Longitudinal Space Charge Amplifier driven by a laser-plasma accelerator

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some results of a feasibility study

Parameters

Methods

Structures

Macro Effects in FODO+BC Lattice (Micro Effects Suppressed)

Principle

Linear One Stage Model (Discrete)

Plasma Oscillations

BCs in FODO Structure

Summary and Conclusion

Parameters

(after discussion with Florian Grüner)

particles:

$q = 40\,\mathrm{pC}$ — 250E6 electrons

$\sigma_{\parallel} = 1.5\,\mu\mathrm{m}$

} $\hat{I} = 3.2\,\mathrm{kA}$

$\mathcal{E}_{\mathrm{av}} = 300\,\mathrm{MeV}$

$\mathcal{E}_{\mathrm{rms}} = 0.6\,\mathrm{MeV}$ — slice energy spread — correlated spread is neglected in the following (small compared to SC induced correlation)

$\varepsilon_{\mathrm{n}} = 0.2\,\mu\mathrm{m}$

$\sigma_{\mathrm{x}} = \sigma_y \approx 0.5\,\mu\mathrm{m}$ — waist $\rightarrow \alpha = 0\,,\;\; \beta = 0.7\,\mathrm{mm}$

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matching to FODO:

to be investigated !!!

example (without SC):

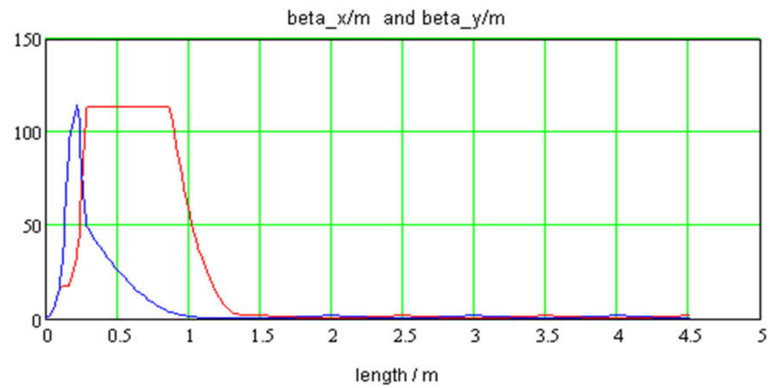
$$\alpha_x = \alpha_y = 0$$

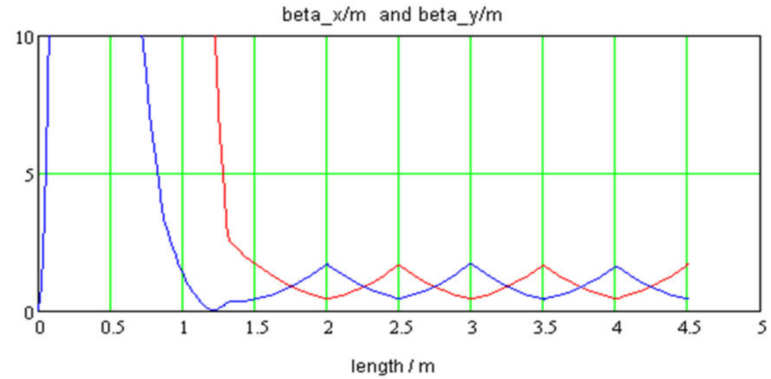
$$\beta_x = \beta_y = 0.7\,\mathrm{mm}$$

to FODO lattice with

$$L_p = 1\,\mathrm{m}$$

$$\varphi = 71\,\mathrm{deg}$$





Methods

linear, 1D impedance

Linear Gain Model (LGM)

useful estimations

infinite bunch length (no shape effects)

chirp generation from macro shape has to be added **artificial**ly

non-linear

Astra: 3D space charge, pseudo steady state

longitudinal and transverse SC effects

slowly with 1E6 particles, < 50E6 particles possible

done: some investigations with 1E6 particles

especially particle dynamics in periodic lattice

in the following

CSRtrack: 1D space charge (pseudo steady state) + 1D-CSR

longitudinal SC effects

simulation with 1E6 in < 10h, < 10E6 particles possible (implementation)

done: many investigations with 1E6 particles

chicanes in FODO, FODO-compressors, …

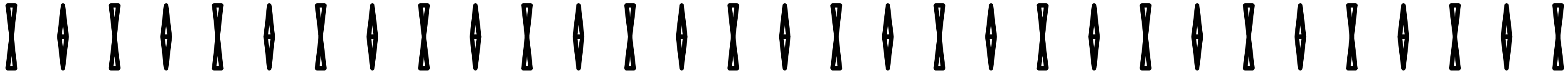
special approach: > 100E6 particles, will be considered later

full electro-dynamics

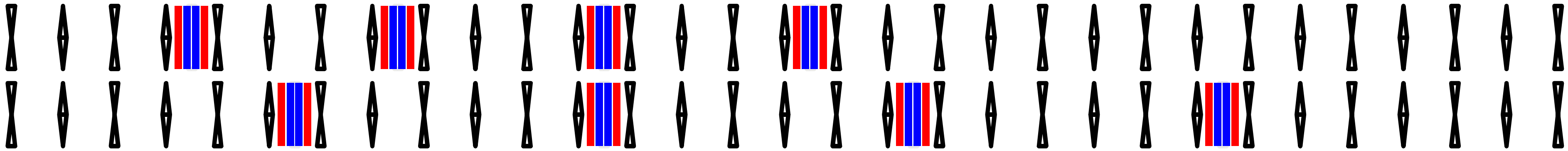
PIC, Lienard Wiechert & co

Structures

FODO structure, period = 1.. 2 m

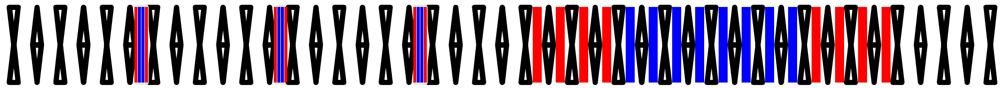


BC-chicanes in FODO structure

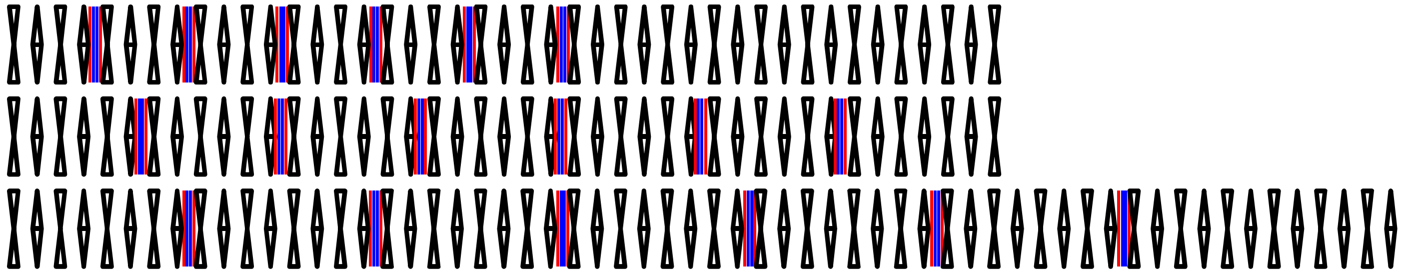


FODO-compressors, FODO period = 40 cm

BC-chicanes + FODO-compressor



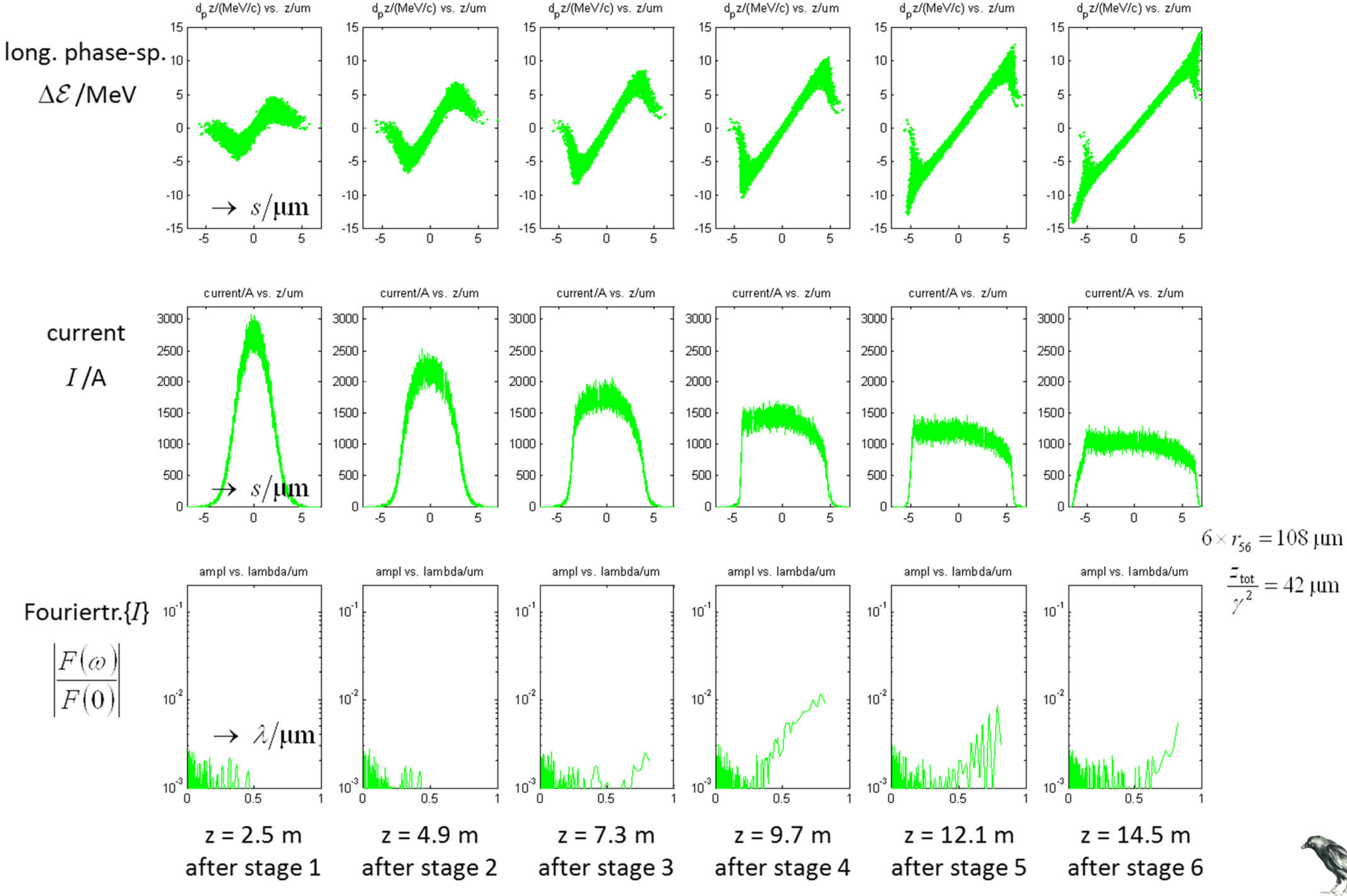
BC-chicanes in FODO structure, period = 0.4 .. 0.8 m



in the following

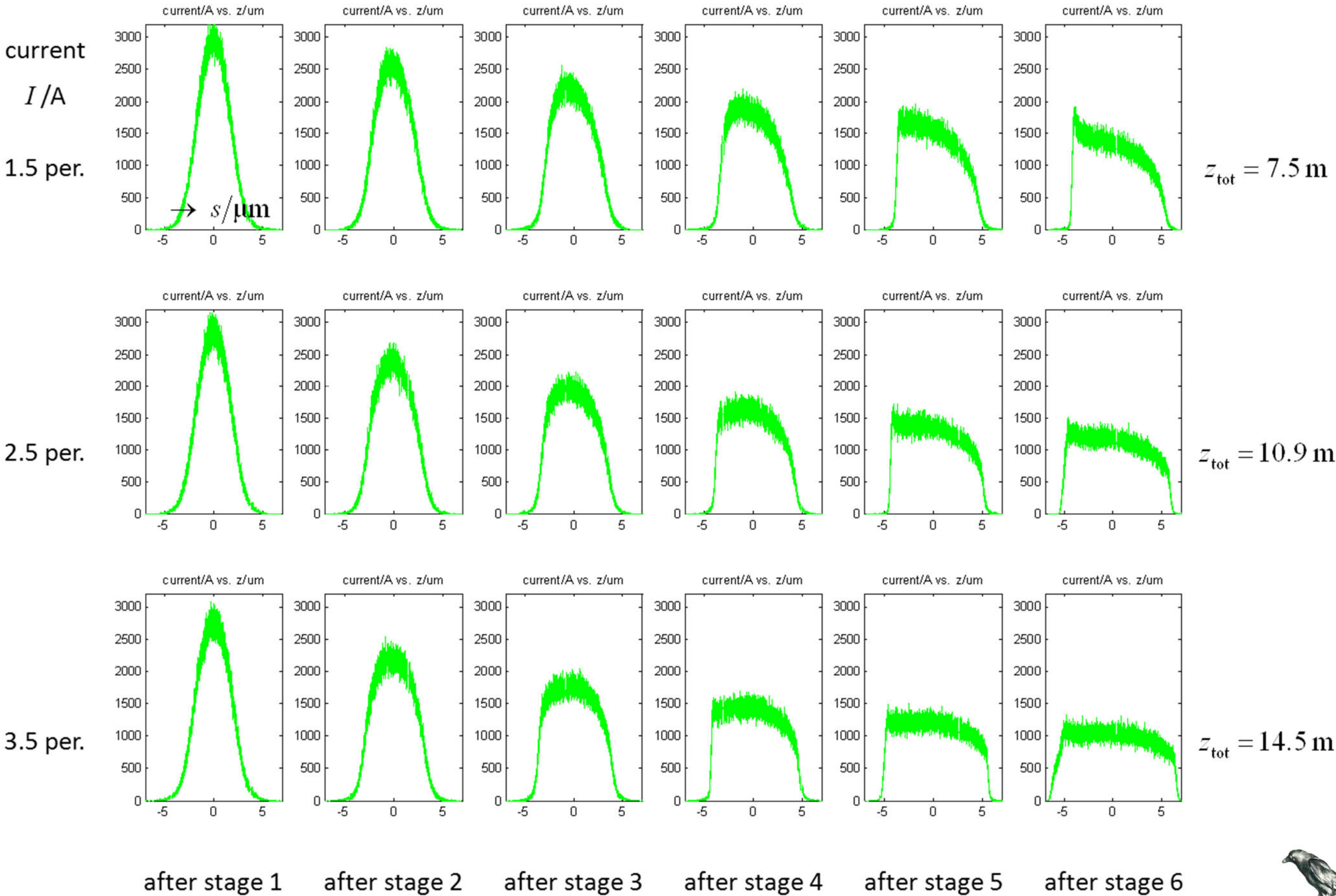
Macro Effects in FODO+BC Lattice (Micro Effects Suppressed)

FODO Lattice Lp=0.6m, 90deg, **6 stages, each = 3.5 FODO periods + 0.5 FODO period with BC (r56 = 18 µm)**



