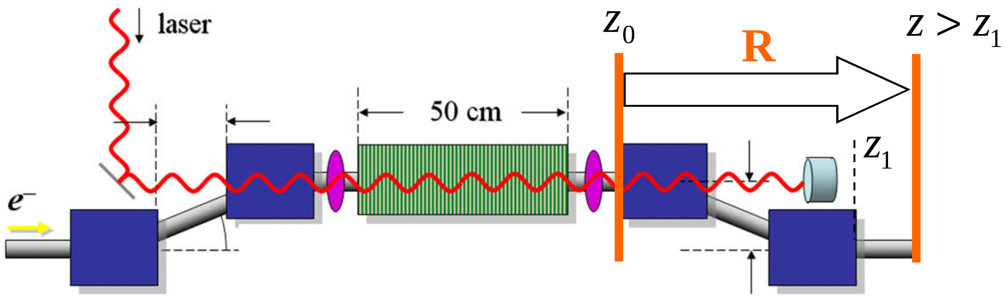
Trickle Heating

theory from SLAC-PUB-13854
Z. Huang et. al.



$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ z_0 \\ \delta_0 \end{pmatrix}$$

$$R_{26} = R_{51} = 0$$

$$R_{16} = R_{52}$$

effect of transverse emittance $\rightarrow$ xz-plane
slice with $z_0 = 0 \quad \delta_0 = 0$

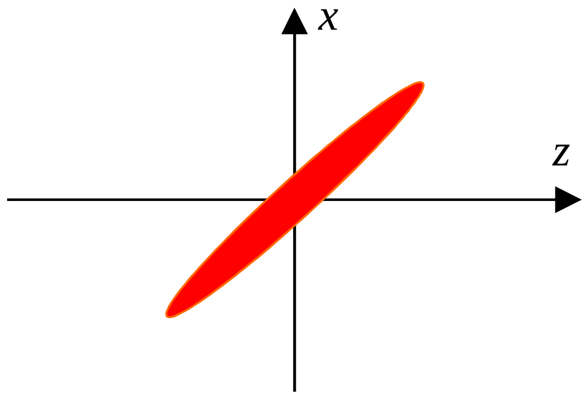
$$\langle xx \rangle = R_{11}^2 \langle x_0 x_0 \rangle + 2 R_{11} R_{12} \langle x_0 x' \rangle + R_{12}^2 \langle x_0' x_0' \rangle$$

$$\langle zz \rangle = R_{52}^2 \langle x_0' x_0' \rangle$$

$$\langle xz \rangle = R_{52} R_{11} \langle x_0 x' \rangle + R_{52} R_{12} \langle x_0' x_0' \rangle$$

slice at $z_0 \rightarrow$ distribution after LH

$$\begin{bmatrix} \langle xx \rangle & \langle xz \rangle \\ \langle xz \rangle & \langle zz \rangle \end{bmatrix} = \varepsilon_x \begin{bmatrix} \beta_{x0} R_{11}^2 - 2\alpha_0 R_{11} R_{12} + \gamma_{x0} R_{12}^2 & \cdots \\ \gamma_{x0} R_{12} R_{52} - \alpha_0 R_{11} R_{52} & \gamma_{x0} R_{52}^2 \end{bmatrix}$$

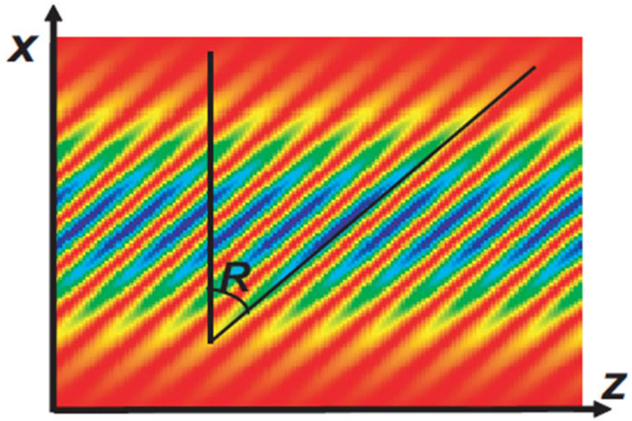


area $\det[\cdots] = \varepsilon_x (R_{11} R_{52})^2$

energy modulation in LH $\rightarrow$ tilted microbunches

tilt angle $R = \dfrac{\langle xz \rangle}{\langle xx \rangle}$

$$= -R_{52} \left(R_{21} + \frac{\alpha}{\beta} R_{11} \right)$$



3D Impedance

generalization (on axis):

