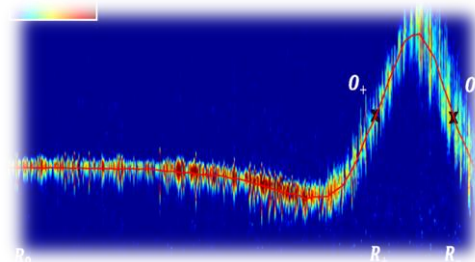


Seeded FLASH spectrum, chirp and pulse duration at once

Armin Azima

26.03.2019

- *The seeded FEL project “sFLASH”*
- *Single XUV pulse THz streaking*
- *A-priori pulse phase shaping results*
- *Amplitude and phase reconstruction of sFLASH pulse*



Supported by BMBF under contract 05K13GU4
and 05K13PE3
DFG GrK 1355
Helmholtz Accelerator R&D



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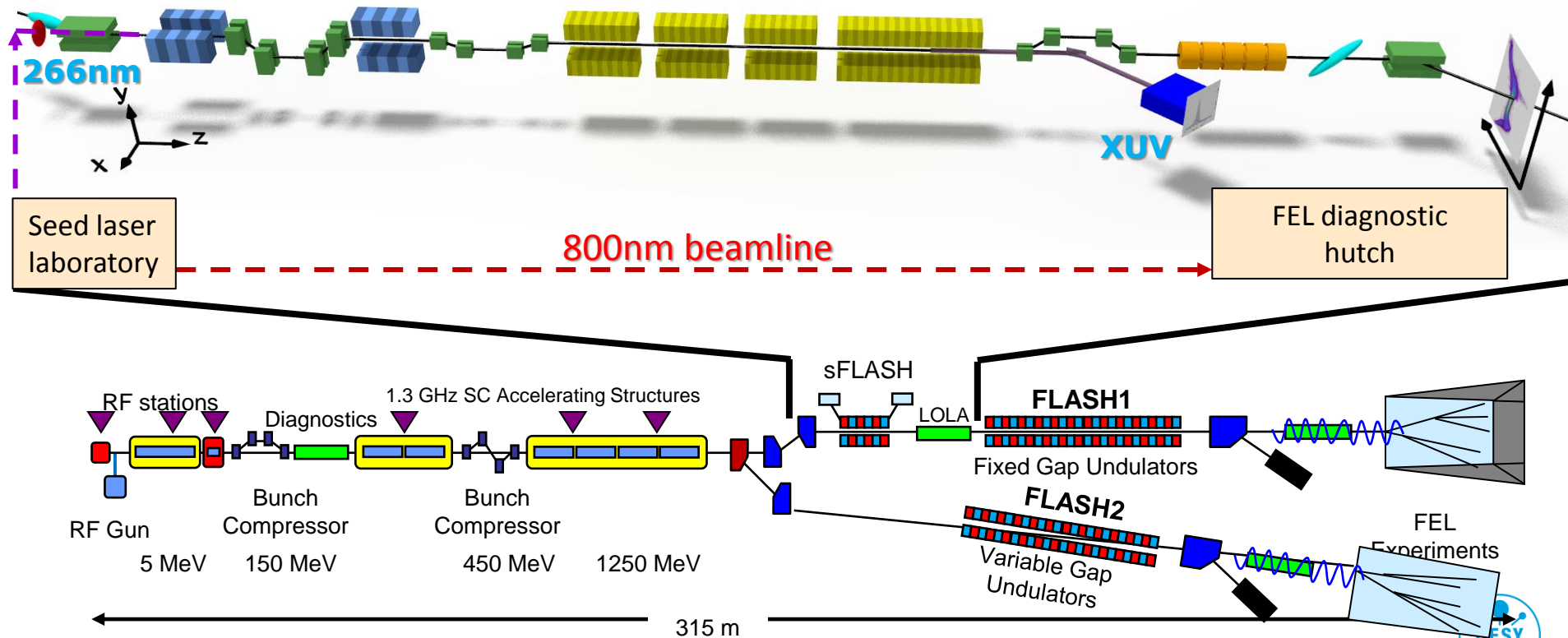
RESEARCH FOR
GRAND CHALLENGES



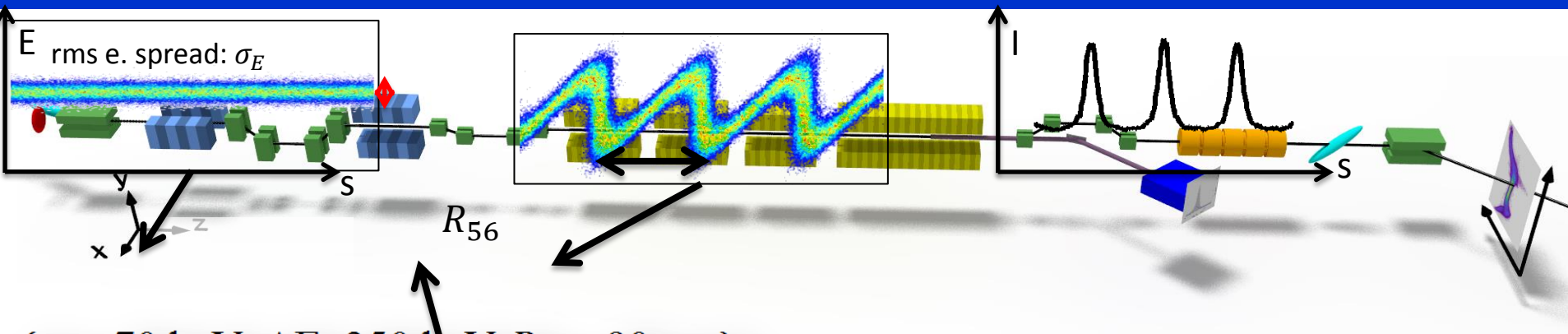
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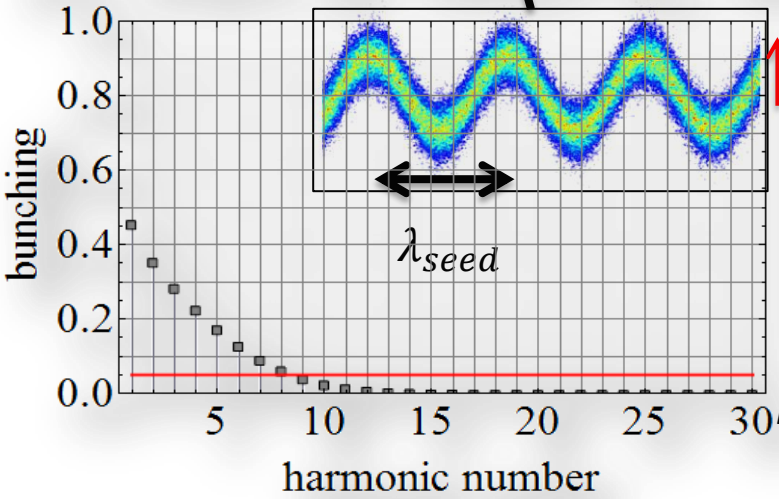
sFLASH layout and operation



High Gain Harmonic Generation (HGHG) seeding



$\{\sigma_E: 70 \text{ keV}, \Delta E: 350 \text{ keV } R_{56}: 90 \mu\text{m}\}$



$$\lambda_{radiator} = \frac{\lambda_{seed}}{n}$$

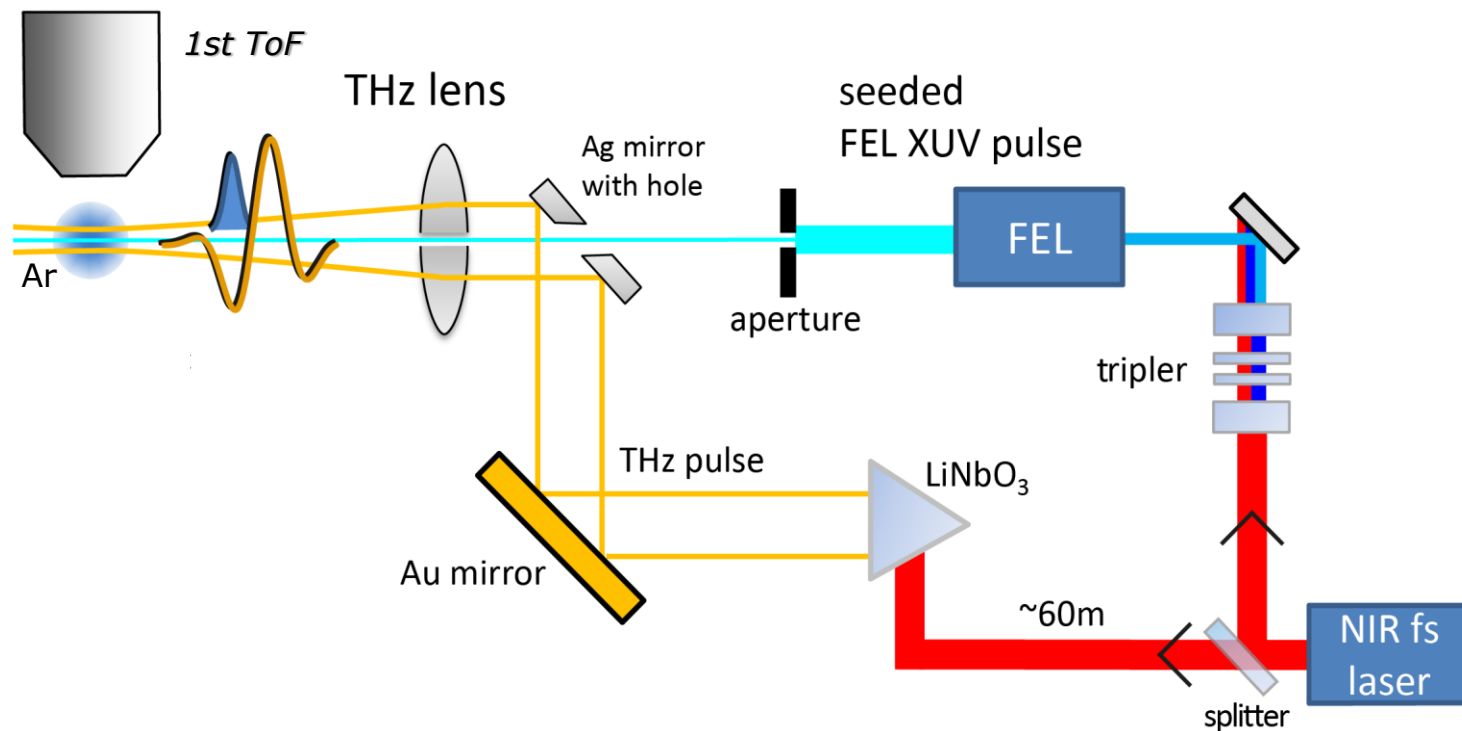
$$n < \frac{\Delta E}{\sigma_E}$$

Bunching factors:

$$b_n = \exp \left[-\frac{1}{2} \cdot \frac{(2\pi)^2 n^2 R_{56}^2 \sigma_E^2}{\lambda_{seed}^2 E_0^2} \right] \cdot J_n \left(\frac{2\pi n \Delta E R_{56}}{\lambda E_0} \right)$$

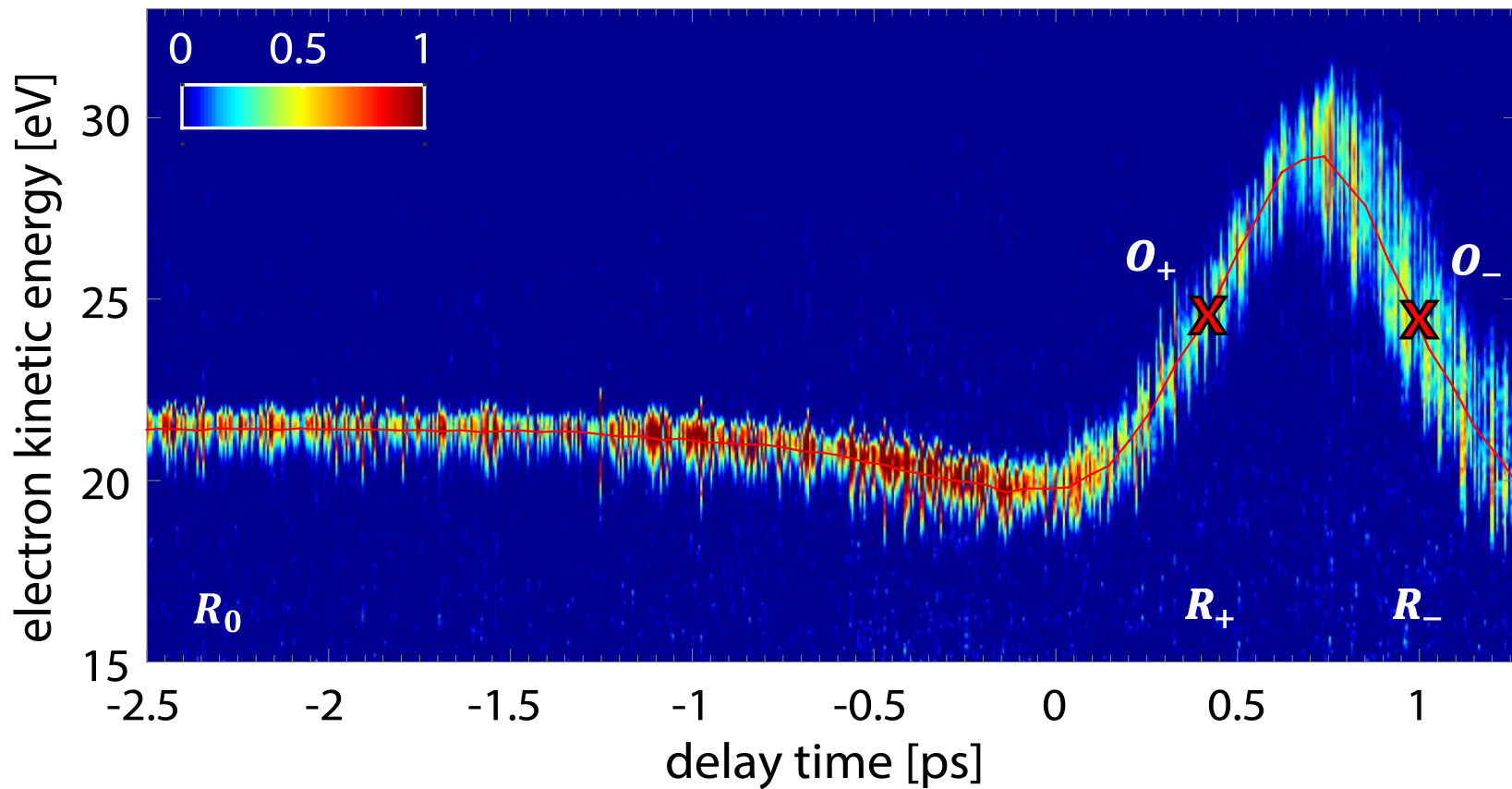
L.H. Yu, Phys.Rev. A 44, 5178 (2001)

THz streaking diagnostic setup



The XUV pulse is directed by a set of screens and apertures into the interaction region. The THz pulse (or optionally a femtosecond 800nm NIR mJ-pulse) is collinearly coupled into the vacuum and focused into the Ar gas flow ($<1e-6$ mbar).

