Beam-Loading Simulations for a TESLA 1.3 GHz Cavity

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Outline

- Motivation
- Computational Modeling
  - Maxwell's Equations
  - Time Evolution of the Electromagnetic Field
- Numerical Results
  - Beam-Loading Simulations
  - Beam-Loading Measurements
- Summary / Outlook

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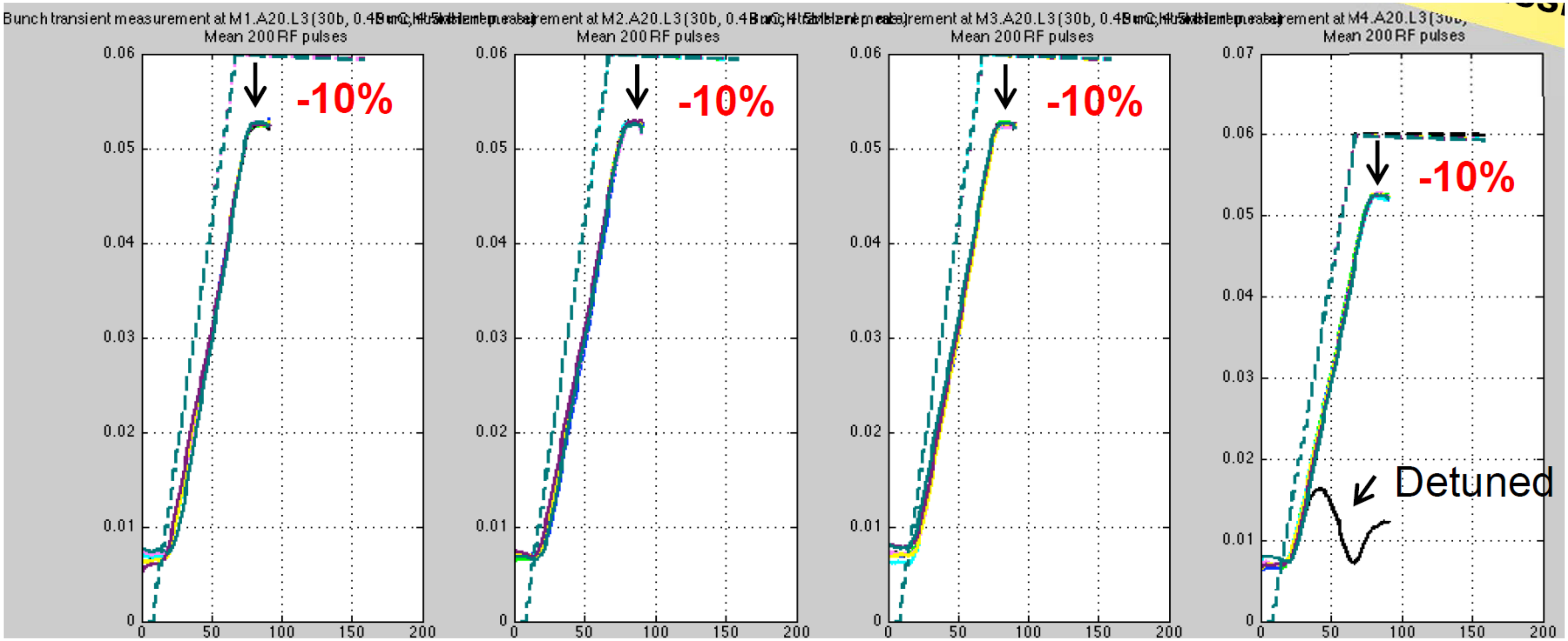
Motivation

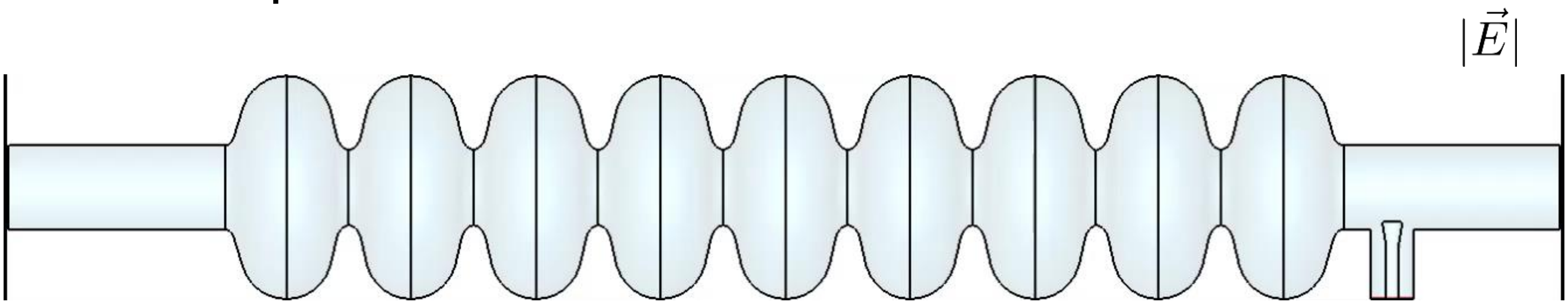
- Beam Loading of a 9 Cell TESLA 1.3 GHz Cavity
  - Comparison of theory with measurement



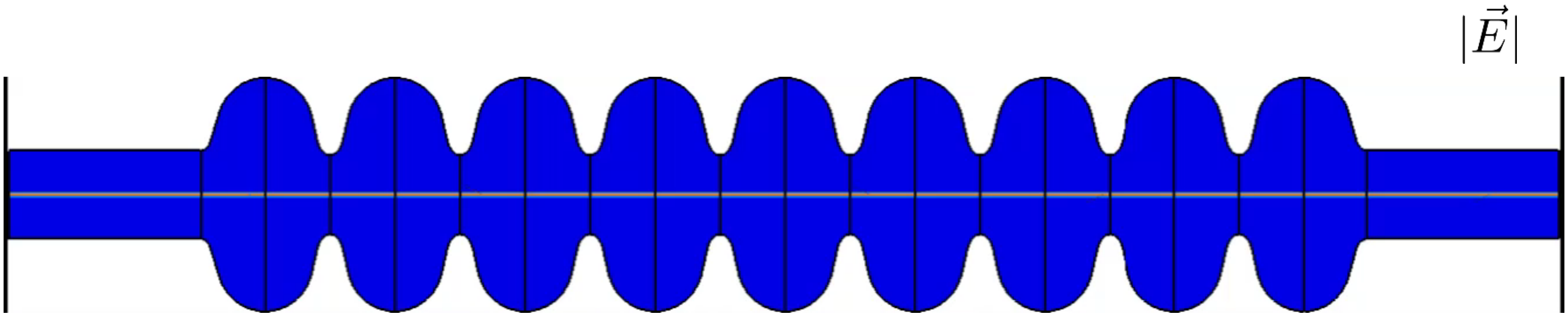
Source: „Precision Control of SRF Cavities“, Sven Pfeiffer, 15.11.2018