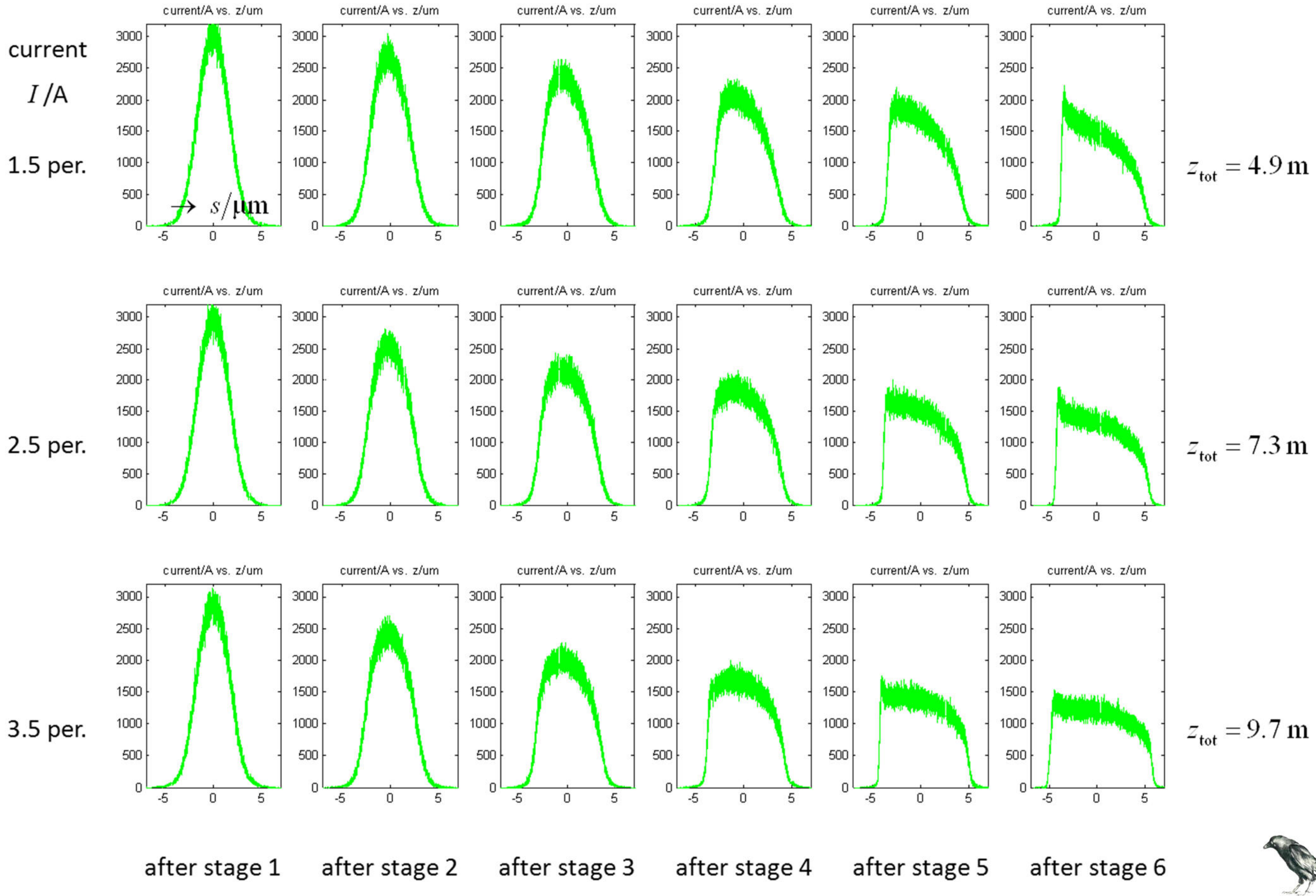
Macro Effects in FODO+BC Lattice (Micro Effects Suppressed)

FODO Lattice Lp=0.6m, 90deg, **6 stages, each = 1.5 / 2.5 / 3.5 FODO periods + 0.5 FODO period with BC (r56 = 18 µm)**



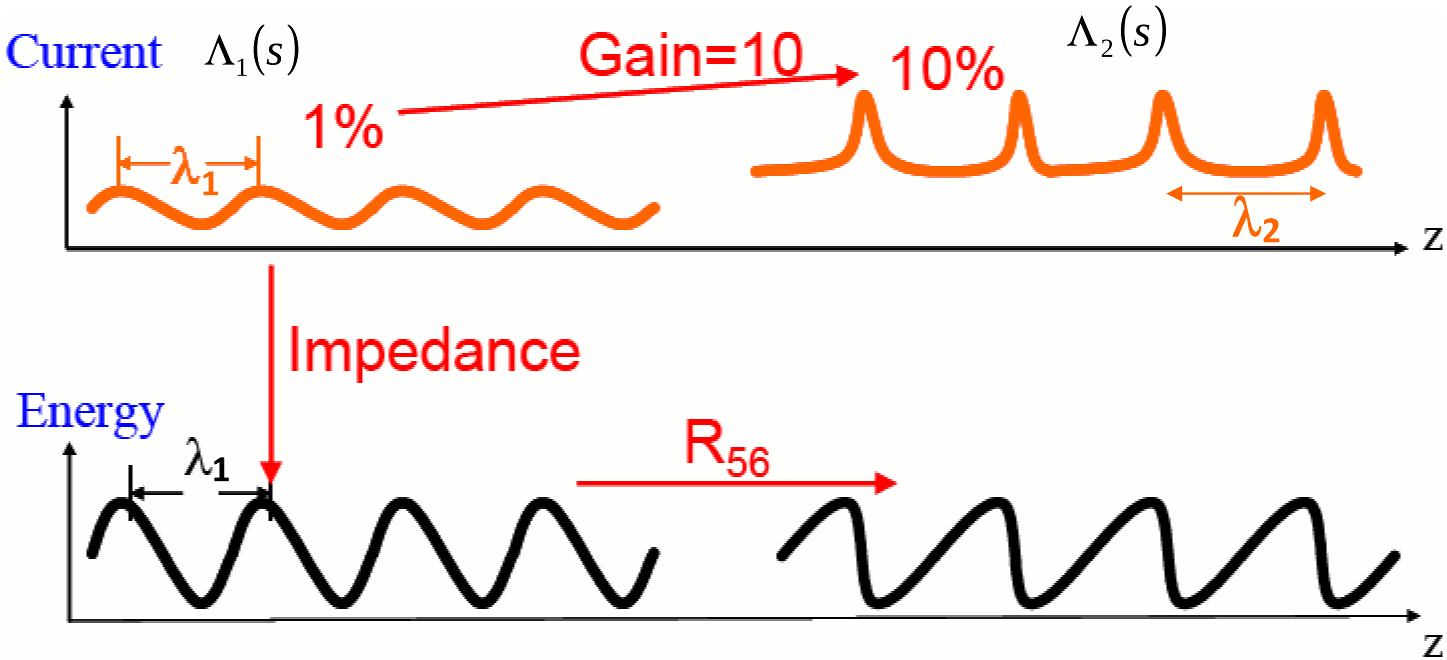
Macro Effects in FODO+BC Lattice (Micro Effects Suppressed)

FODO Lattice Lp=0.4m, 90deg, **6 stages, each = 1.5 / 2.5 / 3.5 FODO periods + 0.5 FODO period with BC (r56 = 18 μm)**



Principle

amplification mechanism



linear gain (LG) regime:

$$\Lambda_1(s) = \overline{\Lambda}_1 \operatorname{Re}\{1 + m \exp(jk_1 z)\} \qquad \Lambda_2(s) = \overline{\Lambda}_2 \operatorname{Re}\{1 + mG \exp(jk_2 z)\}$$

un-modulated, “coasting” beam $\overline{\Lambda}_1$ and $\overline{\Lambda}_2 = C\overline{\Lambda}_1$ with compression C

wavelength of modulation λ_1 and $\lambda_2 = \lambda_1 / C$

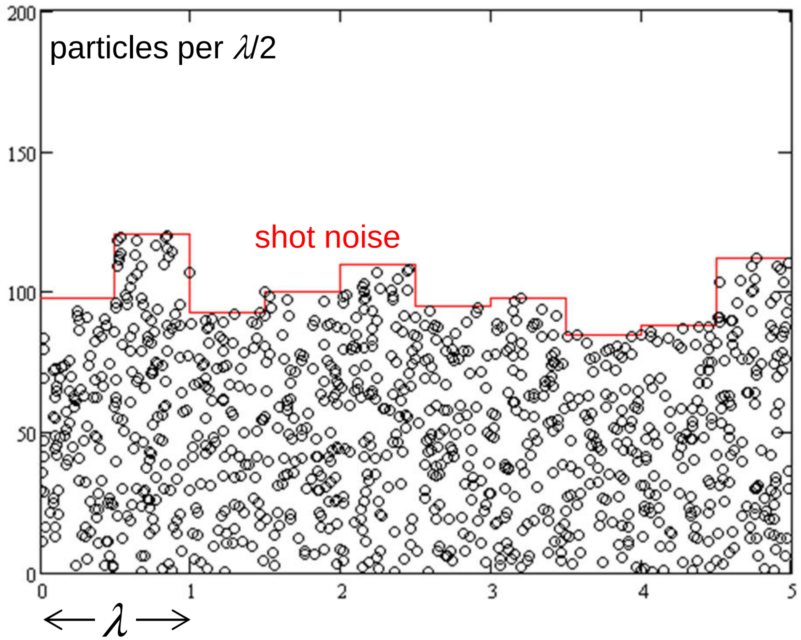
relative modulation amplitude m and $m \cdot G$ with linear gain G

Principle

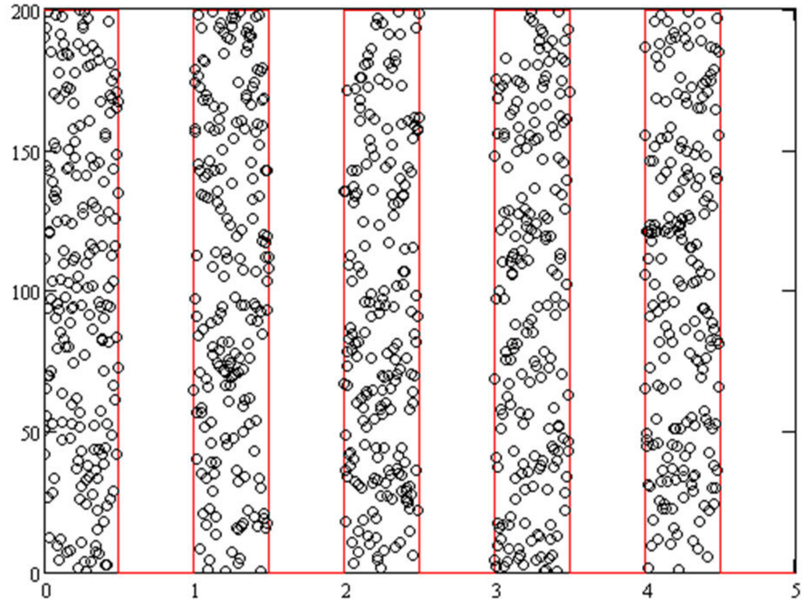
required gain to saturation

simple model for bandwidth ~ center frequency = c/λ

random particle distribution



100% bunched



N_p = ⟨particles per period⟩

$R = \sqrt{N_p}$ = rms fluctuation

$A = N_p$ = amplitude, full modulation

required gain: $G = \frac{A}{R} = \sqrt{N_p}$ our parameters with $\lambda = 250\ \mathrm{nm}$ $G = \sqrt{\frac{\hat{I}\lambda}{ce}} \approx 4000$

Linear One Stage Model (Discrete)

linear gain model (=LGM)

discrete = separate sections, **either** impedance **or** long. dispersion

$$G = \left(1 - j \frac{C r_{56}}{\mathcal{E}_{\text{ref}}/e} \overline{\Lambda}_1 \, c k_1 Z(c k_1) \right) \exp\left(-\frac{1}{2} \left(C k_1 r_{56} \frac{\sigma_{\mathcal{E}}}{\mathcal{E}_{\text{ref}}} \right)^2 \right)$$

wavenumber of initial modulation $k_1 = \frac{2\pi}{\lambda_1}$ compression C

longitudinal dispersion r_{56} of BC at energy $\mathcal{E}_{\text{ref}}$

longitudinal impedance before BC $Z(\omega)$ and current $I = c\overline{\Lambda}_1$

uncorrelated energy spread (Fourier-spectrum of gaussian energy distribution)

Linear One Stage Model (Discrete)

two scales

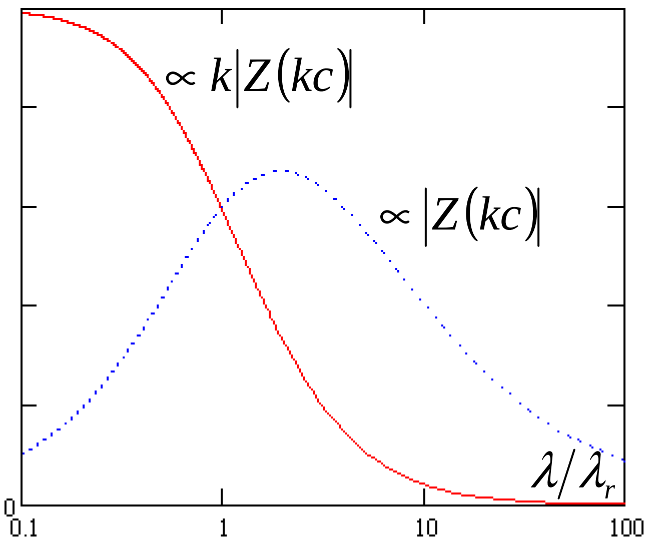
without compression

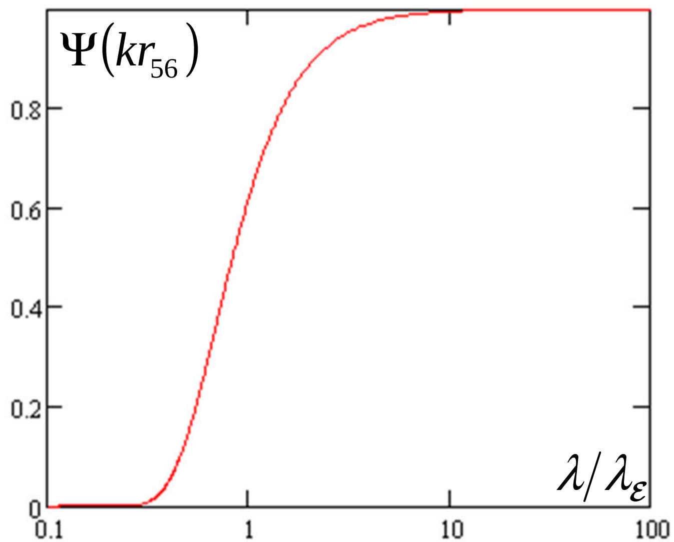
$$ck_1 Z(ck_1)$$

$$\exp\left(-\frac{1}{2}\left(k_1 r_{56}\frac{\sigma_\mathcal{E}}{\mathcal{E}_{\text{ref}}}\right)^2\right)$$

