Macroparticle Noise Reduction in FMM with the Self-Consistent Plummer-Model Near Fields

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Structure

I. Macroparticle Noise

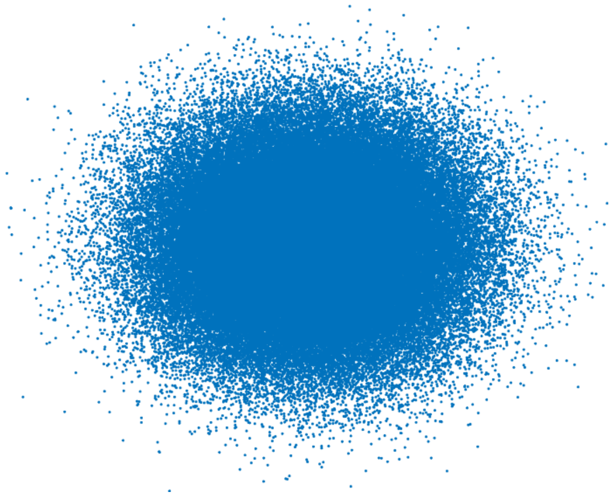
II. Self-Consistent Plummer Fields

III. Modified Near Field Interaction for FMM

IV. Summary and Outlook

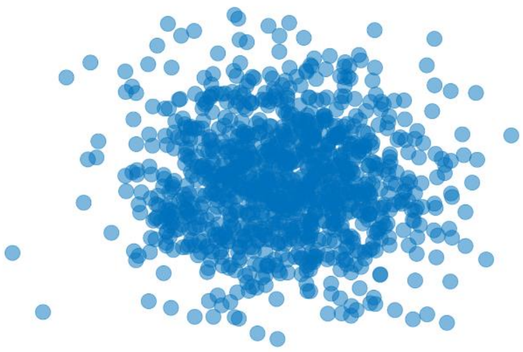
Macroparticle Noise
Macroparticle Approximation

Physical particles:



- Large particle number $N_\mathrm{p} \sim 10^{10}$
- Small particle charge $q_\mathrm{p} \sim 10^{-19}\ \mathrm{C}$
- Point-like particle interaction

Macroparticles :

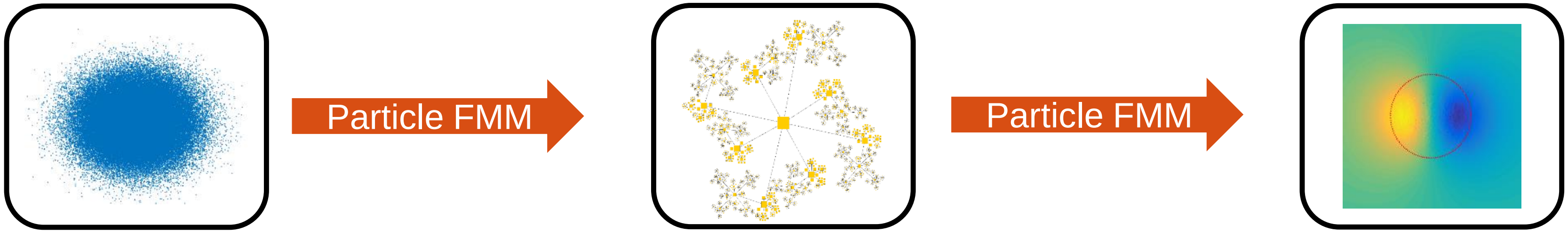


- Reduced particle number $\mathrm{N} \sim 10^{6}$
- Increased particle charge $q_j \sim 10^{-15}\ \mathrm{C}$
- Cloud-like particle interaction

⇒ Collective charge density and space charge field are spatially smooth functions

Macroparticle Noise

Macroparticle Model in FMM



FMM space charge field approximation of macroparticles j located in a tree node k

- Far field region: Multipole moments of a set of **point-like charges**

$$q_{l,m}^{j} \propto r_j^l \, {Y^*}_l^m(\vartheta_j, \varphi_j)$$

- Near field region: Interaction of **homogenously charged spheres** with radius Δ_0

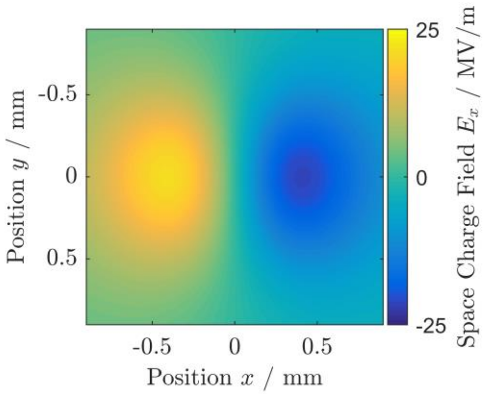
$$\boldsymbol{E}_{\mathrm{P2P}}(\boldsymbol{x}; \boldsymbol{x}_j, q_j) = \frac{q_j}{4\pi\epsilon_0} \frac{\boldsymbol{x}-\boldsymbol{x}_j}{\Delta^3} \text{ with } \Delta = \max\left(\left|\boldsymbol{x}-\boldsymbol{x}_j\right|, \Delta_0\right)$$

Macroparticle Noise
Optimization of Macroparticle Shape

Investigation of particle noise for a Gaussian charge density

$$\rho(\boldsymbol{x}) = \frac{q_0}{\left(\sqrt{2\pi}\sigma_r\right)^3} \, e^{-\frac{\boldsymbol{x}^2}{2\,\sigma_r^2}}$$