Motivation

- Time Evolution of the Electric Field Strength
  - Main-coupler excitation



  - Beam excitation



Outline

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Computational Modeling

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- Maxwell's Equations

$$\operatorname{curl} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\operatorname{curl} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\operatorname{div} \vec{D} = \varrho$$

$$\operatorname{div} \vec{B} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\frac{\partial \varepsilon}{\partial t} = 0$$

$$\frac{\partial \mu}{\partial t} = 0$$

https://de.wikipedia.org

Combine both curl equations:

$$\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \operatorname{curl}(\frac{1}{\mu} \operatorname{curl} \vec{E}) = -\frac{\partial \vec{J}}{\partial t}$$

Ansatz:

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) + \sum_{\nu=1}^{\infty} \alpha_\nu(t)\ \vec{E}_\nu(\vec{r})$$

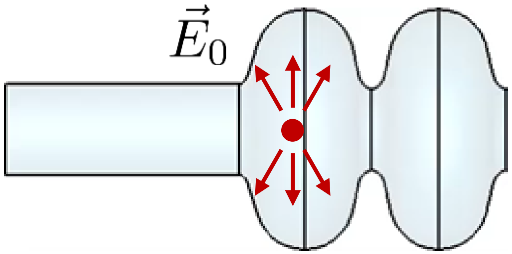
Computational Modeling

- Time Evolution of the Electromagnetic Field
  - Fundamental equation

$$\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mathrm{curl}(\frac{1}{\mu}\,\mathrm{curl}\vec{E}) = -\frac{\partial \vec{J}}{\partial t}$$

  - Ansatz

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \sum_{\nu=1}^{\infty} \alpha_\nu(t)\; \vec{E}_\nu(\vec{r})$$



  - Properties

static:

$$\mathrm{curl}\,\vec{H}_0 = 0 \qquad \mathrm{div}\,\vec{D}_0 = \varrho$$
$$\mathrm{curl}\,\vec{E}_0 = 0 \qquad \mathrm{div}\,\vec{B}_0 = 0$$

dynamic:

$$\mathrm{curl}\,\vec{H}_\nu = +j\omega\vec{D}_\nu \qquad \mathrm{div}\,\vec{D}_\nu = 0$$
$$\mathrm{curl}\,\vec{E}_\nu = -j\omega\vec{B}_\nu \qquad \mathrm{div}\,\vec{B}_\nu = 0$$

Computational Modeling

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- Time Evolution of the Electromagnetic Field
  - Fundamental equation

$$\varepsilon \, \frac{\partial^2 \vec{E}_0(\vec{r},t)}{\partial t^2} + \sum_{\nu=1}^{\infty} \varepsilon \, \frac{\partial^2 \alpha_\nu(t)}{\partial t^2} \, \vec{E}_\nu(\vec{r}) + \sum_{\nu=1}^{\infty} \alpha_\nu(t) \, \mathrm{curl} \left(\frac{1}{\mu} \, \mathrm{curl} \, \vec{E}_\nu(\vec{r}) \right) = -\frac{\partial \vec{J}}{\partial t}$$

eigenvalue equation

$$\varepsilon \, \frac{\partial^2 \vec{E}_0(\vec{r},t)}{\partial t^2} + \sum_{\nu=1}^{\infty} \varepsilon \, \frac{\partial^2 \alpha_\nu(t)}{\partial t^2} \, \vec{E}_\nu(\vec{r}) + \sum_{\nu=1}^{\infty} \alpha_\nu(t) \, \varepsilon \, \omega_\nu^2 \, \vec{E}_\nu(\vec{r}) = -\frac{\partial \vec{J}}{\partial t}$$

$$\varepsilon \, \frac{\partial^2 \vec{E}_0(\vec{r},t)}{\partial t^2} + \sum_{\nu=1}^{\infty} \varepsilon \left(\frac{\partial^2 \alpha_\nu(t)}{\partial t^2} + \alpha_\nu(t) \, \omega_\nu^2 \right) \vec{E}_\nu(\vec{r}) = -\frac{\partial \vec{J}}{\partial t}$$

Computational Modeling

- Time Evolution of the Electromagnetic Field
  - Fundamental equation

$$\varepsilon \, \frac{\partial^2 \vec{E}_0 \cdot \vec{E}_\eta}{\partial t^2} + \sum_{\nu=1}^{\infty} \varepsilon \left(\frac{\partial^2 \alpha_\nu(t)}{\partial t^2} + \alpha_\nu(t)\, \omega_\nu^2 \right) \vec{E}_\nu \cdot \vec{E}_\eta = -\frac{\partial \vec{J}}{\partial t} \cdot \vec{E}_\eta$$

  - Orthogonality

$$\iiint_V \varepsilon \, \vec{E}_0 \cdot \vec{E}_\eta \, \mathrm{d}V = 0$$

$$\iiint_V \varepsilon \, \vec{E}_\nu \cdot \vec{E}_\eta \, \mathrm{d}V = \delta_{\nu,\eta} \, 2W$$

$$\frac{\partial^2 \alpha_\nu(t)}{\partial t^2} + \alpha_\nu(t)\, \omega_\nu^2 = -\frac{1}{2W} \iiint_V \frac{\partial \vec{J}}{\partial t} \cdot \vec{E}_\nu \, \mathrm{d}V$$