impedance

unc. energy spread



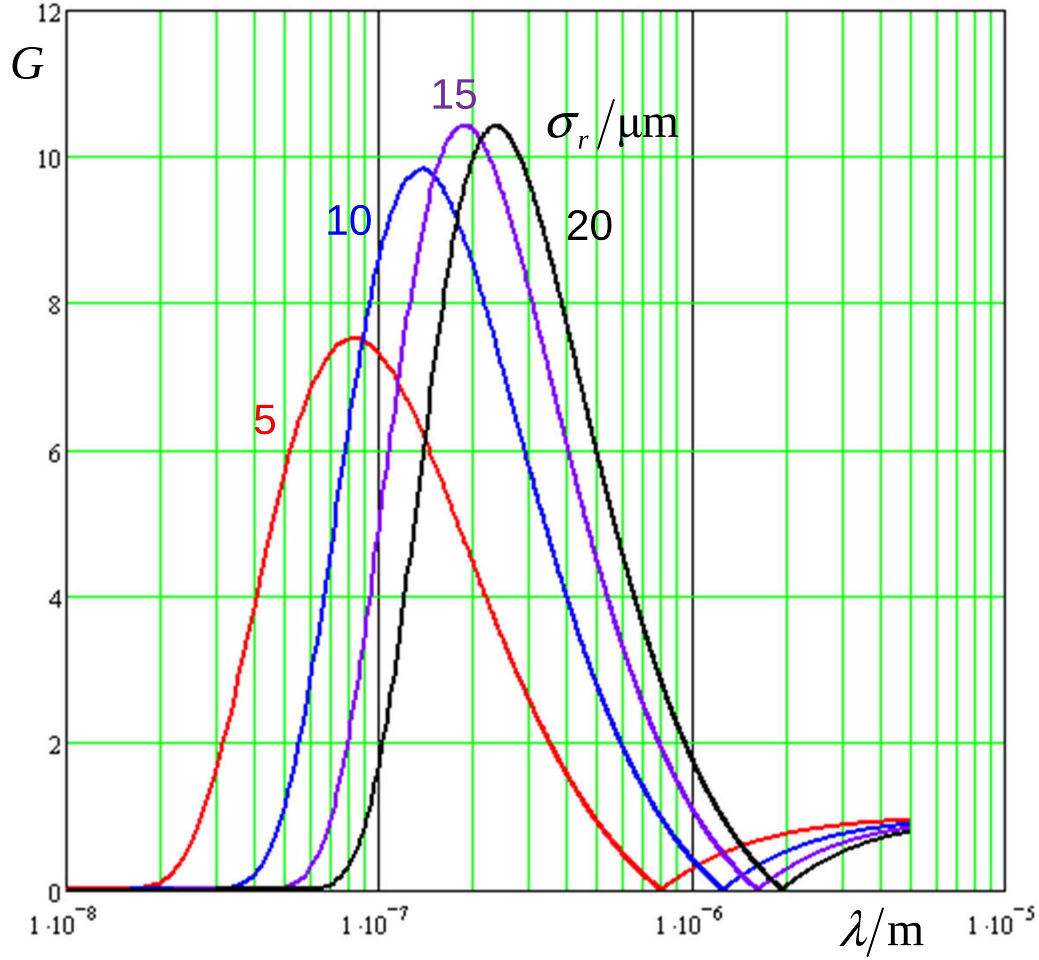


with $\lambda_r = \dfrac{2\pi\sigma_r}{\gamma}$ ⟷ with $\lambda_\mathcal{E} = 2\pi r_{56}\dfrac{\sigma_\mathcal{E}}{\mathcal{E}_{\text{ref}}}$

chose r_{56} so that $\lambda_\mathcal{E} <\approx \lambda_r$ $\quad r_{56} \approx \sigma_r \dfrac{\mathcal{E}}{\sigma_\mathcal{E}}$ $\quad$ our parameters $r_{56} \approx \sigma_r$

Linear One Stage Model (Discrete)

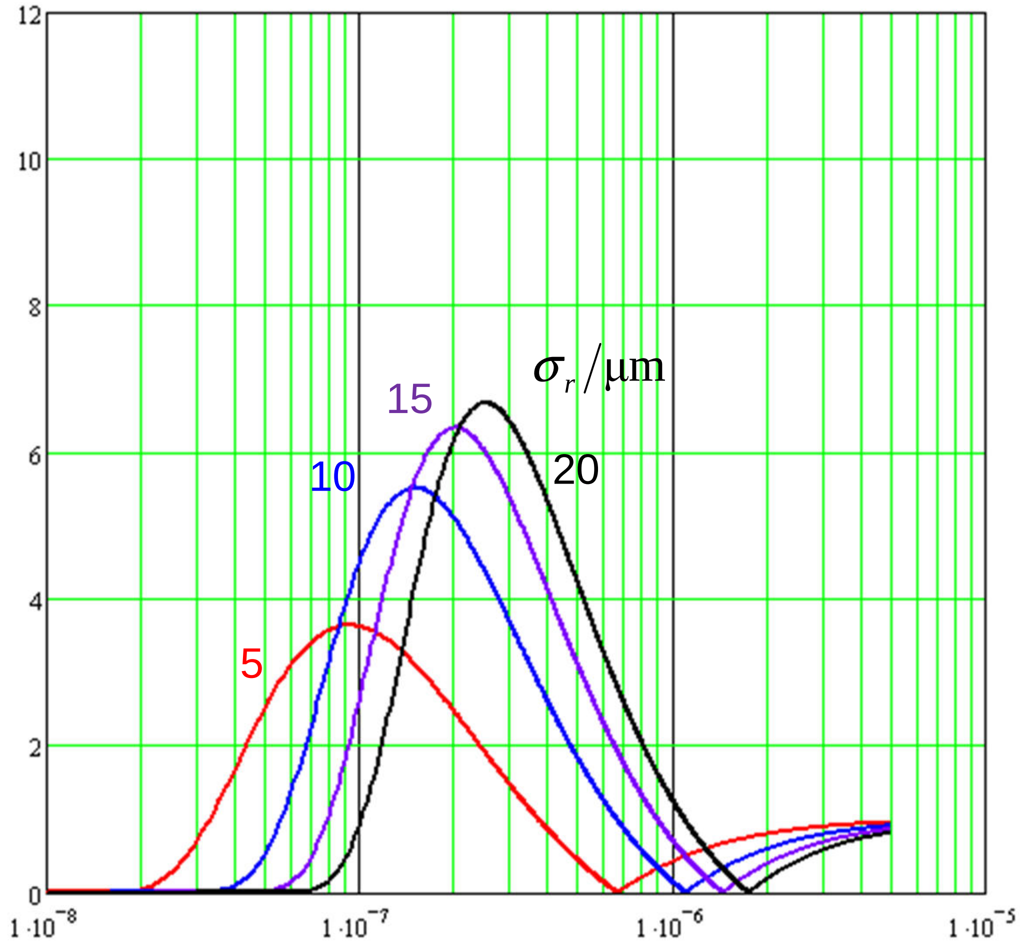
$\sigma_r = 10\,\mu\text{m} \quad L = 1\,\text{m} \quad \hat{I} \approx 3200\,\text{A}$



number of stages to saturation $\dfrac{\ln\left(\sqrt{\dfrac{\hat{I}\lambda_{\max}}{ce}}\right)}{\ln(G_{\max})} =$ 3.9 3.6 3.5 3.5

Linear One Stage Model (Discrete)

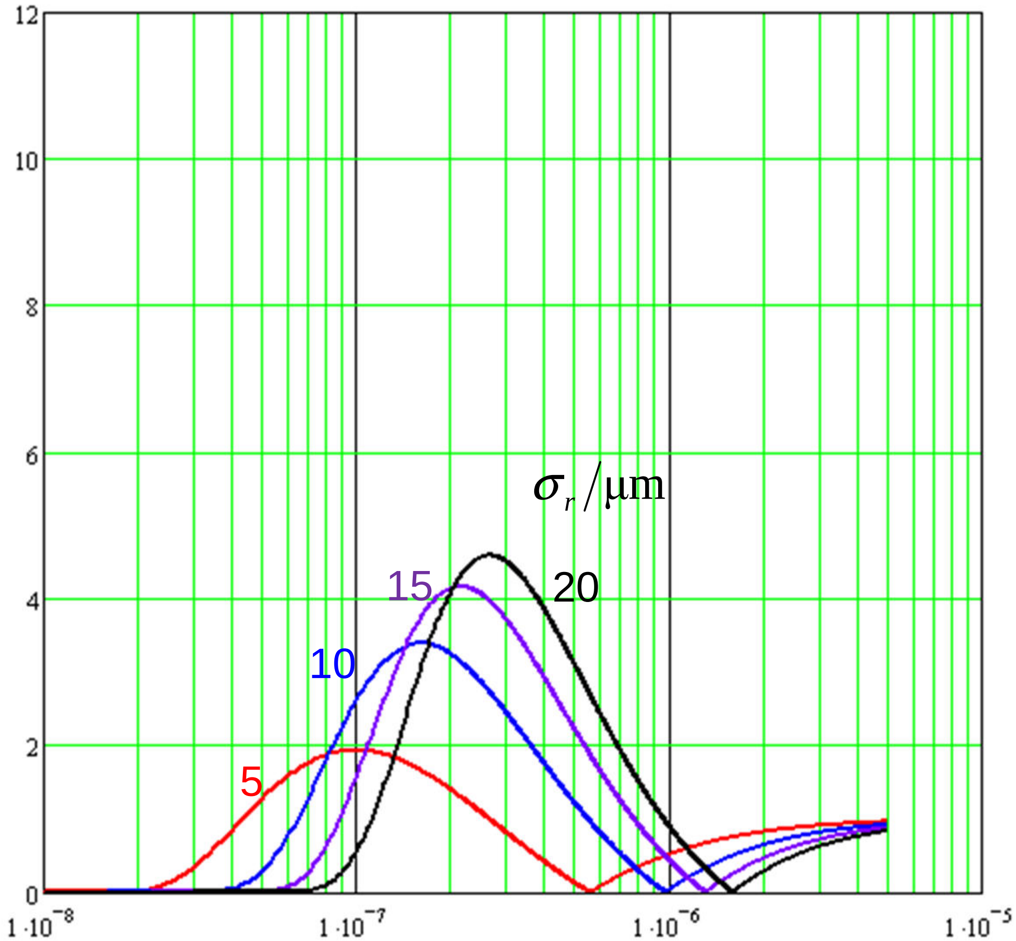
$\sigma_r = 15\,\mu\text{m} \quad L = 1\,\text{m} \quad \hat{I} \approx 3200\,\text{A}$



number of stages to saturation $\dfrac{\ln\left(\sqrt{\dfrac{\hat{I}\lambda_{\max}}{ce}}\right)}{\ln(G_{\max})} =$ 6.1 4.7 4.4 4.4

Linear One Stage Model (Discrete)

$\sigma_r = 20\,\mu\text{m} \quad L = 1\,\text{m} \quad \hat{I} \approx 3200\,\text{A}$



number of stages to saturation $\dfrac{\ln\left(\sqrt{\dfrac{\hat{I}\lambda_{\max}}{ce}}\right)}{\ln\left(G_{\max}\right)} =$ 11.3 6.6 5.7 5.6

Linear One Stage Model (Discrete)

averaged beam radius in FODO lattice

f.i.

E = 300 MeV

Lq = 2 cm

Lp = 40 cm

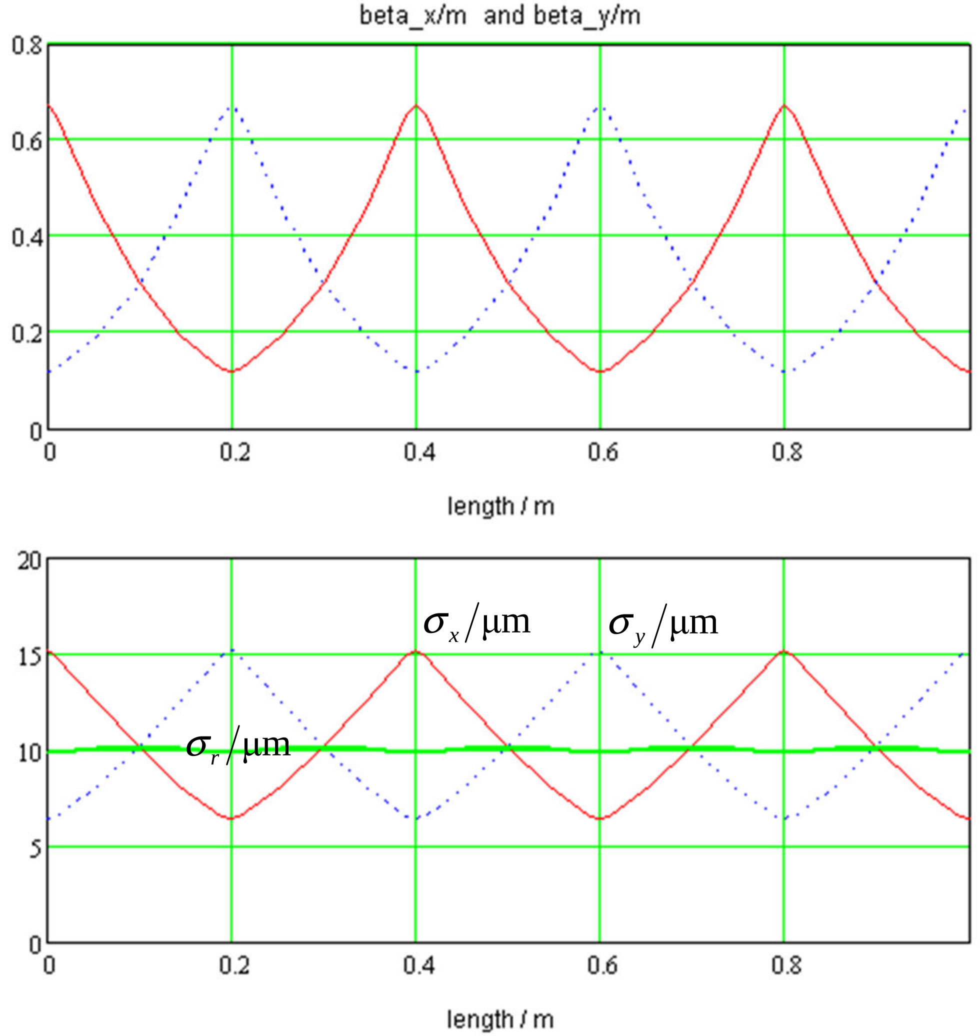
ψ = 90 deg

$$\sigma_r \approx \sqrt{\sigma_x \sigma_y}$$

$$\sigma_{r,av} = \frac{1}{L_p} \int_0^{L_p} \sigma_r(z) dz \approx 10\,\mu\text{m}$$

$$\beta_{r,av} = \frac{\sigma_{r,av}}{\varepsilon} \approx 0.3\,\text{m}$$

$$\frac{L_p}{\psi} \approx 0.26\,\text{m}$$



Linear One Stage Model (Discrete)

