0.001	686.0

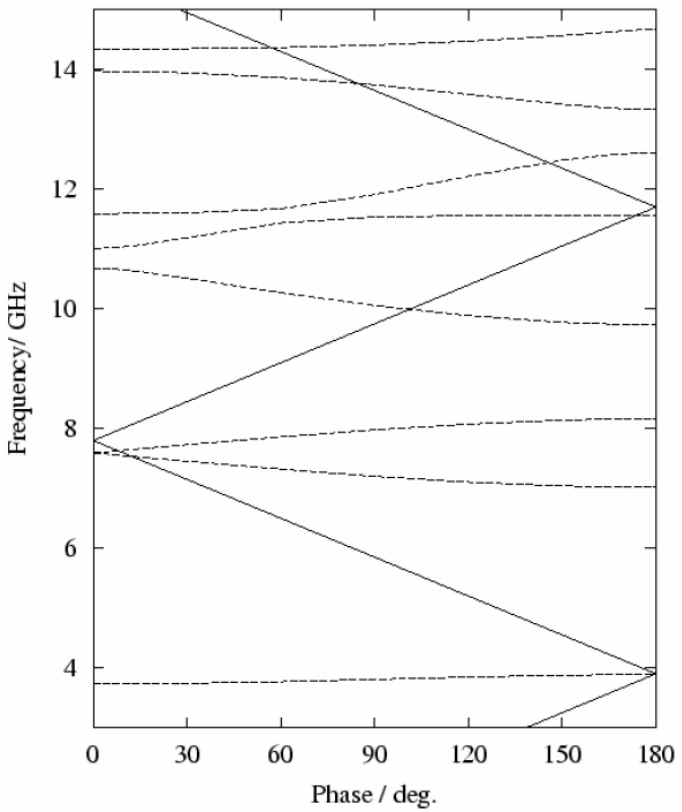


Figure 4: Monopole passbands of a cavity mid-cell.

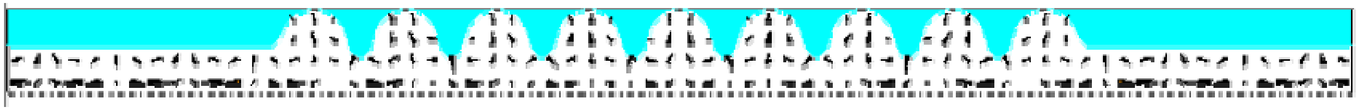
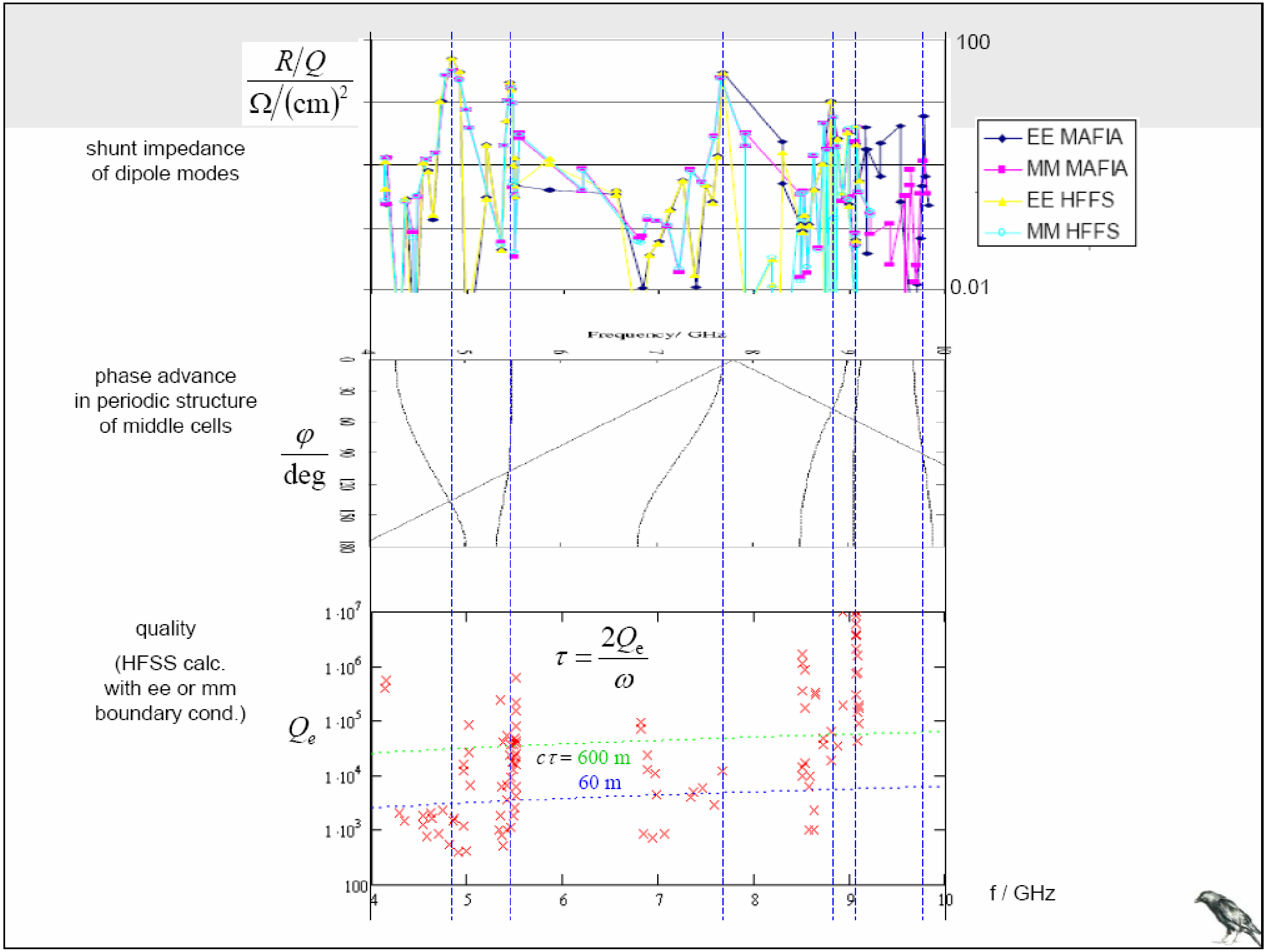