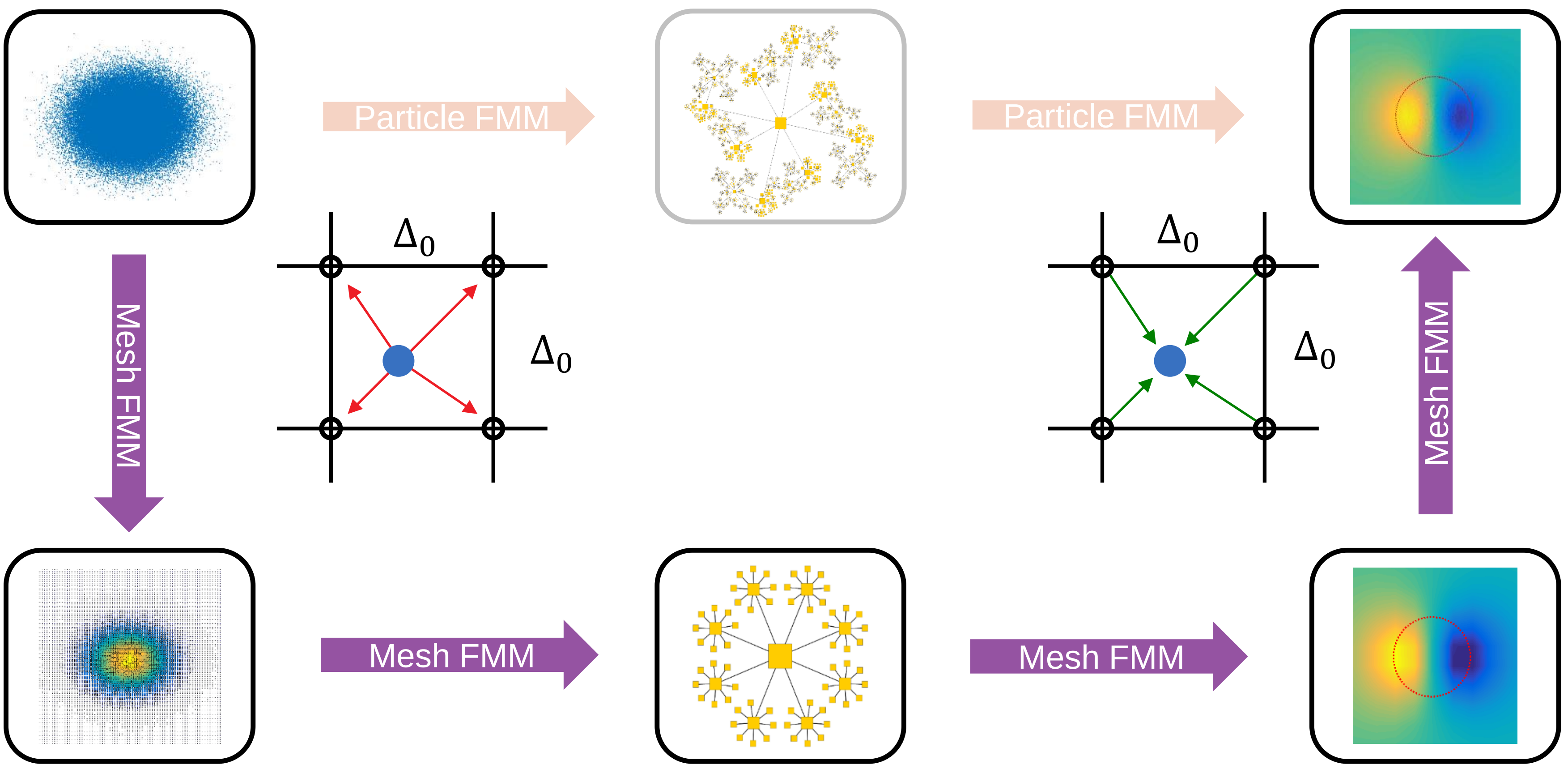
$$(\sigma_r = 0.3\text{ mm},\ q_0 = 1.0\text{ nC})$$



represented by $N = 10^6$ macroparticles with 3 different models:

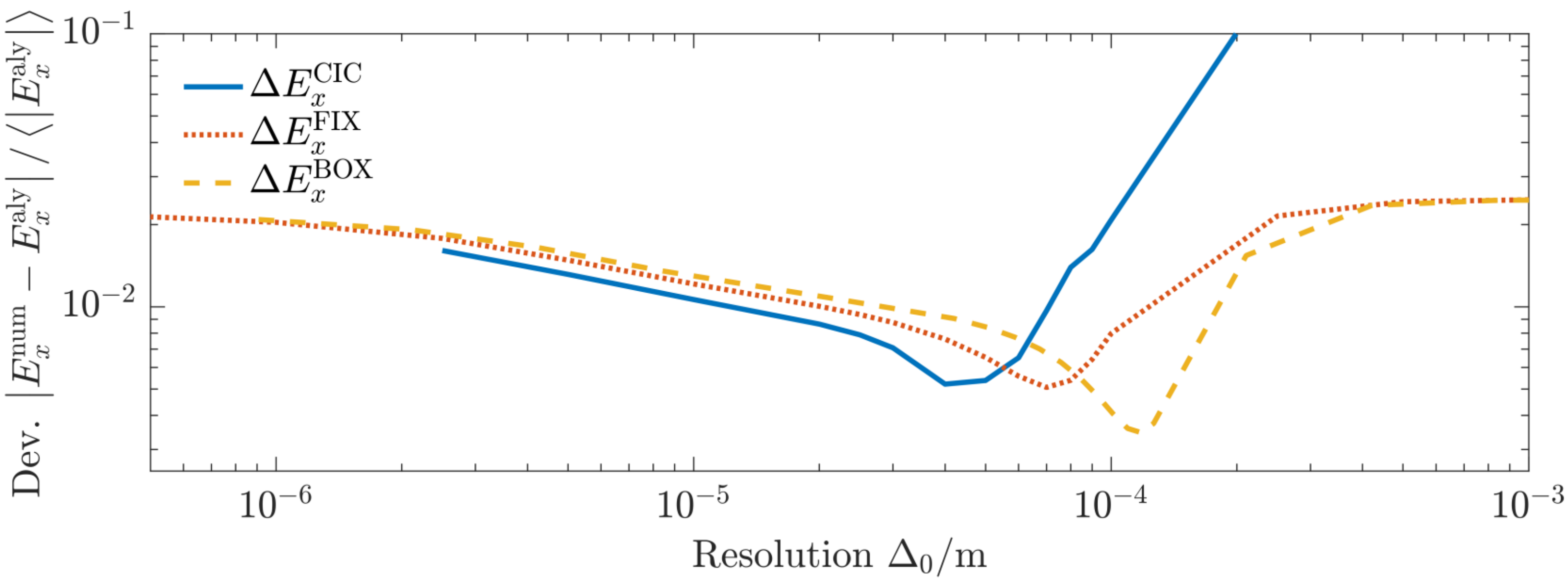
- E_x^{FIX} with fix Δ_0:
  Compute $\boldsymbol{E}_{\mathrm{P2P}}\left(\boldsymbol{x}; \boldsymbol{x}_j, q_j\right)$ with global value for radius Δ_0
- E_x^{BOX} with adaptive Δ_0:
  Macroparticle radius $\Delta_0 \propto r_k$ adapted to radial extension r_k of tree node k
- E_x^{CIC} with cloud-in-cell (CIC) interpolation of $\rho(\boldsymbol{x})$ on grid with spacing Δ_0

Macroparticle Noise
Optimization of Macroparticle Shape



Macroparticle Noise
Optimization of Macroparticle Shape

Deviation of macroparticle approximation from analytic field solution:

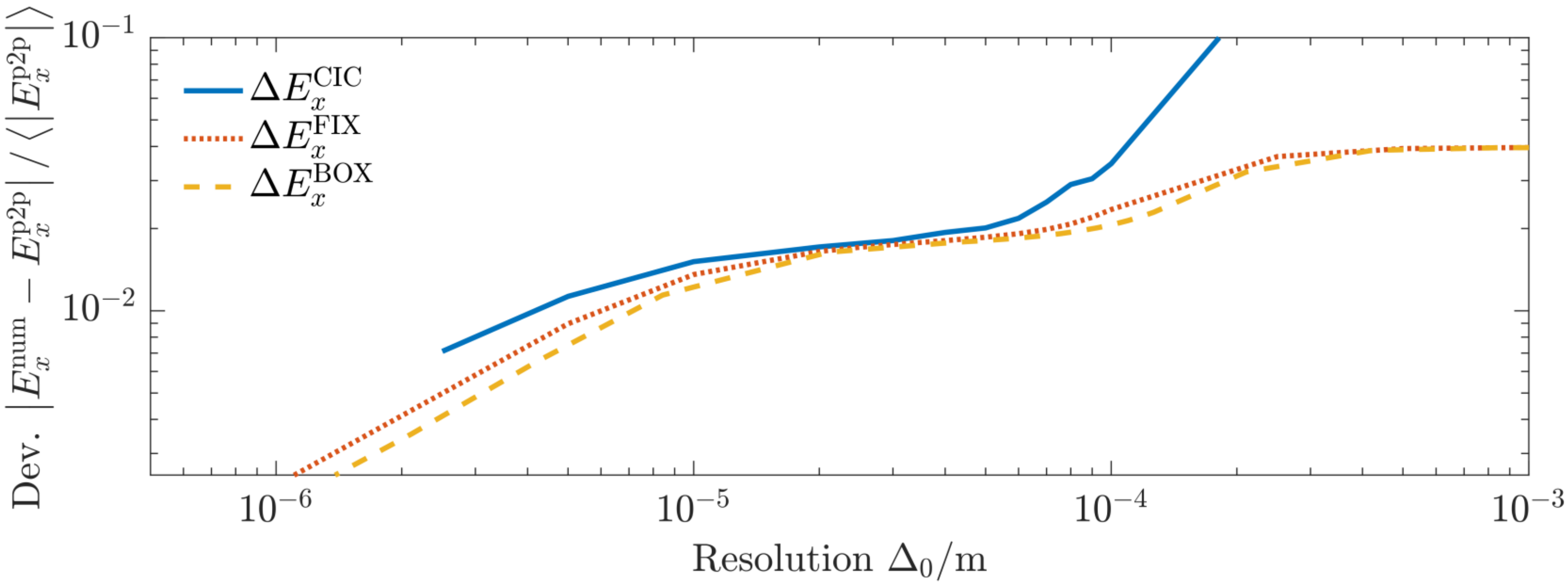


- Best approximation: $\Delta_0^{\mathrm{CIC}} \approx 0.13 \times \sigma_r$
  $\Delta_0^{\mathrm{FIX}} \approx 0.23 \times \sigma_r$
  $\Delta_0^{\mathrm{BOX}} \approx 2.3 \times r_k$

- Upper bound of $\Delta E_x^{\mathrm{FIX}}$, $\Delta E_x^{\mathrm{BOX}}$ for $\Delta_0 \geq 400$ µm ($\boldsymbol{E}_{\mathrm{P2P}} \rightarrow 0$)

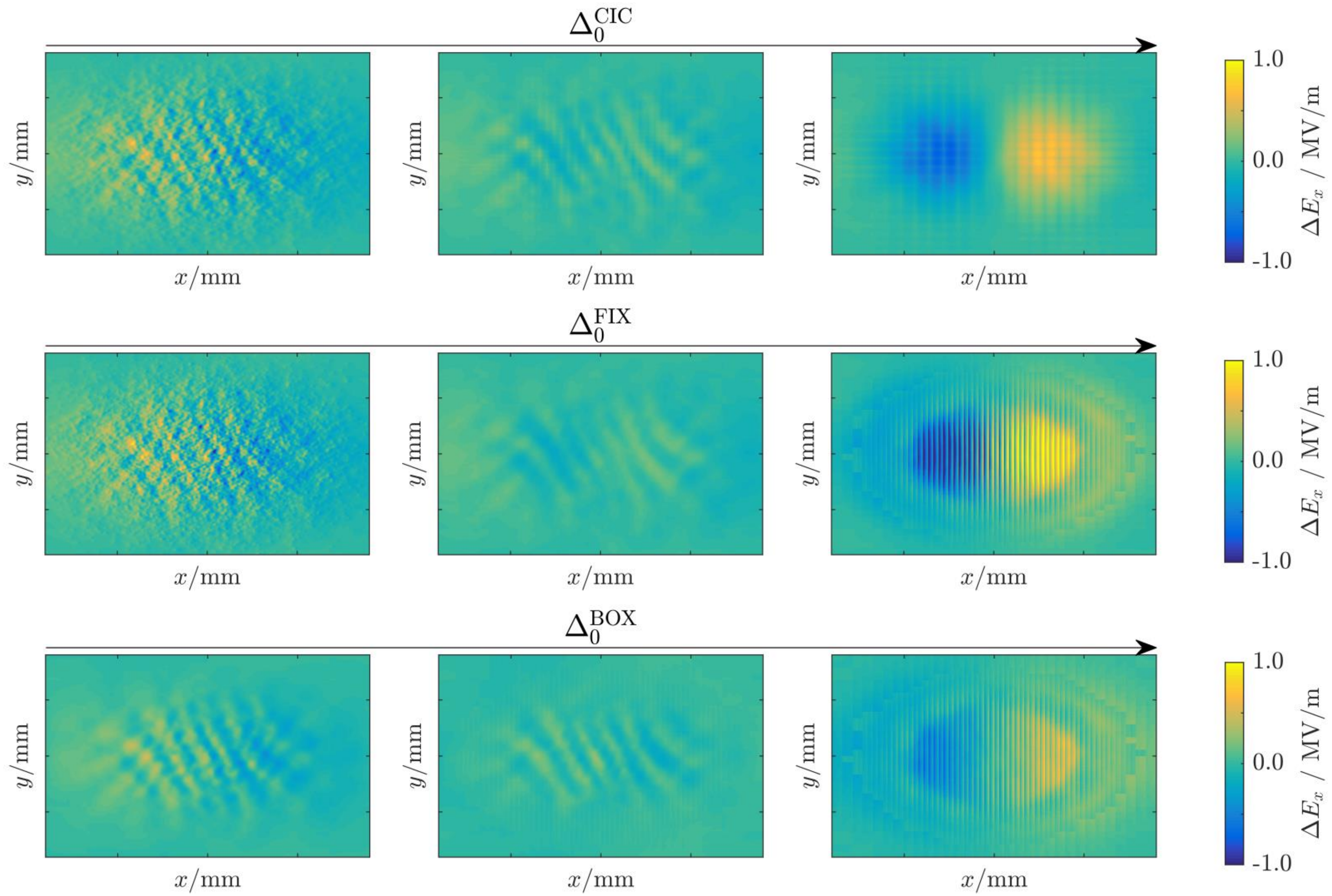
Macroparticle Noise
Optimization of Macroparticle Shape