averaged beam radius in FODO lattice

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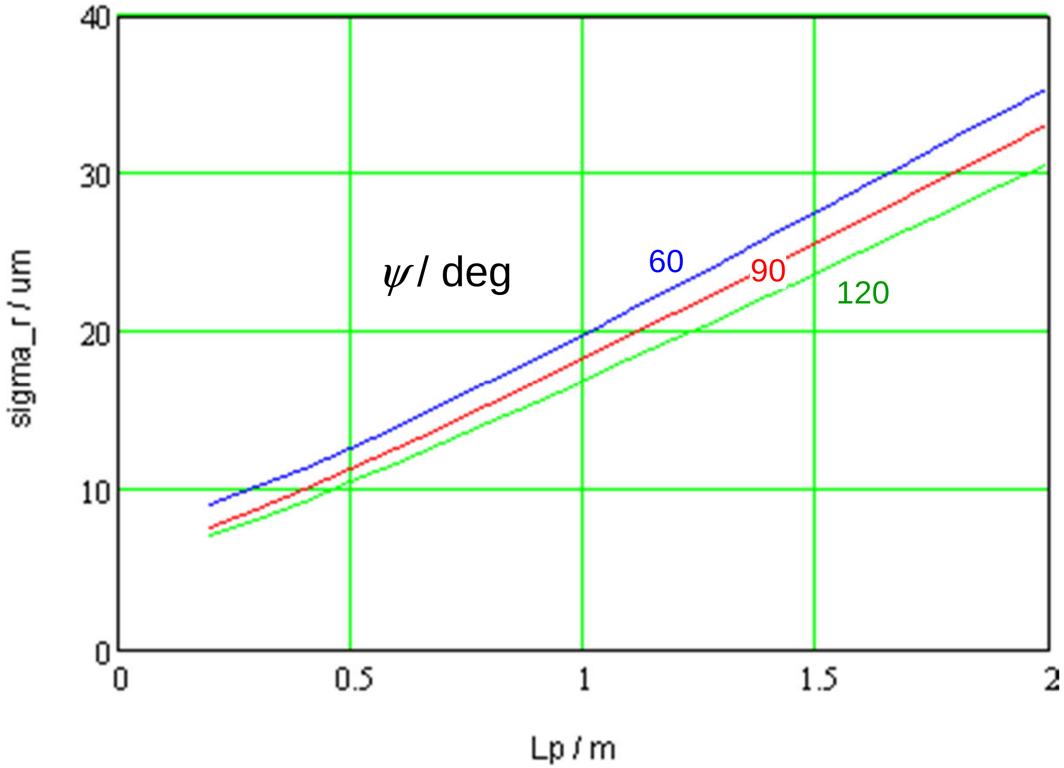
E = 300 MeV

Lq = 2 cm



wavelength of impedance maximum and maximal value scale with $1/\sigma_r$

but $\delta\psi / \delta\mathcal{E}$ scales with phase-advance ψ

$\psi = 90\,\mathrm{deg}$ $\quad L_p = 0.4\,\mathrm{m}, 0.6\,\mathrm{m}, 0.8\,\mathrm{m},$ **in the following**

Plasma Oscillations

linac length of one plasma oscillation (1D theory)

$$L_{\text{plasma}} \approx \frac{2\pi c}{\sqrt{\frac{e}{m_e} \frac{\omega |Z'(\omega)|}{\gamma^3} \frac{I}{c}}} \qquad \text{with} \quad Z' \approx -i \frac{Z_0}{4\pi} \frac{1}{\gamma \sigma_r} \qquad \text{for} \quad \lambda \approx \lambda_r = \frac{2\pi\sigma_r}{\gamma}$$

$$L_{\text{plasma}} \approx 2\pi\sigma_r \sqrt{\gamma^3 \frac{I_{\text{A}}}{I}}$$

f.i. $\sigma_r = 15\,\mu\text{m}$ and $\lambda_r \approx 160\,\text{nm}$ $\hat{I} \approx 3.2\,\text{kA}$ $\rightarrow L_{\text{plasma}} \approx 3\,\text{m}$

length of FODO channel ~ L_{plasma} / 4

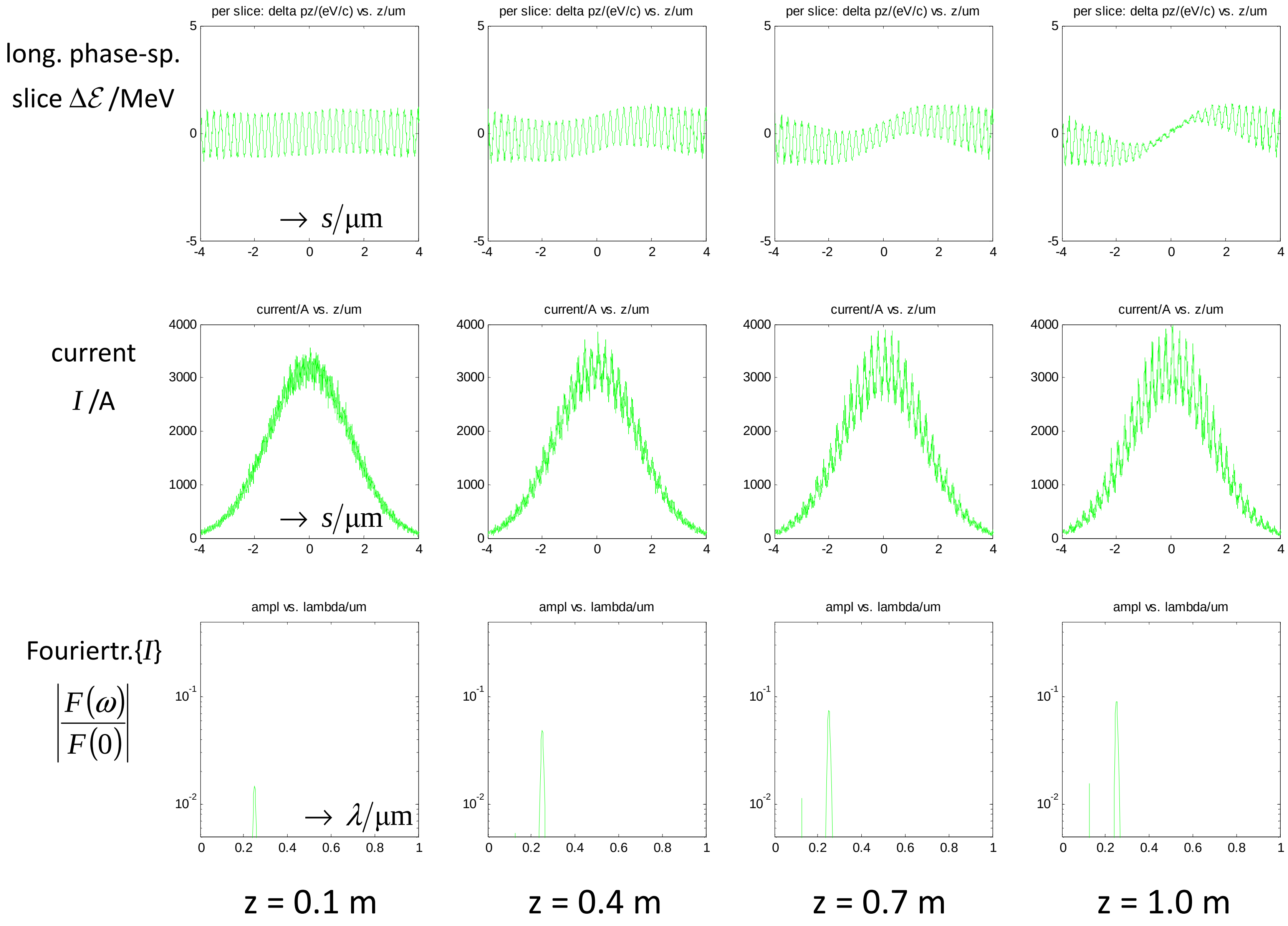
example with macro effects and non linarites:

moderate energy modulation

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 1 MeV**

Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 1 MeV**



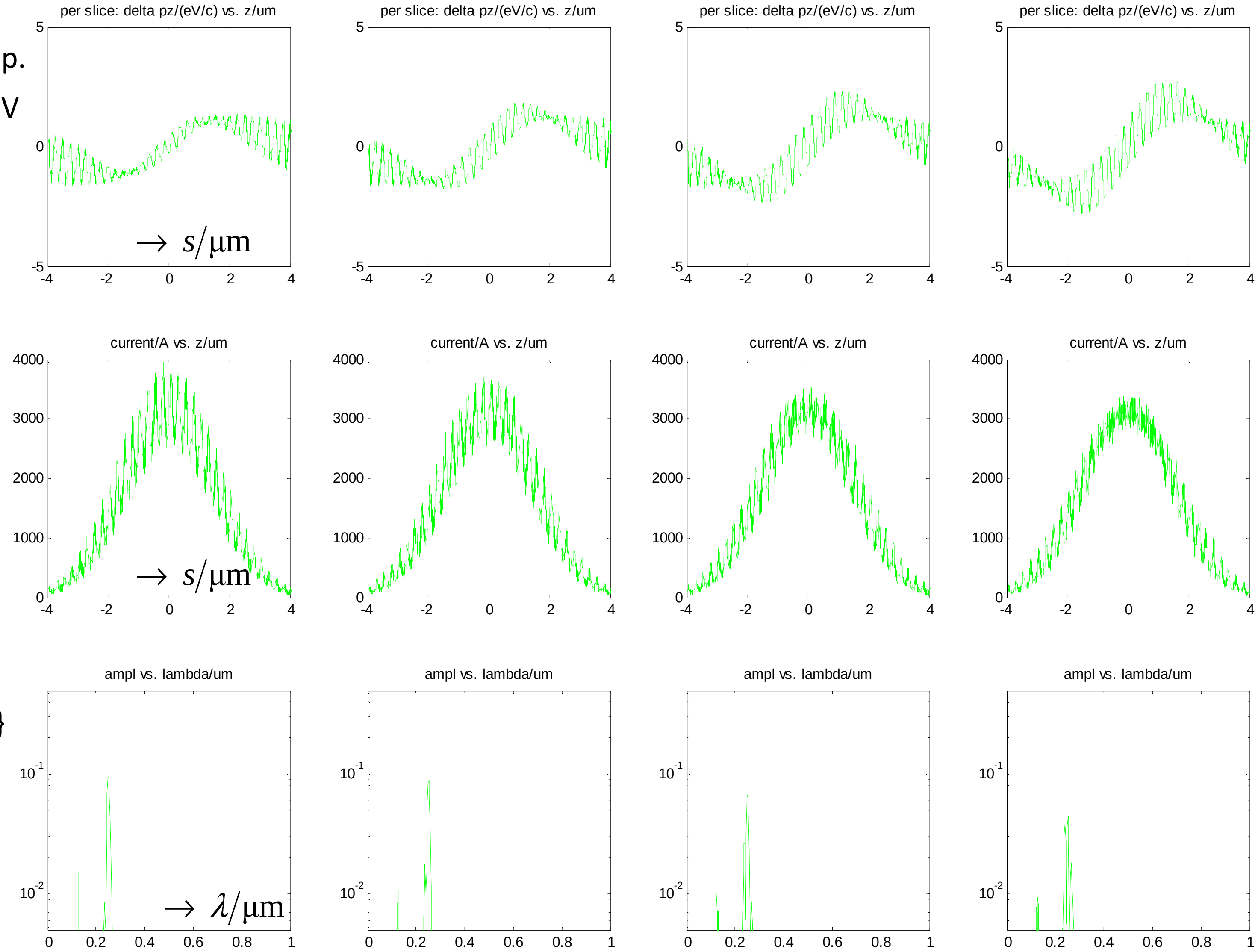
Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 1 MeV**

long. phase-sp. slice $\Delta\mathcal{E}$ /MeV

current I /A

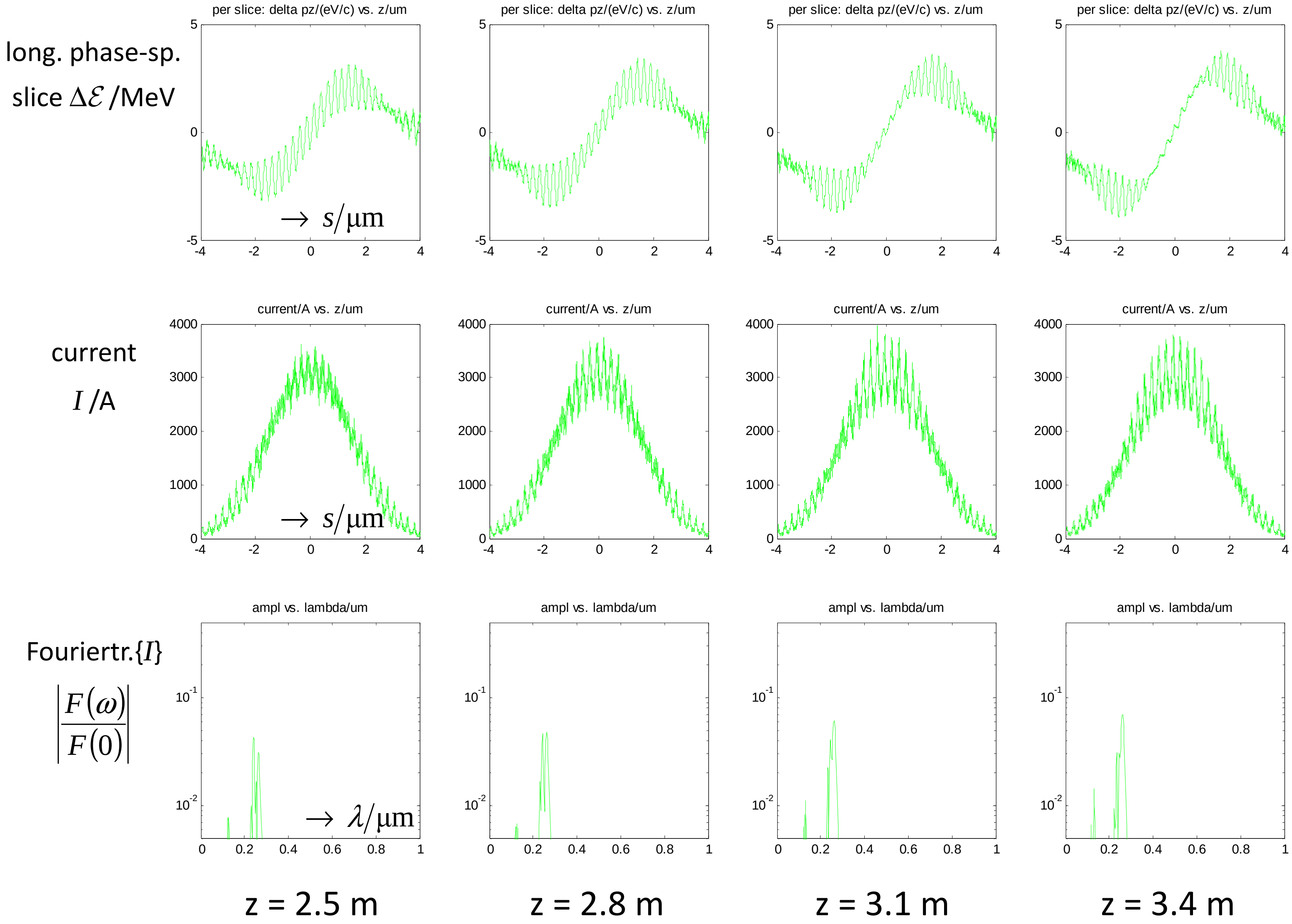
Fouriertr.$\{I\}$ $\left|\frac{F(\omega)}{F(0)}\right|$