ODE to determine the time-dependent modal weight coefficient

Computational Modeling

- Time Evolution of the Electromagnetic Field
  - General solution

$$\frac{\partial^2 \alpha_\nu(t)}{\partial t^2} + \alpha_\nu(t)\,\omega_\nu^2 = f_\nu(t) \qquad\qquad f_\nu(t) = -\frac{1}{2W} \iiint_V \frac{\partial \vec{J}}{\partial t} \cdot \vec{E}_\nu \,\mathrm{d}V$$

LAPLACE

$$p^2 A_\nu(p) + \omega_\nu^2 A_\nu(p) = F_\nu(p)$$

$$A_\nu(p) = \frac{1}{p^2 + \omega_\nu^2} F_\nu(p)$$

$$\alpha_\nu(t) = \frac{1}{\omega_\nu} \sin(\omega_\nu t) \ast f_\nu(t) = \frac{1}{\omega_\nu} \int_0^t f_\nu(\tau)\,\sin(\omega_\nu(t-\tau))\,\mathrm{d}\tau$$

Computational Modeling

- Time Evolution of the Electromagnetic Field
  - General solution (including charge in cavity)

$$\alpha_\nu(t) = \frac{1}{\omega_\nu} \int_0^t f_\nu(\tau)\ \sin(\omega_\nu(t-\tau))\,\mathrm{d}\tau$$

$$= \sin(\omega_\nu t)\,\frac{1}{\omega_\nu}\int_0^t f_\nu(\tau)\cos(\omega_\nu \tau)\,\mathrm{d}\tau - \cos(\omega_\nu t)\,\frac{1}{\omega_\nu}\int_0^t f_\nu(\tau)\sin(\omega_\nu \tau)\,\mathrm{d}\tau$$

  - Steady-state solution (excluding charge in cavity)

$$\alpha_\nu(t) = \sin(\omega_\nu t)\,\frac{1}{\omega_\nu}\int_{-\infty}^{\infty} f_\nu(\tau)\cos(\omega_\nu \tau)\,\mathrm{d}\tau - \cos(\omega_\nu t)\,\frac{1}{\omega_\nu}\int_{-\infty}^{\infty} f_\nu(\tau)\sin(\omega_\nu \tau)\,\mathrm{d}\tau$$

$$= \Re(\underline{c}_\nu\ e^{i\omega_\nu t}) \qquad \text{with} \qquad \underline{c}_\nu = \frac{-i}{\omega_\nu}\int_{-\infty}^{\infty} f_\nu(\tau)\,e^{-i\omega_\nu \tau}\,\mathrm{d}\tau$$

Computational Modeling

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- Time Evolution of the Electromagnetic Field
  - Excited mode magnitude and phase

$$\underline{c}_\nu = \frac{-i}{\omega_\nu} \int_{-\infty}^{\infty} f_\nu(\tau)\, e^{-i\omega_\nu \tau}\, \mathrm{d}\tau \qquad f_\nu(t) = -\frac{1}{2W} \iiint_V \frac{\partial \vec{J}}{\partial t} \cdot \vec{E}_\nu \, \mathrm{d}V$$

  - Simplification using a point-charge excitation

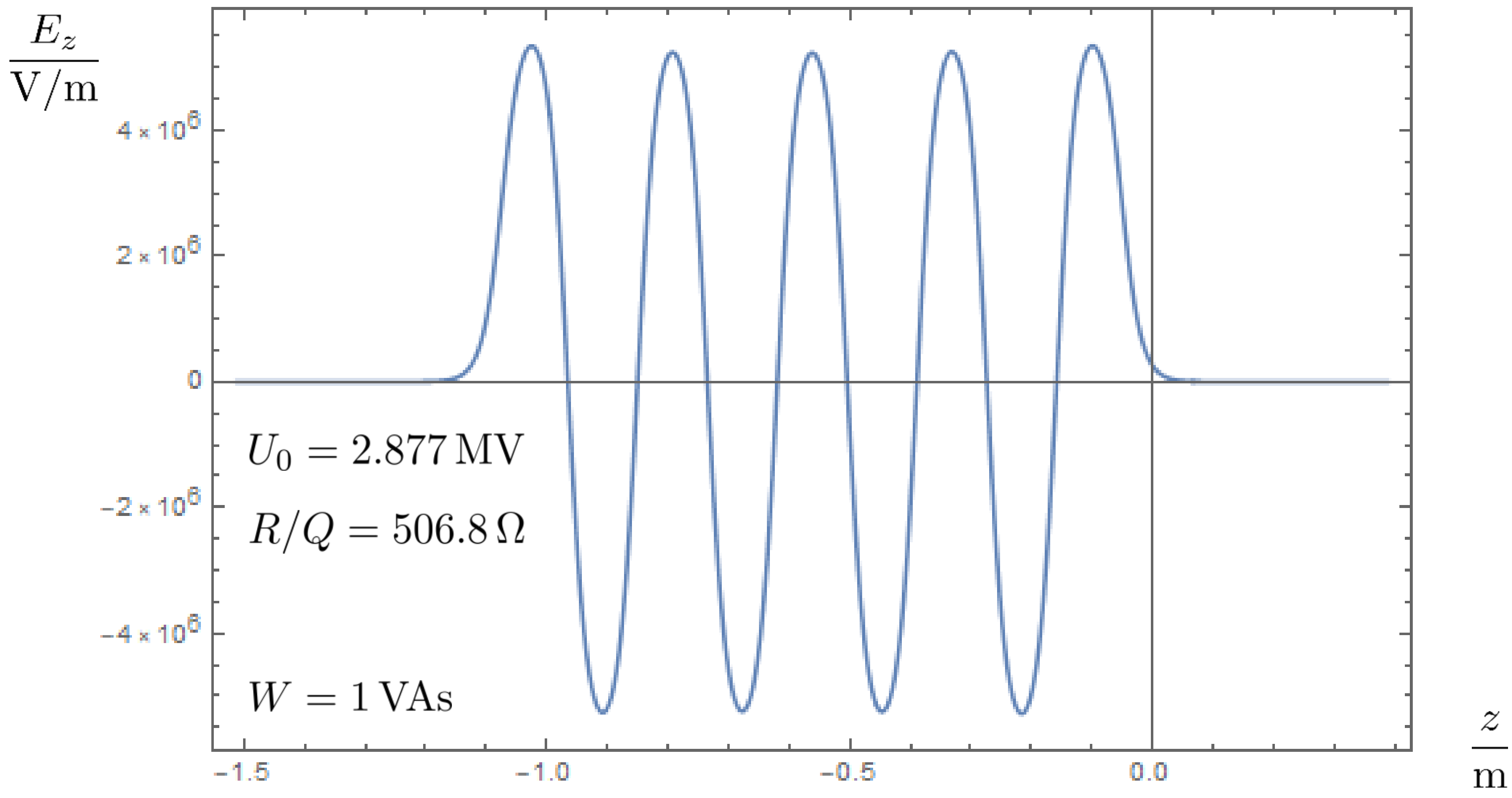
$$\vec{J} = \delta(x - x_0)\, \delta(y - y_0)\, \delta(z - vt)\, j_0\, \vec{e}_z = \varrho\, v\, \vec{e}_z \qquad [j_0] = \mathrm{Am}$$

