Calculation of a SC Gun Cavity with Pickup

the following proposal assumes that we do not need HOM couplers, but that is not certain

an equivalent network

a symmetric 2-port system

equation of motion for external field

field stimulation by driven motion

results / remarks / summary

An Equivalent Network

$$\nabla\times\nabla\times\mathbf{E}+\mu\varepsilon\frac{\partial^2}{\partial t^2}\mathbf{E}=-\frac{\partial}{\partial t}\mu\mathbf{J}$$

$$\nabla\times\nabla\times\mathbf{E}_\nu=\mu\varepsilon\omega_\nu^2\mathbf{E}_\nu$$

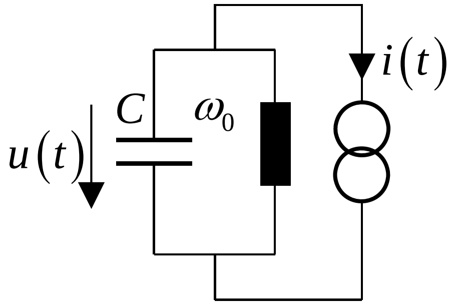
$$\mathbf{E}(\mathbf{r},t)=\sum\alpha_\nu(t)\mathbf{E}_\nu(\mathbf{r})$$

$$\mu\varepsilon\sum\left(\omega_\nu^2+\frac{\partial^2}{\partial t^2}\right)\alpha_\nu(t)\mathbf{E}_\nu(\mathbf{r})=-\frac{\partial}{\partial t}\mu\mathbf{J}$$

$$\int\varepsilon\mathbf{E}_\nu\mathbf{E}_\mu dV=W_\nu\delta_{\nu\mu}$$

$$\rightarrow\left(\omega_\nu^2+\frac{\partial^2}{\partial t^2}\right)\alpha_\nu(t)=-\frac{\partial}{\partial t}\underbrace{\int\mathbf{E}_\nu\mathbf{J}dV}_{g_\nu(t)}$$

$$\mathbf{J}=q\dot{\mathbf{r}}_p(t)\delta\left(\mathbf{r}-\mathbf{r}_p(t)\right)\rightarrow g_\nu(t)=q\dot{\mathbf{r}}_p(t)\cdot\mathbf{E}_\nu\left(\mathbf{r}_p(t)\right)$$



$$\rightarrow\left(\omega_0^2+\frac{d^2}{dt^2}\right)u(t)=-\frac{1}{C}\frac{d}{dt}i(t)$$

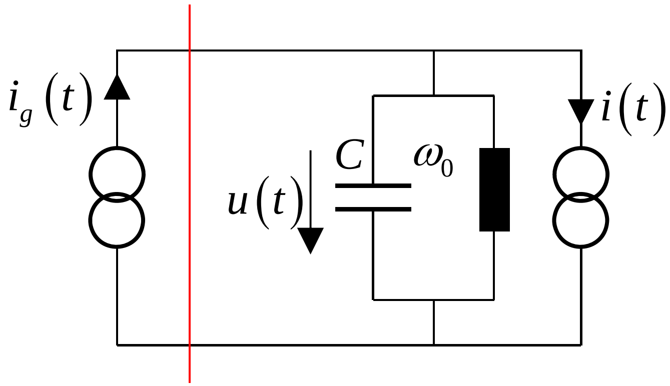
$$u(t)=-\frac{1}{C}\int_0^t i(\tau)\cos\left(\omega_0(t-\tau)\right)d\tau$$

Waveguide Port

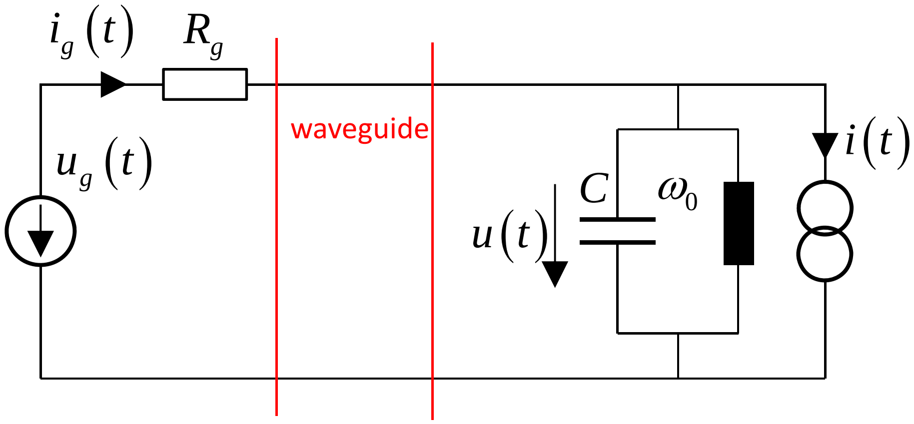
discretize cavity with coupler and (TEM) waveguide

make the waveguide long enough so that higher waveguide modes at the end are negligible $\rightarrow$ description by discrete quantities

calculate eigenmodes with PMC boundary after the waveguide;
consider the port stimulation by a 2d current distribution (proportional to the transverse field pattern of the waveguide mode) $\rightarrow$ eigenmode analysis as before