Figure 10: Electric field of the higher monopole mode #26, f = 7.5765 GHz.

not “real”

damping of dipole modes (single cavity)

(more: s2e March 2007 & TESLA FEL 2003-01)



definition of (R/Q)

$$V_\infty = qk\,\mathrm{Re}\left\{1+2\sum_{\mu=1}^{\infty}\exp(\mu p T_b)\right\} = qk\,\mathrm{Re}\left\{\frac{1+\exp(pT_b)}{1-\exp(pT_b)}\right\}$$

with $p = -\frac{\omega}{2Q} + i\omega$

$$V_{\infty,\mathrm{res}} \approx qk\frac{2}{1-\exp\left(-\frac{\omega}{2Q}T_b\right)} \approx \frac{q}{T}Q\frac{4k}{\omega}$$

Jazek's (R/Q): $V_{\infty,\mathrm{res}} \to I_b Q(R/Q)$

$$(R/Q) = \frac{4k}{\omega}$$

example: $(R/Q) = 10\,\Omega$

$$Q = 10^6$$

$$I = 5\,\mathrm{mA}$$

$$\to V_{\infty,\mathrm{res}} = 50\,\mathrm{kV} = \boxed{10^{-4}}\cdot 500\,\mathrm{MV}$$

TESLA-FEL 2003-01:
(Khabibouline, Solyak, Wanzenberg))

$$(R/Q) = \frac{|V|^2}{2\omega W_{tot}} = \frac{2}{\omega}\frac{|V|^2}{4W_{tot}} = \frac{2k}{\omega}$$

arrival time jitter

chirp $V' \approx \frac{10\,\text{MV}}{2.5\,\text{mm}}$ $\rightarrow$ path length difference (C>>1) $\Delta s \approx 2.5\,\text{mm}$

voltage error (one 3rd harm. cavity) $V_{\infty,\text{res}} = 50\,\text{kV}$ $\rightarrow \boxed{\Delta s \approx 12.5\,\mu\text{m}}$

$\sigma_s \approx 25\,\mu\text{m}$

systematic process!

a) on resonance $\tau = \frac{2Q}{\omega}$ example: $Q = 10^6$

$f = 8\,\text{GHz}$

$\rightarrow \tau \approx 40\,\mu\text{s}$ compensation by LLRF

(fast and accurate energy measurement required)

b) off resonance $$V_M = qk\,\text{Re}\left\{-1 + 2\sum_{\mu=0}^{M} \exp(\mu p T_b)\right\}$$

$$\frac{\Delta V}{\Delta t} = \frac{V_M - V_{M-1}}{T_b} = \underbrace{\frac{qk}{T_b}\exp\left(-M\frac{T_b}{\tau}\right)}\text{Re}\{\exp(j\omega M T_b)\}$$

slope $\left|\frac{\Delta V}{\Delta t}\right|$ depends not on detuning