$$E_z(k_0) = \frac{-eik_0}{2\pi\varepsilon_0\gamma^2\lambda_0}\int dxdydz \times \rho(x,y,z)e^{-ik_0 z}K_0\left(\frac{k_0 r}{\gamma}\right)$$

$$\downarrow$$

$$E_z(k_0) = \frac{-eik_0}{2\pi\varepsilon_0\gamma^2\lambda_0}\int dxdx'dydy'dzd\delta \times f(x,x',y,y',z,z')e^{-ik_0 z}K_0\left(\frac{k_0 r}{\gamma}\right)$$

integration for (nominal) Gaussian transverse phase space:

$$E_z(k_0) \approx \frac{iI_0 Z_0}{2\pi k_0\sigma_r^2}J_1(k_0 R_{56}\delta_L)\exp\left(-\tfrac{1}{2}(k_0 R_{56}\sigma_{\delta 0})^2\right)\exp\left(-\tfrac{\varepsilon}{2\beta}(k_0 R_{52} R_{11})^2\right)\frac{1}{1+\gamma^2 R^2}$$

for $k_0\sigma_r/\gamma >> 1$ with ε,α,β Twiss parameters

$$R = -R_{52}\left(R_{21} + \frac{\alpha}{\beta}R_{11}\right)$$

Trickle Heating

induced energy modulation:

$$\delta_{\mathrm{LSC}} = \frac{1}{\gamma m_0 c^2} \int E_z(k_0) dz$$

$$\delta_{\mathrm{LSC}} = \frac{2i}{k_0 \gamma} \frac{L_{\mathrm{eff}}}{\varepsilon \bar{\beta}} \frac{I_0}{I_A} J_1(k_0 R_{56} \delta_L) \exp\left(-\tfrac{1}{2}(k_0 R_{56} \sigma_{\delta 0})^2\right)$$

with $L_{\mathrm{eff}} = \int dz \times \frac{\varepsilon \bar{\beta}}{\sigma_r^2} \exp\left(-\tfrac{\varepsilon}{2\beta}(k_0 R_{52} R_{11})^2\right) \frac{1}{1+\gamma^2 R^2}$

$\bar{\beta}$ typical beta function

$\sigma_r^2 \approx \sigma_x \sigma_y$ (round beam)

rms energy spread with trickle heating:

$$\sigma_\delta \approx \sqrt{\sigma_{\delta 0}^2 + 0.5(\delta_L(\delta_0))^2 + 2|\delta_{LSC}(\delta_L(\delta_0))|}$$

with $\delta_L(\delta_0) = \sqrt{s_f}\,\delta_0$

$s_f \approx 1$ shape factor (electron/photon beam size)

$\sigma_{\delta 0}$ uncorr. spread before heater

Trickle Heating II

for: $\sigma_{\delta 0} << \sigma_\delta$

$$\sigma_\delta \approx \sqrt{A\delta_0^2 + \left(BJ_1\left(C\delta_0\right)\right)^2}$$

with

$$A = 0.5 s_f$$

$$B = \frac{2\sqrt{2}}{k_0 \gamma} \frac{L_{\mathrm{eff}}}{\varepsilon \overline{\beta}} \frac{I_0}{I_A} \exp\left(-\tfrac{1}{2}\left(k_0 R_{56} \sigma_{\delta 0}\right)^2\right)$$

$$C = \sqrt{s_f}\, k_0 R_{56}$$

example LCLS:

$R_{56} = 3.9\,\mathrm{mm}$ (last two magnets of chicane)