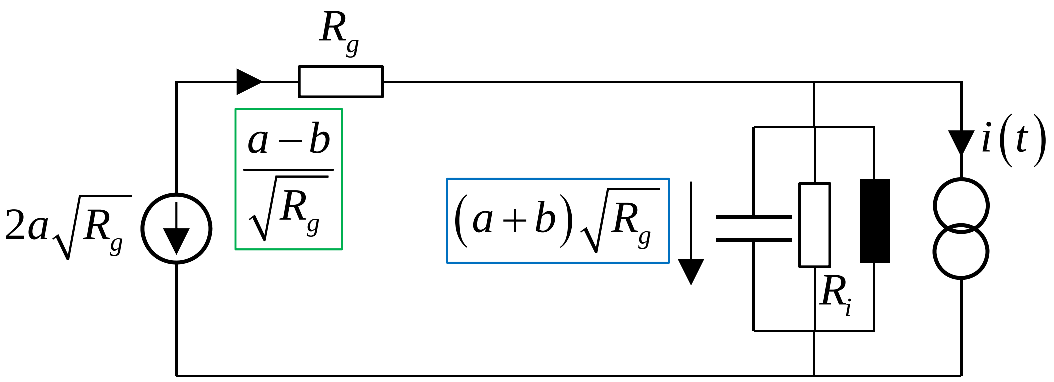


calculate the generator current with help of an external network



waveguide parameters:

$$i_g = \frac{a-b}{\sqrt{R_g}} \qquad u = (a+b)\sqrt{R_g}$$



this is usually done in frequency domain: $i(t) = I_{dc}\sum_{\alpha=-\infty}^{\infty}\delta(t-\alpha T) = I_{dc} + \underbrace{2I_{dc}}_{I_{ac}}\sum_{\beta=1}^{\infty}\cos(\beta 2\pi t/T)$!

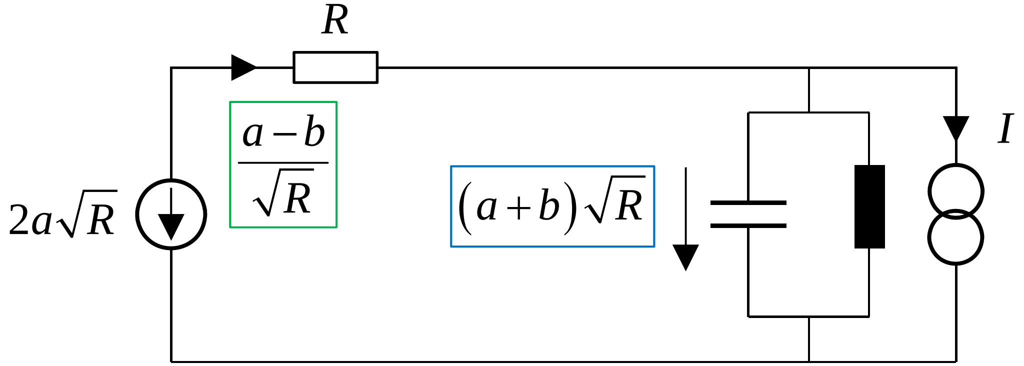
$$Z = \frac{V}{I_{ac}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R_i}}$$

$$\underbrace{(a+b)\sqrt{R_g}}_{V} = Z\left(\frac{a-b}{\sqrt{R_g}} - I_{ac}\right)$$

port quantities

beam quantities

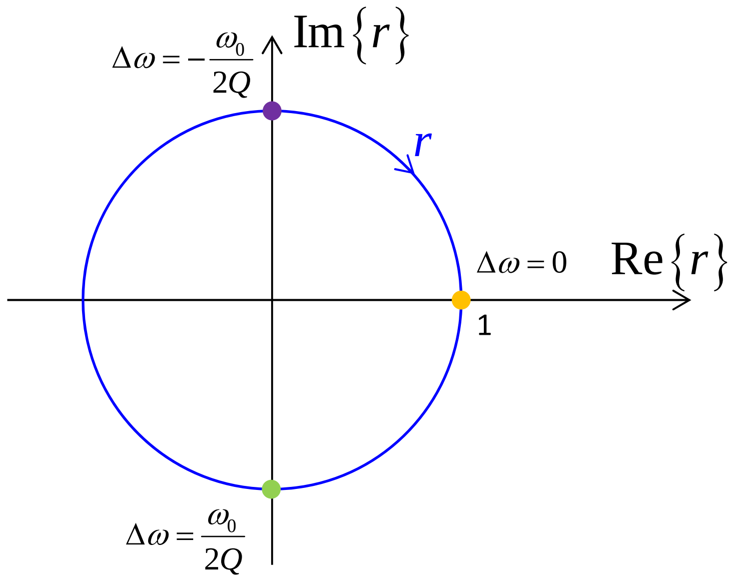
in the following: $\frac{1}{R_i} = 0 \qquad R_g \to R \qquad I_{ac} \to I$



$$Z = \frac{1}{C}\frac{j\omega}{\omega_0^2 - \omega^2} \approx \frac{-j}{2\Delta\omega C}$$