First streaking data of seeded FLASH (8th Harm., 37.2eV in Argon)



Evaluation of chirp and pulse duration of seeded FLASH

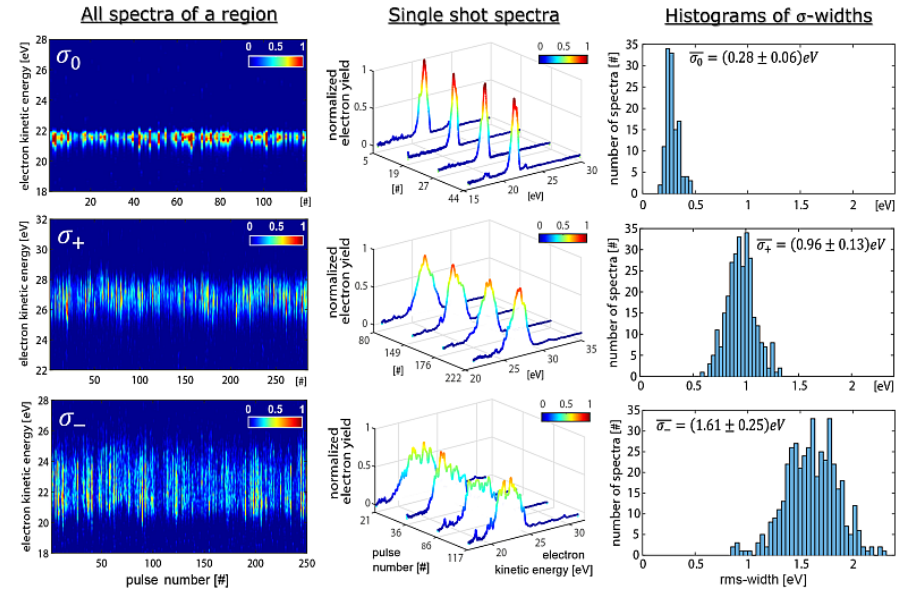
$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 \cdot s_- + \sigma_{-,decon}^2 \cdot s_+}{(s_+ + s_-) \cdot s_+ s_-}}$$

$$c = \frac{\sigma_{+,decon}^2 \cdot s_-^2 - \sigma_{-,decon}^2 \cdot s_+^2}{4s_+ s_- \cdot (s_+ + s_-) \cdot \tau_{XUV}^2}$$

with $\sigma_{\pm,decon} = \sqrt{\sigma_{\pm}^2 - \sigma_0^2}$, s_{\pm} widths of the deconvolved streaked spectra and for the streaking speeds (i.e. the slopes) each measured at the inflexion points O_{\pm} and unstreaked O_0 respectively as defined before

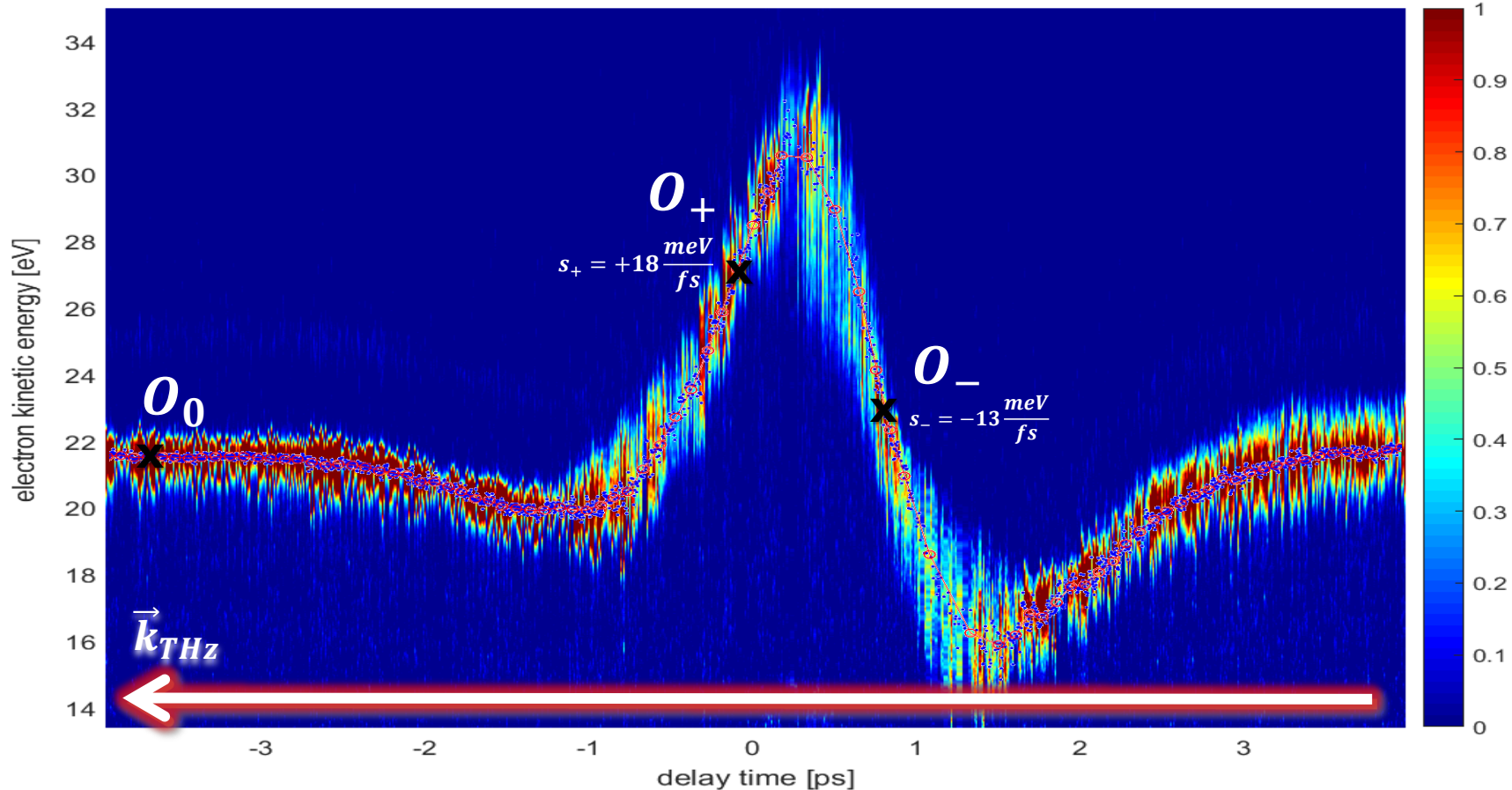
Using the averaged values for $\sigma_{\pm,0}$, we have deduced¹

$$\tau_{XUV,rms} = (58 \pm 7.5) \text{ fs} \quad c = (-1.9 \pm 0.8) \frac{\text{THz}}{\text{fs}}$$

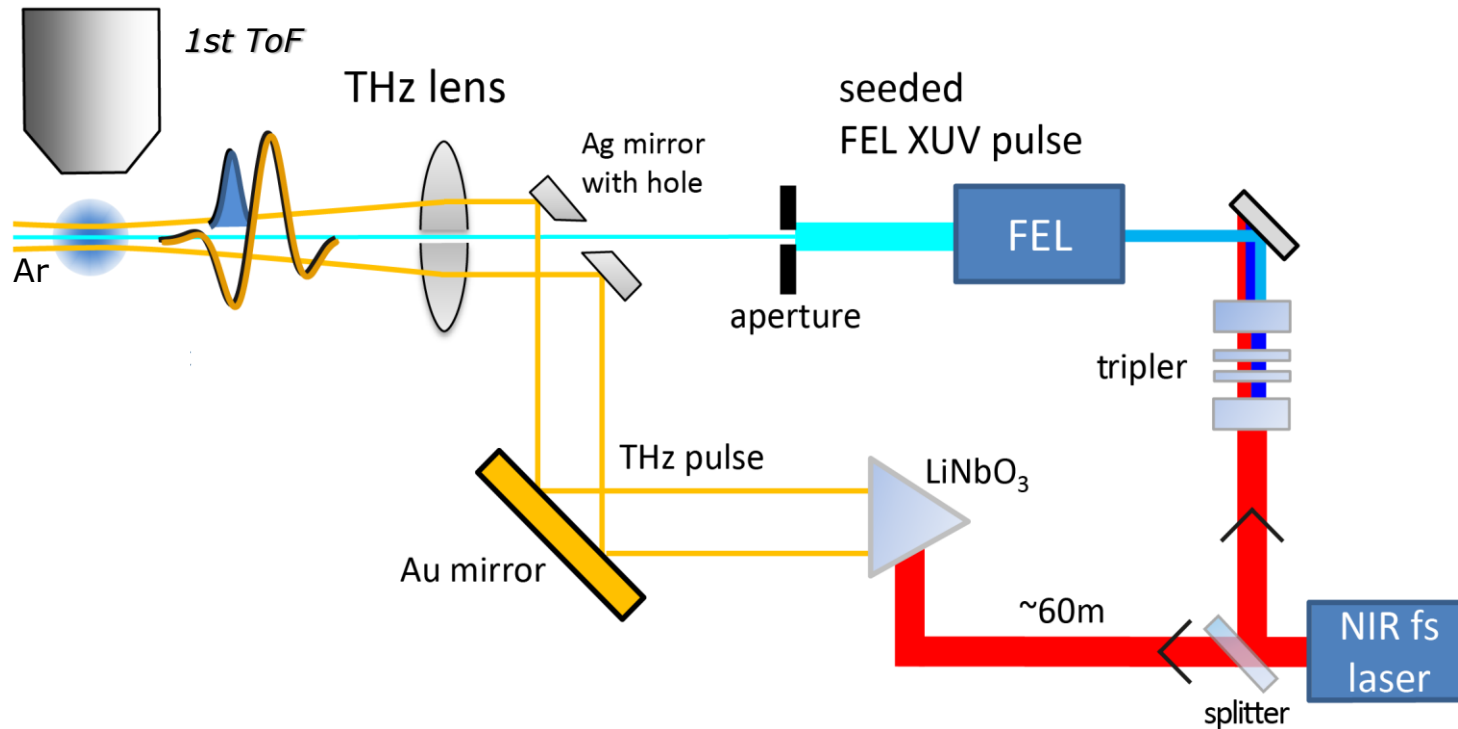


¹A. Azima et al, *New J. Phys.* **20** 013010 (2018)

Another THz streaking with seeded pulses on 8th harmonic (37.2eV)

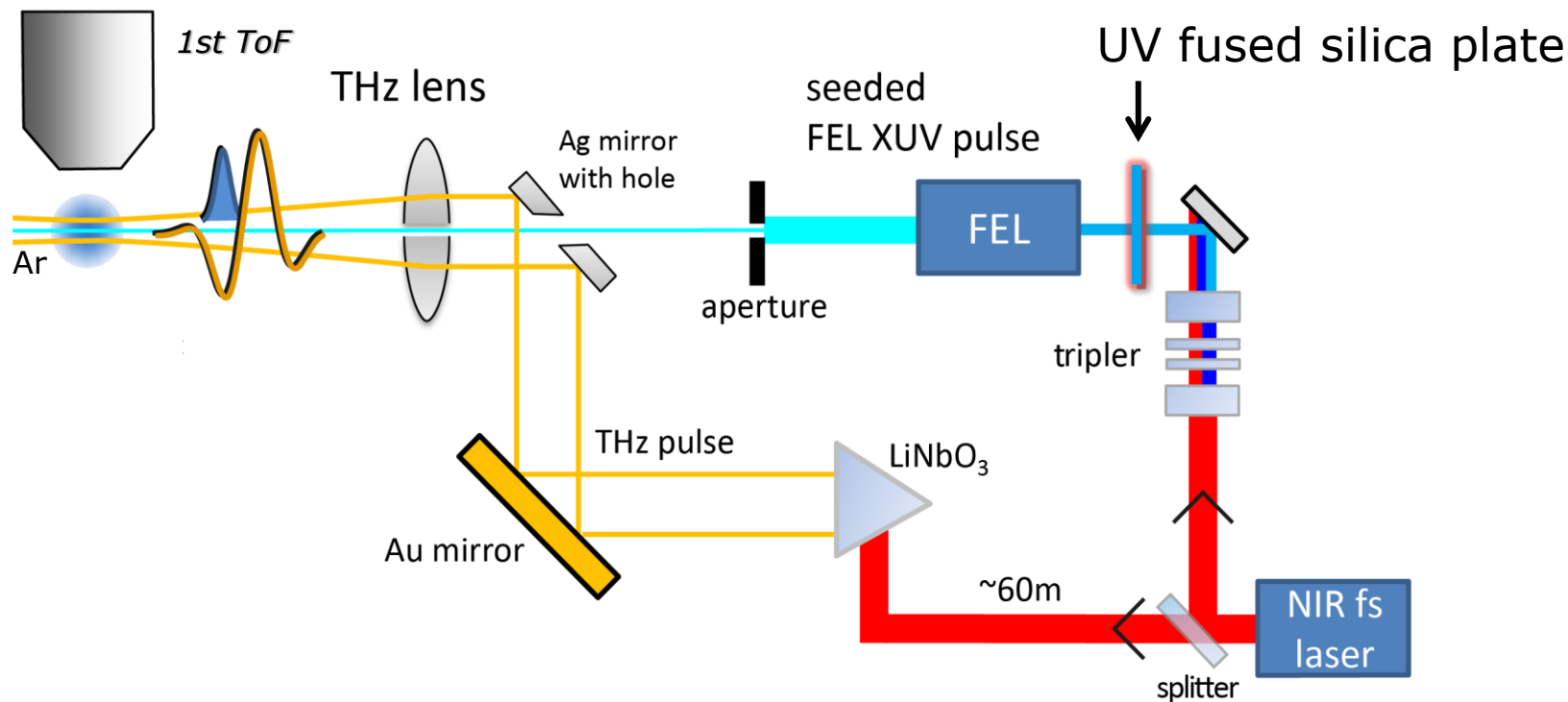


Beam transport and experimental setup



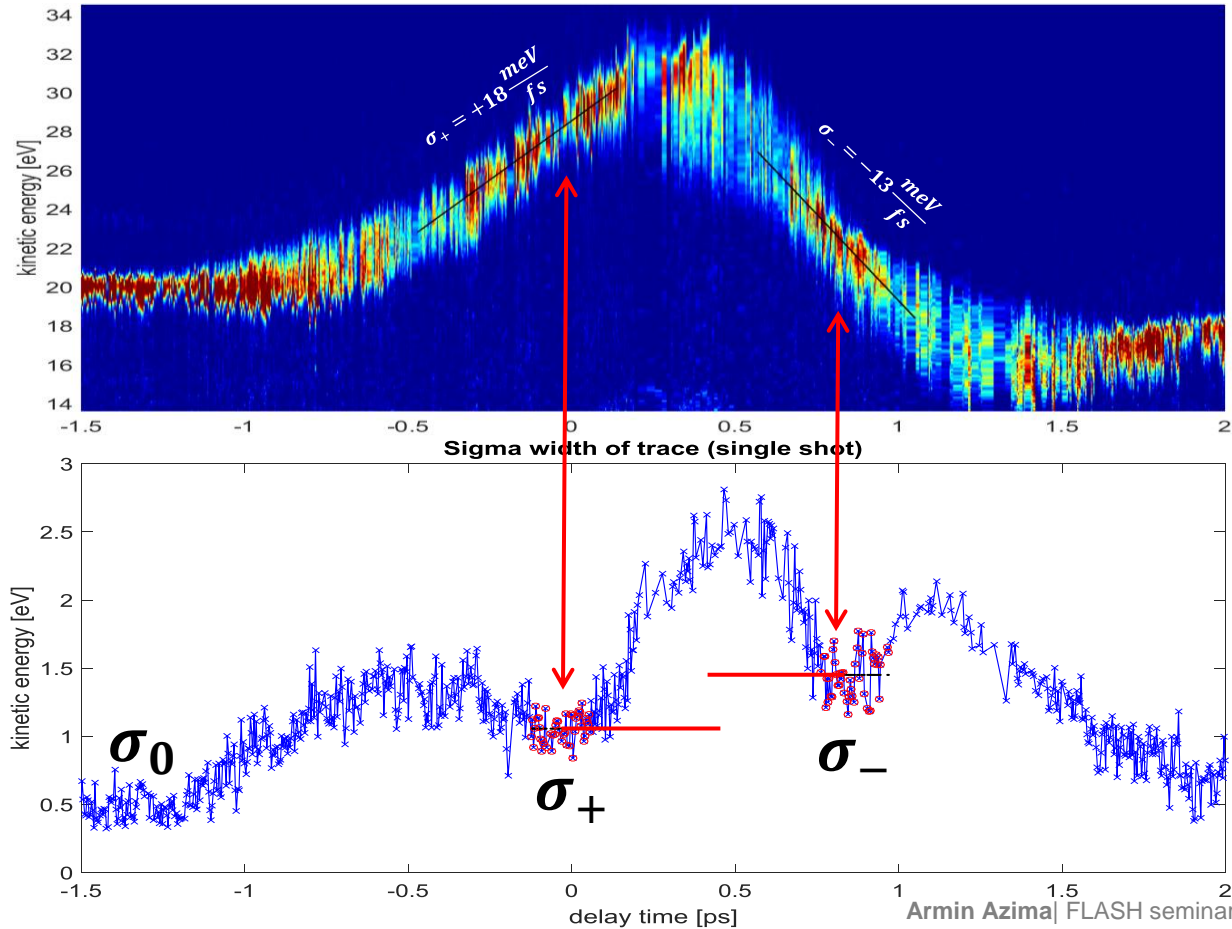
The XUV pulse is directed by a set of screens and apertures into the interaction region. The THz pulse (or optionally a femtosecond 800nm NIR mJ-pulse) is collinearly coupled into the vacuum and focused into the Ar gas flow ($<1e-6\text{mbar}$).

A-priori chirping of the seeded FEL



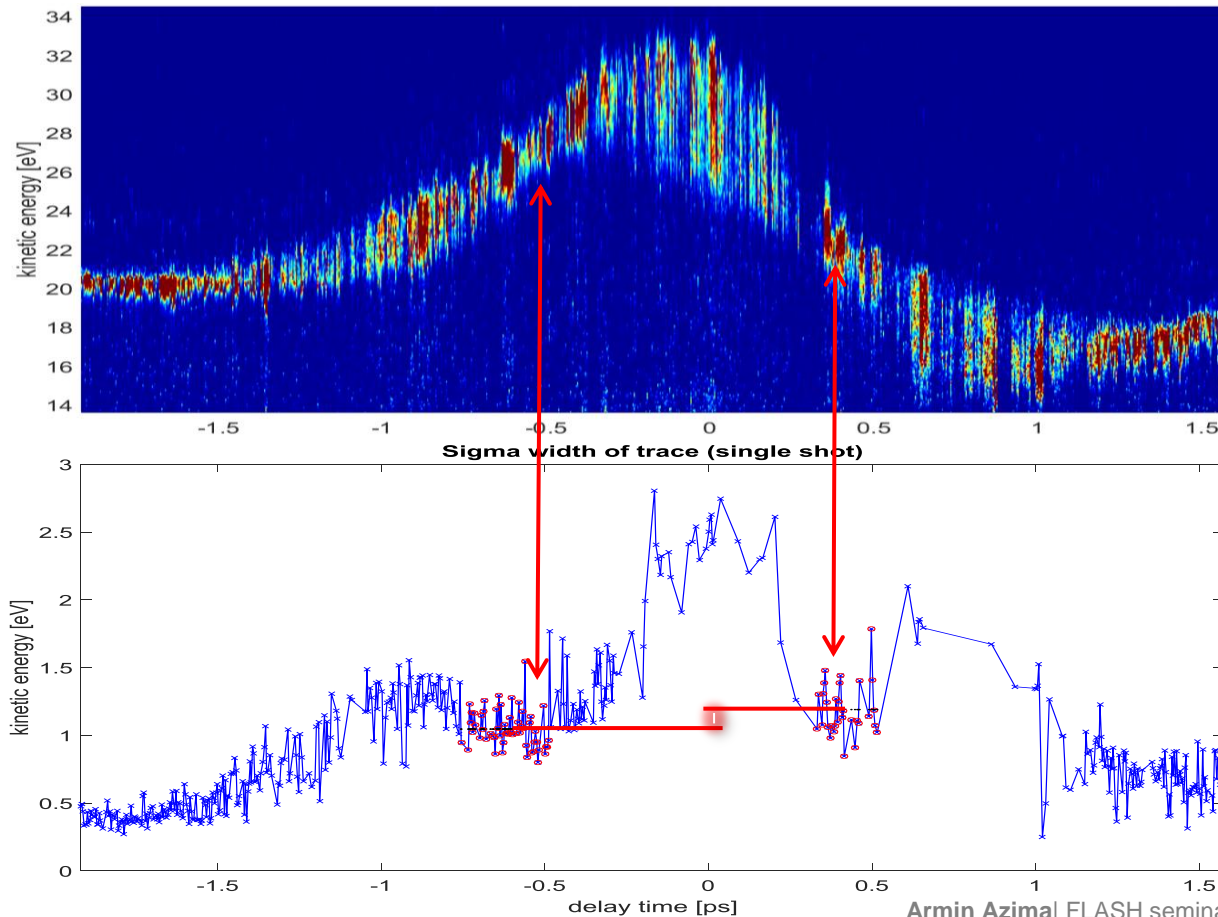
The XUV pulse is directed by a set of screens and apertures into the interaction region. The THz pulse (or optionally a femtosecond 800nm NIR mJ-pulse) is collinearly coupled into the vacuum and focused into the Ar gas flow ($<1e-6$ mbar).