$\lambda_0 = 758\,\mathrm{nm}$

$I_0 = 37\,\mathrm{A}$

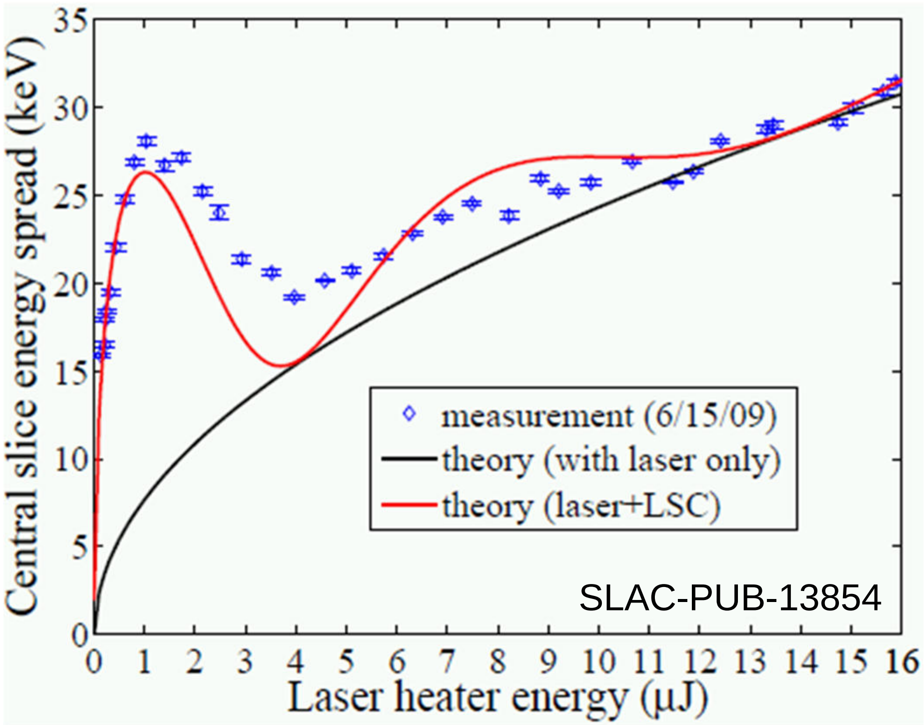
$\gamma\varepsilon = 0.4\,\mu\mathrm{m}$

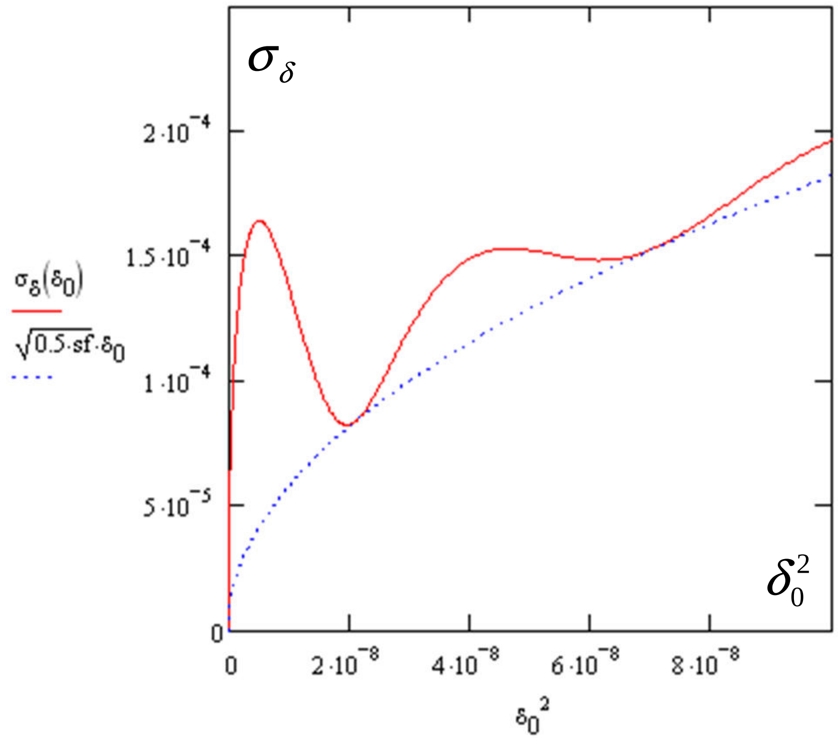
$m_0 \gamma c^2 = 135\,\mathrm{MeV}$ $\rightarrow$ $A \approx 0.331$