Deviation to space charge field of point-like macroparticles ($\Delta_0^{p2p} = 10\ \mathrm{nm}$) :



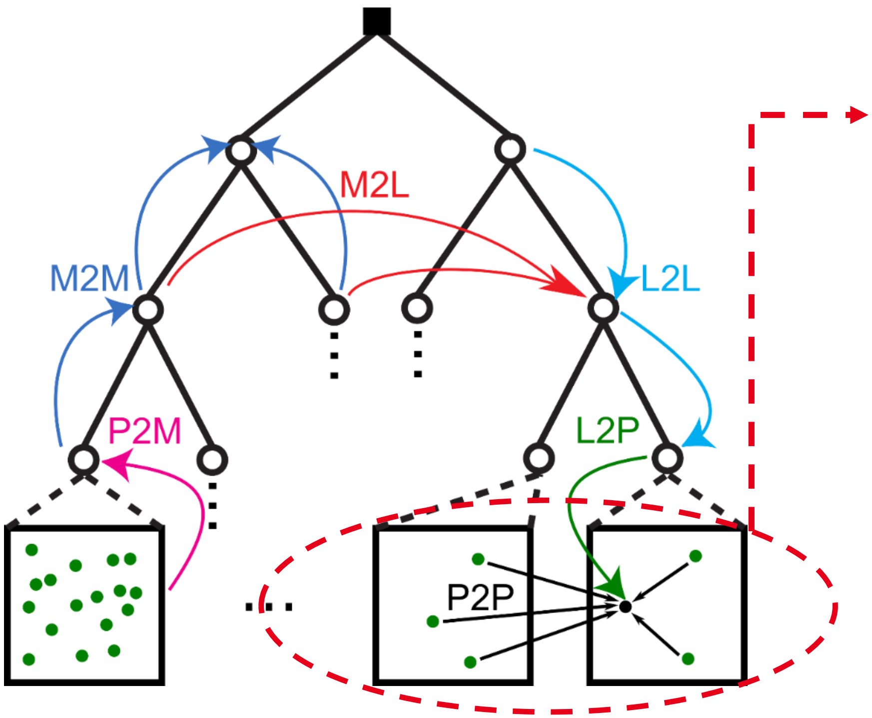
- Macroparticle noise dominates for $\Delta_0 \leq 30\ \mu\mathrm{m}$

Macroparticle Noise
Optimization of Macroparticle Shape



Macroparticle Noise
Inconvenience of Particle FMM

- Evaluation of $\boldsymbol{E}_{\mathrm{P2P}}(\boldsymbol{x}_i; \boldsymbol{x}_j, q_j)$ inefficient



If $|x_i - x_j| > \Delta_0$

$$\boldsymbol{E}_{\mathrm{P2P}} \propto \sum_{j=1}^{N_k} \frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{|\boldsymbol{x}_i - \boldsymbol{x}_j|^3} \Rightarrow O(N_k \times N_k)$$

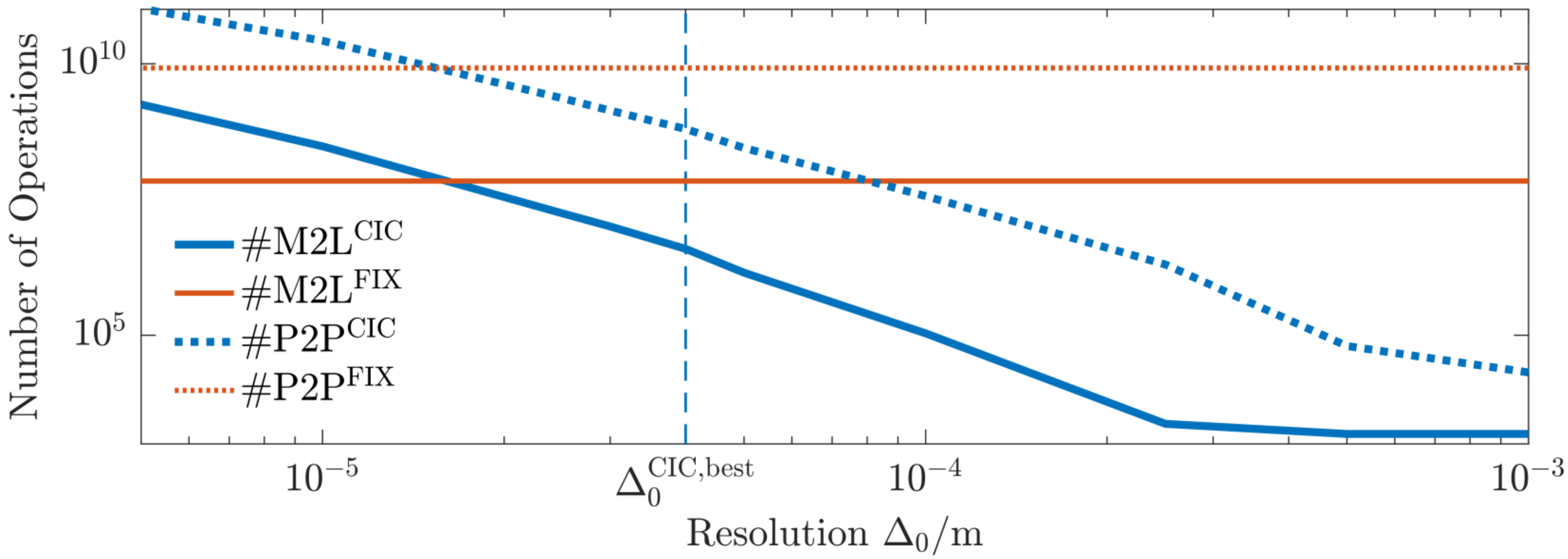
however for $|x_i - x_j| \leq \Delta_0$

$$\boldsymbol{E}_{\mathrm{P2P}} \propto N_k\, x_i - \underbrace{\sum_{j=1}^{N_k} \boldsymbol{x}_j}_{\boldsymbol{c}_k} \Rightarrow O(2 \times N_k)$$

(Plot based on: R. Yokota, ExaFMM User's Manual, 2011)