Un-chirped seed pulse (266nm)



No glass plate

CHIRPED seed pulse (266nm)



with 4.4mm
UV fused silica
inserters in the
optical seed pulse
adding 870fs^2 of GDD

Evaluation of chirp with and without pre-chirping

$$\tau_{XUV} = \sqrt{\frac{s_-(\sigma_{+,decon}^2) + s_+(\sigma_{-,decon}^2)}{(s_-s_+)(s_- + s_+)}}$$

$$c = \frac{(\sigma_{+,decon}^2)s_-^2 - (\sigma_{-,decon}^2)s_+^2}{4(s_-s_+)(s_- + s_+)\tau_{XUV}^2}$$

$$s_+ = 12.3 \frac{meV}{fs}, \quad s_- = 17.7 \frac{meV}{fs},$$

no glass plate

$$\begin{aligned} \sigma_+ &= (1.056 \pm 0.10) eV \\ \sigma_- &= (1.45 \pm 0.17) eV \\ \sigma_0 &= (0.51 \pm 0.11) eV \end{aligned}$$

$$\tau_{XUV,RMS} = (72 \pm 9) fs$$

$$c_{XUV} = (-0.1 \pm 0.2) \frac{THz}{fs}$$

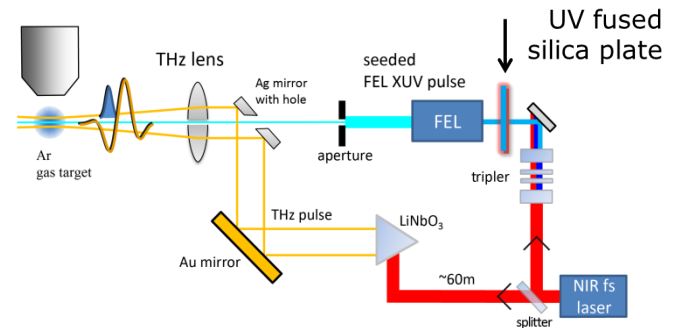
with 4.4mm glass plate

$$\begin{aligned} \sigma_+ &= (1.08 \pm 0.14) eV \\ \sigma_- &= (1.18 \pm 0.2) eV \\ \sigma_0 &= (0.39 \pm 0.07) eV \end{aligned}$$

$$\tau_{XUV,RMS} = (67 \pm 8) fs$$

$$c_{XUV} = (+0.27 \pm 0.2) \frac{THz}{fs}$$

$$GDD = (300^{+650}_{-120}) fs^2$$



The normal chirp of the optical seed pulse is transferred to the seeded FEL.

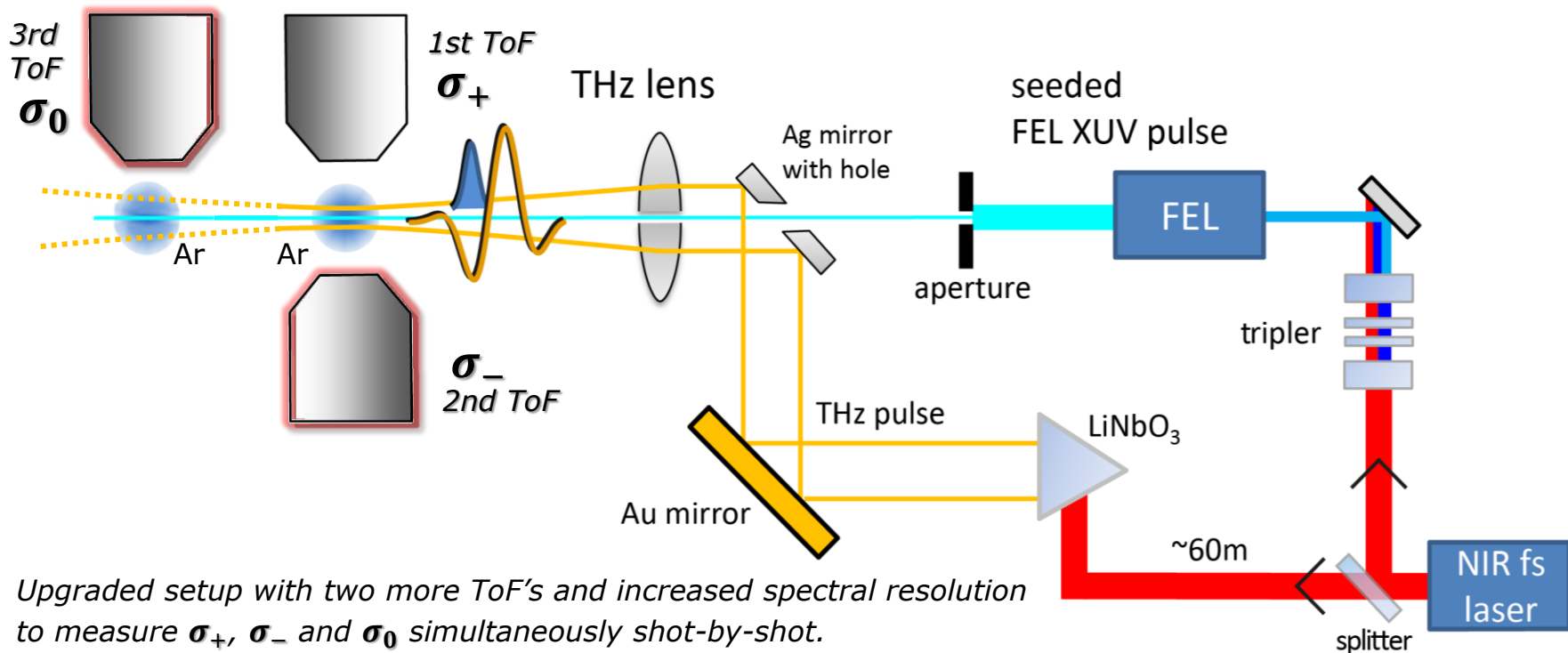
And the seeded FEL pulse becomes slightly shorter with glass.

Recent results

- Observed first indication for successful a-priori phase control of seeded FLASH FEL pulses and wanted to repeat it in a systematic way.

➔ No showing recent results.

Upgrade to three ToF's finished

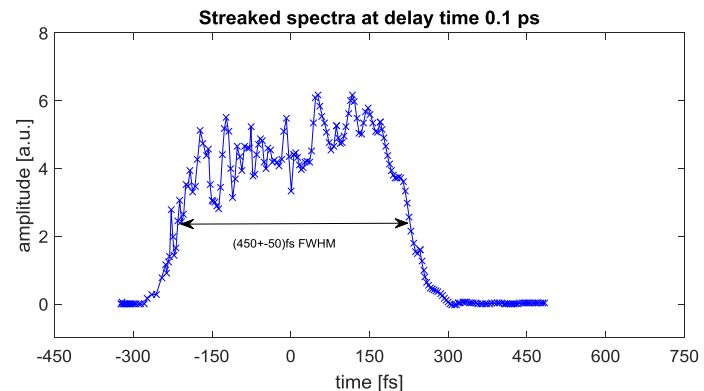
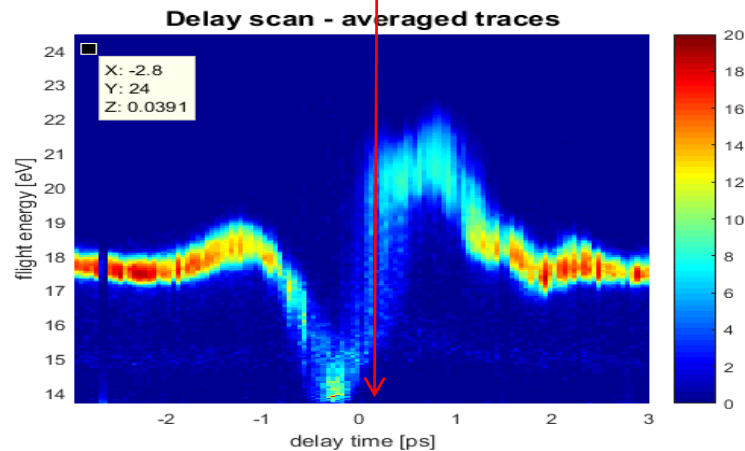
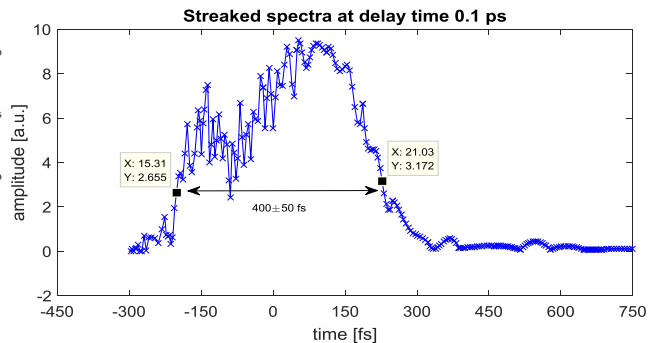
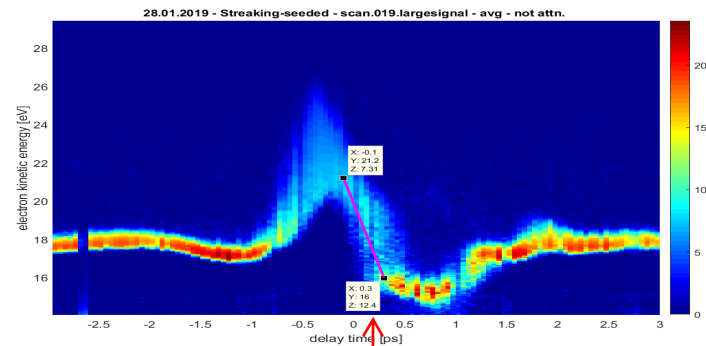


Upgraded setup with two more ToF's and increased spectral resolution to measure σ_+ , σ_- and σ_0 simultaneously shot-by-shot.

Thus, measuring τ_{XUV} and chirp c **online** and **shotwise**

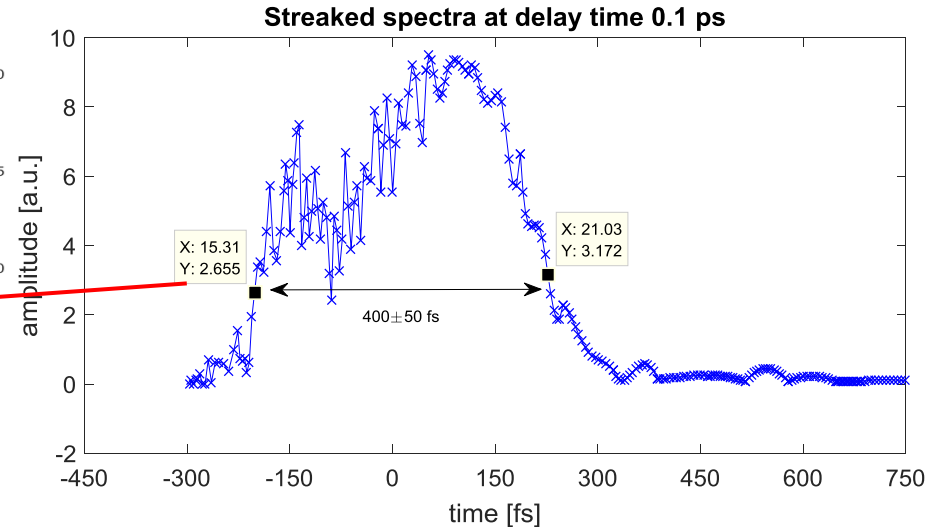
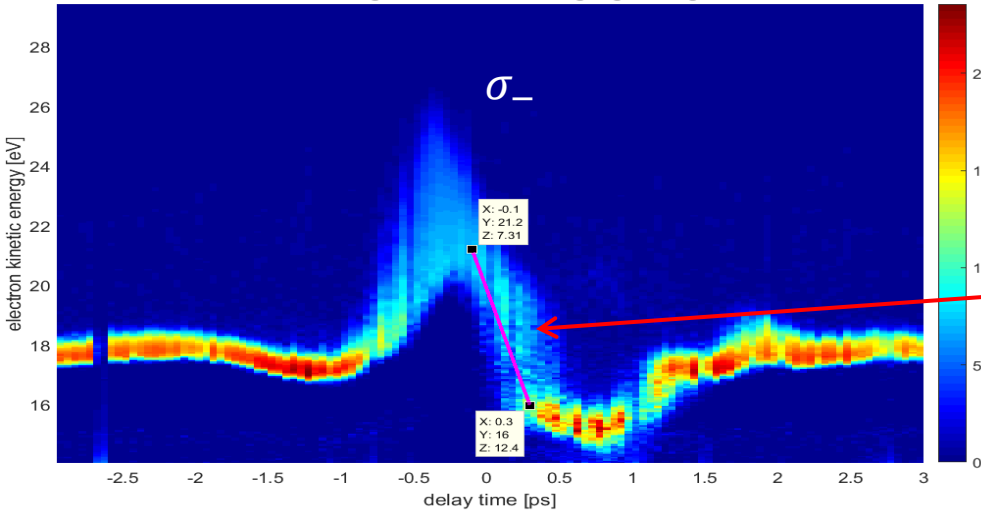
THz streaking with seeded pulses on 7th harmonic (32.5eV)

Simultaneously measured streaking traces with both sign.



THz streaking with seeded pulses on 7th harmonic (32.5eV)

28.01.2019 - Streaking-seeded - scan.019.largesignal - avg - not attn.



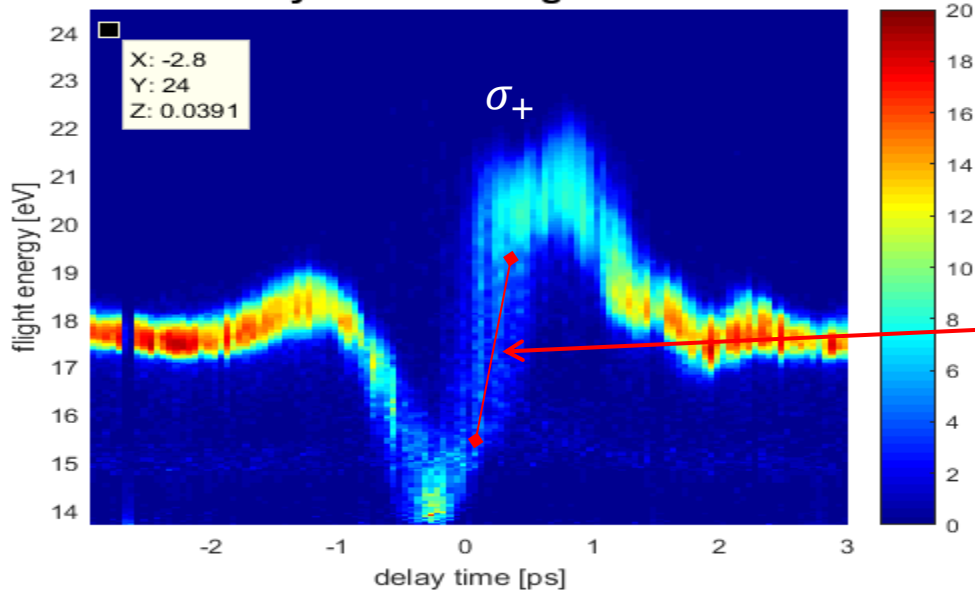
1st ToF (A)

~ 400 fs FWHM

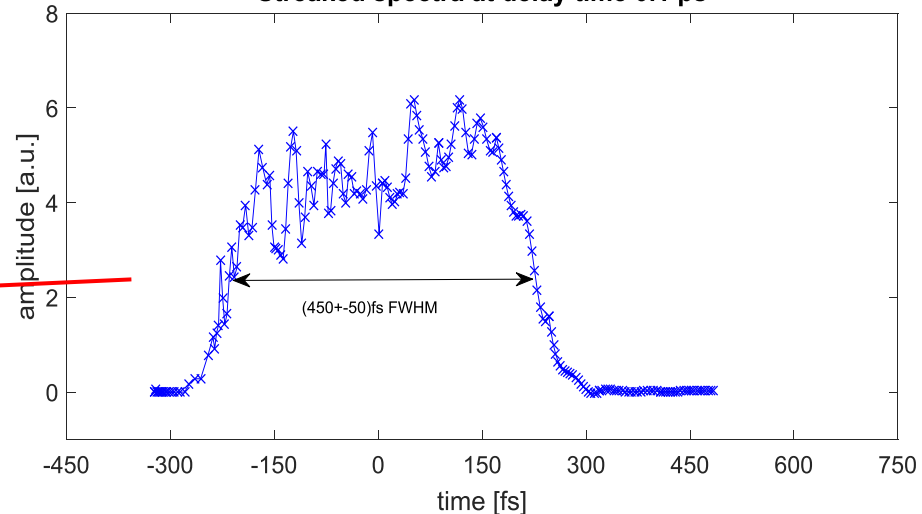
$\Leftrightarrow 170$ fs RMS $\gg 70$ fs RMS

THz streaking with seeded pulses on 7th harmonic (32.5eV)

Delay scan - averaged traces



Streaked spectra at delay time 0.1 ps



450fs FWHM ??