$L_{\mathrm{eff}} = 0.81\,\mathrm{m}$ for $\overline{\beta} = 5\,\mathrm{m}$ $\qquad B \approx 2.73 \cdot 10^{-4}$ (to spectrometer)

$s_f = 0.662$ $\qquad C \approx 2.63 \cdot 10^4$

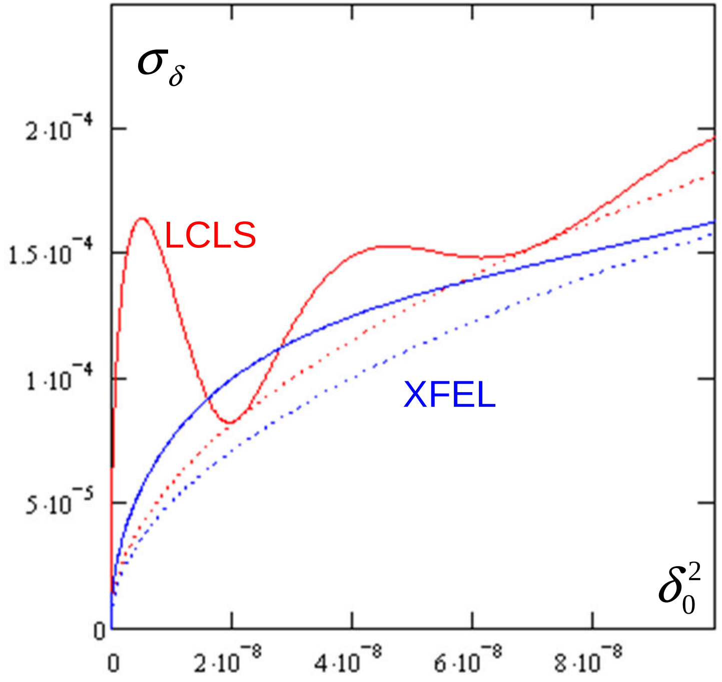
Example LCLS





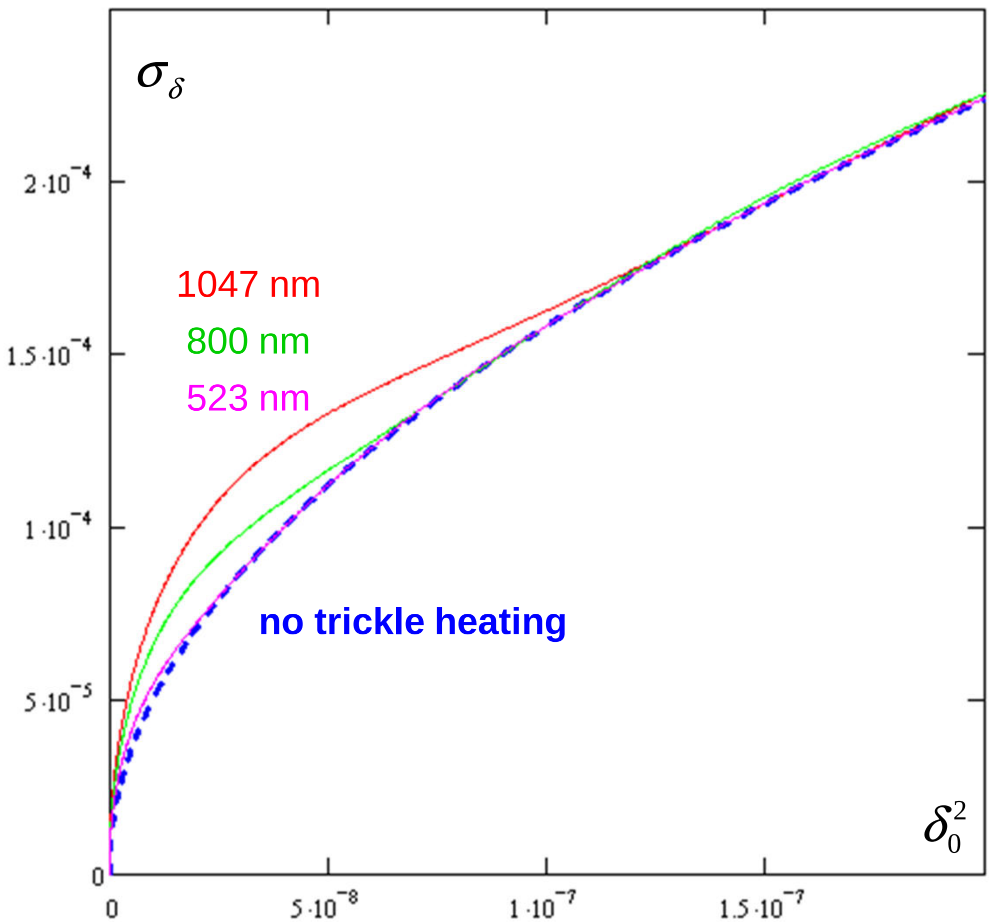
Comparison XFEL, LCLS

XFEL (1nC)	LCLS
$R_{56} = 2.3\,\mathrm{mm}$	$R_{56} = 3.9\,\mathrm{mm}$
$\lambda_0 = 1047\,\mathrm{nm}$	$\lambda_0 = 758\,\mathrm{nm}$
$I_0 = 50\,\mathrm{A}$	$I_0 = 37\,\mathrm{A}$
$\gamma\varepsilon = 1.0\,\mu\mathrm{m}$	$\gamma\varepsilon = 0.4\,\mu\mathrm{m}$
$m_0\gamma c^2 = 130\,\mathrm{MeV}$	$m_0\gamma c^2 = 135\,\mathrm{MeV}$
$L_{\mathrm{eff}} = 0.48\,\mathrm{m}$	$L_{\mathrm{eff}} = 0.81\,\mathrm{m}$ for $\bar{\beta} = 5\,\mathrm{m}$