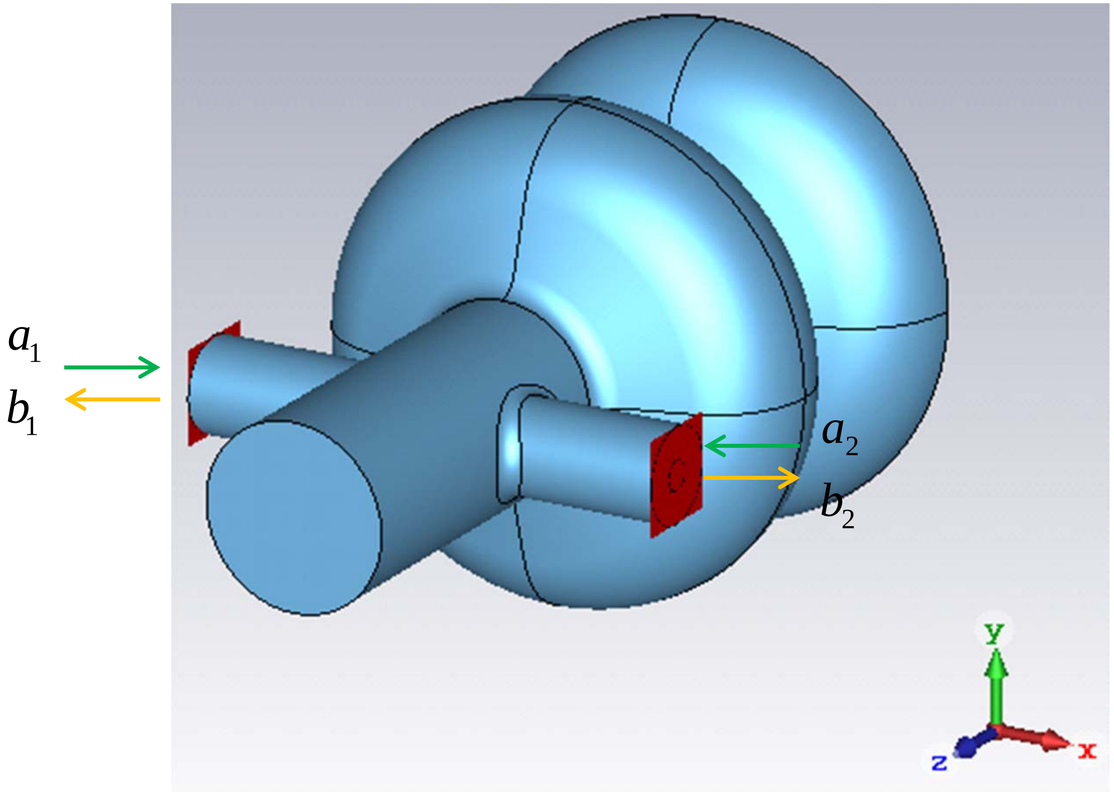
$$\Delta\omega = \omega - \omega_0$$

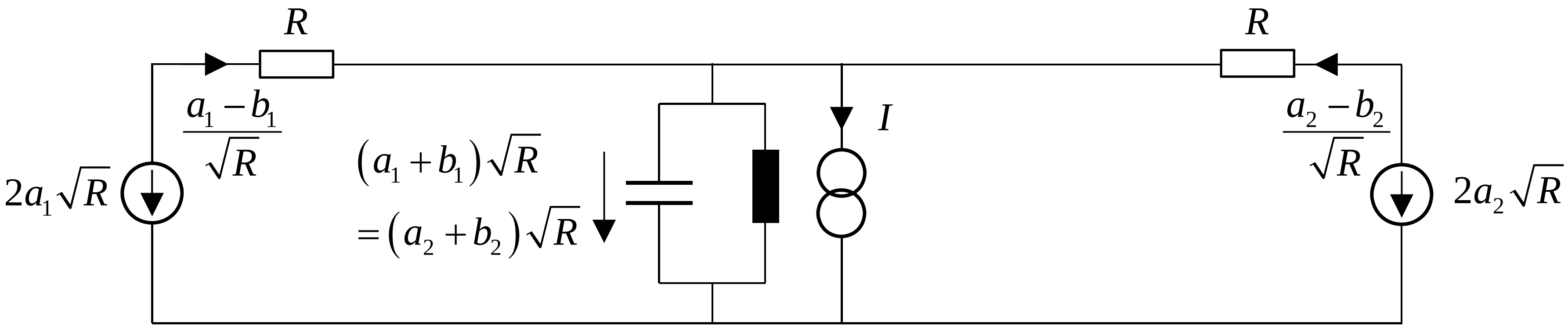
$$b = \underbrace{\frac{Z-R}{Z+R}}_{r} a - \frac{Z\sqrt{R}}{Z+R} I$$



$$r = \frac{1 - j2Q\dfrac{\Delta\omega}{\omega_0}}{1 + j2Q\dfrac{\Delta\omega}{\omega_0}} \qquad Q = \omega_0 RC$$

Symmetric Two Port System





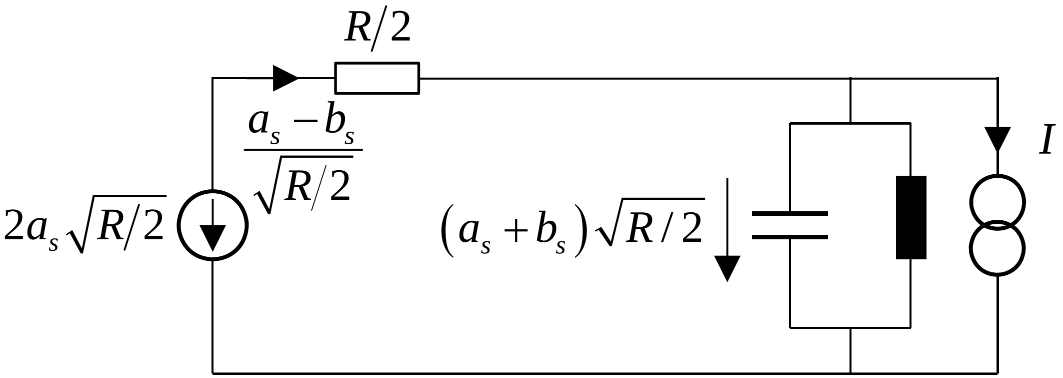
symmetric and antisymmetric operation (without beam)

$$\begin{pmatrix} a_s \\ a_a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\begin{pmatrix} b_s \\ b_a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} b_s \\ b_a \end{pmatrix} = \begin{pmatrix} r_s & 0 \\ 0 & r_a \end{pmatrix} \begin{pmatrix} a_s \\ a_a \end{pmatrix}$$

symmetric:



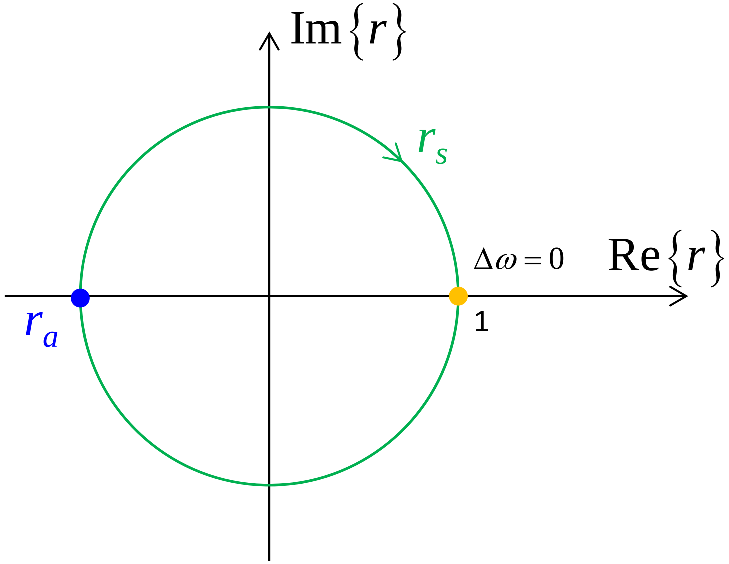
same network as for one-port,
but $R \to R/2$ and $Q \to Q/2$

$$r_s = \frac{Z - R/2}{Z + R/2}$$

anti-symmetric: $(a_1 + b_1)\sqrt{R} = (a_2 + b_2)\sqrt{R} \to a_a = -b_a \to r_a = -1$

s/a system

$$\begin{pmatrix} b_s \\ b_a \end{pmatrix} = \begin{pmatrix} r_s & 0 \\ 0 & r_a \end{pmatrix} \begin{pmatrix} a_s \\ a_a \end{pmatrix}$$

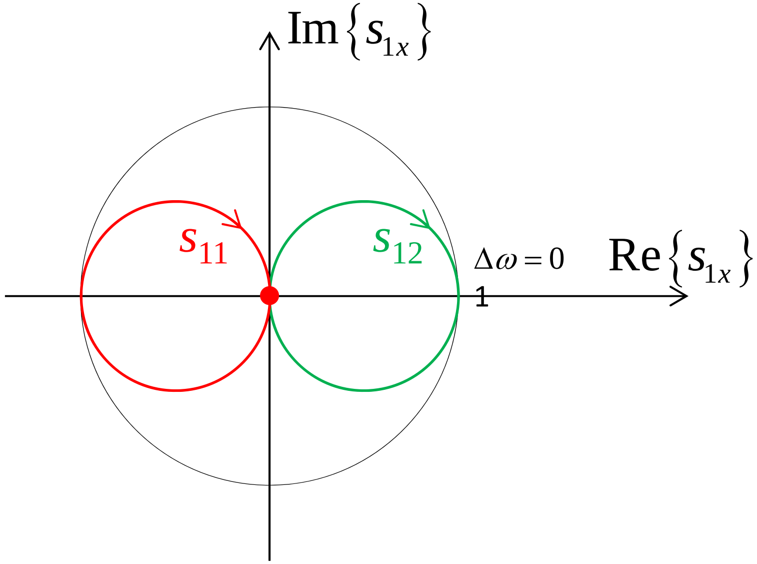


1/2 system

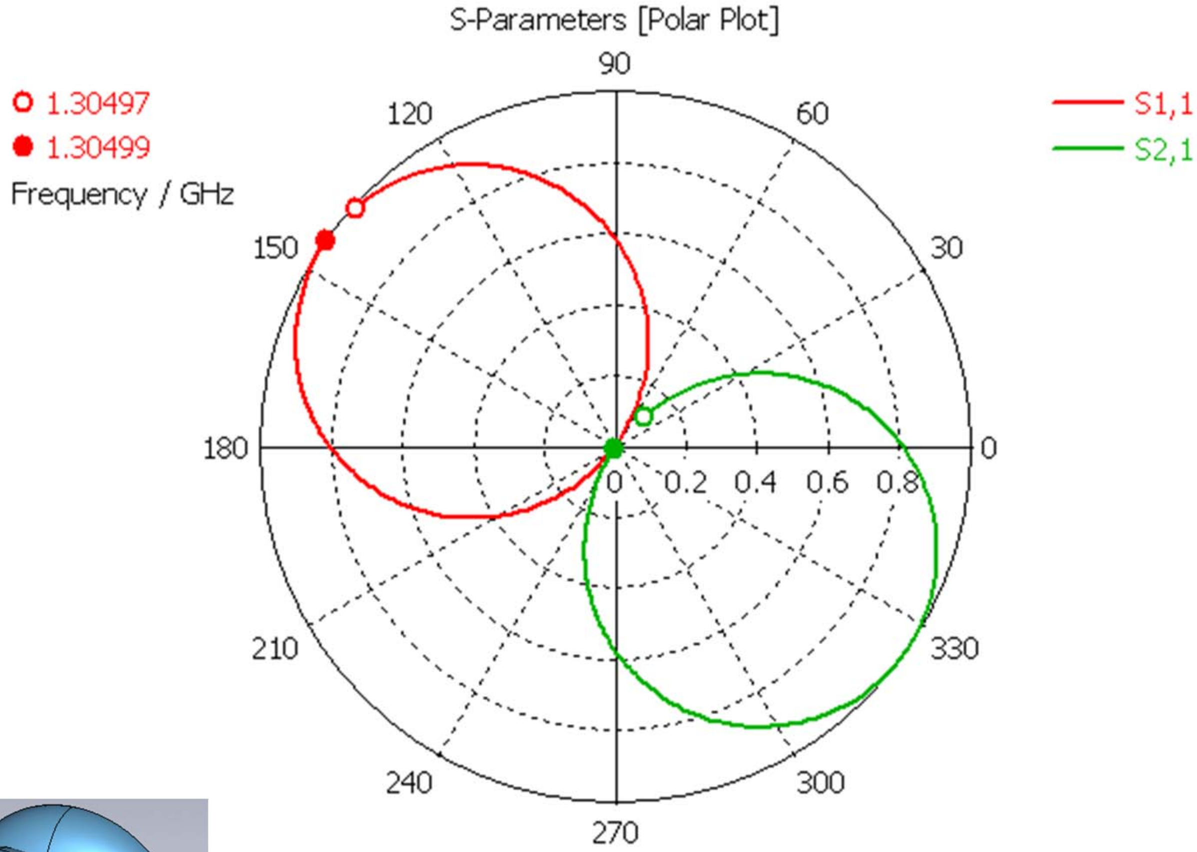
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{12} & s_{11} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