$$\varrho = \delta(x - x_0)\, \delta(y - y_0)\, \delta(z - vt)\, q_0 \qquad [q_0] = \mathrm{As}$$

$$\underline{c}_\nu = \frac{q_0}{2W} \int_{-\infty}^{\infty} (\vec{e}_z \cdot \vec{E}_\nu)\, e^{-i\omega_\nu \frac{z}{v}}\, \mathrm{d}z = q_0 \frac{\underline{U}_0}{2W} \qquad [\underline{c}_\nu] = 1$$

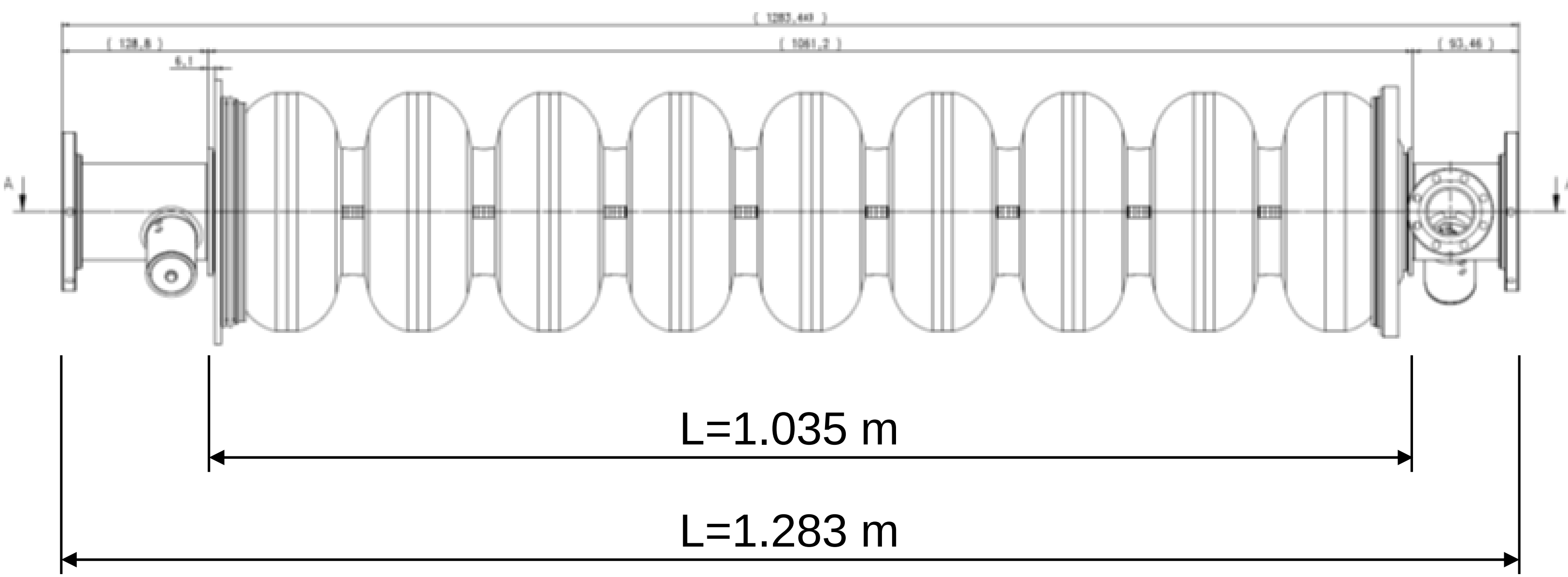
Computational Modeling

- Concentration on the Accelerating Mode
  - Field profile on the axis for a TESLA 1.3 GHz cavity



Computational Modeling

- Properties of the Accelerating Cavity
  - Mean accelerating gradient



$$E_{\text{mean}} = \frac{U_0}{L}$$

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Numerical Results

- Beam-Loading Simulations
  - Cavity parameter

$$
\begin{aligned}
f_{\mathrm{res}} &= 1.299\ \mathrm{GHz} \\
Q_{\mathrm{res}} &= 5.407 \cdot 10^6 \\
U_0 &= 2.877\ \mathrm{MV} \\
R/Q &= 506.8\,\Omega
\end{aligned}
$$

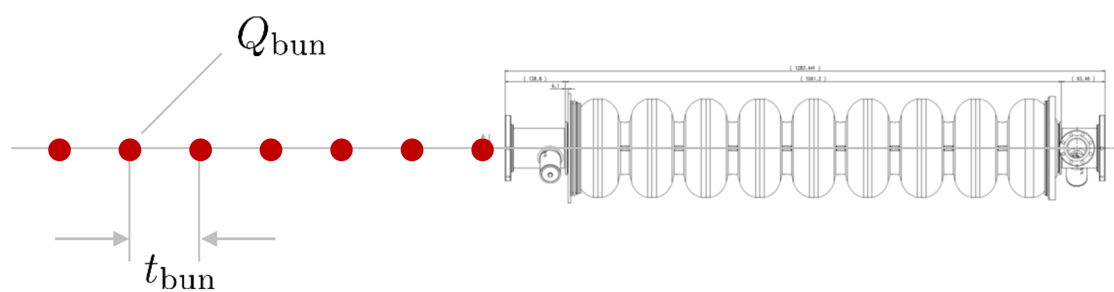
Time scales

$$
\begin{aligned}
t_{\mathrm{bun}}/t_{\mathrm{res}} &= 289 \\
t_{\mathrm{qua}}/t_{\mathrm{res}} &= 1.7 \cdot 10^6 \\
t_{\mathrm{qua}}/t_{\mathrm{bun}} &= 5958.2
\end{aligned}
$$