$s_f = 0.5$	$s_f = 0.662$
$A \approx 0.25$	$A \approx 0.331$
$B \approx 1.29 \cdot 10^{-4}$	$B \approx 2.73 \cdot 10^{-4}$
$C \approx 0.99 \cdot 10^{4}$	$C \approx 2.63 \cdot 10^{4}$



$$\sigma_\delta \approx \sqrt{A\delta_0^2 + \left(B J_1(C\delta_0)\right)^2}$$

XFEL 1nC, λ = 1047 nm, 800nm, 523nm



Simplified

$$\left(\frac{\sigma_\delta}{\delta_0}\right)^2 = A + \left(B\frac{J_1(C\delta_0)}{\delta_0}\right)^2 \approx A\left(1+\left(\boxed{\frac{BC}{2\sqrt{A}}}\right)^2\right) \qquad \text{for} \qquad C\delta_0 <\approx 1$$

enhancement factor:

$$F = \frac{BC}{2\sqrt{A}} = \frac{2}{\gamma}\frac{L_{\mathrm{eff}}R_{56}}{\varepsilon\overline{\beta}}\frac{I_0}{I_A}\underbrace{\exp\left(-\tfrac{1}{2}\left(k_0 R_{56}\sigma_{\delta 0}\right)^2\right)}_{\text{weak}}$$