2nd ToF (B)

Evaluation of Non-Gaussian profile with Gaussian profile based model

$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 + \sigma_{-,decon}^2}{2s^2}}$$

$$c = \frac{(\sigma_{+,decon}^2) - (\sigma_{-,decon}^2)}{8s\tau_{XUV}^2}$$

$$s_+ = 13 \frac{meV}{fs}, \quad s_- = 13 \frac{meV}{fs},$$

no glass plate

$$\sigma_+ = (5.2 \pm 0.60) eV$$

$$\sigma_- = (5.9 \pm 0.6) eV$$

$$\sigma_0 = (0.8 \pm 0.1) eV$$

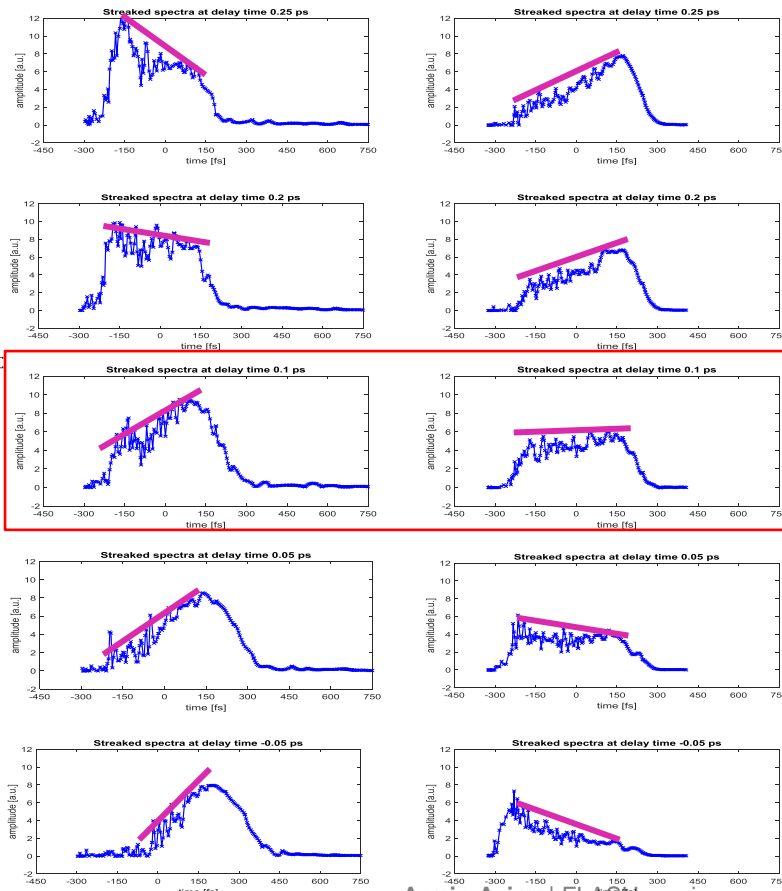
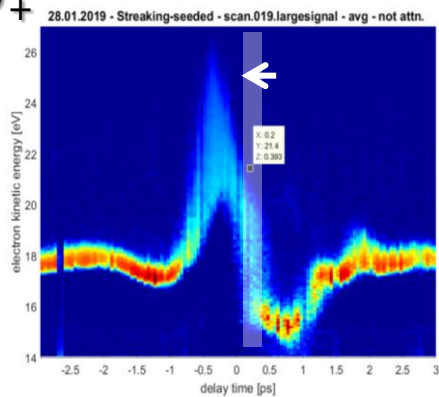
$$\tau_{XUV,RMS} = (423 \pm 40) fs$$

$$c_{XUV} = (-4.1 \pm 2) \frac{meV}{fs} = (-640 \pm 300) \frac{THz}{ps}$$

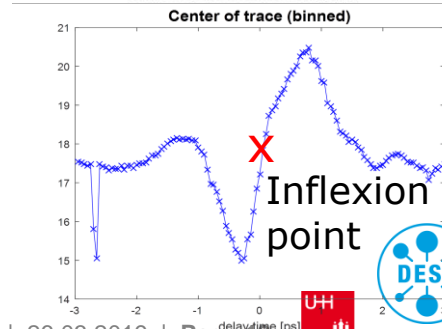
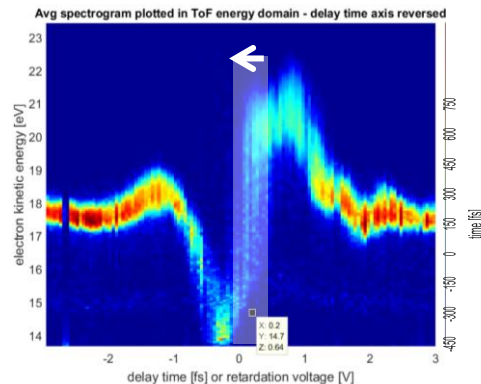
Non-Gaussian pulse profile in the seeded FEL pulse

0mm glass

σ_+

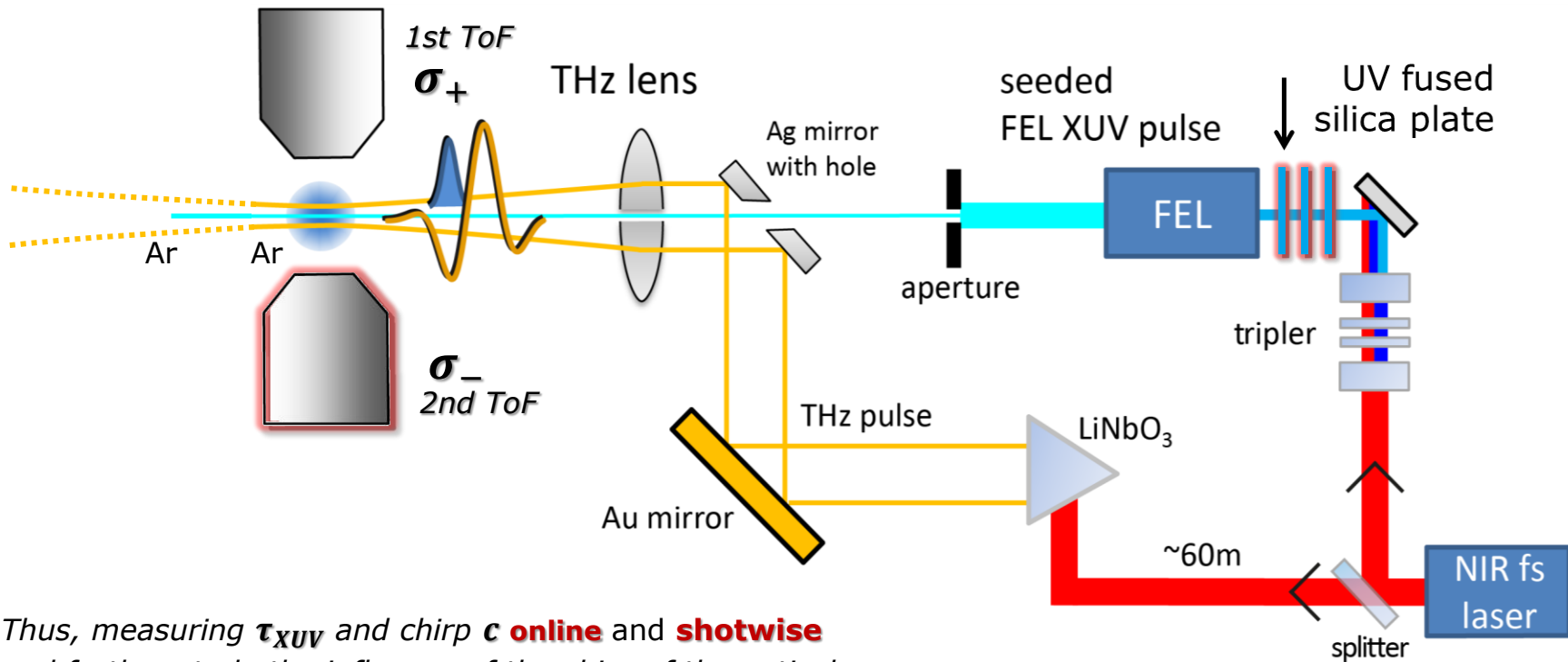


σ_-



Which pair of profiles is the "correct" pulse profile ?

UV enhanced glass plate „a-priori“ chirping measurements

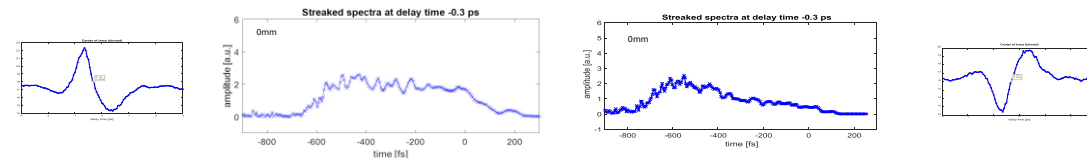


Thus, measuring τ_{XUV} and chirp c **online** and **shotwise** and further study the influence of the chirp of the optical seed pulse \rightarrow (“coherent control”)

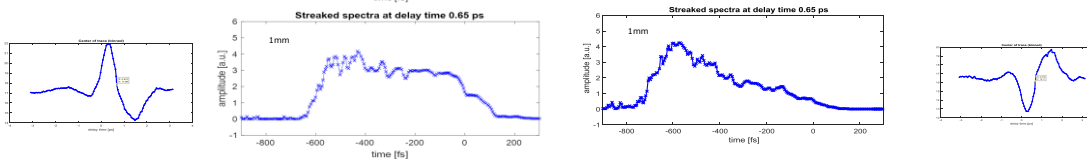
UV enhanced glass plate „a-priori“ chirping measurements

First: 0mm, 1mm, 3mm and 5mm with a good quality signal measured at inflex. point

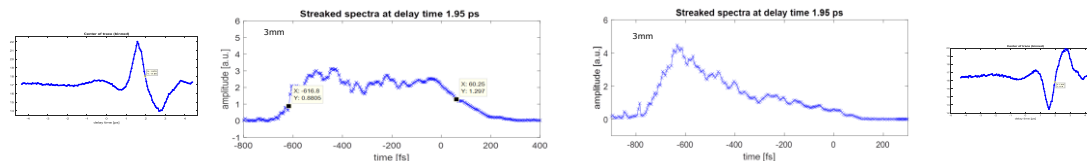
0mm glass



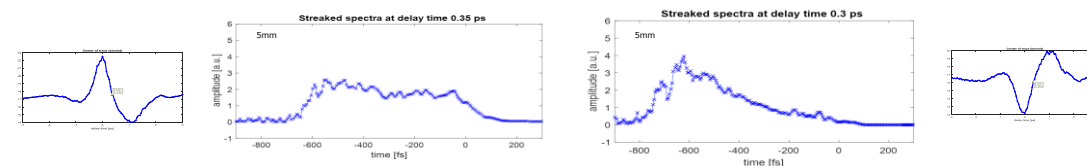
1mm glass



3mm glass



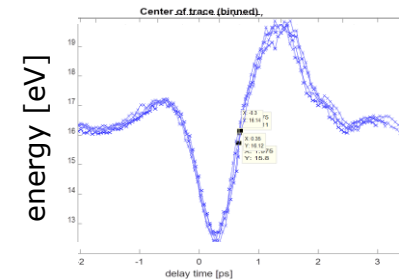
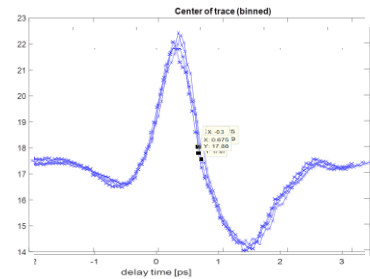
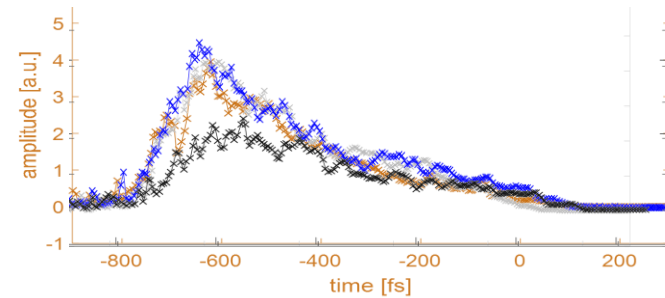
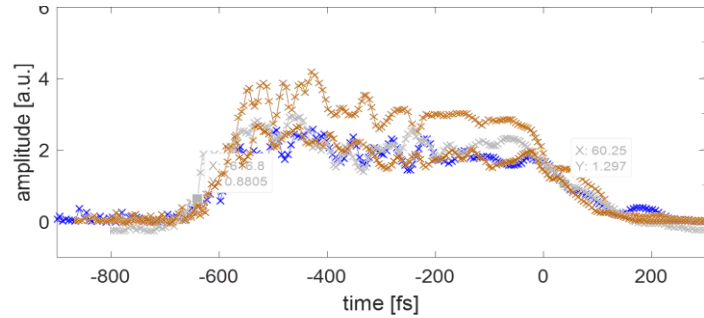
5mm glass



The signal power increased after inserting the first glass plate !

But no systematic changes observable !

UV enhanced glass plate „a-priori“ chirping measurements



Also a closer look reveals no significant systematic changes with inserted glass plates!

Summary of glass plate a-priori chirping measurements

A systematic dependence of the pulse duration or shape from the applied chirp to the optical seed couldn't been observed.

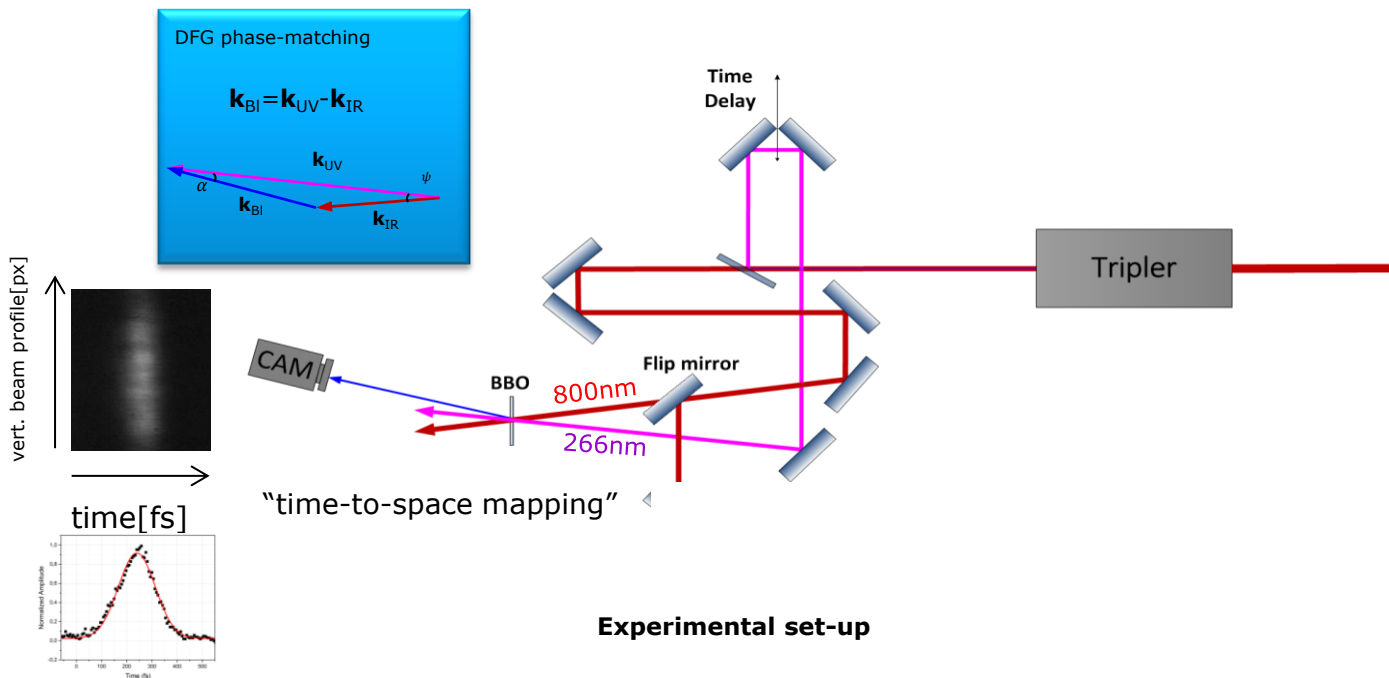
Reasons are:

- The XUV pulse duration was very high, up to 400fs FWHM and the applied chirp of only 8mm of UV-SQ glass didn't induce strong enough modulations on the optical seed as compared with the too large pulse duration.
We had chosen only up to 8mm of glass, since in a previous experiment the power of the seeded FEL dropped to zero for 12mm of glass.
- Calculations and measurements with LOLA traces have indicated, that 8mm of a-priori chirp should have been enough, if the FEL pulse duration were as short as had been measured in previous campaigns to only 130fs FWHM.