z = 1.3 m | z = 1.6 m | z = 1.9 m | z = 2.2 m

Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 1 MeV**



Plasma Oscillations

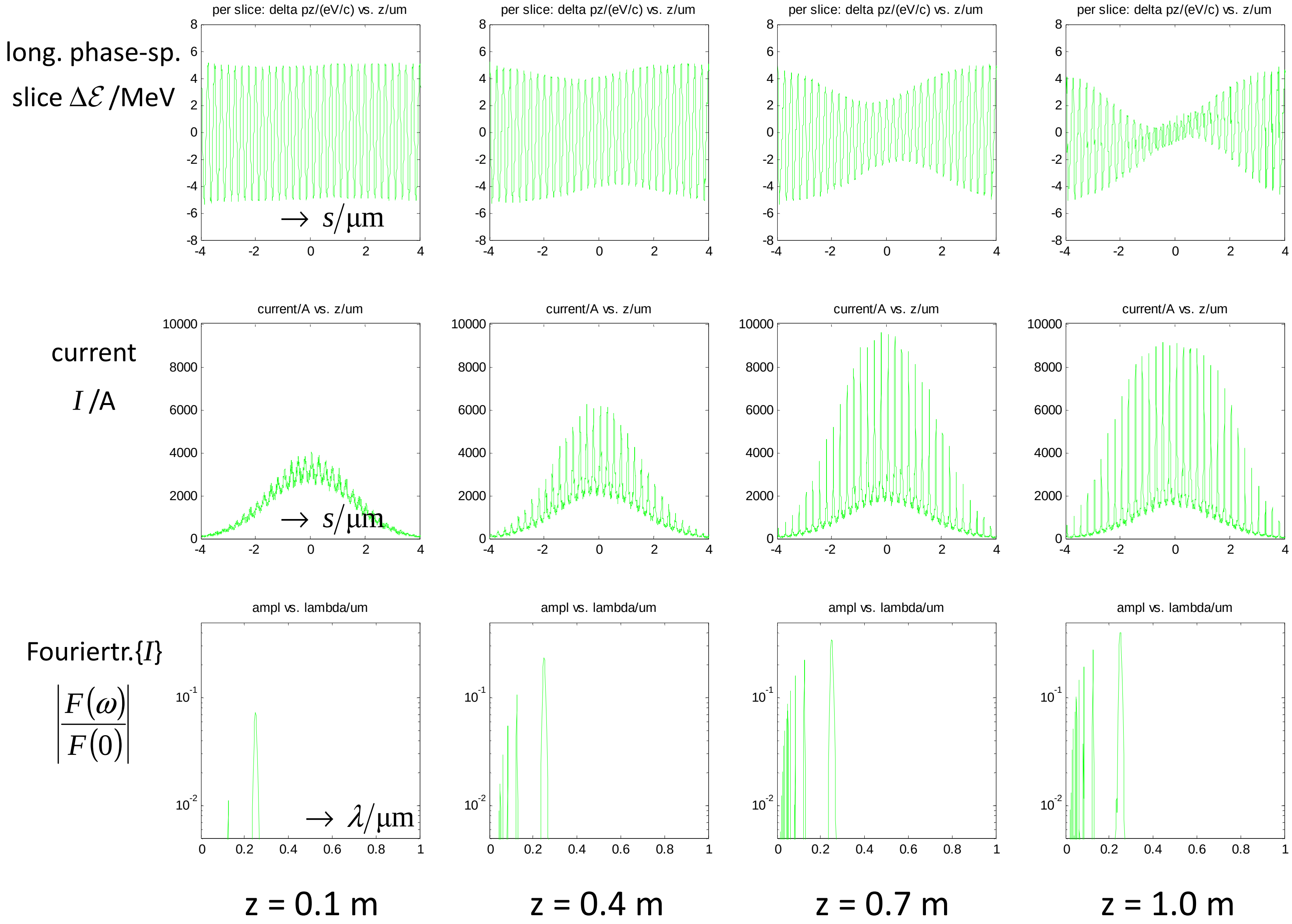
example with macro effects and non linarites:

strong energy modulation

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 5 MeV**

Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 5 MeV**



Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 5 MeV**

long. phase-sp.

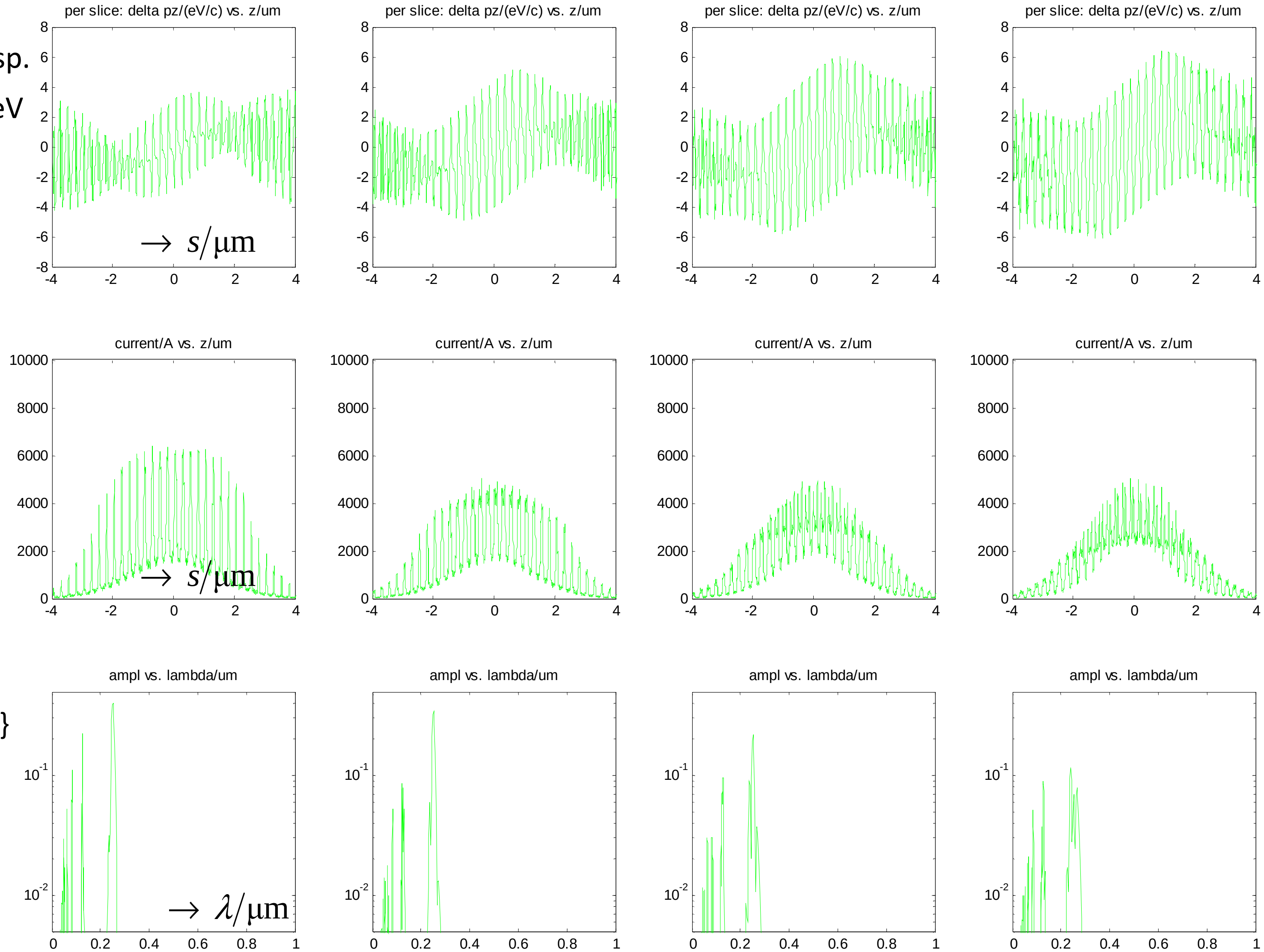
slice $\Delta\mathcal{E}$ /MeV

current

I /A

Fouriertr.$\{I\}$

$\left|\frac{F(\omega)}{F(0)}\right|$



z = 1.3 m | z = 1.6 m | z = 1.9 m | z = 2.2 m

Plasma Oscillations

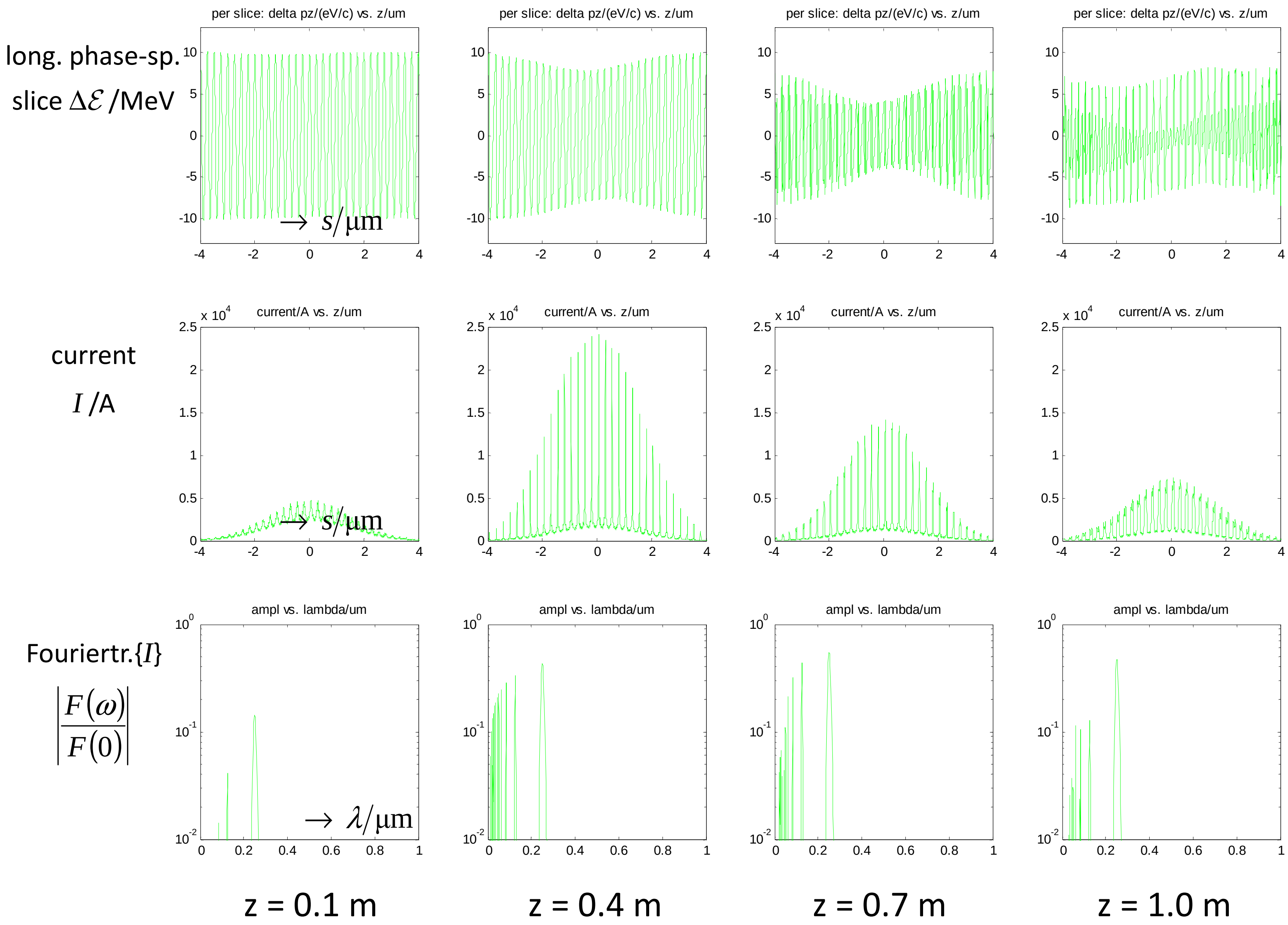
example with macro effects and non linarites:

very strong energy modulation

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 10 MeV**

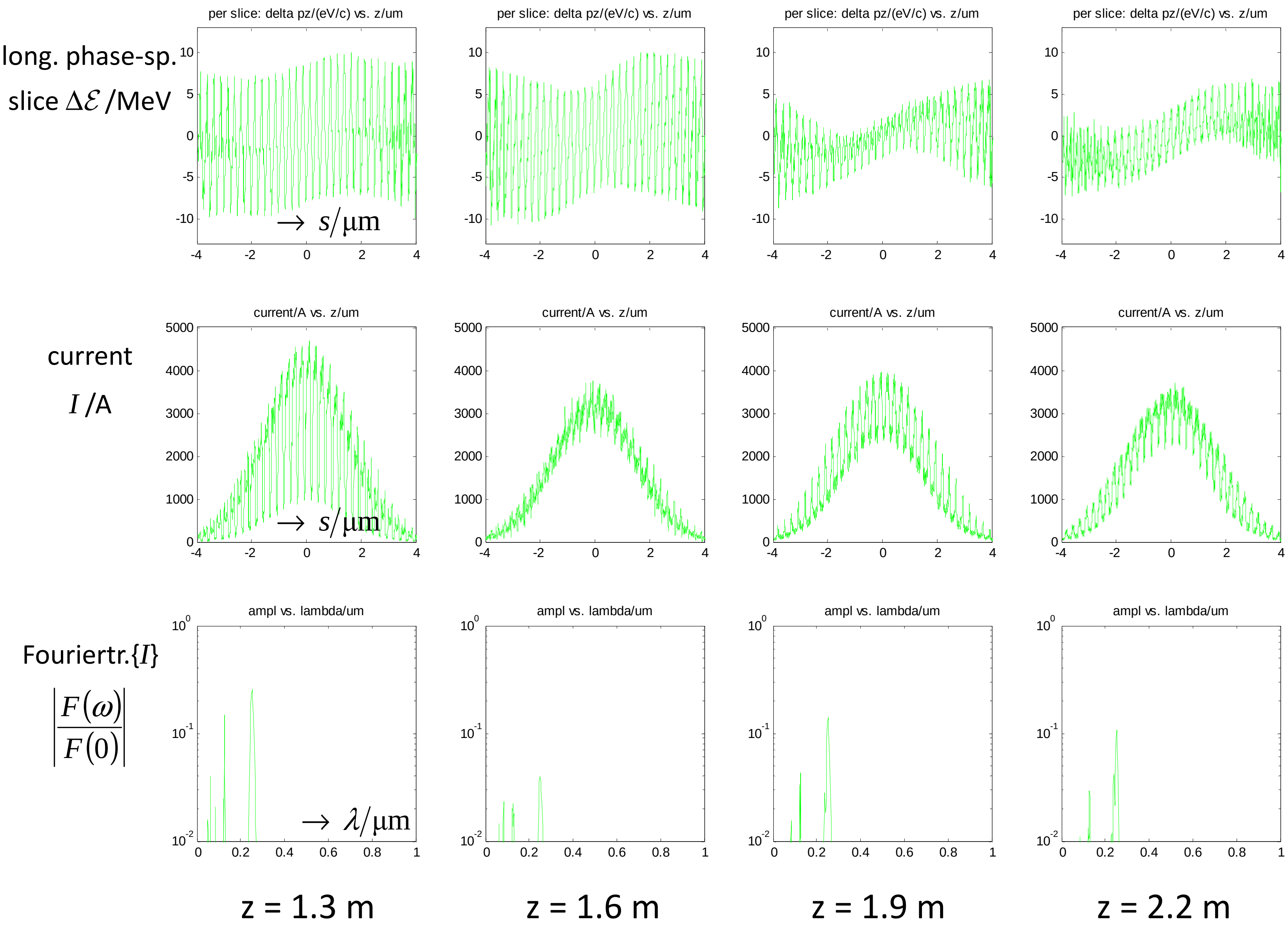
Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 10 MeV**