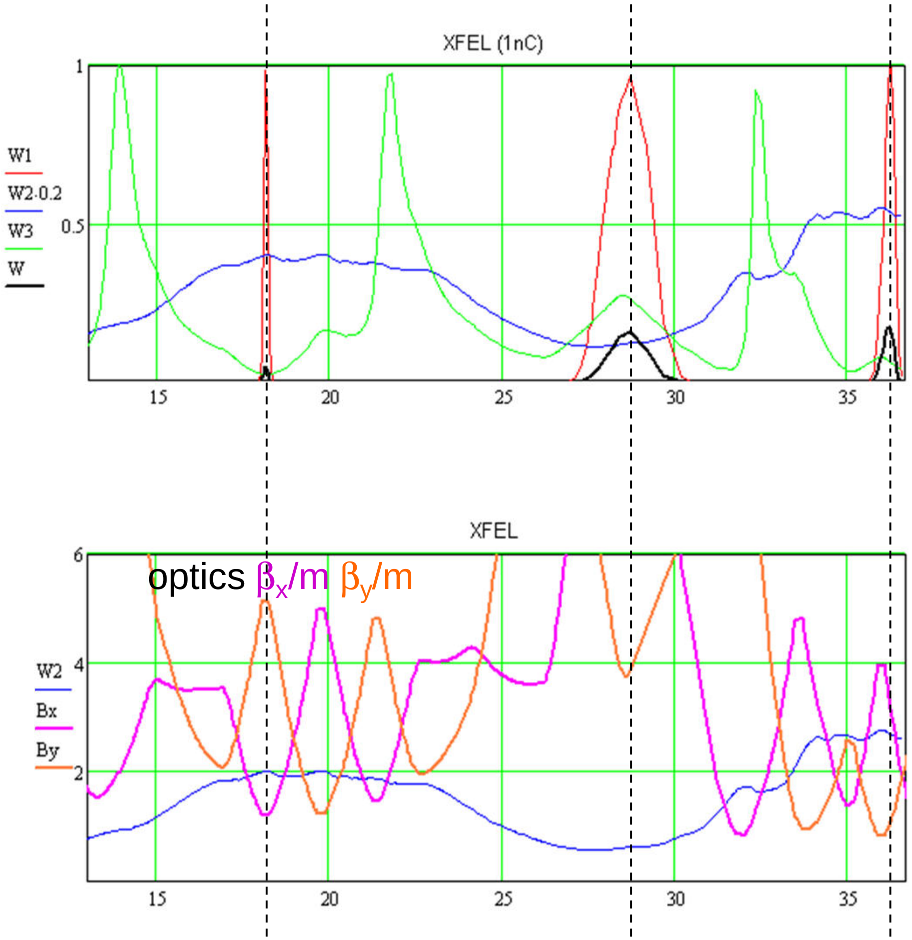
… see summary

Effective length

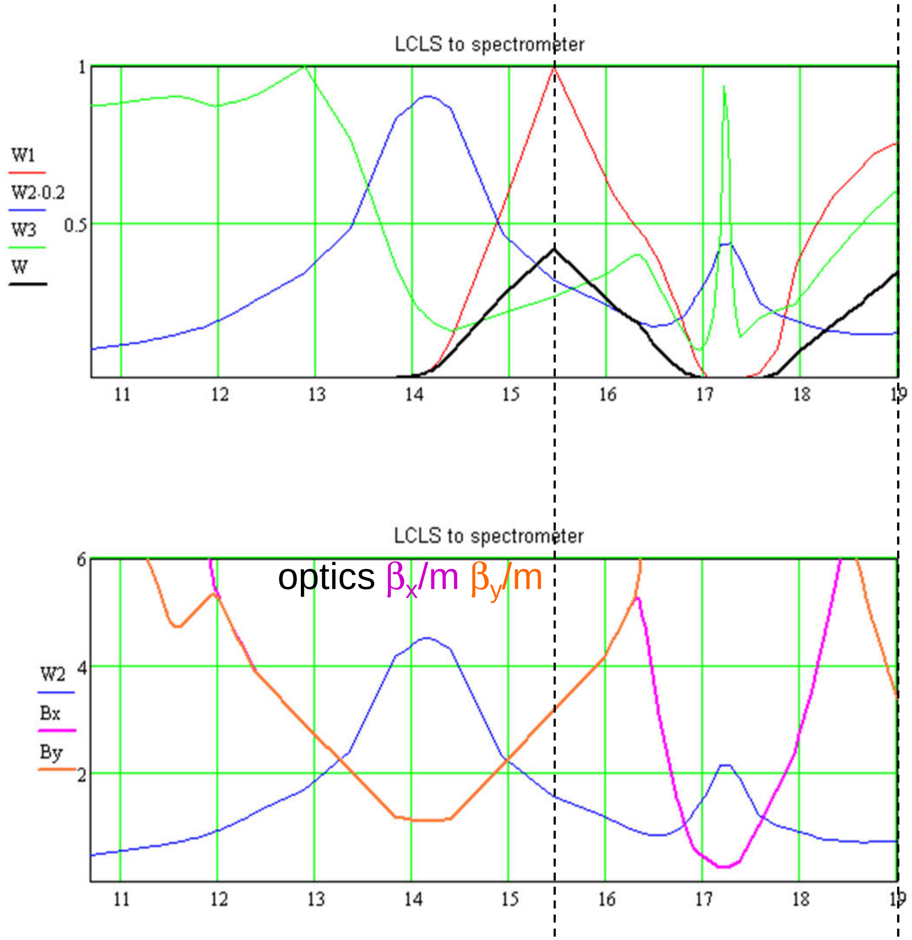
$$L_{\mathrm{eff}} = \int dz \times \underbrace{\exp\left(-\frac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right)}_{W_1} \underbrace{\frac{\varepsilon\bar{\beta}}{\sigma_r^2}}_{W_2} \underbrace{\frac{1}{1+\gamma^2 R^2}}_{W_3}$$

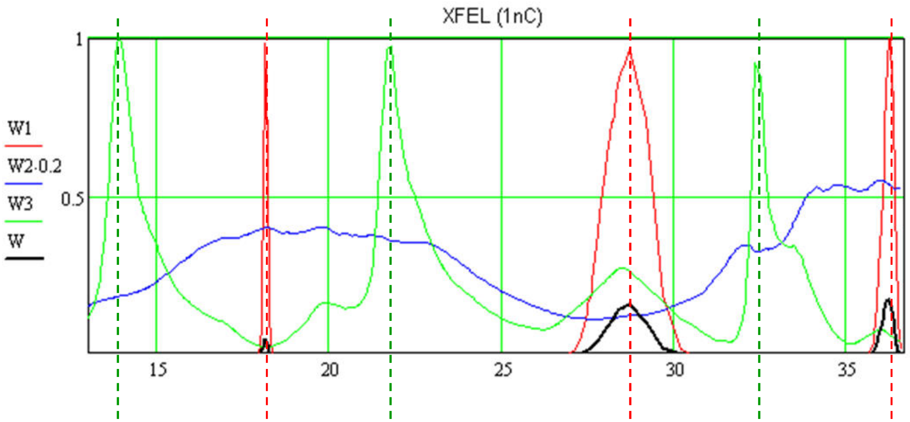
$$W = W_1 W_2 W_3$$

XFEL (1nC) $L_{\mathrm{eff}} = 0.48\,\mathrm{m}$



LCLS to spectrometer $L_{\mathrm{eff}} = 0.81\,\mathrm{m}$





$$R_{11}(z)=\sqrt{\frac{\beta}{\beta_0}}\left(\cos\psi+\alpha_0\sin\psi\right)$$

$$R_{12}(z)=\sqrt{\beta\beta_0}\,\sin\psi$$

$R_{11}(z)=0$ $\quad\rightarrow \cos\psi+\alpha_0\sin\psi=0$

$R(z)=0$ $\quad\rightarrow \langle xz\rangle=\varepsilon R_{52}\left(\gamma_0 R_{12}-\alpha_0 R_{11}\right)$ $\quad\rightarrow \sin\psi-\alpha_0\cos\psi=0$