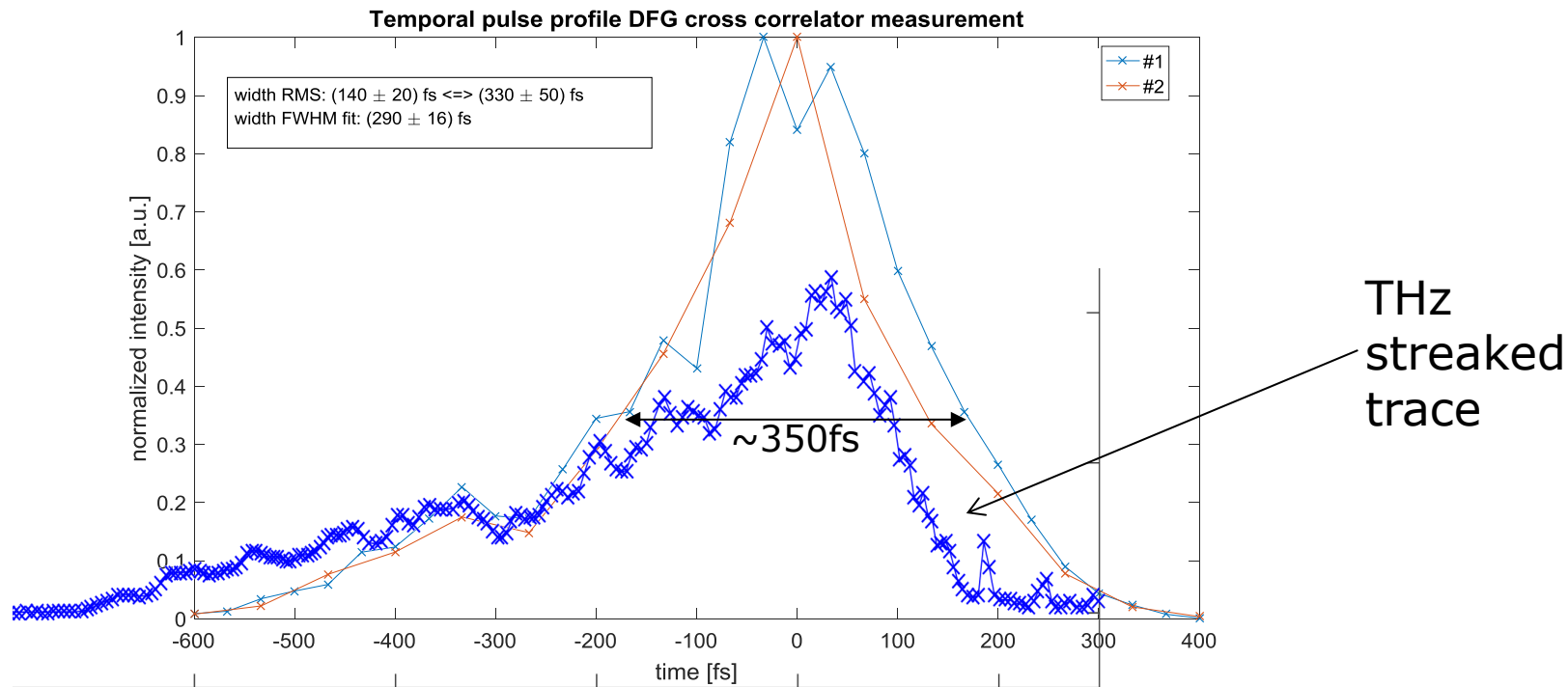
But why is the FEL pulse duration that large ?

Measuring the 266nm optical seed temporal pulse profile

- Measurement of the optical seed pulse properties using the CAMP DFG single-shot cross-correlator scheme.



The optical seed pulse temporal profile



5 weeks delay between both measurements !

Treating Non-Gaussian XUV pulses

- The current theoretical model to determine pulse duration and chirp using two ToF's simultaneously bases on the assumption that the XUV pulse is mainly formed by a Gaussian temporal pulse profile only chirped with second order GVD.
- The recent measurements now demonstrate, that this assumptions is generally not justified.

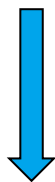
How to deal with Non-Gaussian XUV pulses ?

... look into the attosecond-community.

classical FROG-Spectrogram

$$\tilde{S}(\omega, \tau) = \left| \int_{-\infty}^{\infty} P(t)G(t+\tau)e^{i\omega t} dt \right|^2$$

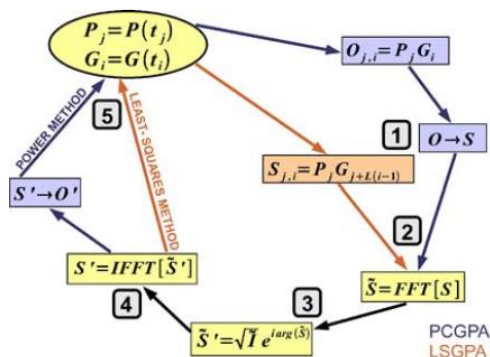
$P(t)$: field of XUV pulse
 $G(t)$: field of gate



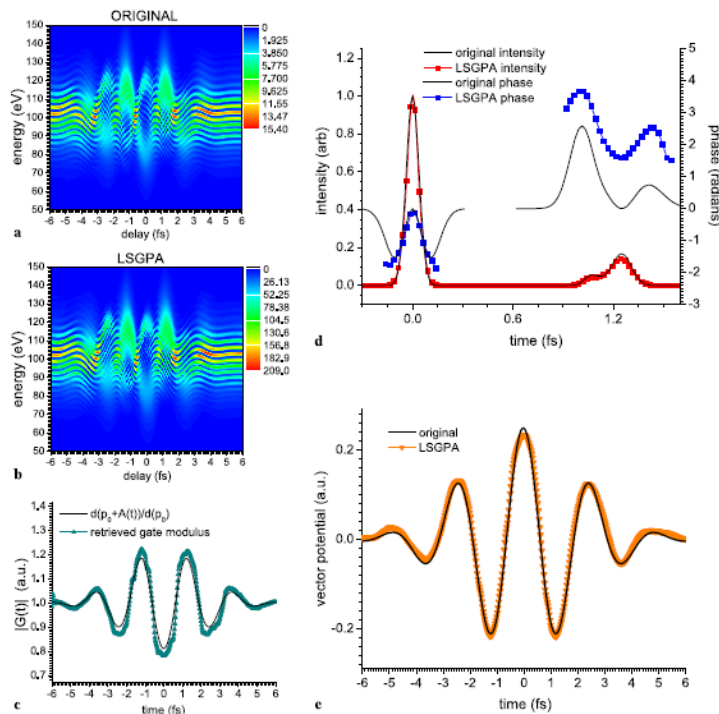
Streaking-Spectrogram

$$\hat{S}(p, \tau) = \left| \int_{-\infty}^{\infty} E_X(t)G(t+\tau)e^{\frac{i}{2}p^2 t} dt \right|^2$$

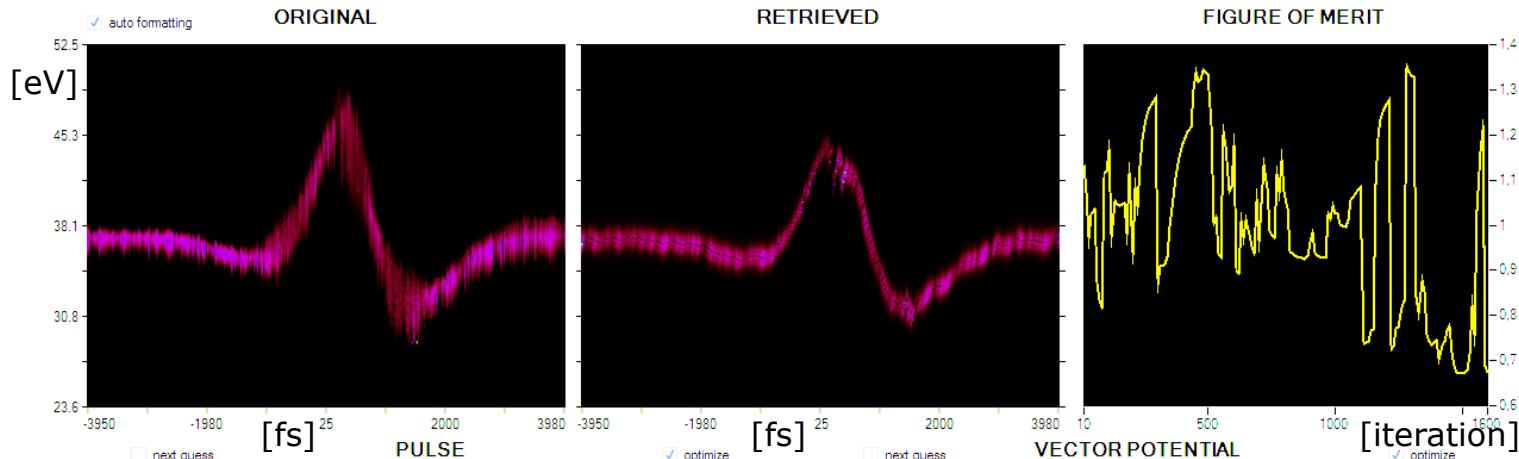
$$G(t) = \frac{d(p_0 + A_L(t))}{d(p_0)} e^{-i \int_t^{\infty} (p_0 A_L(t') + \frac{1}{2} A_L^2(t')) dt'}$$



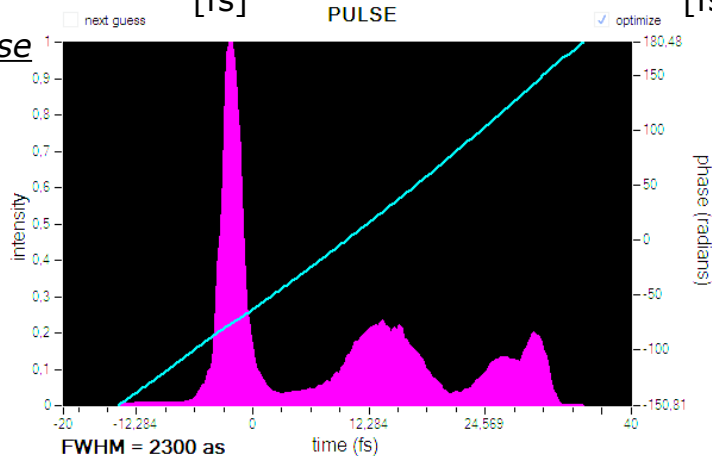
Gagnon et al., Appl. Phys. B, 92, 25–32 (2008)



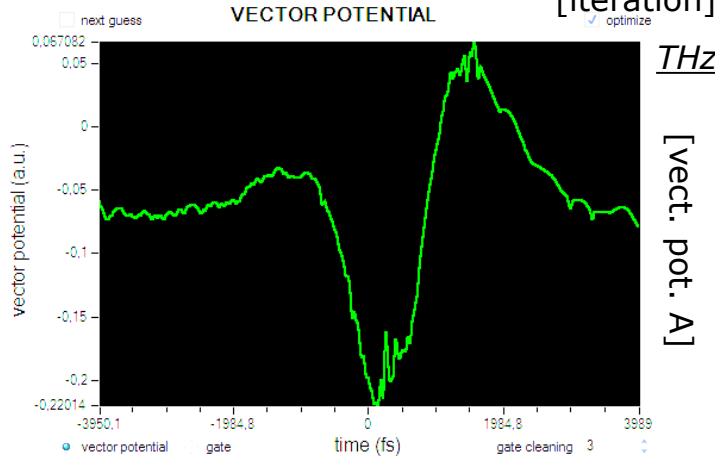
Tried retrieval with LSPGA code ... trial #1