  - Bunch parameter

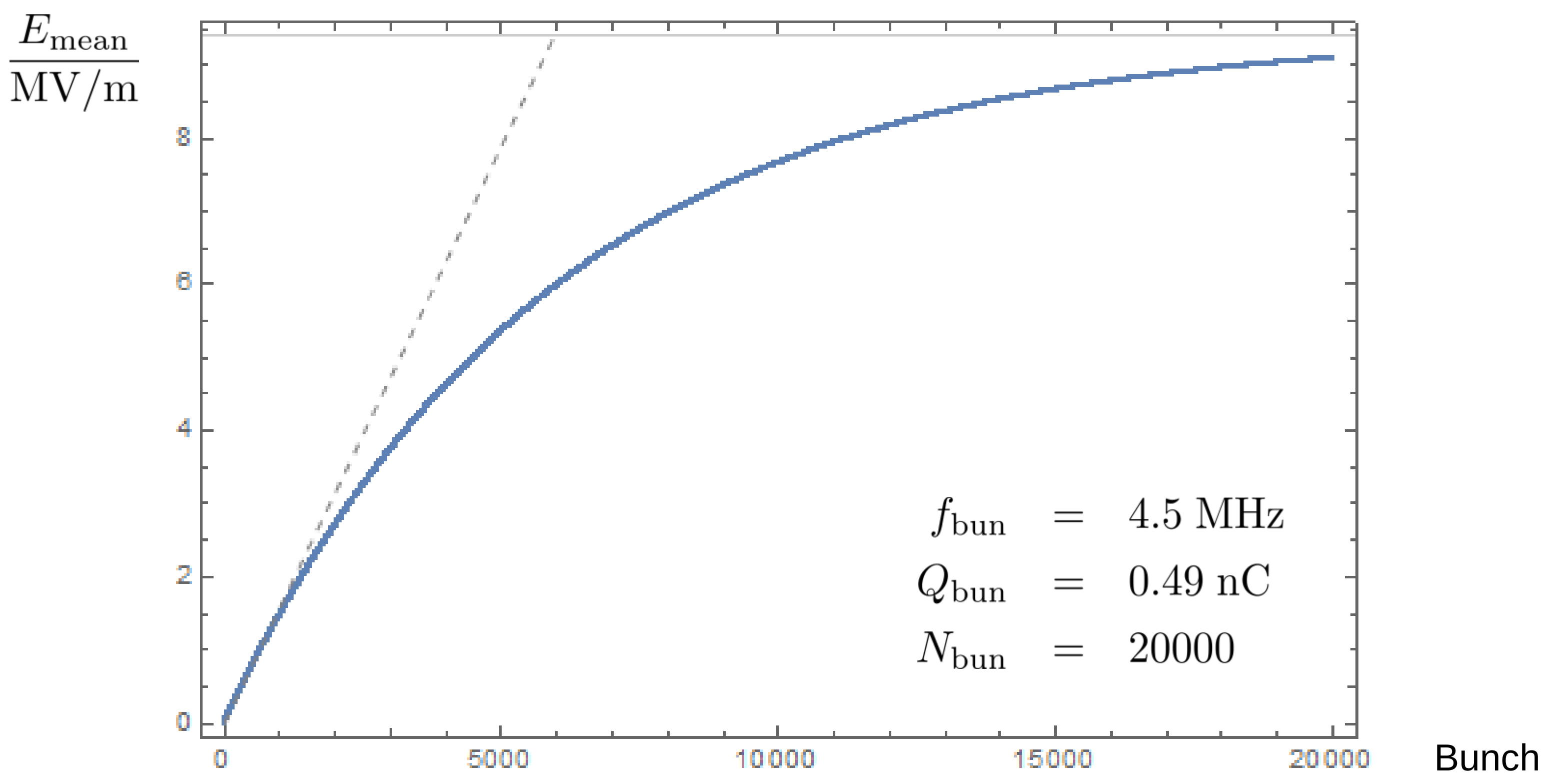
$$
\begin{aligned}
f_{\mathrm{bun}} &= 4.5\ \mathrm{MHz} \\
Q_{\mathrm{bun}} &= 0.49\ \mathrm{nC} \\
N_{\mathrm{bun}} &= 30
\end{aligned}
$$