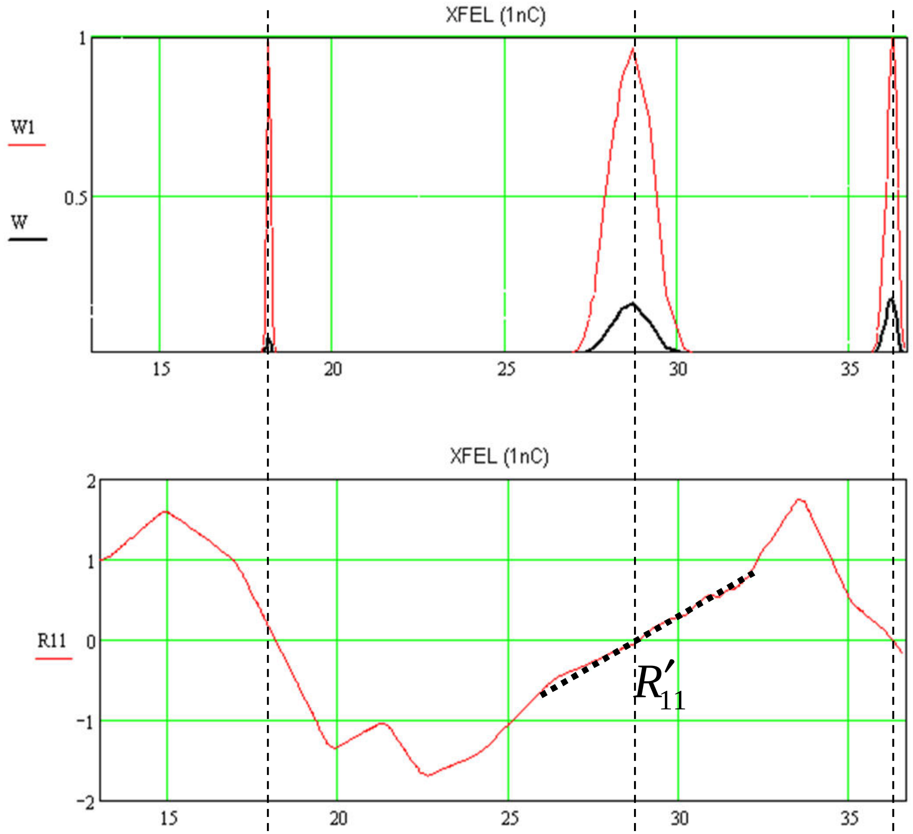
therefore: 90 deg phase shift between both conditions

$$L_{\text{eff}} = \int dz \times \underbrace{\boxed{\exp\left(-\tfrac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right)}}_{W_1} \frac{\varepsilon\bar{\beta}}{\sigma_r^2}\,\frac{1}{1+\gamma^2 R^2}$$

$$\int dz \times \exp\left(-\tfrac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right) \approx \frac{\sqrt{2\pi\,\beta/\varepsilon}}{k_0 R_{52} R'_{11}}$$

$$L_{\text{eff}} \approx \sqrt{2\pi}\,\frac{\bar{\beta}}{k_0 R_{52}} \sum_{\nu} \left(\frac{1}{R'_{11}} \frac{\sigma_x}{\sigma_r^2} \frac{1}{1+\gamma^2 R^2} \right)_{z=z_\nu^{(R11=0)}}$$

$$\boxed{L_{\text{eff}} \propto \frac{1}{k_0 R_{52}\sqrt{\varepsilon}}}$$



$z_1^{(R11=0)}$ $z_2^{(R11=0)}$ $z_3^{(R11=0)}$

Summary

enhancement factor

wavelength / nm

charge / nC

XFEL

charge / nC	523.5	801	1047
0.02	0.214	0.265	0.317
0.1	0.383	0.486	0.554
0.25	0.517	0.679	0.782
0.5	0.634	0.862	1.007
1	0.728	1.058	1.272

for $I \propto q,\ \varepsilon \propto \sqrt{q},\ \sigma_{\delta 0} \propto q$

LCLS 6.24

XFEL (1nC)