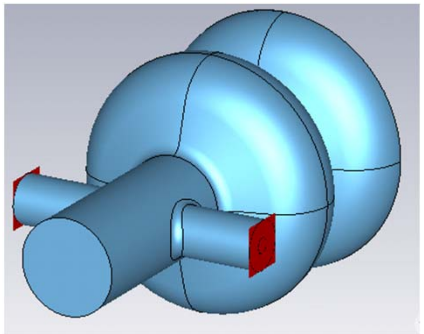
$$s_{11} = \frac{r_s + r_a}{2}$$

$$s_{12} = \frac{r_s - r_a}{2}$$



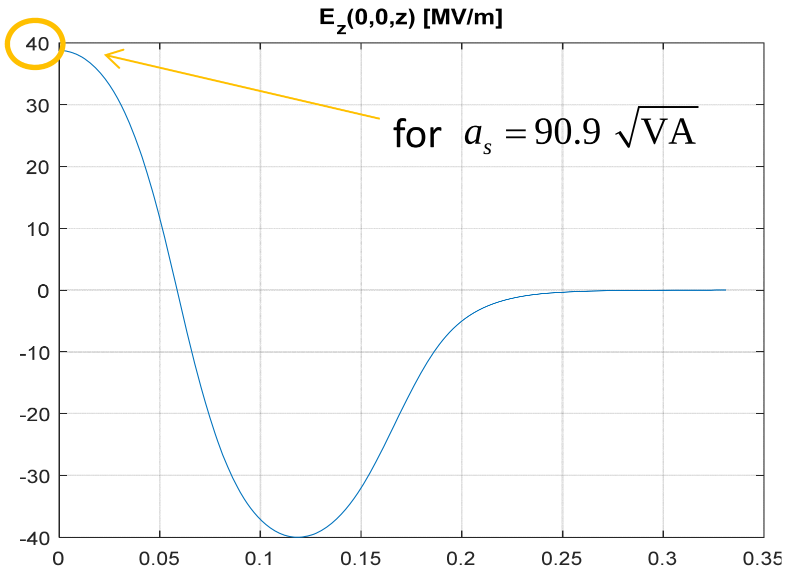
calculation by MWS $\rightarrow Q = 5 \cdot 10^6$