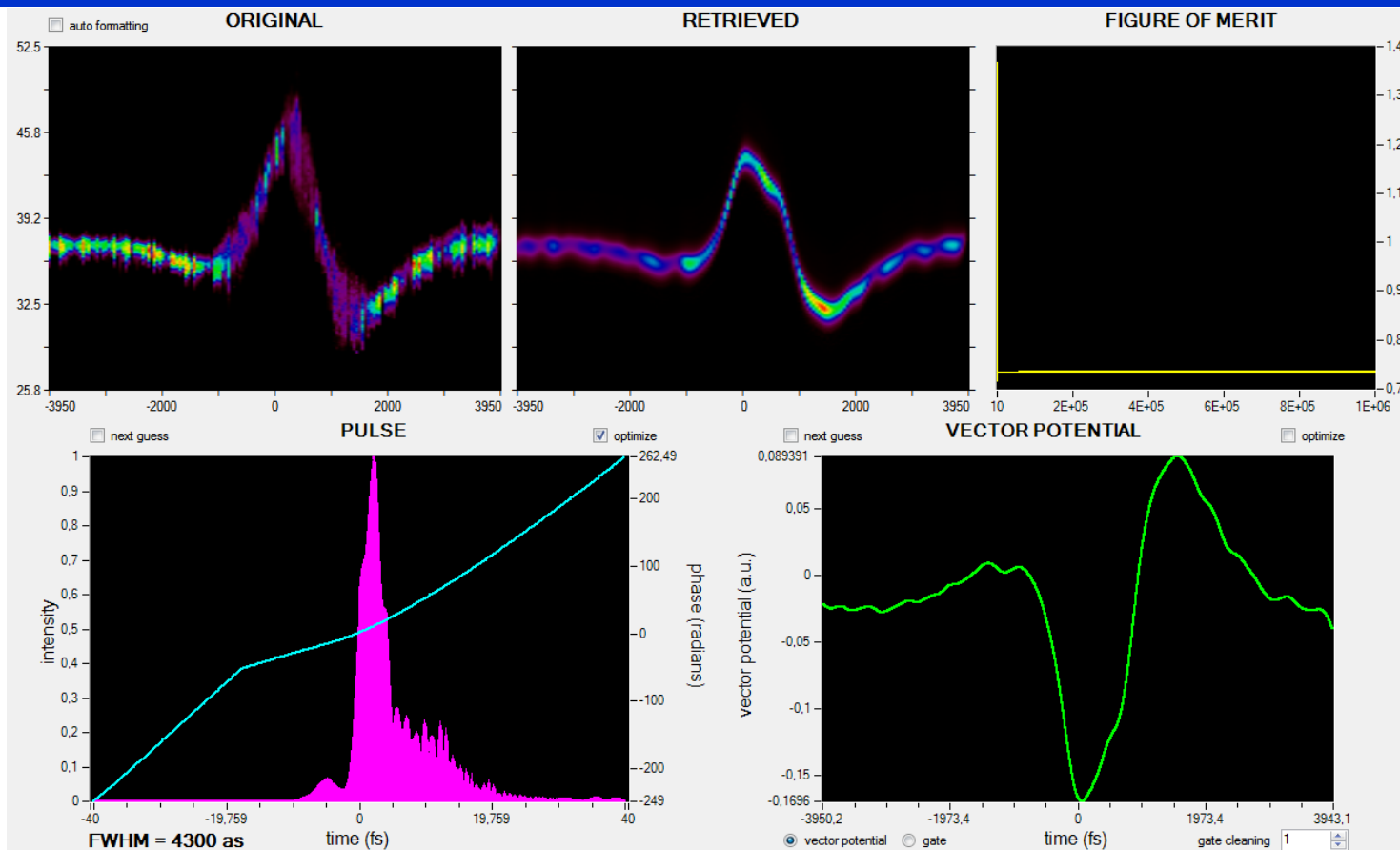


XUV pulse profile

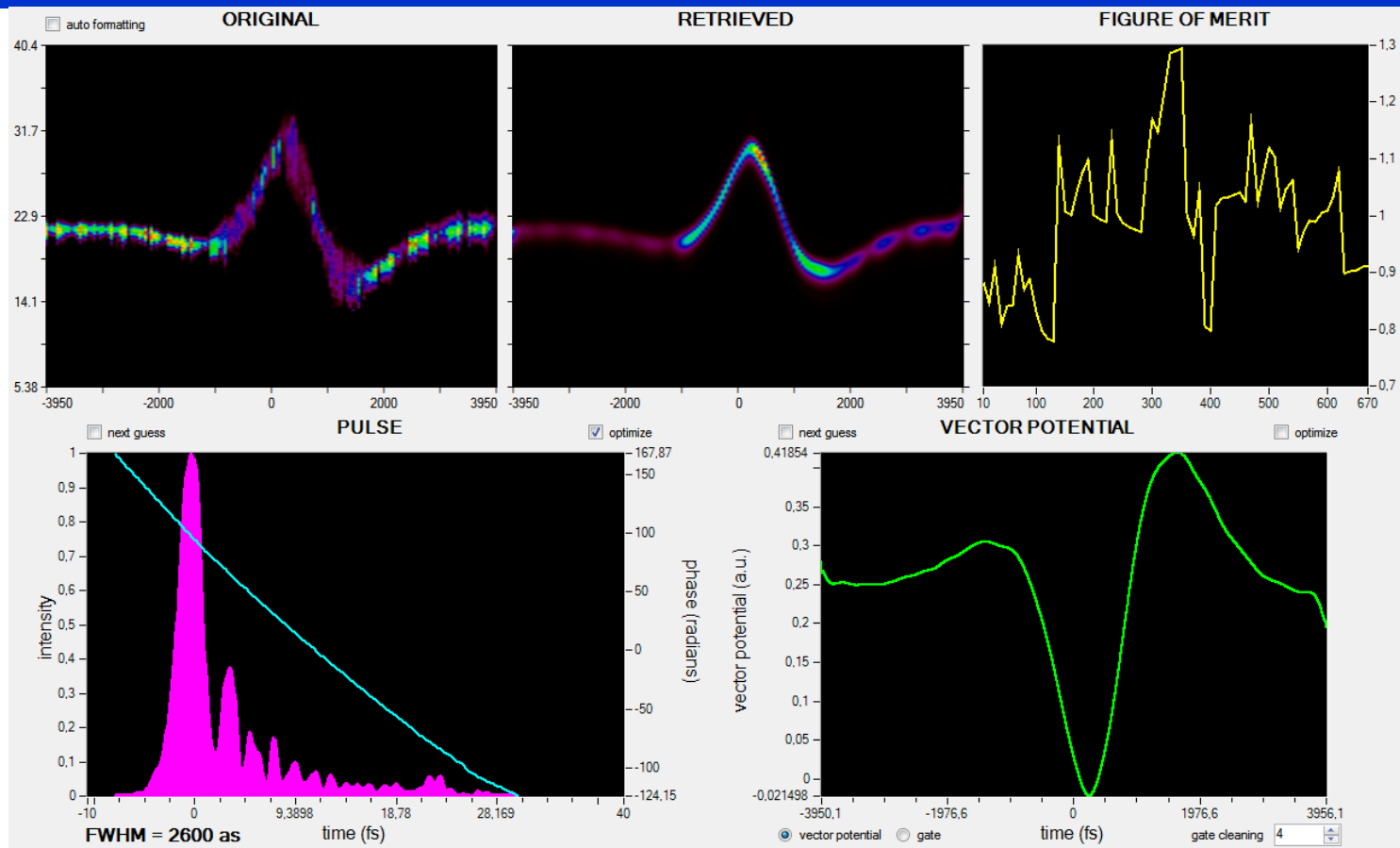


THz pulse

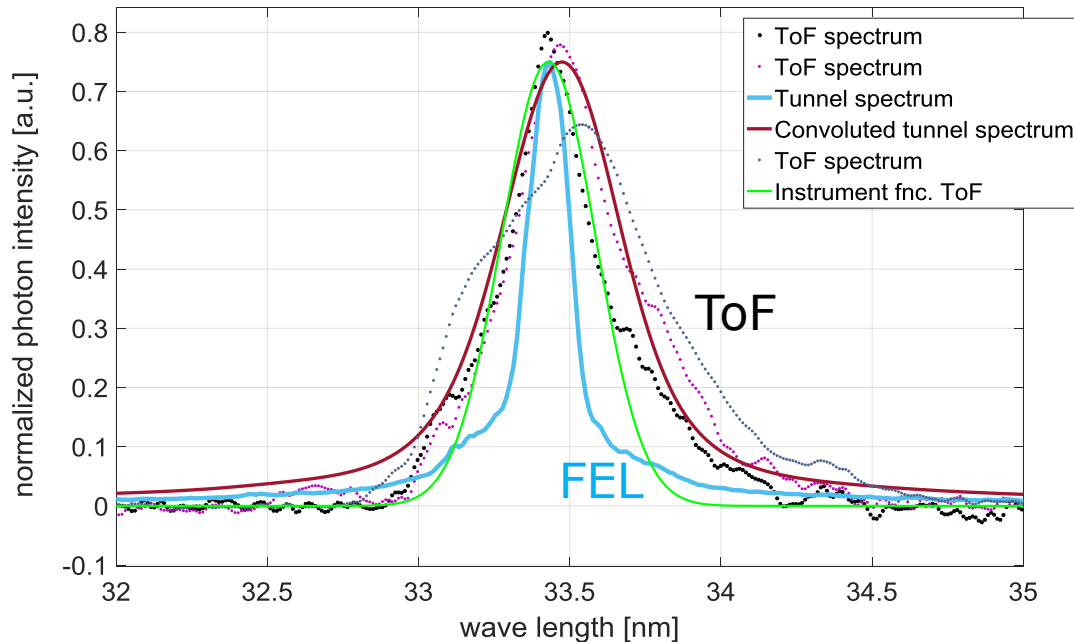




... #3, but without real convergence.



Problem of limited ToF spectral resolution



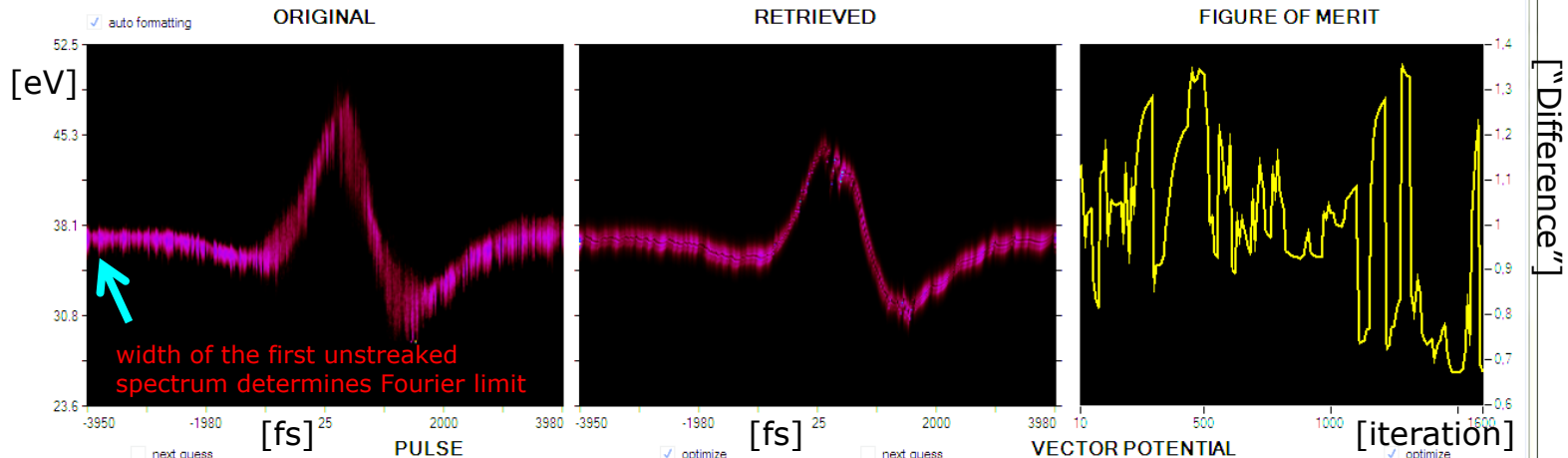
Resolution of
ToF spectrometer
 $0.45\text{nm} \Leftrightarrow$
(former) 0.5 eV FWHM

ToF resolution
not high enough

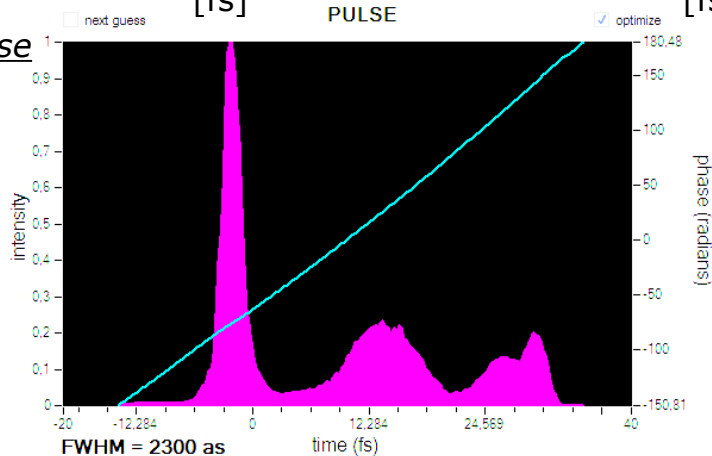
→ LSPGA code assumes
too large bandwidth \Leftrightarrow
too short pulses

$$\overline{\Delta\lambda_{FEL}} = (0.2 \pm 0.05) \text{ nm} \Leftrightarrow \Delta t_{FEL,Fourier} = 8.3_{-1.5}^{+3} \text{ fs} \quad (\text{FWHM})$$

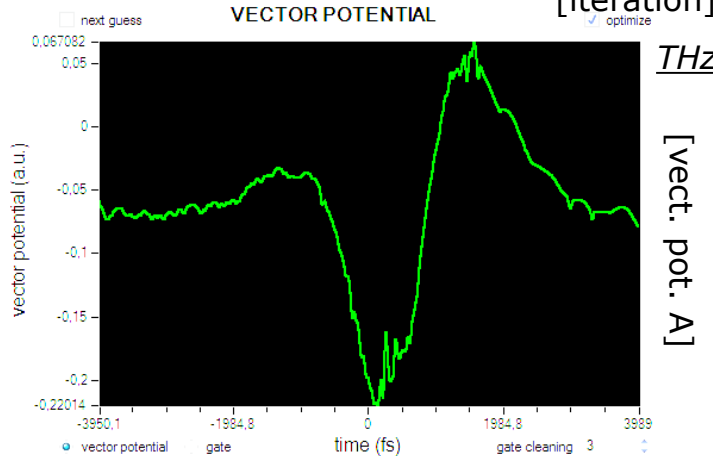
Tried retrieval with LSPGA code "Attogram" (J. Gagnon)



XUV pulse profile



THz pulse



Numerical deconvolution of spectrometer function

- In the year 1963, two Russian scientists have first time found a general solution for the “deconvolution” problem
 - called the “Tikhonov regularization for linear problems” - Tikhonov and Arsenin (1963 and 1977)

Method:

$$h(t) = f(t) \circ g(t) \quad \Leftrightarrow \quad H(\omega) = F(\omega) * G(\omega)$$

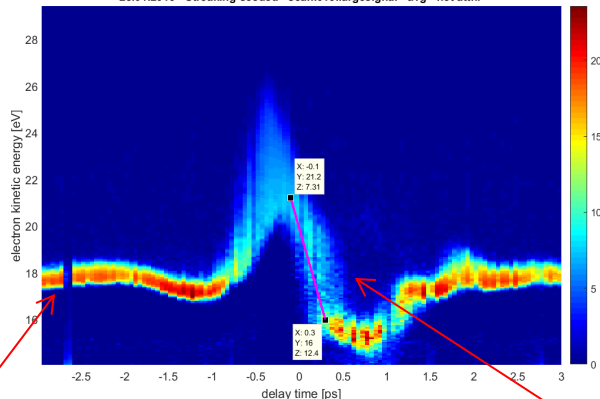
$$\rightarrow F(\omega) = H(\omega) / G(\omega)$$

- problems with zeros of denominator $G(\omega)$ treated by a special approach of preparing the denominator (Hann window filtering, noise handling)

Numerical deconvolution of spectrometer function

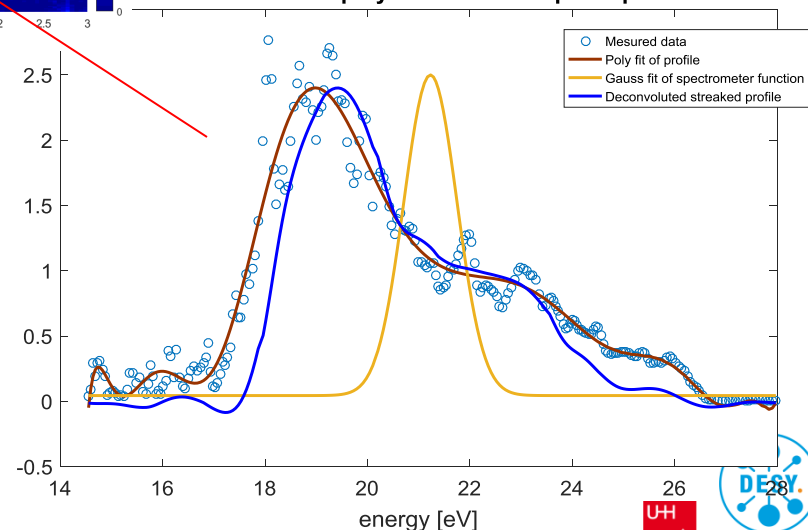
Works **NOT** with the unstreaked spectra !!

28.01.2019 - Streaking-seeded - scan.019.largesignal - avg - not attn.

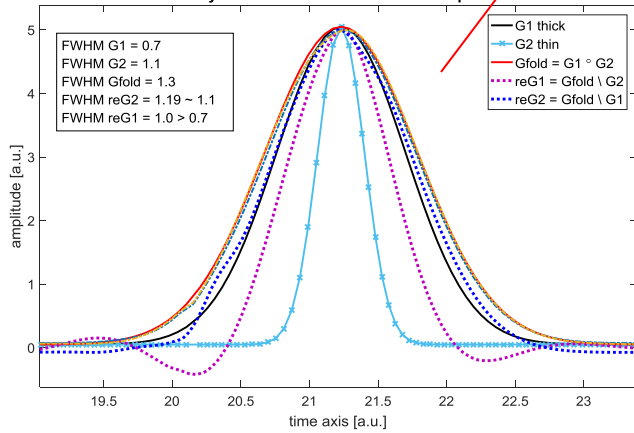


Deconvolution **WORKS** in the streaked regime

Deconvolution of polyfitted streaked pulse profile



Numerical study of deconvolution in the Fourier space / limitations



Summary

> Positive

- > Observed first indication for successful a-priori phase control of seeded FLASH FEL pulses.
- > Upgraded THz setup with two more ToF's to measure pulse duration and chirp in a single shot.
- > Observed a Non-Gaussian XUV pulse profile and tried to reconstruct amplitude and phase from THz streaking spectrograms.

> Negative

- > A systematic analysis and repetition of the a-priori chirp control hasn't worked yet, mainly because of too long XUV pulses due to too long 266nm optical seed pulses !
- > The Non-Gaussian pulse reconstruction does not yet lead to stable and conclusive results, but the reasons for this non-convergence are identified !

➔ Next steps

Next steps - for a success of the “a-priori phase control” project

- Reduce the pulse duration of the **266nm optical seed pulse**, as its **too large pulse duration** seems to be the reason for the too long seeded FEL pulses.
- Compare the THz streaked XUV pulse profiles with LOLA electron bunch modulation profiles to find differences or analogies ²
- Redo streaking with 8th harmonic and use more fused silica (up to 30mm) if XUV pulses stay that long.
- Retry the amplitude/phase reconstruction with “de-convoluted” spectra taken with ToF’s spectral resolution at optimal design value of 0.2 eV FWHM with maximum retardation voltage ↔ lower signal-to-noise ratio

BMBF project evaluation (maybe) leads to the grant of one PhD student position to focus on

“Amplitude and phase retrieval of
EEHG seeded FEL pulses using the
THz streaking method”

In case of a positive decision, the student will further optimize and reprogram the LSPGA code to match it to the seeded HGHG FLASH situation (mainly implement a limited ToF spectral resolution and make it 64bit ready) and apply it to the future EEHG seeded FEL pulses at 10th or 11th harmonic order to fully reconstruct phase and amplitude of the coherent seeded FLASH pulses !!

END

> The LSPGA code in detail

Recursion equations for generating the retrieved pulse and gate

$$P_j = \frac{\sum_m S_{j,m} G_{j+L(m-1)}^*}{\sum_m |G_{j+L(m-1)}|^2},$$

$$G_k = \frac{\sum_n S_{k-L(n-1),n} P_{k-L(n-1)}^*}{\sum_n |P_{k-L(n-1)}|^2},$$

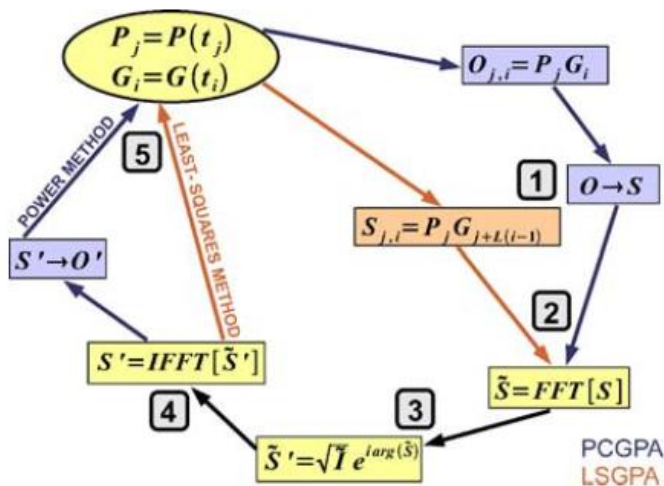
with

$$j = 1 \dots N_\varepsilon, \quad k = 1 \dots N_\varepsilon + L(N_\tau - 1),$$

$$m = \text{Max} \left(1, \left\lceil \frac{R-j+1}{L} \right\rceil + 1 \right)$$

$$\dots \text{Min} \left(N_\tau, \left\lfloor \frac{R-j}{L} \right\rfloor + N_\tau \right), \quad R = N_\varepsilon/2$$

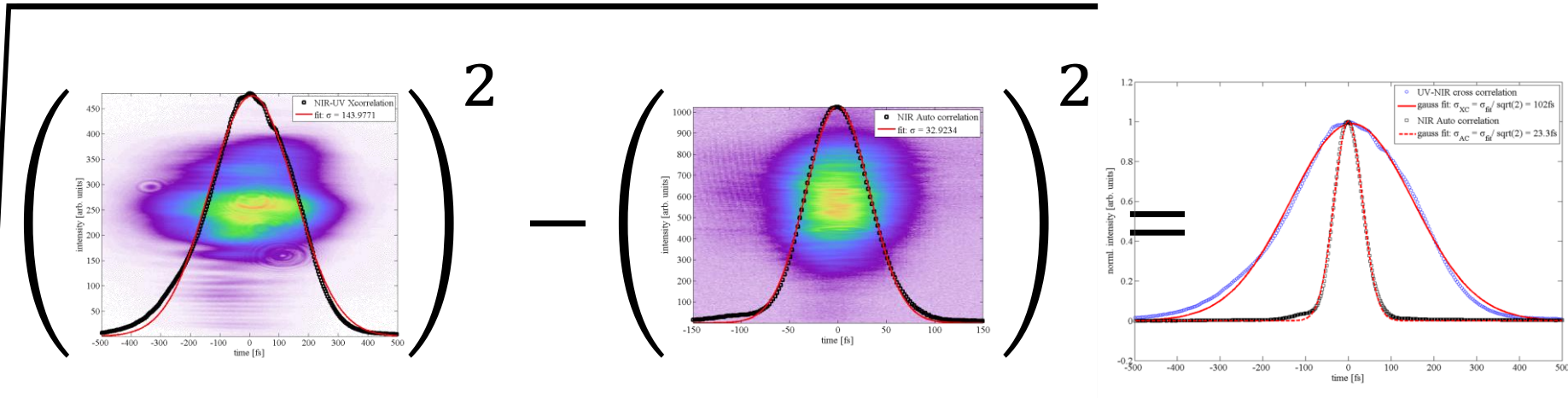
$$n = \text{Max} \left(1, \left\lceil \frac{k-N_\varepsilon}{L} \right\rceil + 1 \right) \dots \text{Min} \left(N_\tau, \left\lfloor \frac{k}{L} \right\rfloor \right)$$



> Former DFG cross-correlation measurement

The optical seed pulse - 2016

- Pulse duration measurement of the 266nm seed pulse using a single-shot line DFG cross-correlator setup combined with a 800nm autocorrelator.