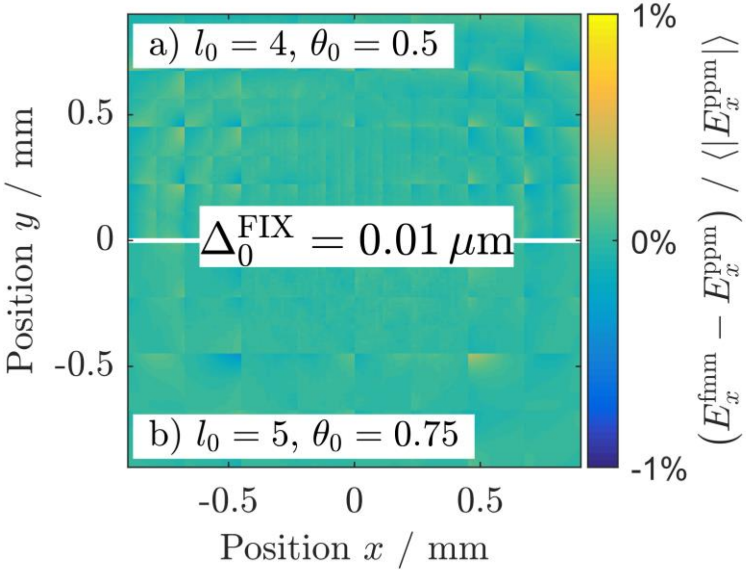
Macroparticle Noise
Inconvenience of Particle FMM

- Advantages of FMM for $\Delta_0 \to 0$ cannot be utilized for $\Delta_0^{\mathrm{OPT}} \sim 50\ \mu\mathrm{m}$

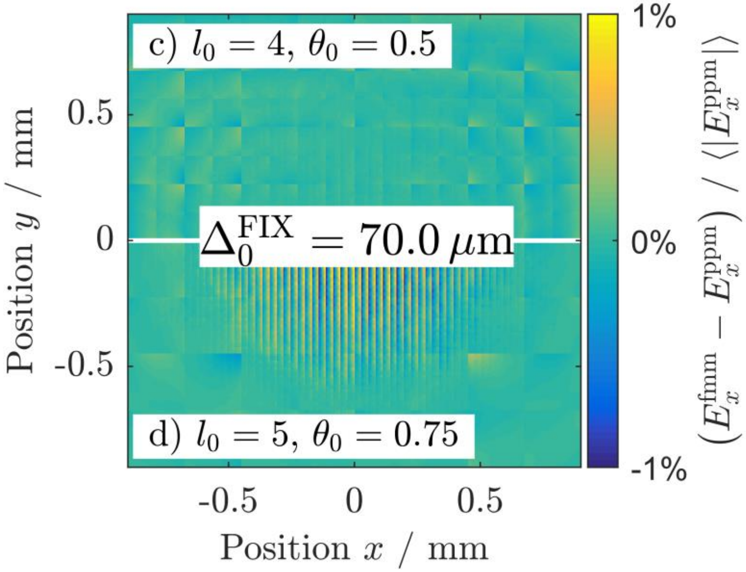


Macroparticle Noise
Inconvenience of Particle FMM

- Radial extension $r_k + \Delta_0$ limits the admissibility parameter to $\theta_0 \leq \left(1 + \frac{\Delta_0}{r_k + r_{k'}}\right)^{-1}$



For small Δ_0 a more restrictive admissibility parameter $\theta_0 = 0.5$ in (a) and higher order moments $l_0 = 5$ in (b) yield consistent results



For large Δ_0 the near field of extended macroparticles in c) and the far field approximation based on point charges in (d) are not identical

Self-Consistent Plummer Fields
Self-Consistent Field Methods

Representation of space charge potential

$$\phi(\boldsymbol{x}) = \sum_{n,l,m} p_{n,l,m}^{\text{SCF}} \times \phi_{n,l,m}^{\text{SCF}}(\boldsymbol{x})$$

as moments $p_{n,l,m}^{\text{SCF}}$ of a bi-orthogonal set of functions $\phi_{n,l,m}^{\text{SCF}}, \rho_{n,l,m}^{\text{SCF}}$ which

$$\Delta\, \phi_{n,l,m}^{\text{SCF}}(\boldsymbol{x}) = \frac{1}{\epsilon_0}\, \rho_{n,l,m}^{\text{SCF}}(\boldsymbol{x}).$$

Ansatz based on the Plummer model with smoothing parameter a_0

$$\phi_{0,0,0}^{\text{SCF}}(\boldsymbol{x}; a_0) \propto (r^2 + a_0^2)^{-\frac{1}{2}}$$

provides the generalized basis set [1]

$$\phi_{n,l,m}^{\text{SCF}}\left(\frac{r}{a_0}, \vartheta, \varphi; a_0\right) = -\frac{\tilde{r}^l}{(1+\tilde{r}^2)^{l+\frac{1}{2}}}\, C_{n,l+1}\left(\frac{\tilde{r}^2 - 1}{\tilde{r}^2 + 1}\right) \sqrt{4\pi}\; Y_l^m(\vartheta, \varphi).$$

Self-Consistent Plummer Fields

Self-Consistent Field Properties

- Fully factorized model for near field interactions

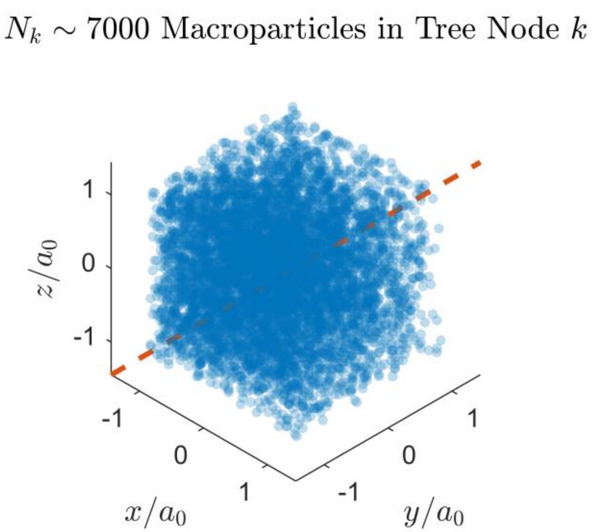
$$O_{P2P}(N_k \times N_k) \Rightarrow O_{SCF}(N_k \times n_0 \times l_0^2)$$

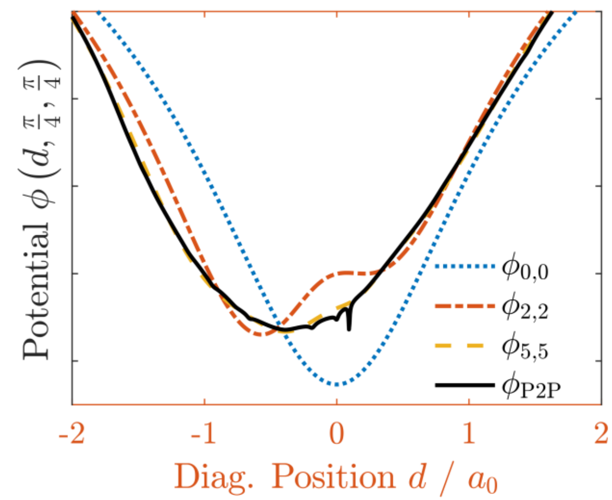
- Spatially smooth potential functions

$$\phi_{0,0,0}^{\mathrm{SCF}}(r \to 0; a_0) = \frac{1}{a_0} \text{ and } \phi_{0,0,0}^{\mathrm{SCF}}(r \to \infty; a_0) = \frac{1}{r}$$