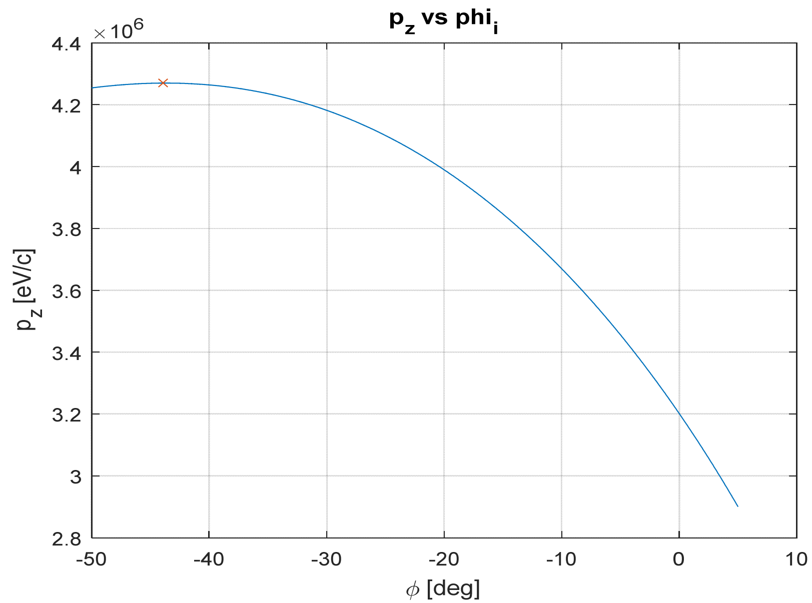


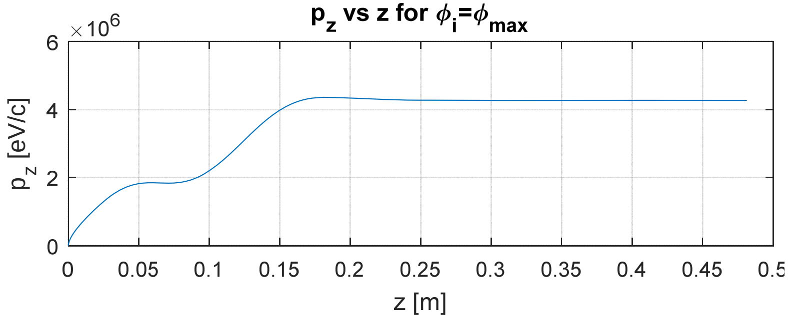


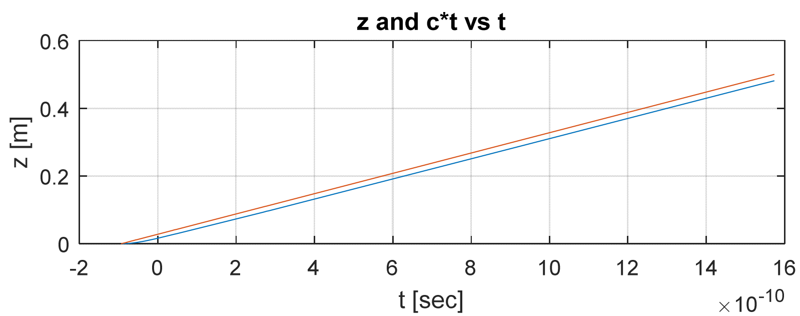
this corresponds to the theoretical result after a shift of the reference plane

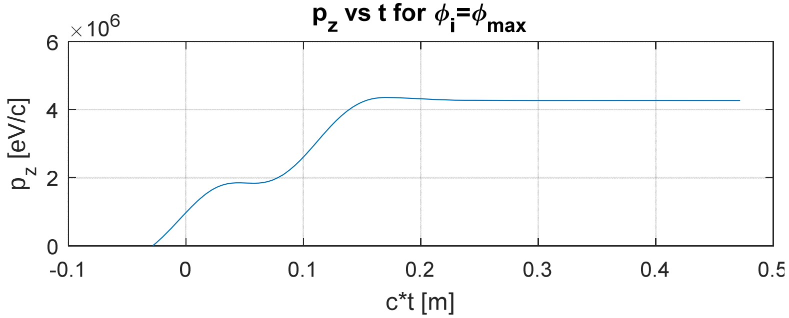
EoM for External Field $\rightarrow \mathbf{r}_p(t)$

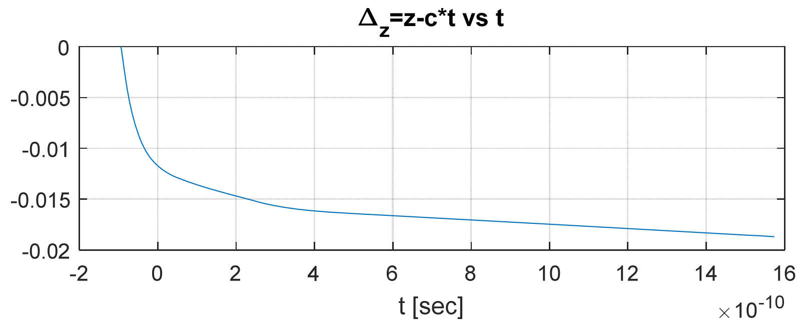












Voltages for Driven Motion

we are interested in the following “voltage” integrals for driven particle motion:

$$V = \frac{\mathcal{E}_{\mathrm{kin}}}{q} = \int E_z(z)\exp\left(j\omega_0 t_p(z)\right)dz \qquad \rightarrow p = \frac{\beta}{c}\left(\mathcal{E}_{\mathrm{kin}} + mc^2\right)$$

$$V_\perp = \frac{c}{q}\Delta p_x = \frac{1}{q}\int F_x\left(z_p(t)\right)\exp\left(j\omega_0 t\right)c\,dt \qquad \frac{\Delta p_x}{p} = \frac{V_\perp}{\beta\left(V + mc^2/q\right)}$$

with $z_p(t)$ and $t_p(z)$ the motion in due to external fields

f.i.: $a_s = 90.9\ \sqrt{\mathrm{VA}} \rightarrow p = 4.27\ \mathrm{MeV/c} \rightarrow V = 3.79\ \mathrm{MV}$

$$V_\perp = 0\ \mathrm{V}$$

or: $a_a = 1\ \sqrt{\mathrm{VA}} \rightarrow V = 0\ \mathrm{V}$

$$V_\perp = -0.830\ \mathrm{V}$$

↓

we know all what we need!

putting the pieces together

operation with beam, on resonance ($Z \to \infty$)

one port system:

$$b = a - I\sqrt{R}$$

$$V = a2\sqrt{R} - IR$$

$$V_\perp = aX_{a1} - IX_{I1}$$

$$R \approx \frac{1}{6} Q_1 \left(\frac{R}{Q}\right)_{TESLA} = 8.7 \cdot 10^8 \text{ Ohm}$$

$$Q_1 = 10 \cdot 10^6$$

two port system:

$$b_s = a_s - I\sqrt{R/2}$$

$$b_a = -a_a$$

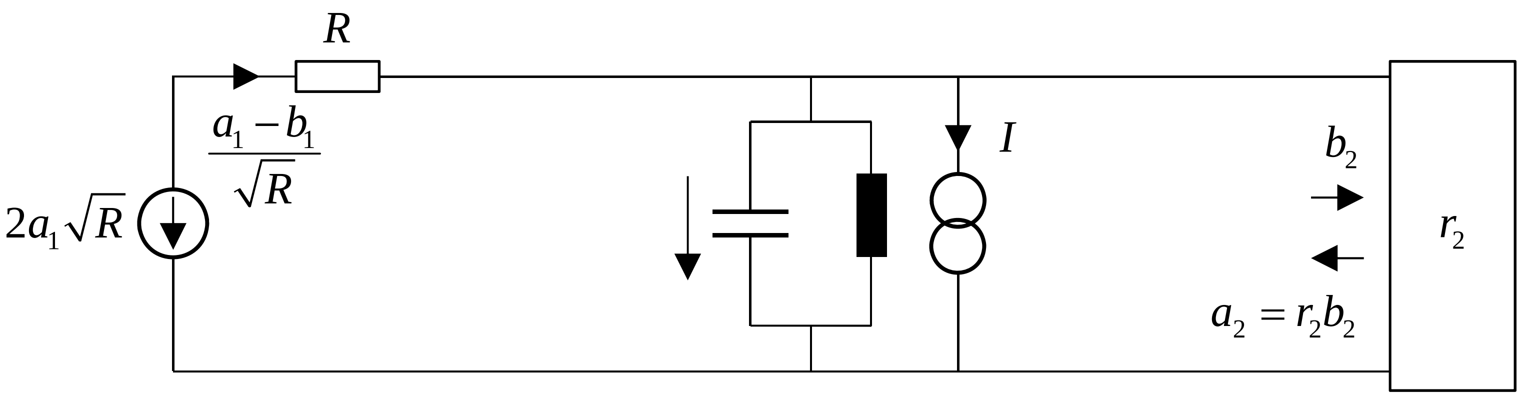
$$V = a_s\sqrt{2R} - I\,R/2 \qquad \to R = \frac{1}{2}\left(\frac{V(a_s,0)}{a_s}\right)^2 = 2\frac{V(0,I)}{I}$$

$$V_\perp = a_a X_{a2} \qquad \to X_{a2} = \frac{V_\perp(a_a)}{a_a}$$

$$X_{I2} = 0$$

$$R = \frac{1}{2}\left(\frac{3.79 \text{ MV}}{90.9\sqrt{\text{VA}}}\right)^2 = 8.7 \cdot 10^8 \text{ Ohm} \qquad X_{a2} = -\frac{0.830 \text{ V}}{1\sqrt{\text{VA}}} = -0.830\sqrt{\text{Ohm}} \qquad Q_2 = 5 \cdot 10^6$$

New One-Port System



$$V_{\parallel} = \sqrt{R}\left(1+r_2\right)a_1 - \frac{R}{2}\left(1+r_2\right)I$$

$$b_1 = r_2 a_1 - \frac{1+r_2}{2}\sqrt{R}I$$

$$V_{\perp} = X_{a2}\frac{1-r_2}{\sqrt{2}}a_1 + \frac{X_{a2}\sqrt{R}}{2\sqrt{2}}r_2 I$$

$$b_2 = a_1 - \frac{I_s\sqrt{R}}{2} = \frac{V_{\parallel}}{\left(1+r_2\right)\sqrt{R}}$$

for $r_2 = 1$ the new system behaves exactly like the original one-port system with Q = 10E6

no transverse field is stimulated by the klystron (a_1); the stimulation by the beam (I) is not avoidable; this is the same for any passive compensation

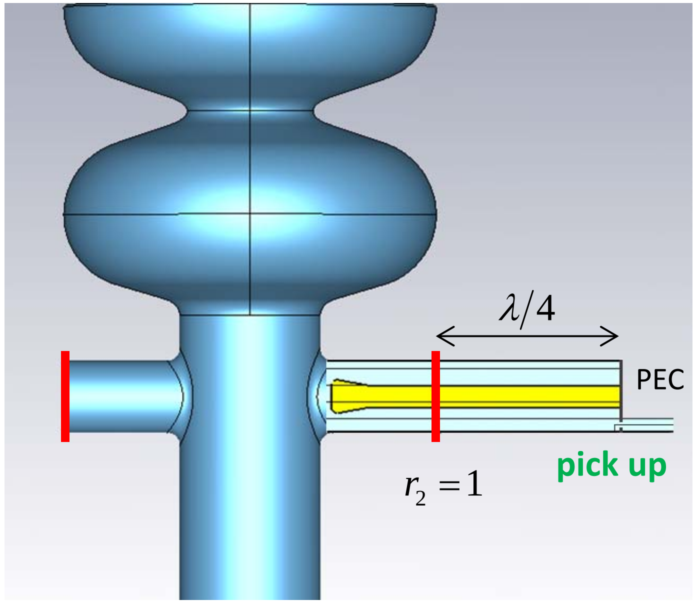
b_2 is direct proportional to the cavity voltage; a small part of b_2 could be coupled out and used as **pickup** for the cavity voltage!

in numbers: $Q = 10 \cdot 10^6$

$R = 8.7 \cdot 10^8$ Ohm

$X_{a2} = -0.830\ \sqrt{\text{Ohm}}$

$V_{\|} = 3.79$ MV



I_{dc} / mA	0	2.2	0.1
P_{1f} / kW	2.06	8.26	**2.16**
P_{1r} / kW	2.06	0	1.78
$V_{\perp}$ / V	0	-37.7	-1.7
$\Delta p_x / p$	0	-8.8E-6	**-0.4E-6**