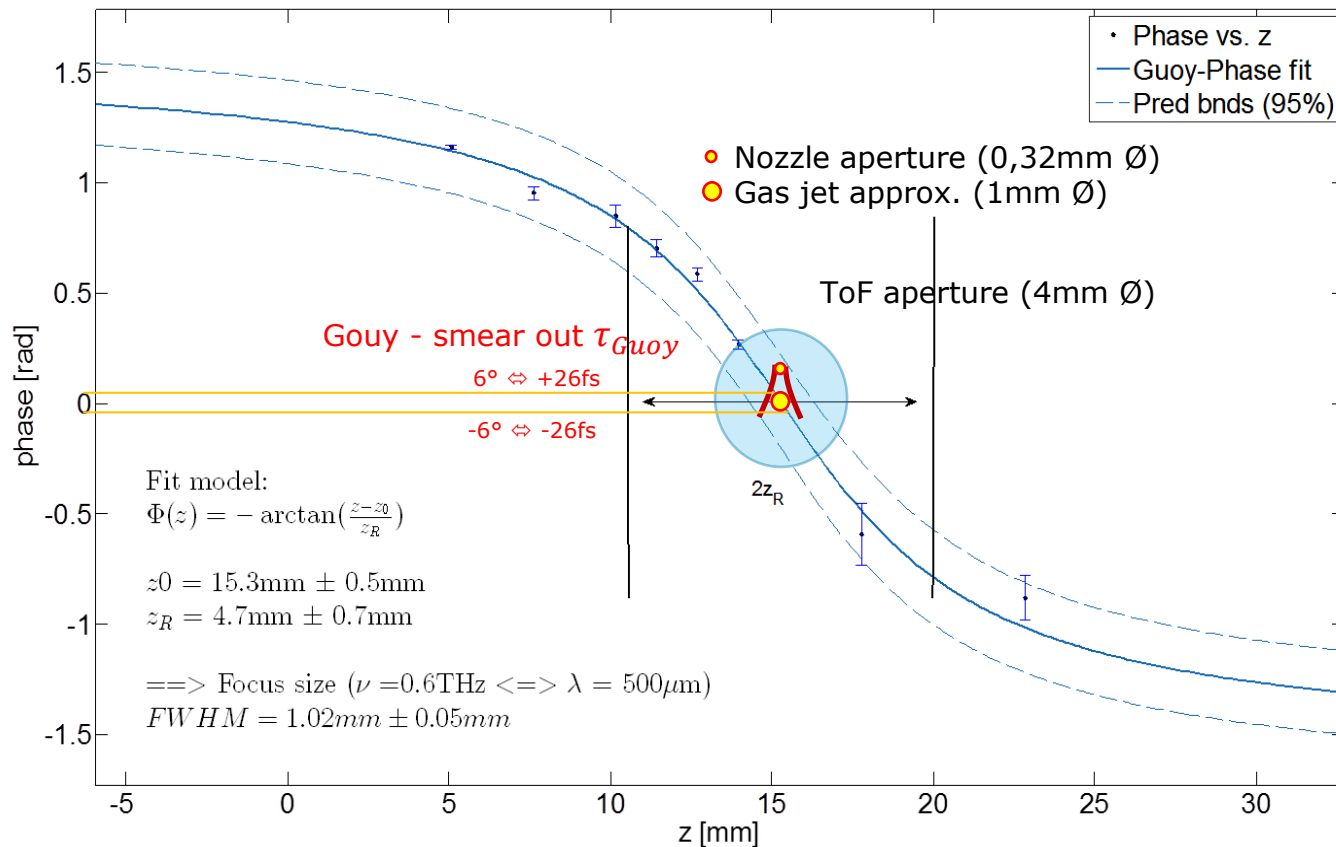


$$\sqrt{(205\text{fs})^2 - (72\text{fs})^2} \sim 200\text{fs FWHM} \pm 10\% > 136\text{fs FWHM of XUV (from THz streak.)}$$

> Guoy phase shift

Influence of the Guoy phase shift

Guoy phase measurement of THz focus



$$E(t) = A * \exp\left[-\left(\frac{t-t_0}{w_0t}\right)^2 \cdot \cos(2\pi\nu_{THz} * (t+t_0) + \phi_{off})\right]$$

$$A = 10.4 \text{ MV/m} \pm 7.5 \%$$

$$w_0t = 600 \text{ fs} \pm 8.6\%$$

$$\nu_{THz} = 0.64 \text{ THz} \pm 5\%$$

$$\phi_{off} = 1 \pi \pm 19\%$$

$$t_0 = 1.8 \text{ ps} \pm 30\text{fs}$$

Include the spectral
Guoy broadening

$$\sigma_{\pm,real} = \sqrt{\sigma_{\pm,meas}^2 - s^2 * \tau_{Guoy}^2}$$



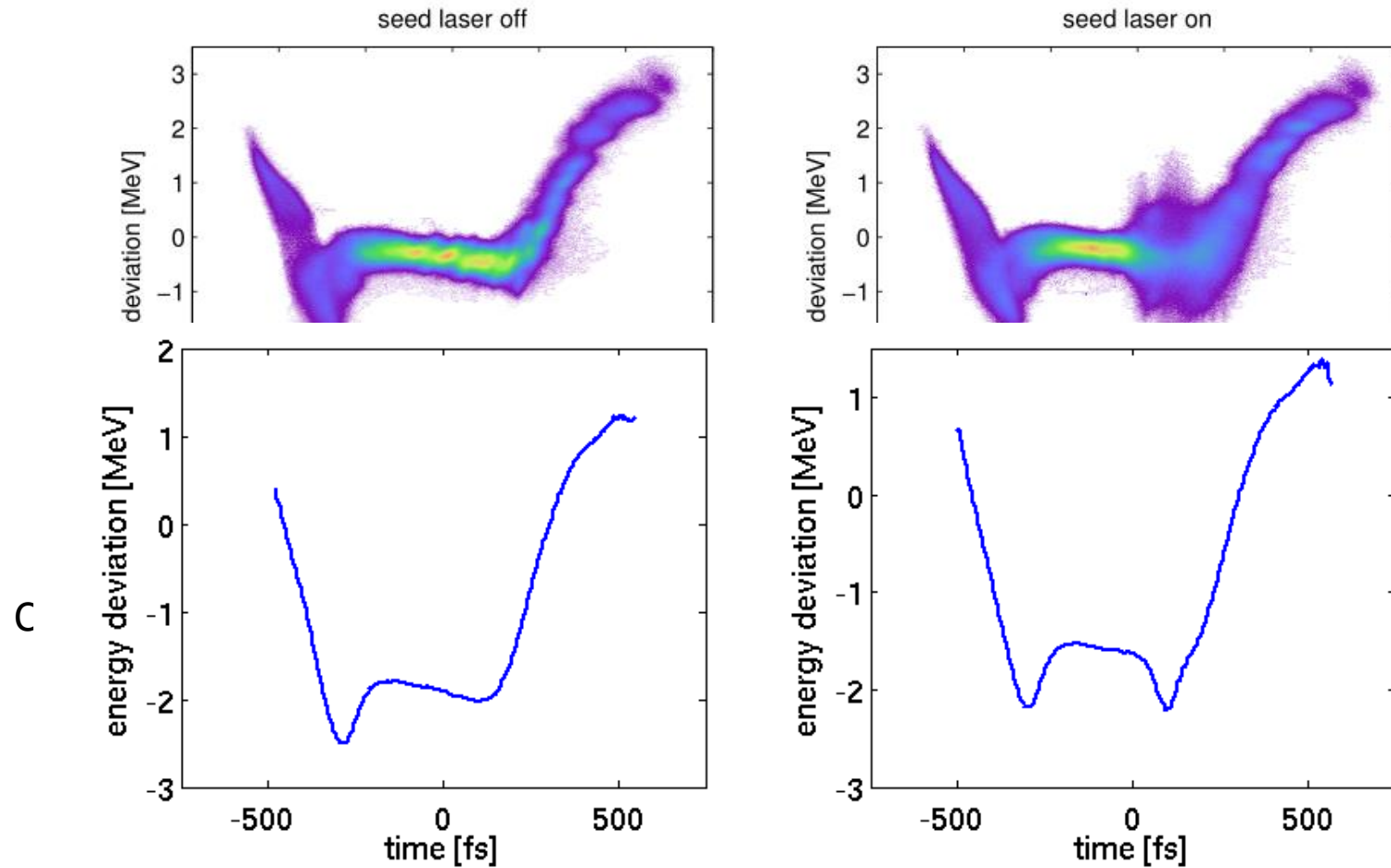
$$\tau_{XUV,RMS} = 59\text{fs} \pm 11\%$$

$$\tau_{XUV,FWHM} = 138\text{fs}$$

$$c_{XUV} = -3.2 \frac{\text{meV}}{\text{fs}} \pm 46\%$$

> Electron bunch based measurement method

Transverse Deflecting Structure

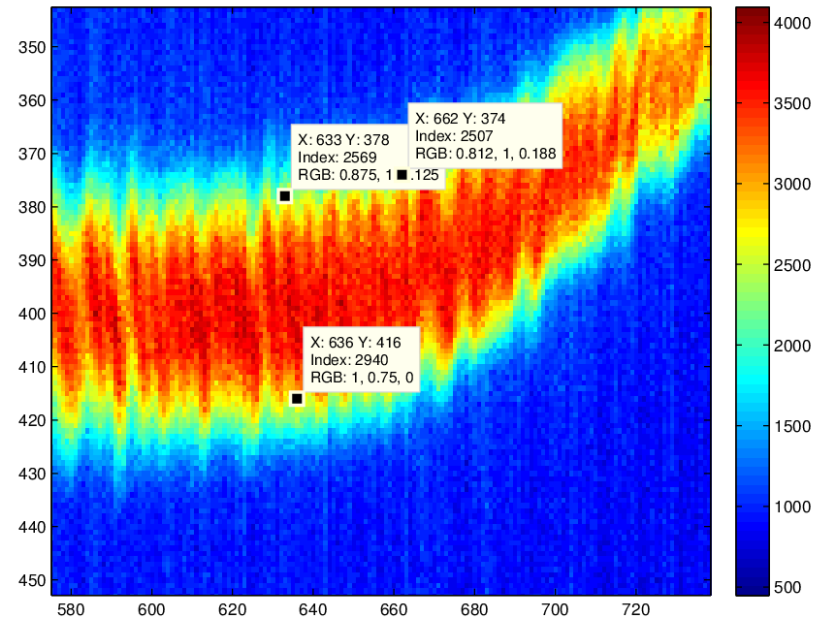
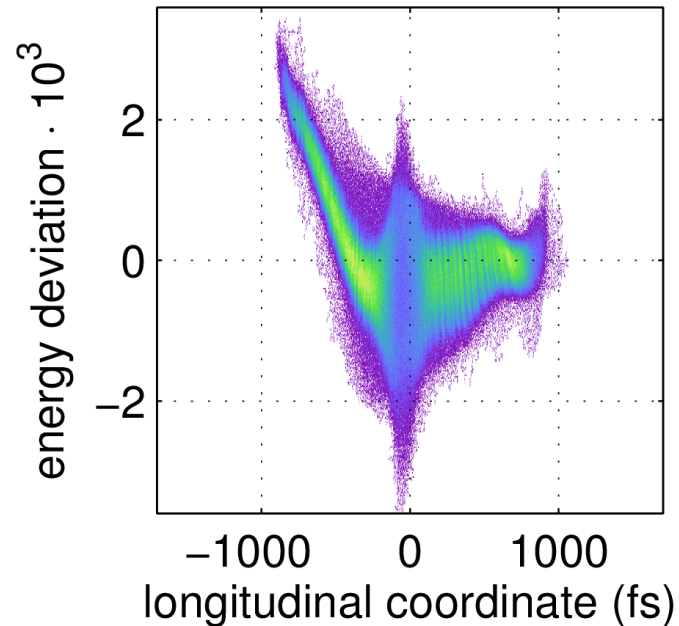


file:



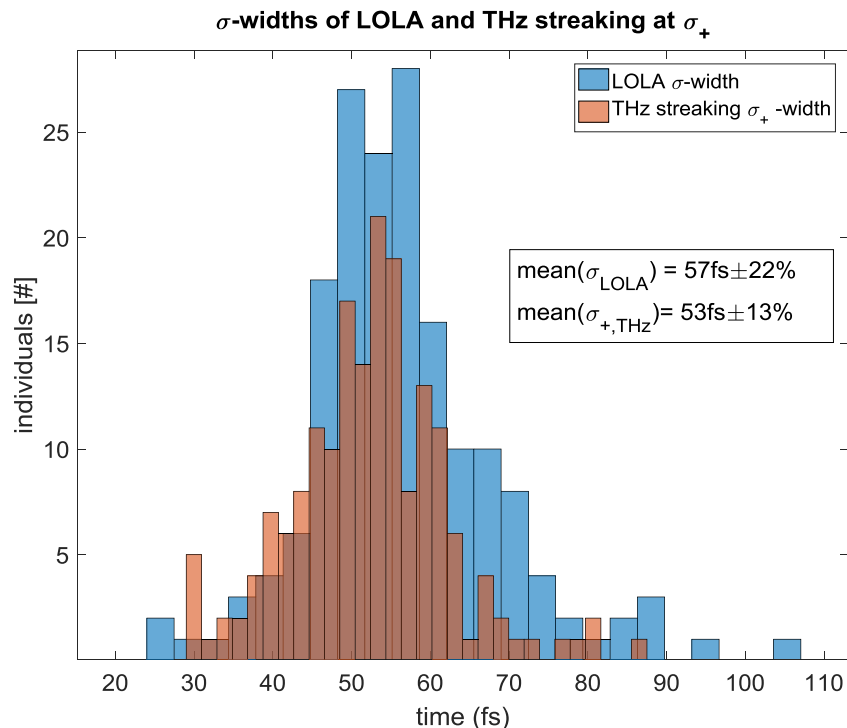
Microbunch instability

- Very poor and instable HGHG operation ($< 1\mu\text{J}$)



Comparison with TDS measurement on electron bunch

- Last seminar we presented a method how to estimate the XUV light pulse duration from a TDS electron bunch energy streak trace.