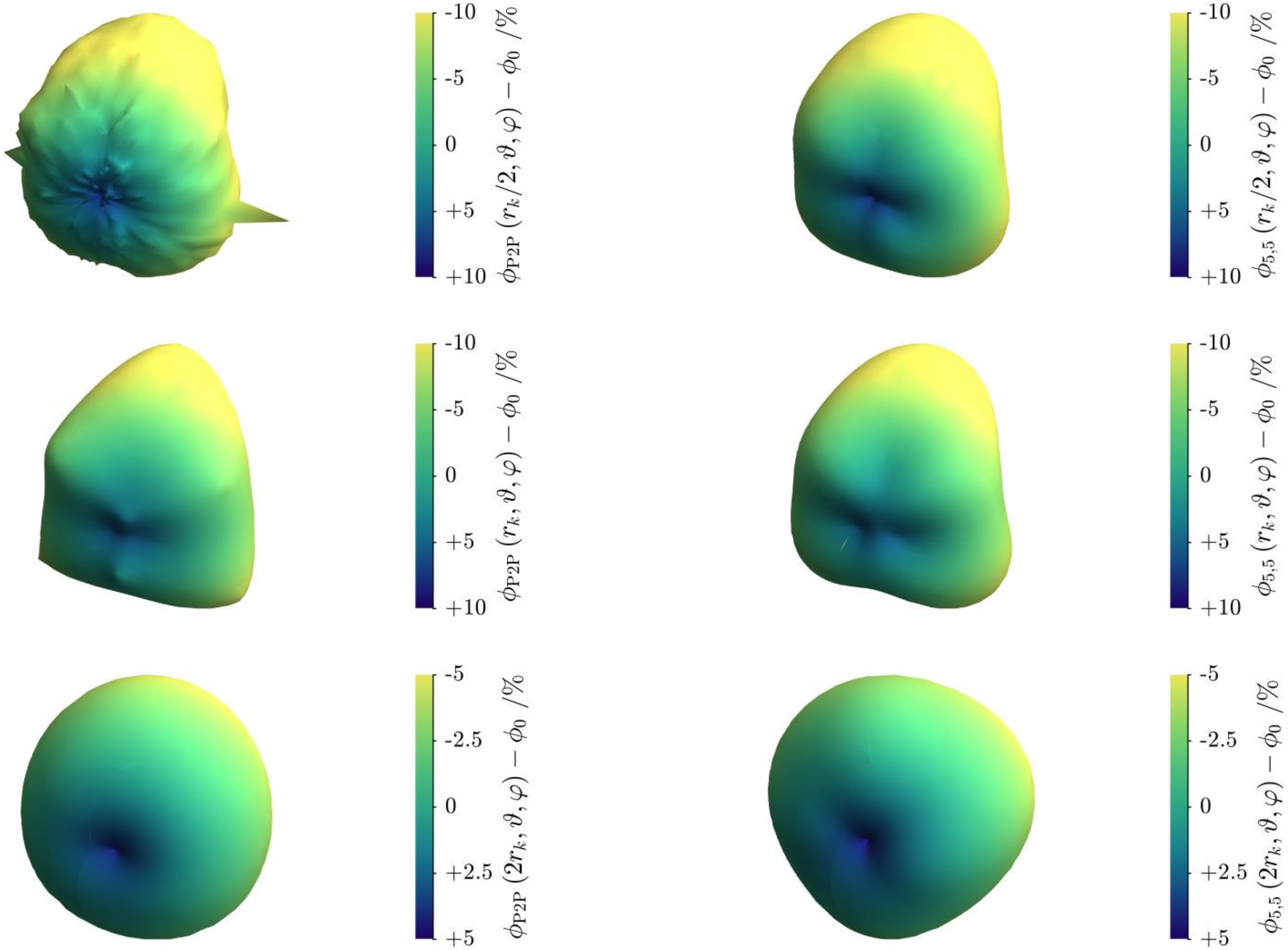
- Computationally cheap evaluation of coefficients

$$p_{n,l,m}^{\mathrm{SCF},j} \propto q_j \, {\phi^{\mathrm{SCF}}}^*_{n,l,m}(r_j, \vartheta_j, \varphi_j; a_0) \propto q_{l,m}^j$$