$$t_{\mathrm{bun}} \cdot c_0 = 66.6\,\mathrm{m}$$



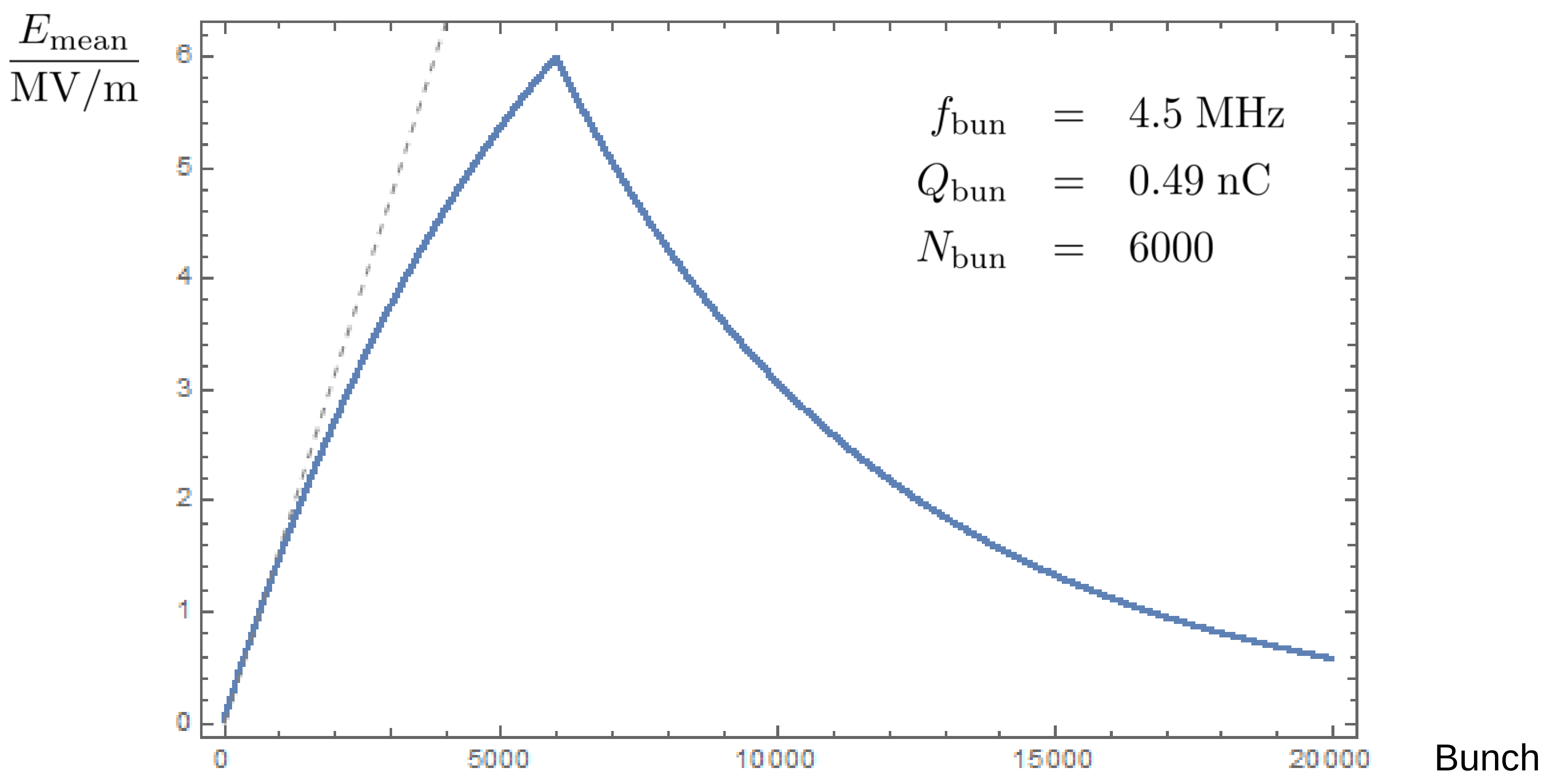
Numerical Results

- Beam-Loading Simulations
  - Example



Numerical Results

- Beam-Loading Simulations