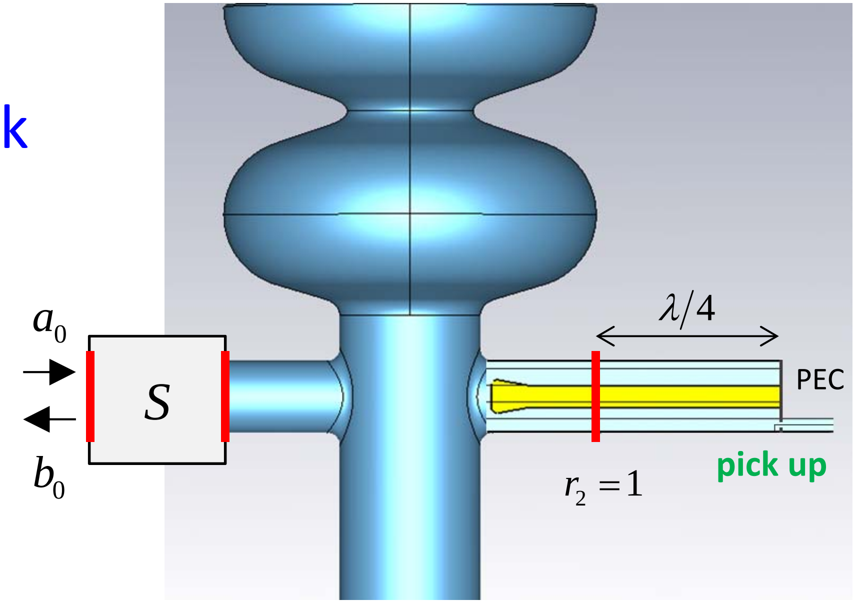
operation with 100 pC, 1MHz
only 379 W go to the beam
increase Q to decrease P_{1f}

method 1: decrease coupling, pull antennas back

Method 2:
use (external) matching network

$$\begin{pmatrix} b_0 \\ a_1 \end{pmatrix} = \underbrace{\frac{1}{1+h^2}\begin{pmatrix} 1-h^2 & 2h \\ 2h & h^2-1 \end{pmatrix}}_{S}\begin{pmatrix} a_0 \\ b_1 \end{pmatrix}$$

f.i. lambda/4 transformer



$$Q_0 = h^2 Q_1$$

$$b_0 = a_0 - h\sqrt{RI}$$

$$V_\parallel = 2h\sqrt{R}a_0 - h^2 RI$$

$$V_\perp = \frac{X_{a2}\sqrt{R}}{2\sqrt{2}} I$$

for $V_\parallel = 3.79$ MV, h = 2 and Q_0 = 40E6:

I_{dc} / mA	0	0.1
P_{1f} / W	516	**723**
P_{1r} / W	516	344
$V_\perp$ / V	0	-1.7
$\Delta p_x / p$	0	**-0.4E-6**

Remarks

the **used** discrete network is not empirical but based on Maxwell's equations

one mode model: the accelerating mode is strongly dominant; therefore we neglected all other modes

in general the excitation of modes by the beam and by ports is a **coupled problem**:

$$\left(\omega_\nu^2 + \frac{\partial^2}{\partial t^2}\right)\alpha_\nu(t) = -\frac{\partial}{\partial t}\left(g_\nu(t) + h_\nu(t)\right)$$

$$g_\nu(t) = \int \mathbf{E}_\nu \mathbf{J} \, dV$$ beam

$$h_\nu(t) \sim (a - b) \int \mathbf{E}_\nu \mathbf{E}_{\perp,\mathrm{port}} \, dA$$ port

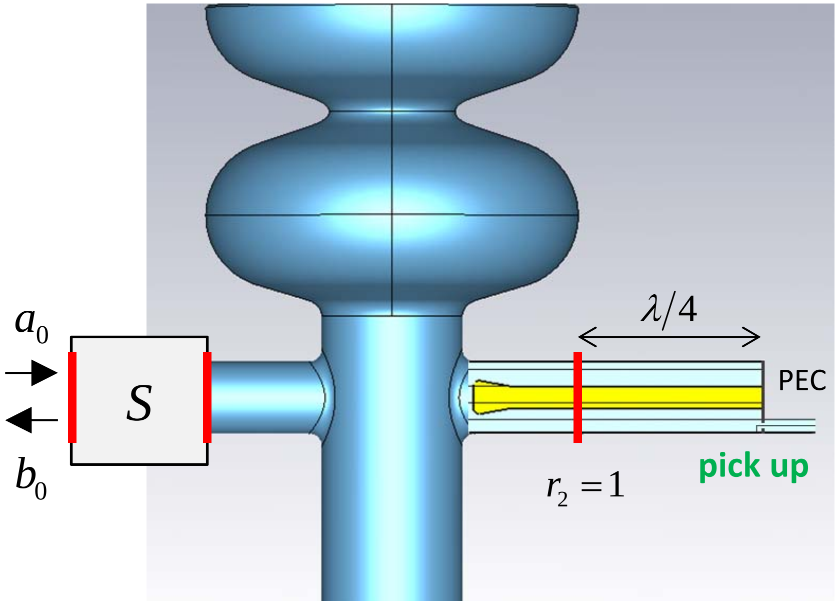
excitation

$$a + b \sim \sum_\nu \alpha_\nu \int \mathbf{E}_\nu \mathbf{E}_{\perp,\mathrm{port}} \, dA$$ coupling

usual equivalent networks for multi-cell cavities **are empirical**

Summary

symmetrical "inner" geometry with asymmetric outer network



one-port coupler

pickup in the compensation stub $\rightarrow$ no interference with the cavity geometry or peak fields; no crosstalk from main port

perfect symmetry due to external stimulation (klystron)

very good symmetry due to beam stimulation

antenna position is fixed, Q_{ext} can be controlled by outer network (this is possible with a moderate standing wave ratio and reasonable peak fields)

if we need HOM couplers: try to keep the symmetry loop coupling