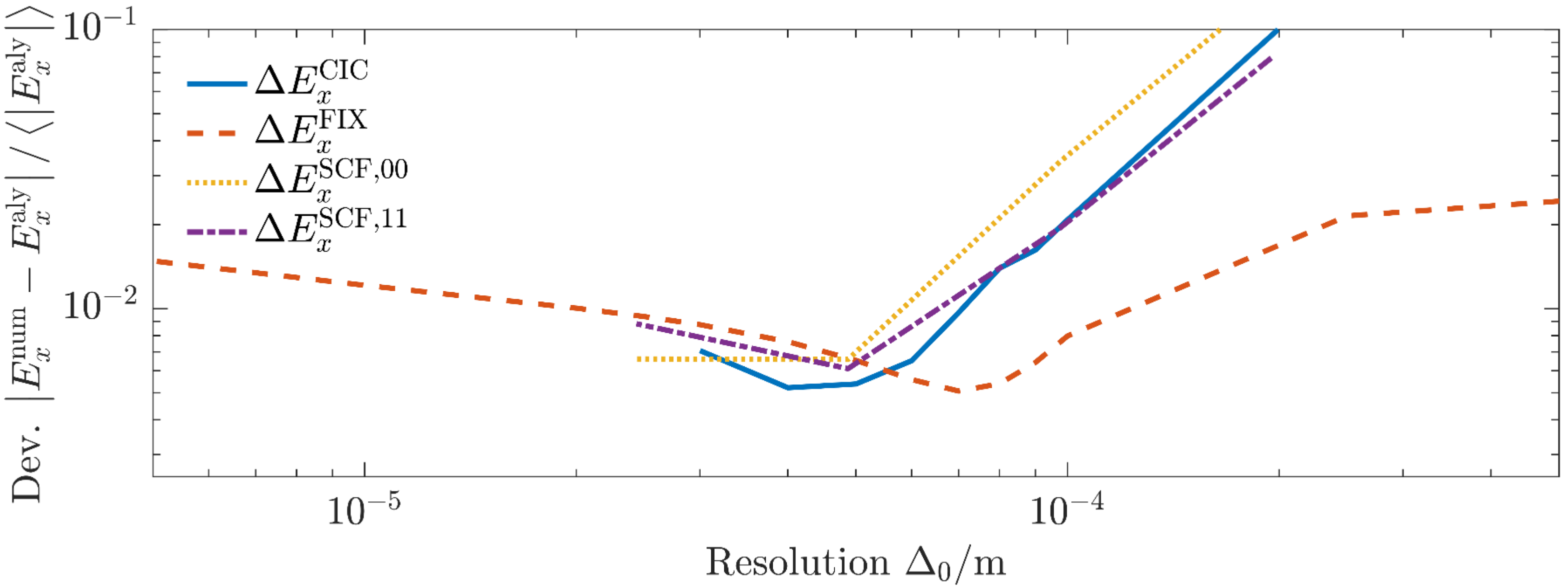


Self-Consistent Plummer Fields
Self-Consistent Field Properties



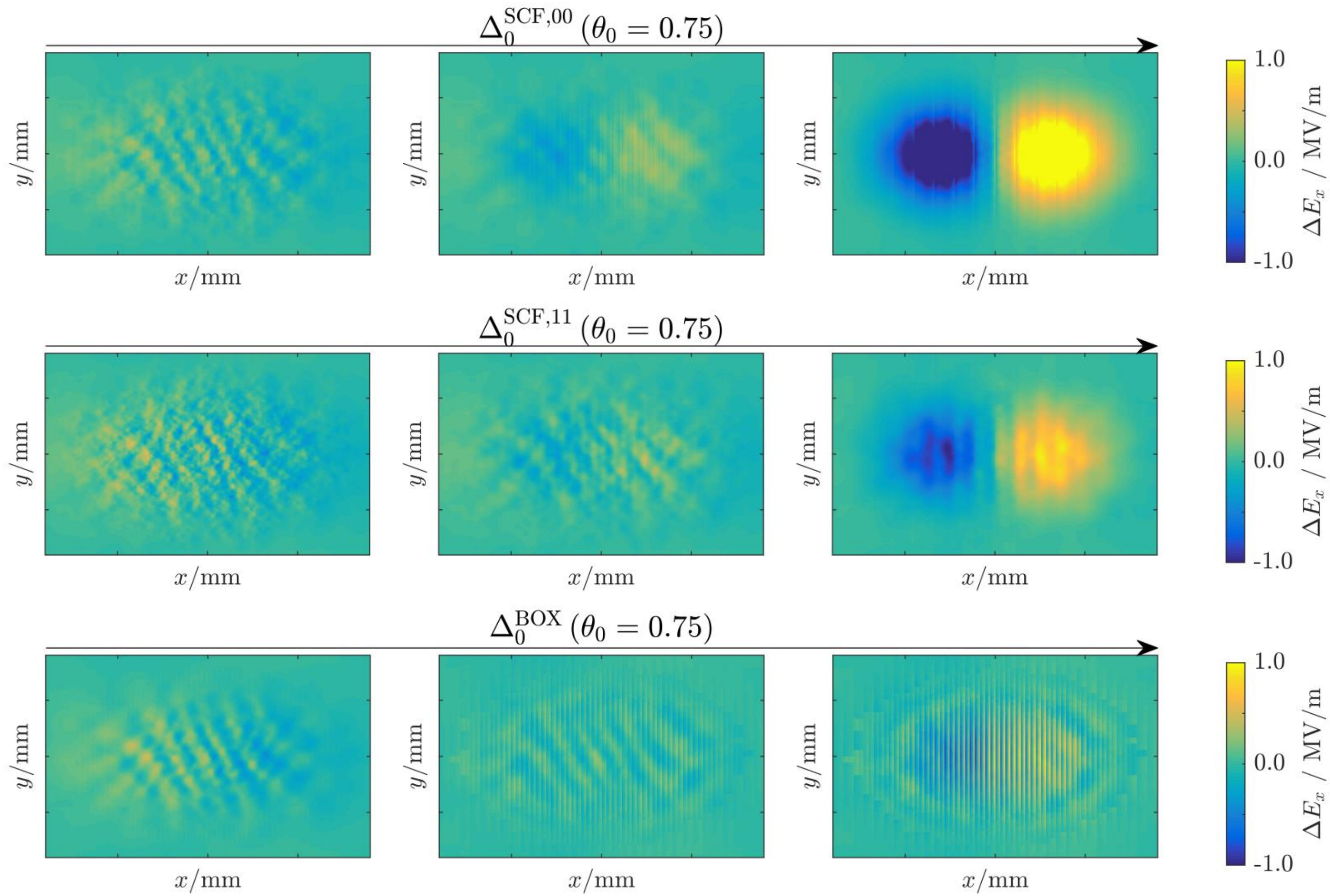
Modified Near Field Interaction for FMM
Self-Consistent Near Field Model

Deviation of macroparticle approximation from analytic field solution:



- $E_x^{\mathrm{SCF,n_0l_0}}$ with radial n_0 and angular l_0 order best at $\Delta_0 \approx 50\ \mu\mathrm{m}$
- $E_x^{\mathrm{SCF,11}}$ similar to E_x^{CIC} method

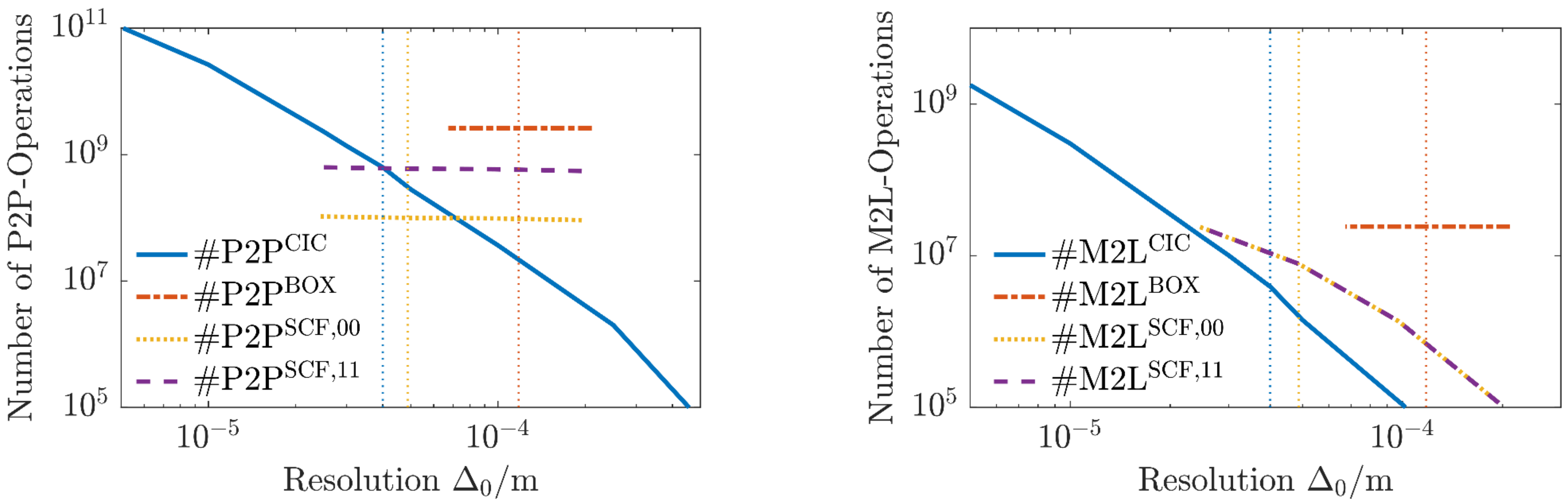
Modified Near Field Interaction for FMM
Self-Consistent Near Field Model