Plasma Oscillations

FODO Lattice Lp=0.6m, 90deg, **initial energy modulation with λ = 250 nm, ΔE = 10 MeV**



BCs in FODO Structure

real particels: 40 pC $\longrightarrow$ **250×10^6** electrons

CSRtrack simulation: **10^6** macro particles

shot noise is increased by factor $\sqrt{250} \approx 16$

saturation too early

but **all** stages have to be simulated (right macro effect)

solutions: quiet start technique: difficult for large band-with !!!

reduce gain of micro amplification in some stages

linear gain not quite right

$G(\omega) \approx (G_1(\omega))^6$ (high gain approximation)

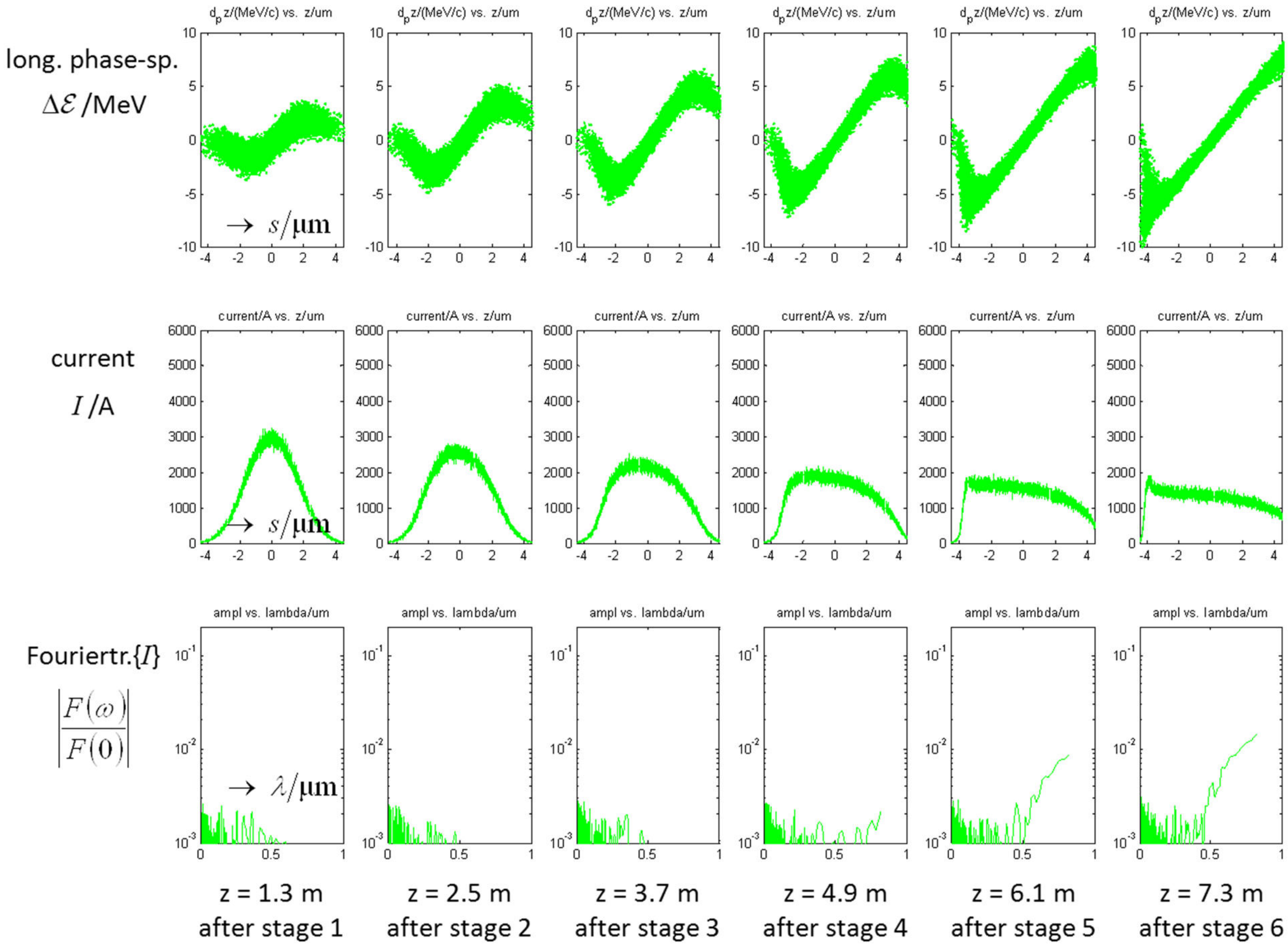
f.i.: $G(\omega) \approx \sqrt{250}(G_1(\omega))^4$ (h.g. in 4 stages)

more particles ($\longrightarrow$ 250×10^6): next attempt

for the following: suppressed μ effects in stages 1 and 2

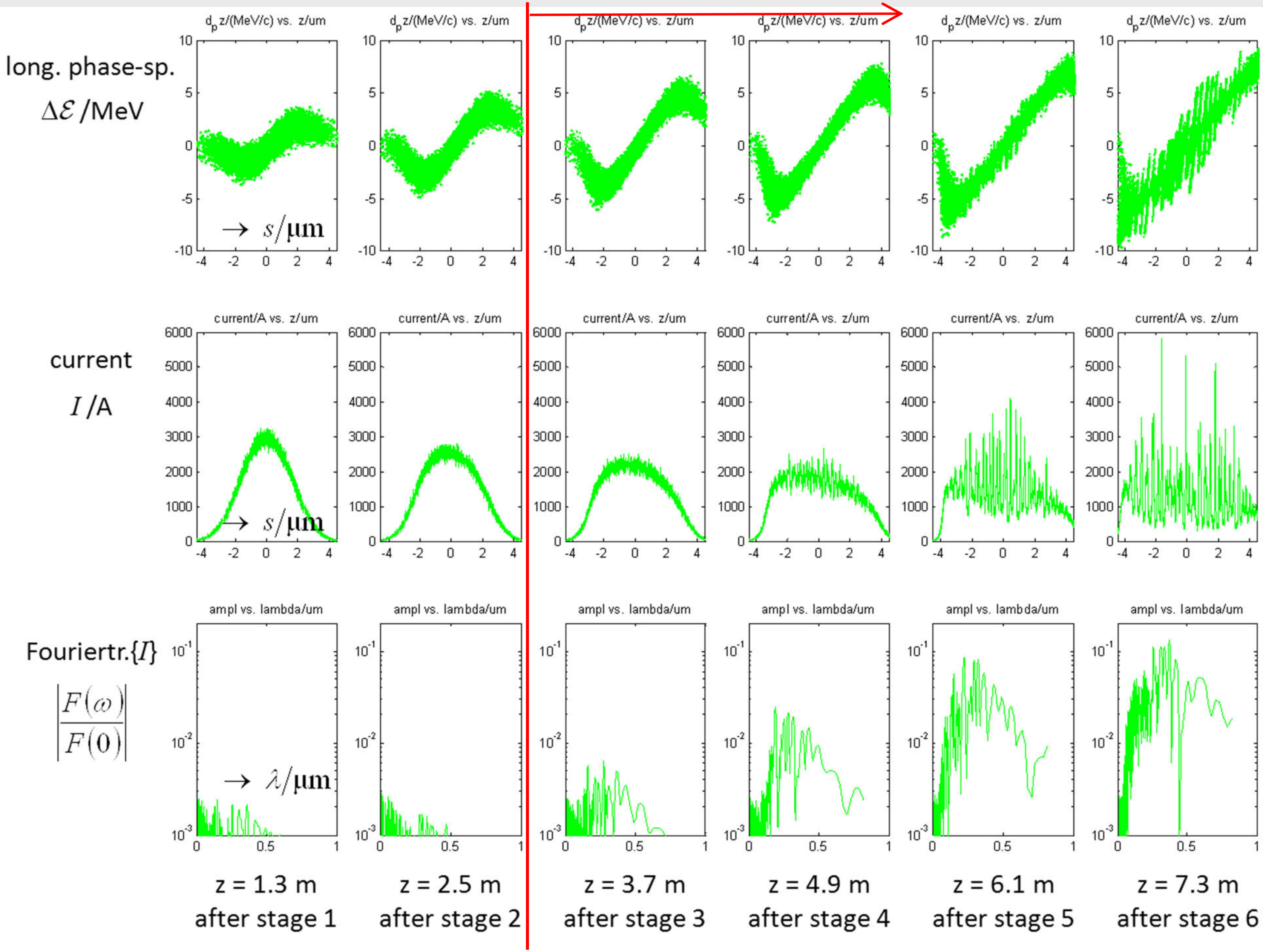
BCs in FODO Structure: µ-effects suppressed

FODO Lattice Lp=0.6m, 90deg, **6 stages, each = 1.5 FODO periods + 0.5 FODO period with BC (r56 = 18.9 µm)**



BCs in FODO Structure: with µ effects in stage 3..6

FODO Lattice Lp=0.6m, 90deg, **6 stages, each = 1.5 FODO periods + 0.5 FODO period with BC (r56 = 18.9 µm)**



BCs in FODO Structure

the next 4 slides: always 6 stages, always after stage 6
followed by a closer look to cases 1 & 2

FODO: Lp=0.6m, 90 deg
between compressors: 1.5 FODO periods $\rightarrow L_{sc}$ = 0.9 m, L_{tot} = 7.3 m (with BCs)

r56 = 18.9 μm	r56 = 14.0 μm	r56 = 10.8 μm	r56 = 8.6 μm

FODO: Lp=0.6m, 90 deg
between compressors: 2.5 FODO periods $\rightarrow L_{sc}$ = 1.5 m, L_{tot} = 10.9 m (with BCs)

r56 = 18.9 μm	r56 = 14.0 μm	r56 = 10.8 μm [1]	r56 = 8.6 μm

FODO: Lp=0.4m, 90 deg
between compressors: 1.5 FODO periods $\rightarrow L_{sc}$ = 0.6 m, L_{tot} = 4.9 m (with BCs)

r56 = 18.9 μm	r56 = 14.0 μm	r56 = 10.8 μm	r56 = 8.6 μm

FODO: Lp=0.4m, 90 deg
between compressors: 2.5 FODO periods $\rightarrow L_{sc}$ = 1.0 m, L_{tot} = 7.3 m (with BCs)

r56 = 18.9 μm	r56 = 14.0 μm	r56 = 10.8 μm	r56 = 8.6 μm [2]

BCs in FODO Structure

FODO: Lp=0.6m, 90deg
6 x {1.5 FODO periods + 0.5 FODO period with BC}
r56 = 18.9 .. 8.6 µm