  - Example



Numerical Results

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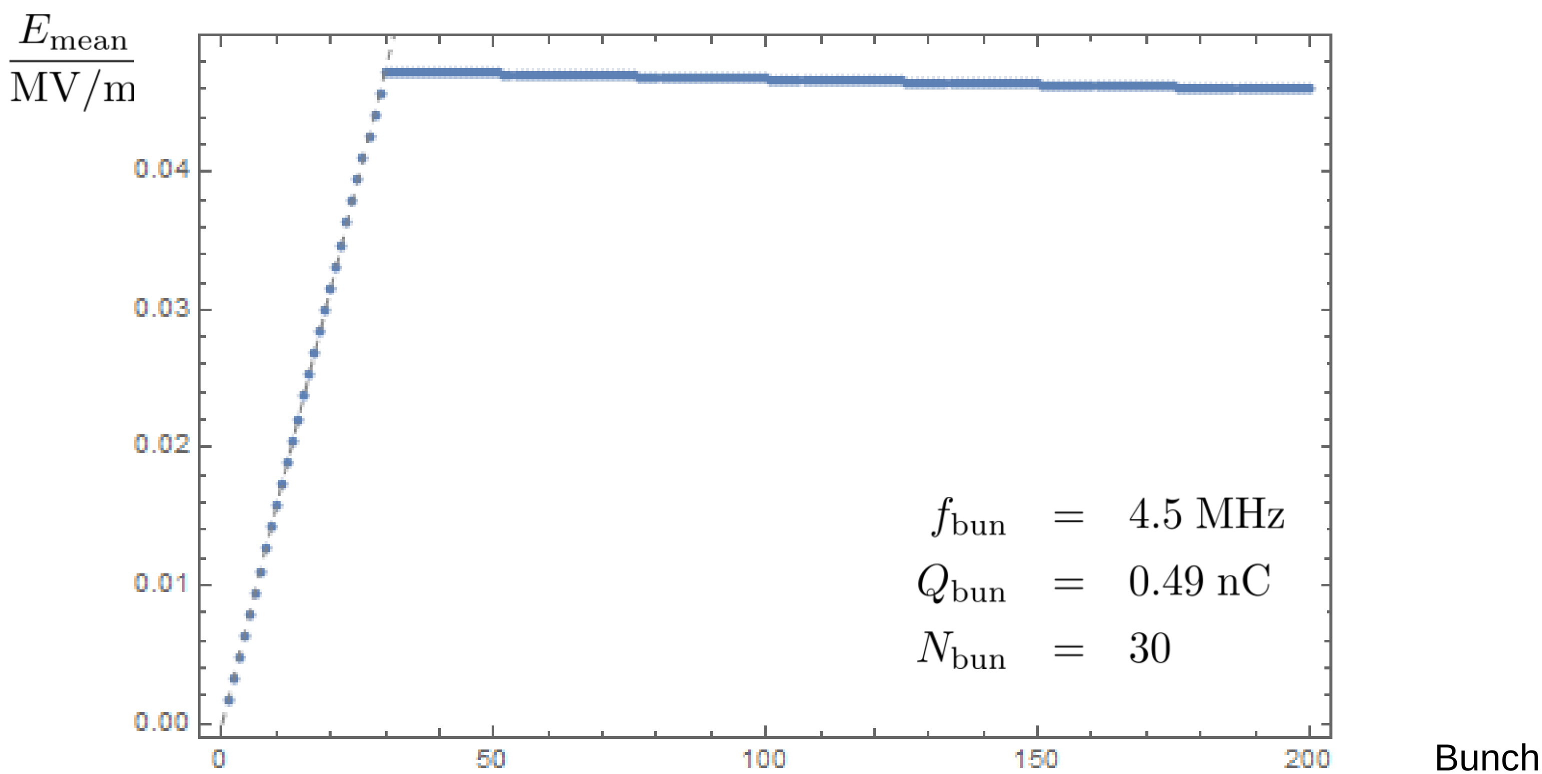
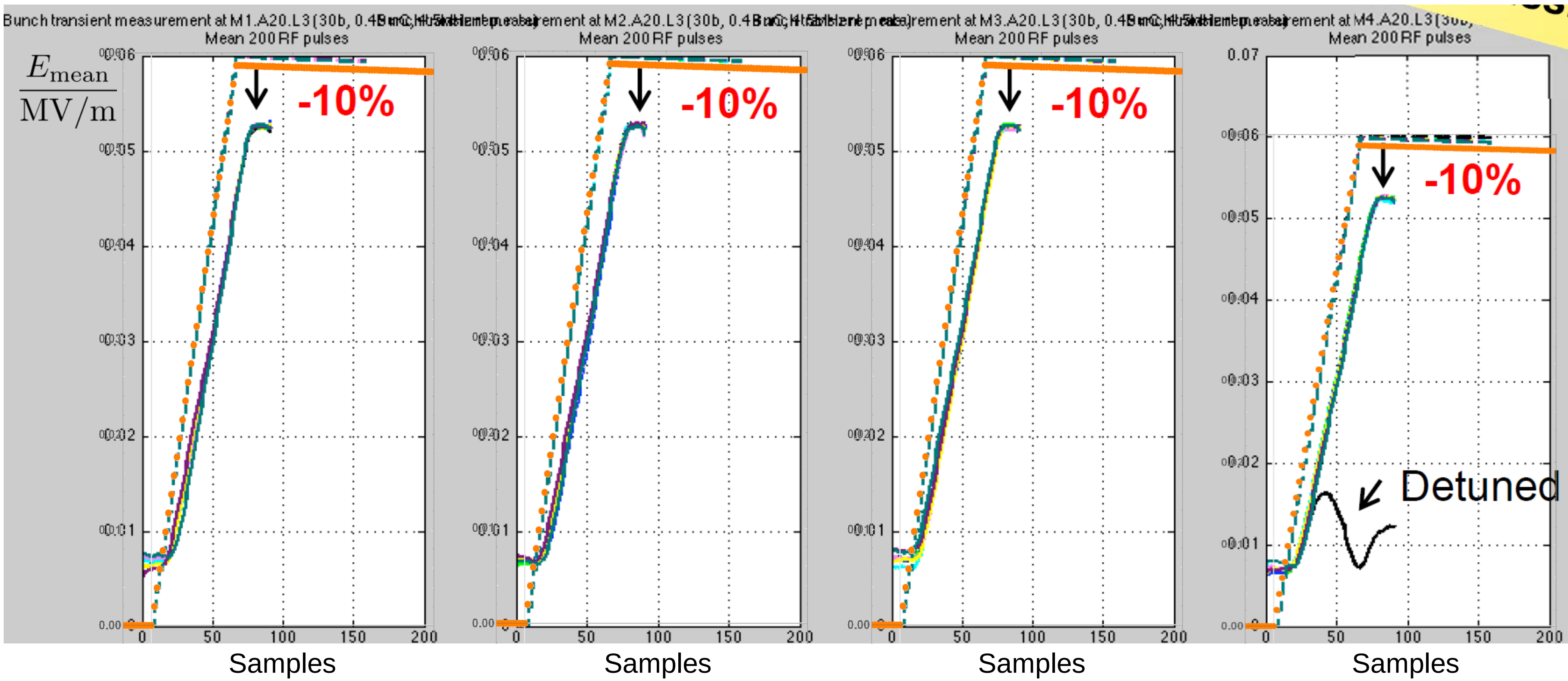
- Beam-Loading Simulations
  - Example