Modified Near Field Interaction for FMM

Self-Consistent Near Field Model

Self-consistent fields reduce noise and num. cost of space charge computation:



- Efficiency gain from reduced spatial resolution of space charge field
- Simplified tree structure for FMM computation

Summary and Outlook

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Summary:

- Comparison of three methods for smoothing macroparticle noise in FMM
- Inconvenience of particle based FMM if small spatial resolution is needed
- Utilization of self-consistent basis functions for smoothing near fields in FMM

Outlook:

- Investigate influence of smoothing on beam dynamics simulations
- Check alternative bi-orthogonal basis sets (e. g. Bessel function solutions)

References:

[1] Hernquist, L. et al., A Self-Consistent Field Method For Galactic Dynamics

SCF Space Charge Computation

Direct Application of Plummer Basis Functions

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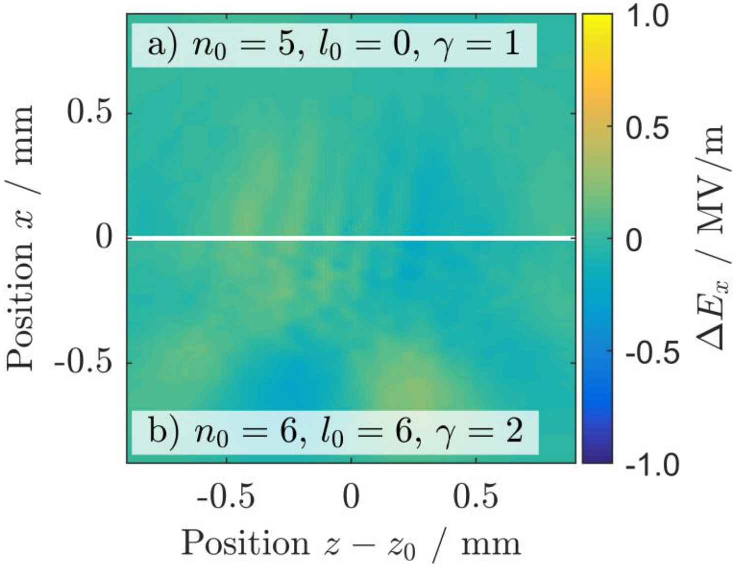
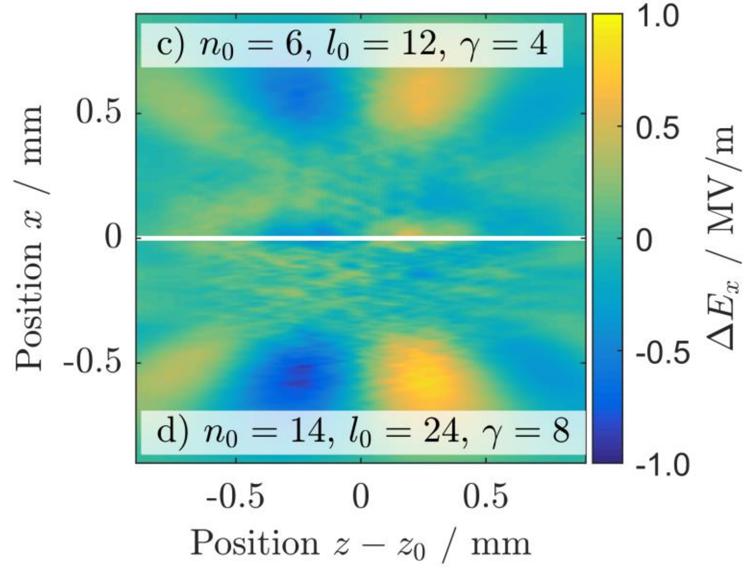
Approximation of particle beam space charge fields with Plummer Basis Functions

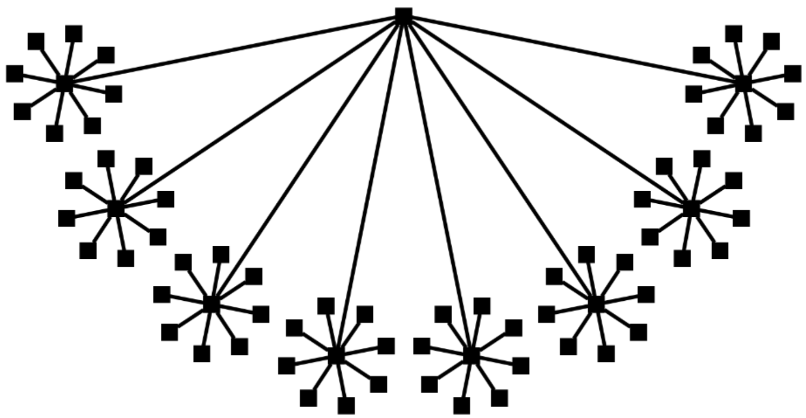




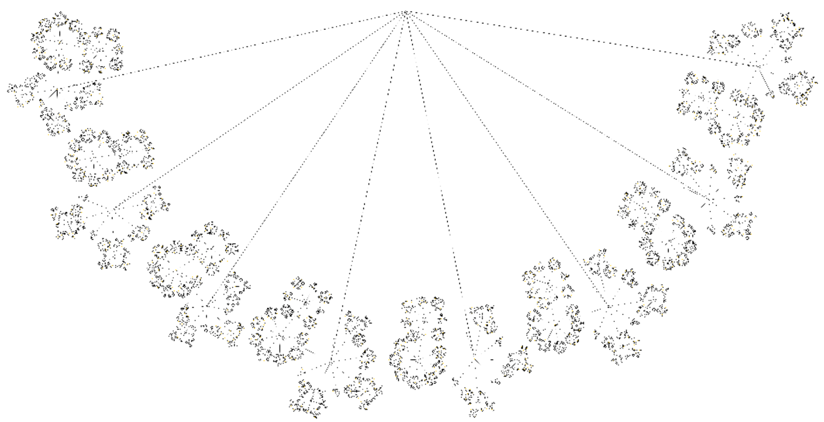
- Very efficient for rotationally symmetric charge distributions such as a) and b)
- Inefficient for description of elongated charge density of relativistic beams $\gamma \geq 4$

Macroparticle Noise in FMM
Inconvenience of Particle FMM

- CIC interpolation of $\rho(\boldsymbol{x})$ simplifies FMM tree structure significantly



Tree of E_x^{CIC} computation



Tree of E_x^{FIX} computation