after stage 6
z=7.3m

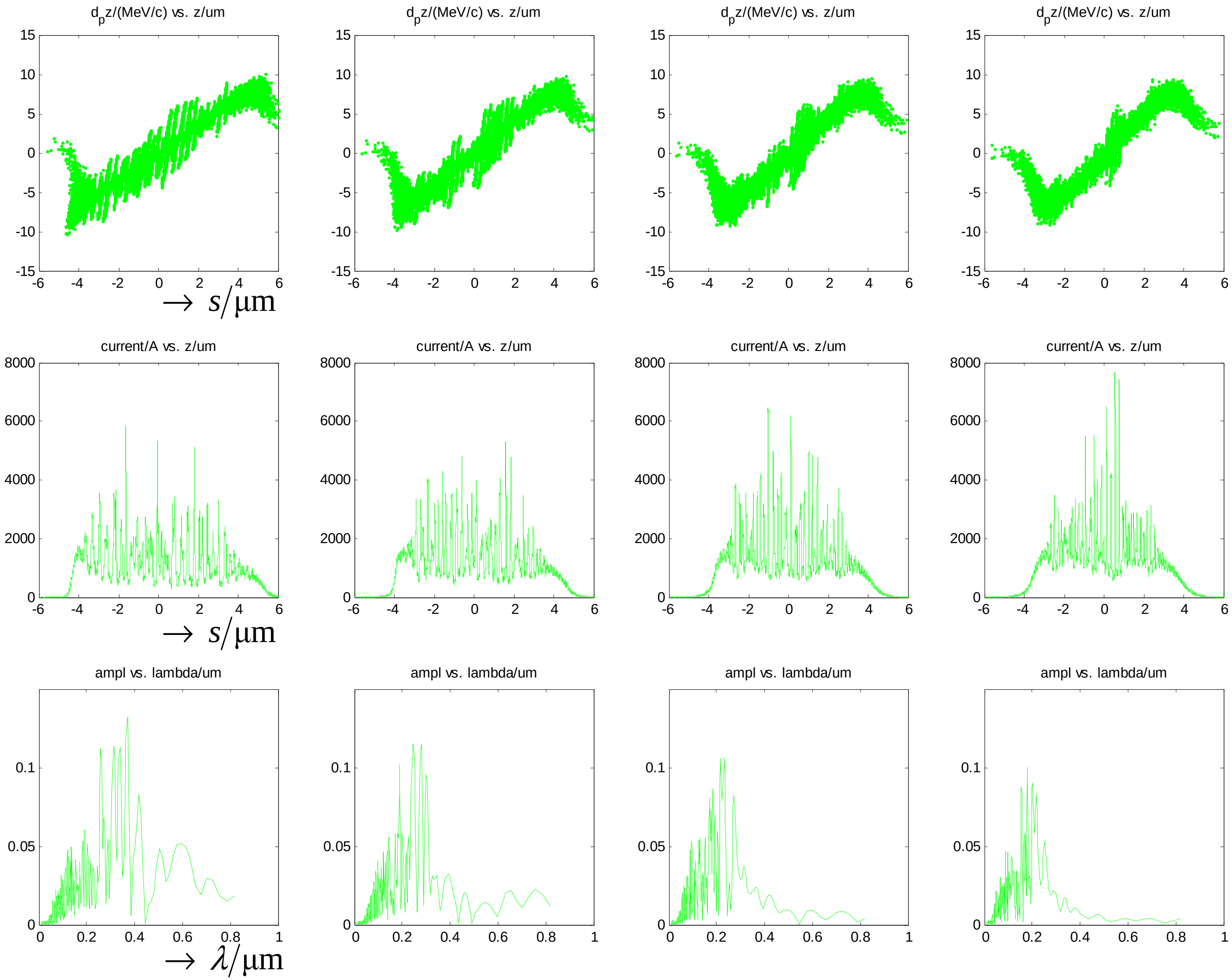
$\Delta\mathcal{E}$ /MeV

current

I /A

Fouriertr.$\{I\}$

$$\left|\frac{F(\omega)}{F(0)}\right|$$



r56 = 18.9 µm

r56 = 14.0 µm

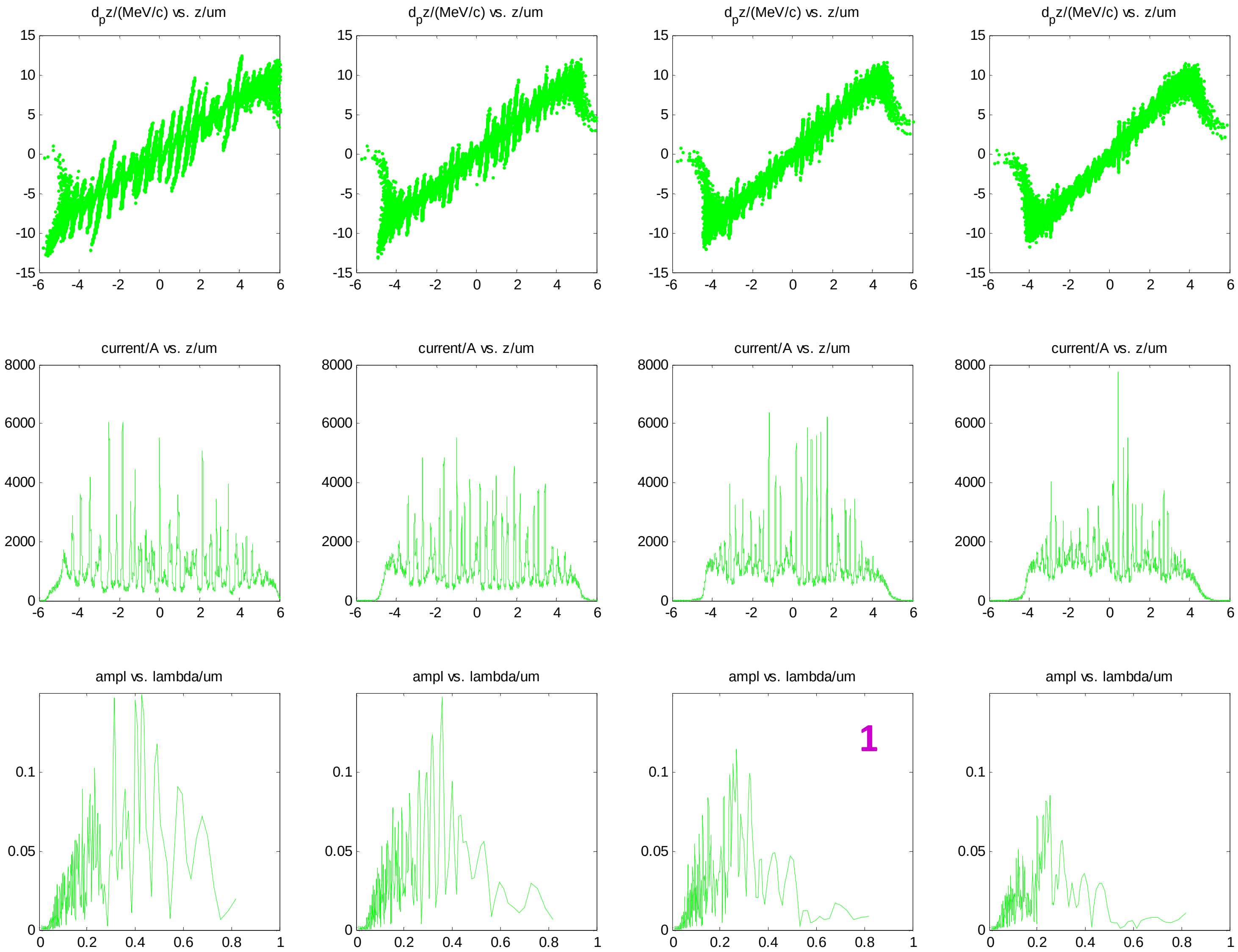
r56 = 10.8 µm

r56 = 8.6 µm

BCs in FODO Structure

FODO: Lp=0.6m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 18.9 .. 8.6 µm

after stage 6
z=10.9m

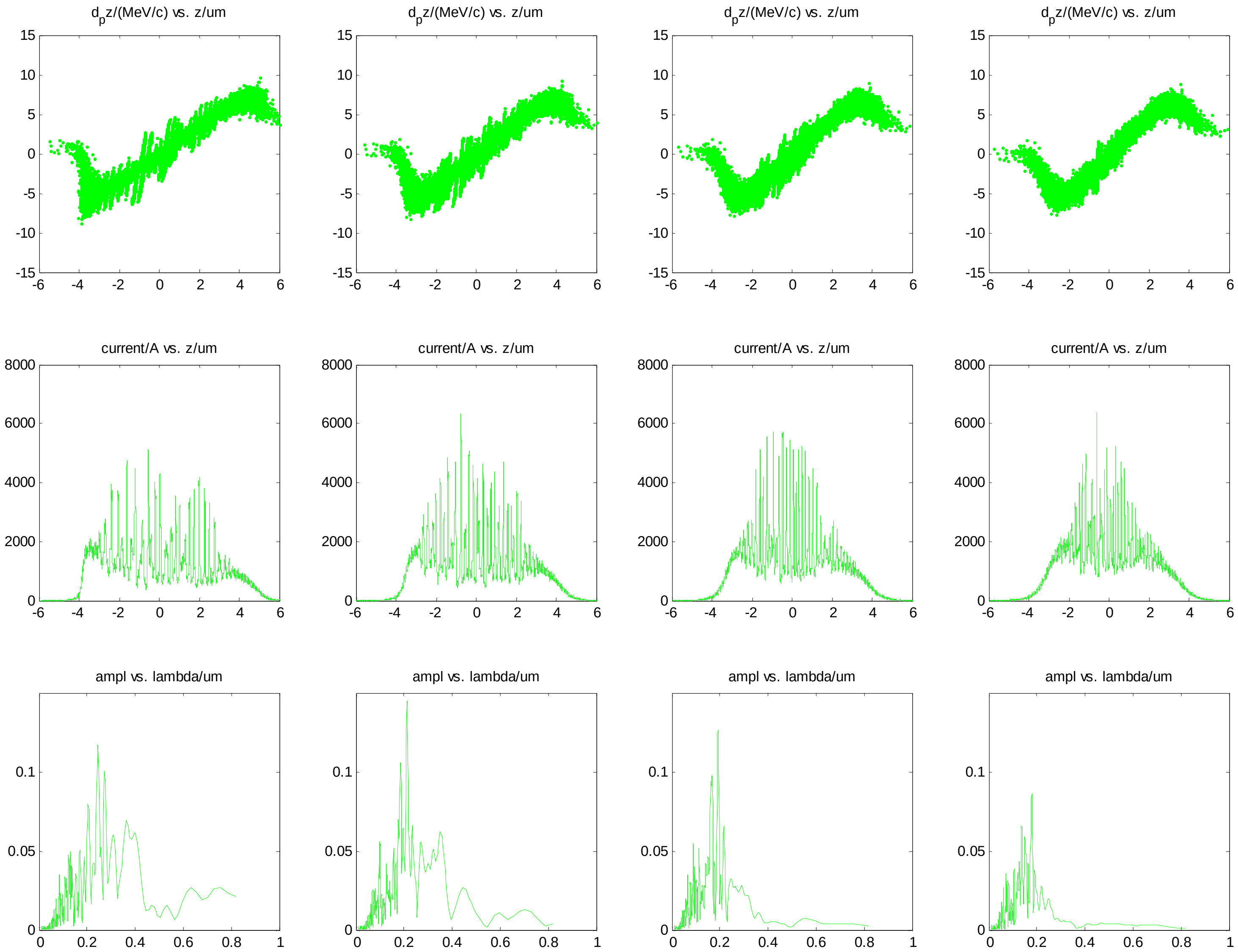


r56 = 18.9 µm   r56 = 14.0 µm   r56 = 10.8 µm   r56 = 8.6 µm

BCs in FODO Structure

FODO: Lp=0.4m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 18.9 .. 8.6 µm

after stage 6
z=4.9m

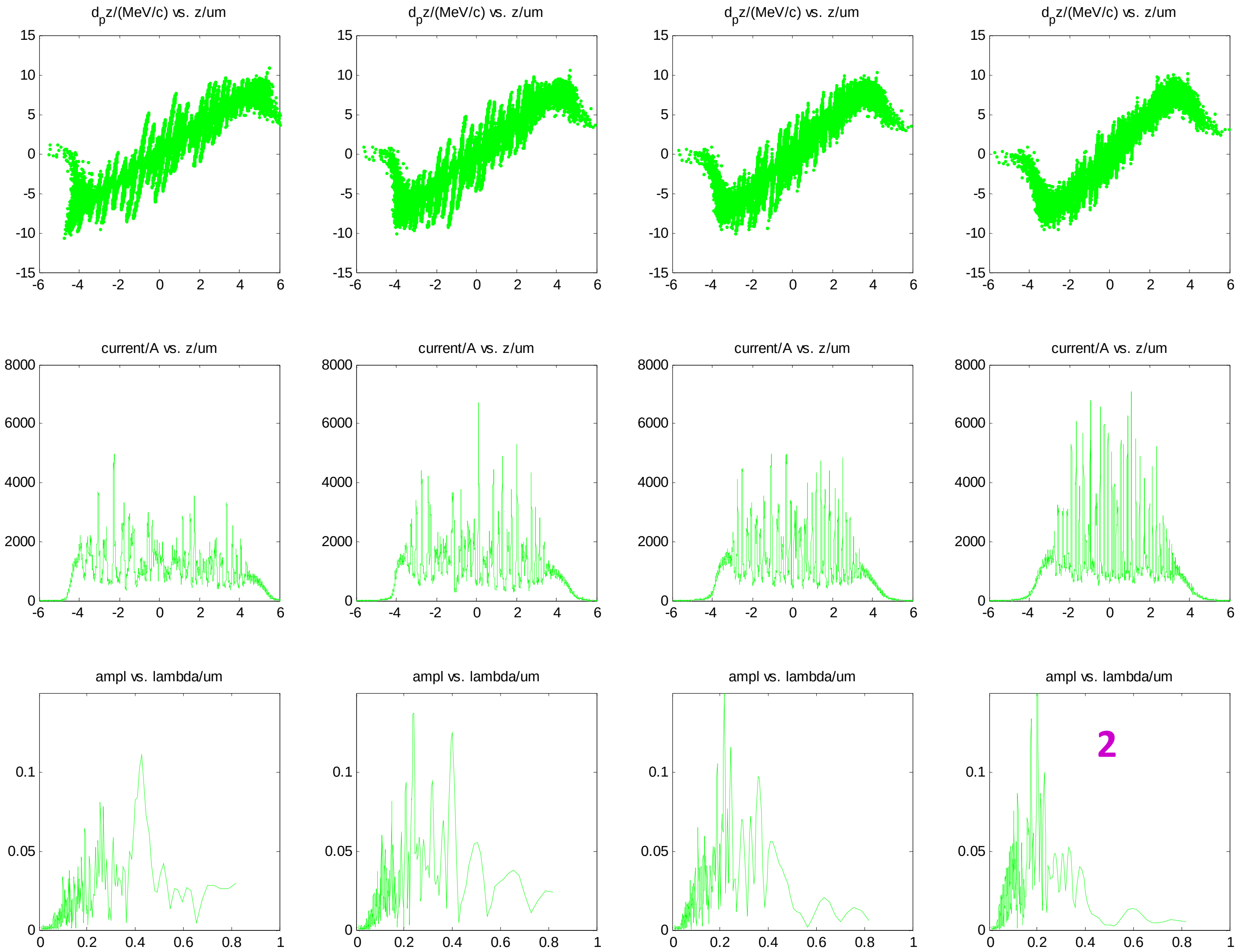


r56 = 18.9 µm | r56 = 14.0 µm | r56 = 10.8 µm | r56 = 8.6 µm

BCs in FODO Structure

FODO: Lp=0.4m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 18.9 .. 8.6 μm