Numerical Results

- Beam-Loading Measurements

L=1.035 m, Q = 0.49 nC

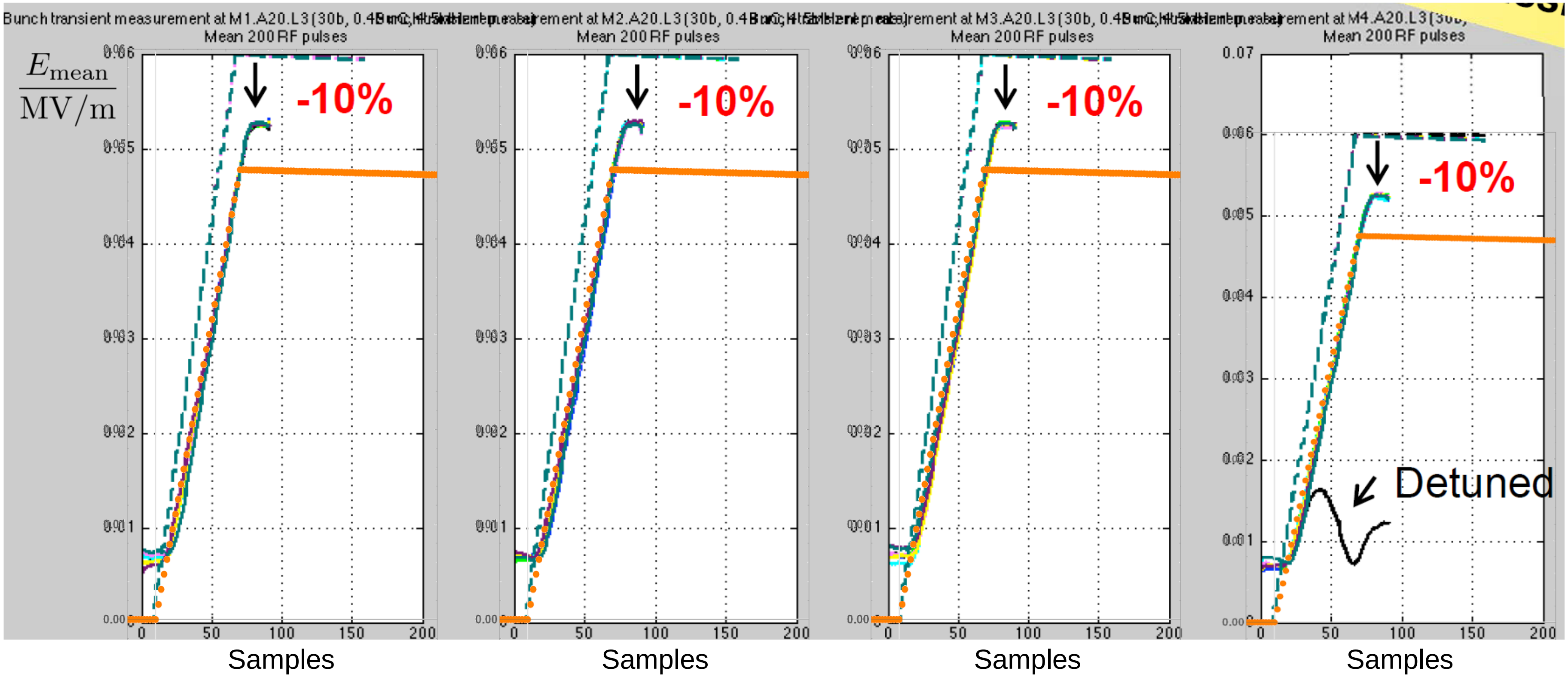


$f_{\text{sampling}} = 9.0\,\text{MHz}$

Numerical Results

- Beam-Loading Measurements

L=1.2834 m, Q = 0.49 nC



$f_{\text{sampling}} = 9.0\,\text{MHz}$

Numerical Results

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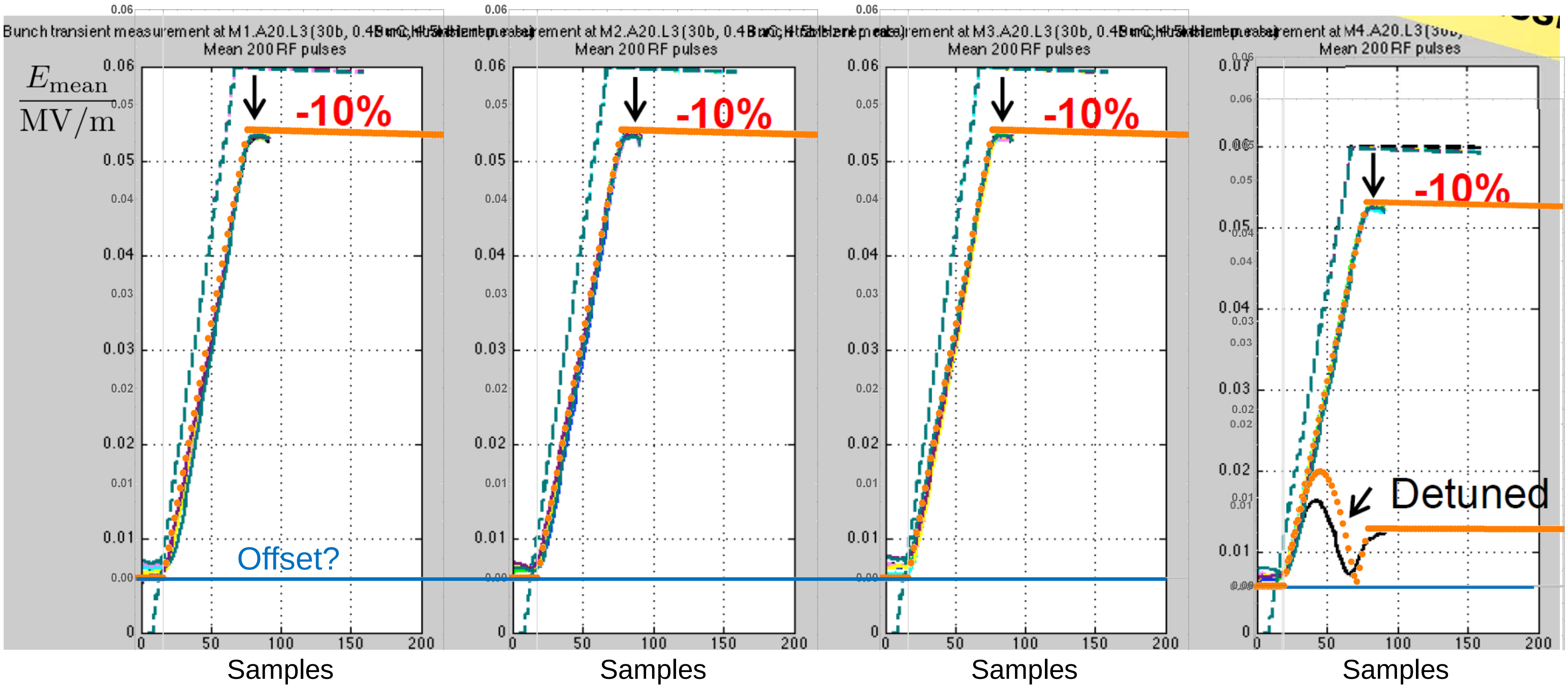
- Beam-Loading Measurements

L=1.2834 m, Q = 0.49 nC



$f_{\mathrm{sampling}} = 9.0\,\mathrm{MHz}$

Discussion

- Possible Sources for the Observed Mismatch
  - Bunch repetition frequency?
  - Bunch charge?
  - Number of Bunches?
  - Longitudinal voltage of the fundamental mode?
    (loss parameter, R/Q)
  - Influence of the bunch length?

  - Measurement of modes other than the fundamental one?
    (signal processing for the pickup voltage, offset)
  - Mean gradient definition?

Summary / Outlook

- Summary
  - Modeling of the beam-loading process

    Derivation from first principles.

    Concentration on the fundamental mode and a point charge excitation.
  - Comparison to measurements

    The measured data can be reproduced on a trial basis with a modified mean gradient and an offset of the pickup signal, but another set of parameters may lead to similar results. Further discussions required.
- Outlook
  - Rule out step-by-step any possible source of errors