after stage 6
z=7.3m



r56 = 18.9 μm

r56 = 14.0 μm

r56 = 10.8 μm

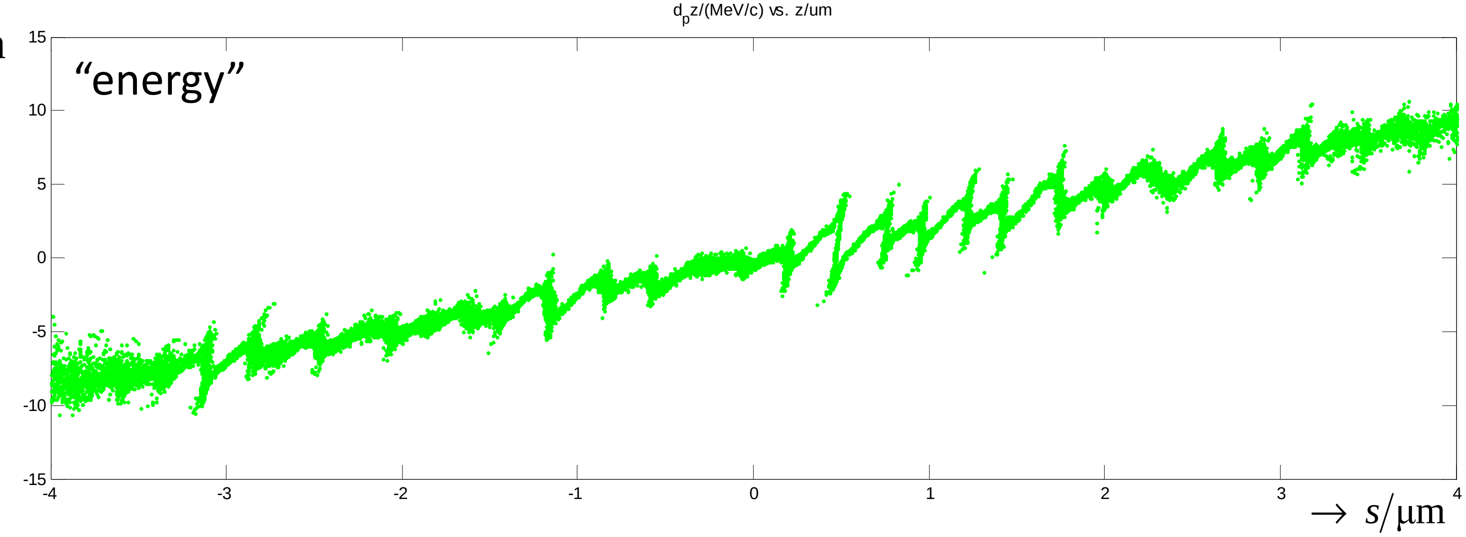
r56 = 8.6 μm

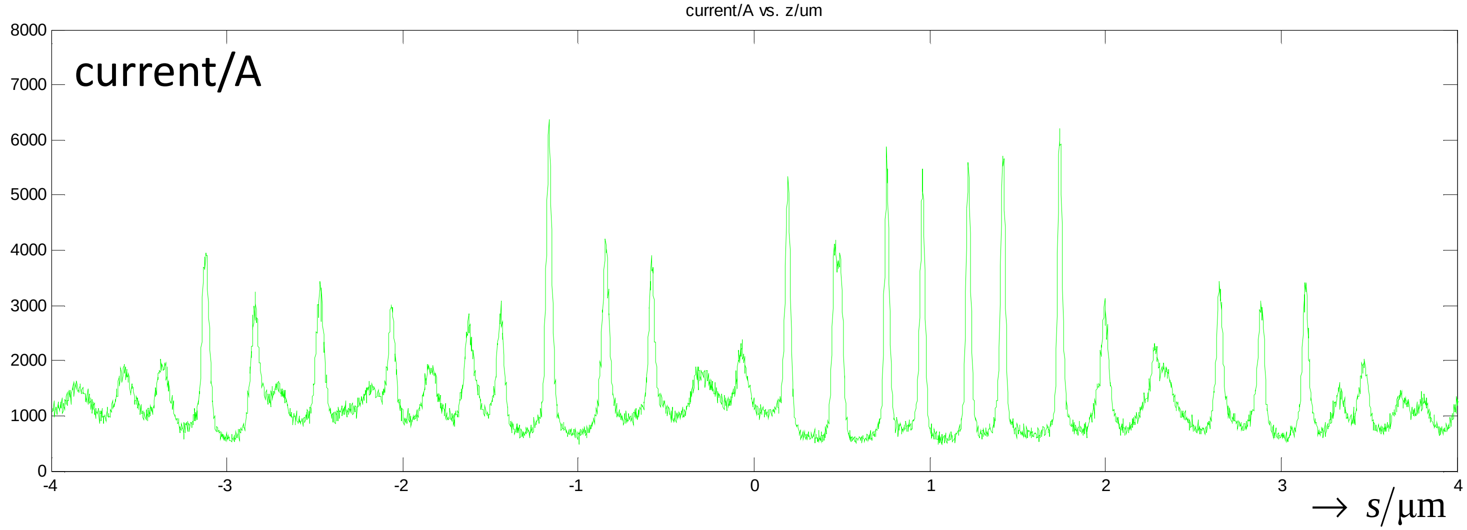
BCs in FODO Structure

1

FODO: Lp=0.6m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 10.8 µm

$L_{tot} = 10.9\ \mathrm{m}$



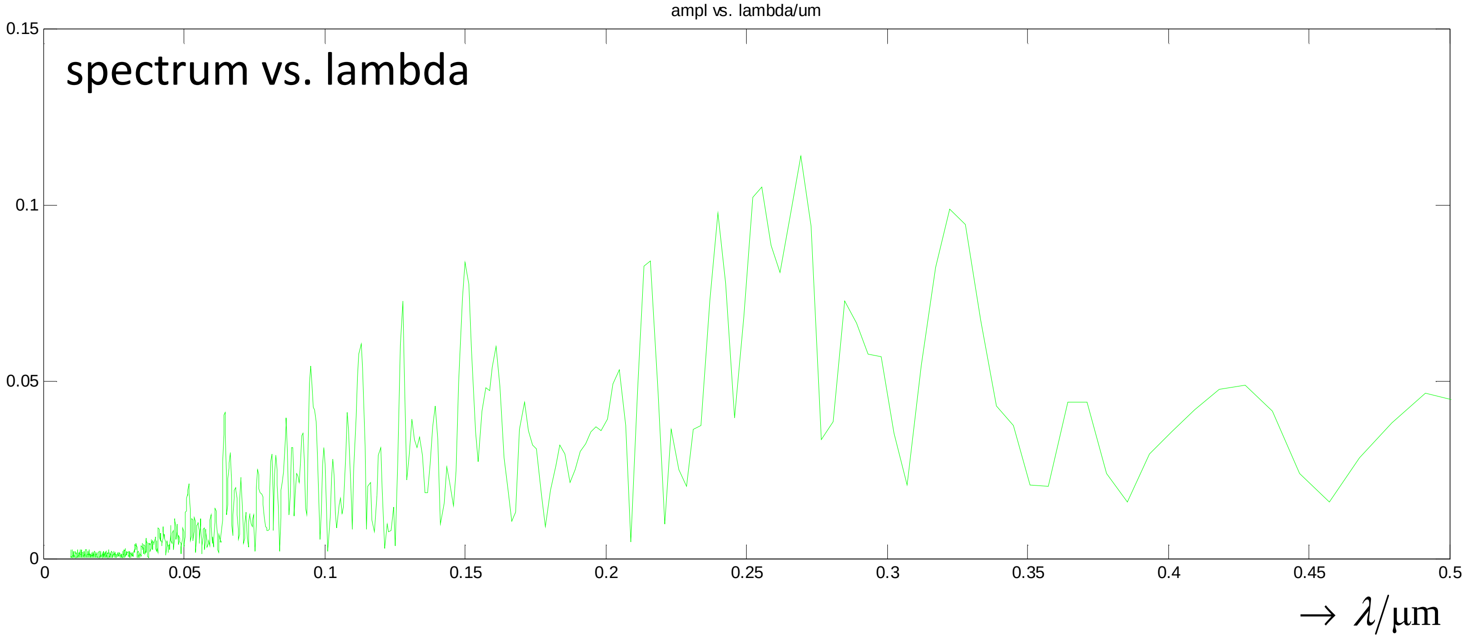


BCs in FODO Structure

1

FODO: Lp=0.6m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 10.8 µm

$L_{\text{tot}} = 10.9\ \text{m}$

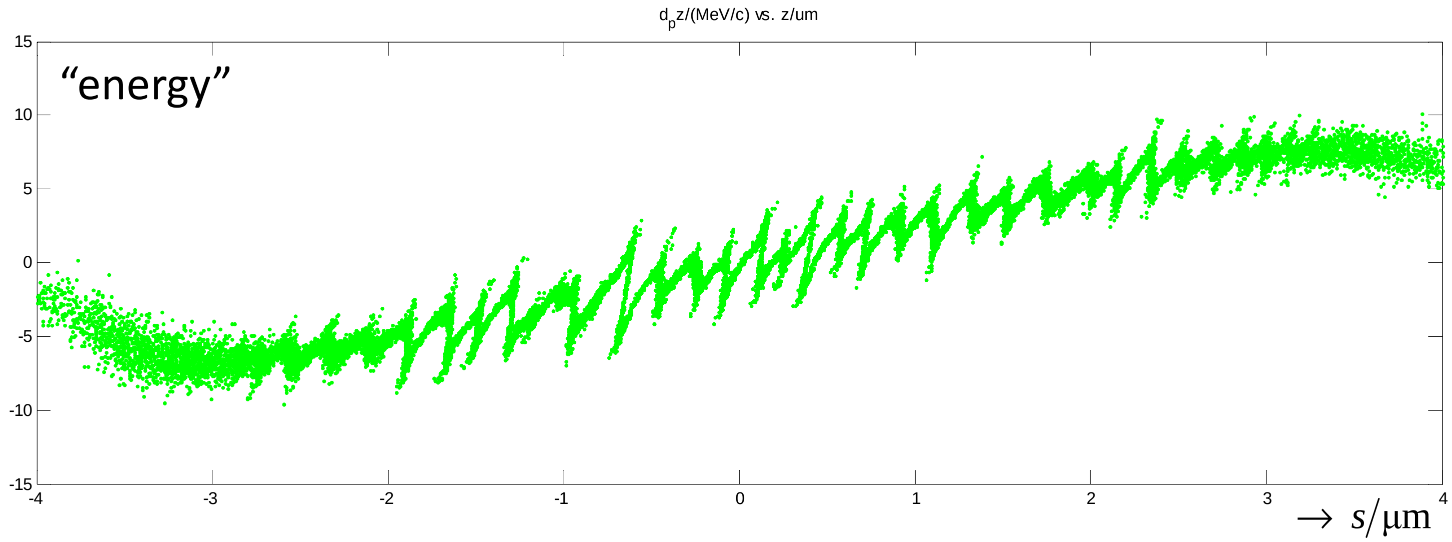


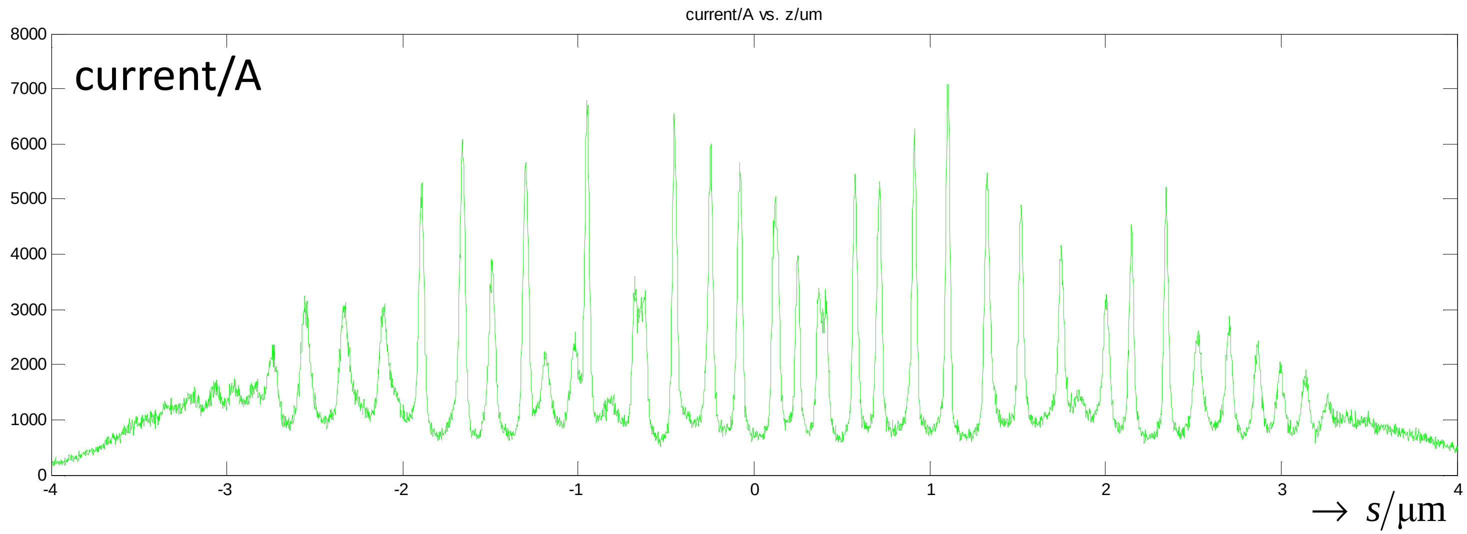
BCs in FODO Structure

2

FODO: Lp=0.4m, 90deg
6 x {2.5 FODO periods + 0.5 FODO period with BC}
r56 = 8.6 µm

$L_{\text{tot}} = 7.3\,\text{m}$





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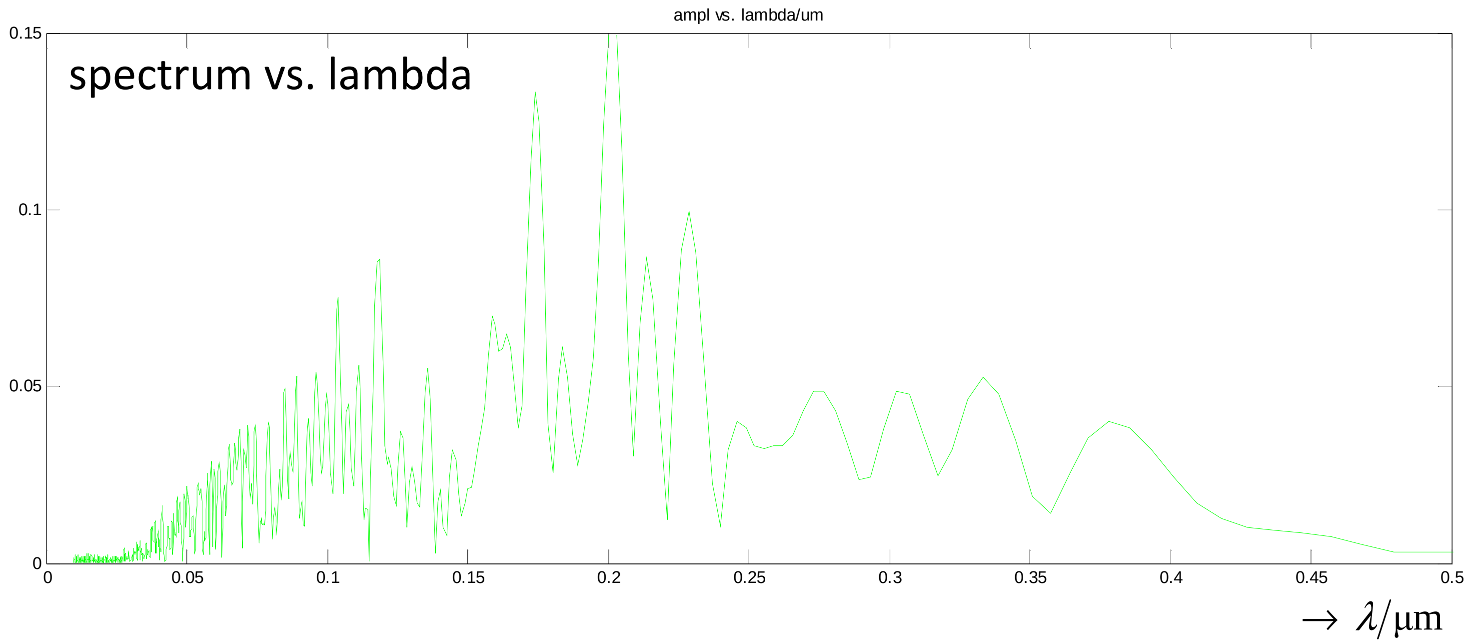

Summary and Conclusion

parameters from Florian Grüner for plasma acceleration

significant macro effects: induced chirp + lengthening

plasma oscillations: $L_{\text{plasma}}/4 \sim L_{\text{sc}}$

LGM: expected wavelength ~ 250 nm, required gain for sat. ~ 4000

promising: scheme with C-chicanes in FODO lattice, 6 stages
FODO: L_p = 0.4 .. 0.6 m, ψ = 90 deg, $L \approx$ 1 m
ultra short C-chicanes, $r_{56} \approx$ 10 µm

higher harmonics observed (simulated) ~100 nm and below

to be investigated:
- simulation of radiator
- matching from source to FODO lattice with SC
- more simulations with transverse SC (Astra)
- 250E6 particles
- electrodynamics beyond Poisson approach (and 1D CSR)
- no conclusions (so far) for FODO compressors