TDS: $\tau_{XUV,RMS} = 57\text{fs} \pm 22\%$

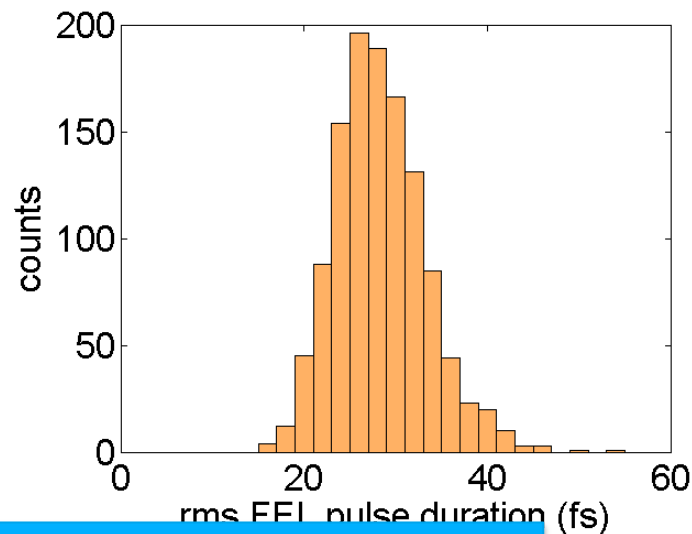
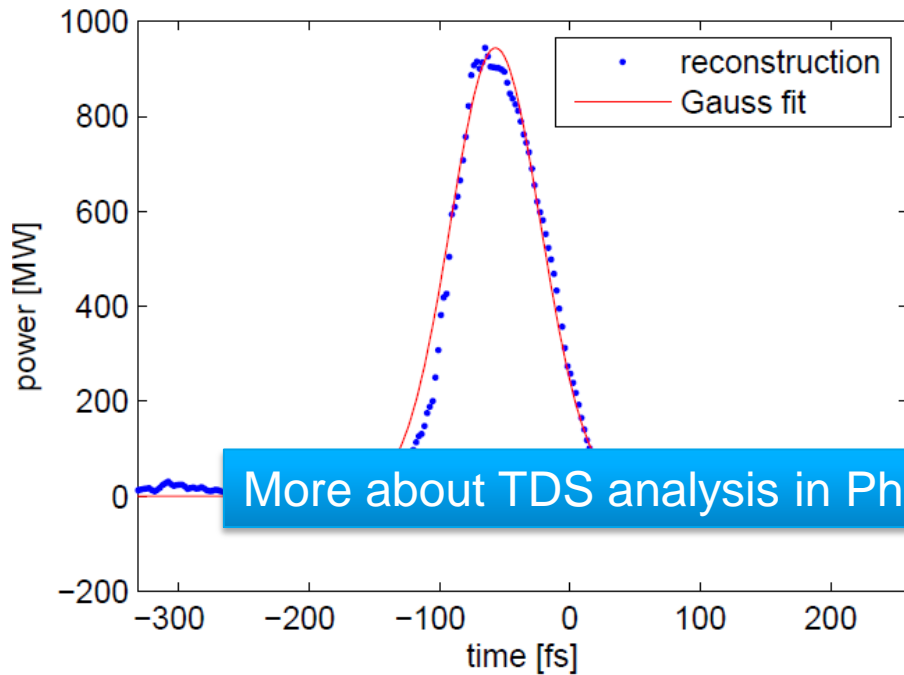
THz: $\tau_{XUV,RMS} = 53\text{fs} \pm 13\%$

A direct comparison of the widths histograms shows comparability of both measurement methods.

(But a single shot correlation didn't succeed)

Transverse Deflecting Structure

$$P(t_i) = \Delta E(t_i) \cdot I(t_i) / e$$



More about TDS analysis in PhD defense by Tim Plath

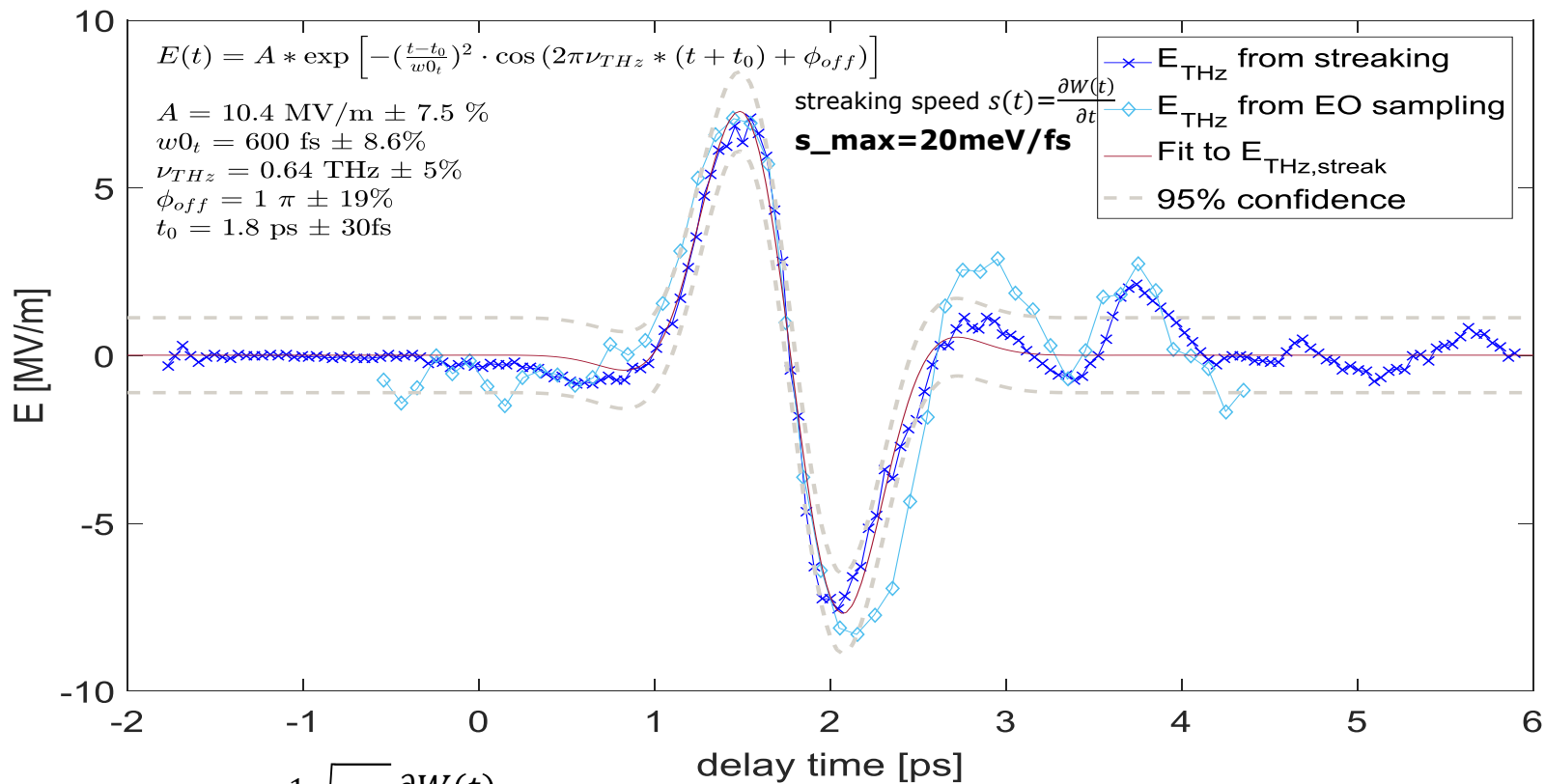
$$\langle \tau_{\text{FEL}} \rangle = 28.4 \pm 5.6 \text{ fs}$$

From this calculate: Pulse duration, peak power and energy.

T. Plath et al. accepted at Scientific Reports

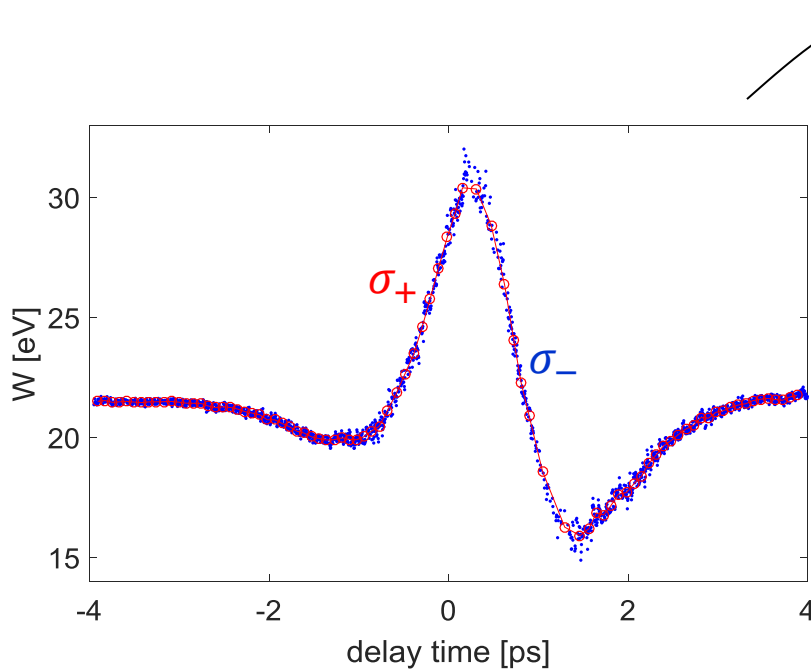
> Vector potential determination

... with electro optical sampling

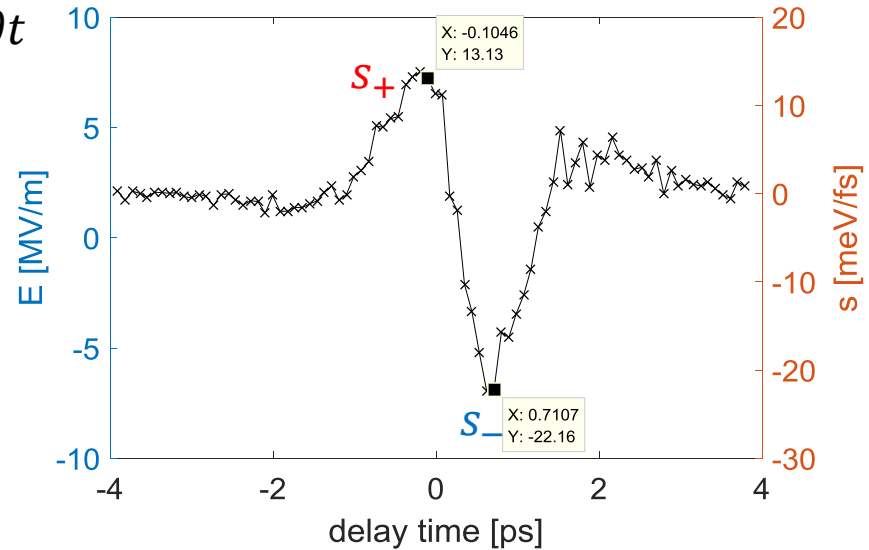


$$E_{THz}(t) = \frac{1}{e} \sqrt{\frac{m_e}{2W_0}} \frac{\partial W(t)}{\partial t}$$

THz streaking with seeded pulses on 8th harmonic (33.5eV)



$$\frac{\partial}{\partial t}$$



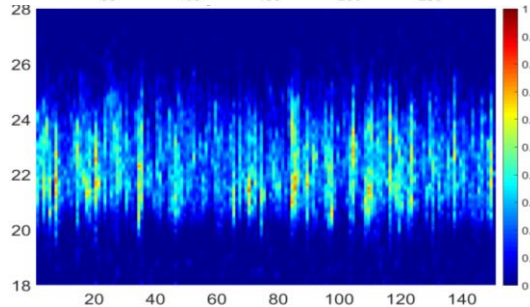
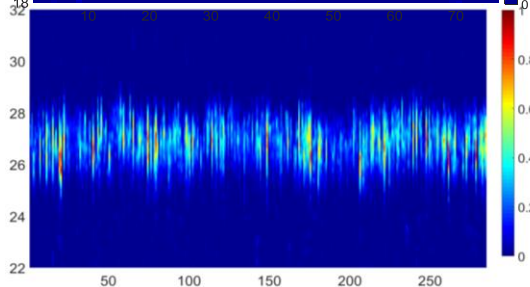
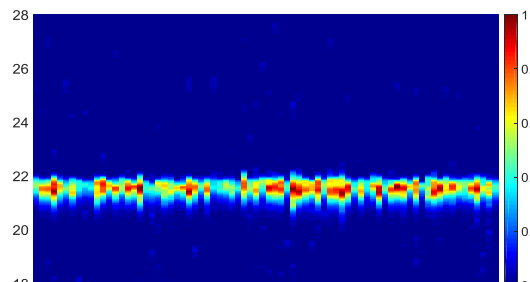
$$E_{THz}(t) = \frac{1}{e} \sqrt{\frac{m_e}{2W_0}} s(t)$$

, with streaking speed $s(t) = \frac{\partial W(t)}{\partial t}$

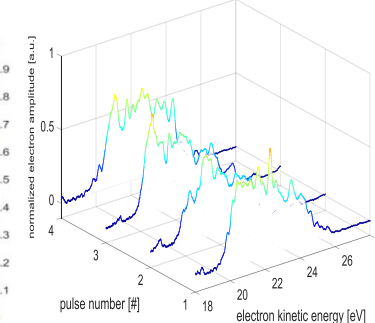
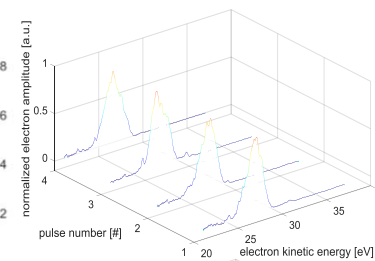
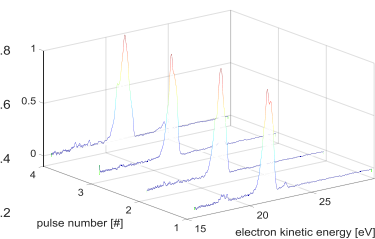
> Single shot results

Single shot analysis – overview

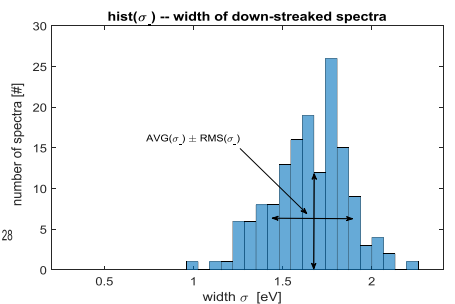
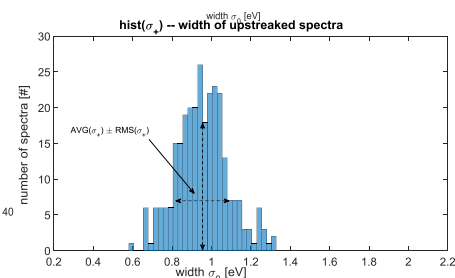
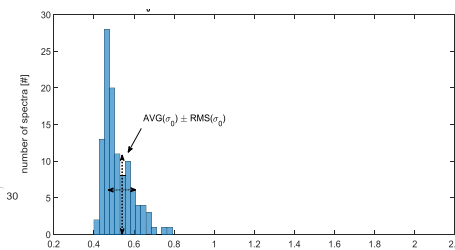
All spectra of a series



Single spectra



Histograms of the σ -widths



$$\sigma_0 = 0.37\text{eV} \pm 12\%$$

$$\sigma_+ = 0.95\text{eV} \pm 14\%$$

$$\sigma_- = 1.65\text{eV} \pm 13\%$$

> More on theory

Streaking theory – semiclassical model

W is the energy modulation from the quiver field, which can be calculated analytically:

$$W = \frac{\vec{p}^2}{2m} = \frac{\left(\vec{p}_0 + e \int_{t_i}^{\infty} \vec{E}_L\right)^2}{2m} \quad \text{with } E_L(t) = E_{THz}(t) = E_{0,THz}(t) \cos(\omega_{THz}t + \varphi)$$

$$\Rightarrow W(t_i) = W_0 \pm \sqrt{8W_0 U_p(t_i) \sin(\varphi_i)}$$

$$\text{Ponderomotive energy } \hbar\omega_{THz} \ll U_p(t) = \frac{e^2 E_{0,THz}^2(t)}{4m_e \omega_{THz}^2} \ll W_0 \quad (!)$$

$$\text{Unstreaked kinetic energy } W_0 = \frac{\vec{p}_0^2}{2m} = \hbar\omega_{XUV} - I_{pot} \gg 0 \quad (!!)$$

(Polarization of \vec{E}_L parallel to \vec{v} electrons)

Streaking theory – semiclassical model

Inserting $U_p(t) = \frac{e^2 E_{0,THz}^2(t)}{4m_e \omega_{THz}^2}$ into $W(t) = W_0 \pm \sqrt{8W_0 U_p(t)} \sin(\omega_{THz} t)$

leads to $W(t) \approx W_0 \pm e \sqrt{\frac{2W_0}{m_e}} \frac{E_{0,THz}(t)}{\omega_{THz}} \sin(\omega_{THz} t) = W_0 \pm e \sqrt{\frac{2W_0}{m_e}} A(t)$ *

→

$$E_{THz}(t) = \frac{1}{e} \sqrt{\frac{m_e}{2W_0}} s(t) \quad , \text{ with streaking speed } s(t) = \frac{\partial W(t)}{\partial t}$$

Streaking theory – Gaussian pulse with linear chirp

- > a quantum mechanical model assuming a linearly chirped XUV pulse

$$E_{XUV}(t) = E_{XUV}^0 e^{-a(t-t_0)^2} e^{i(\omega_0(t-t_0)+c(t-t_0)^2)}$$

with the two unknown parameters chirp parameter c and pulse duration $\tau_{XUV} = \frac{1}{2\sqrt{a}}$ leads to the relation

$$\sigma_{\pm} = \sqrt{\sigma_0^2 + \tau_{XUV}^2 s_{eff}^2}$$

with effective streaking speed

$$s_{eff} = \sqrt{s^2 \pm 4cs}$$

with the widths of the spectra σ_{\pm} at each inflexion point of the streaking trace $W(t)$.

Streaking theory – Gaussian pulse with linear chirp

From the two formula $\sigma_{\pm} = \sqrt{\sigma_0^2 + \tau_{XUV}^2 (s^2 \pm 4cs)}$ one obtains two equations to determine τ_{XUV} and chirp parameter c

$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 + \sigma_{-,decon}^2}{2s^2}}$$

$$c = \frac{(\sigma_{+,decon}^2) - (\sigma_{-,decon}^2)}{8s\tau_{XUV}^2}$$

with the de-convoluted widths

$$\sigma_{\pm,decon} = \sqrt{\sigma_{\pm}^2 - \sigma_0^2}$$

Streaking theory – Temporal resolution

For the simple case of equal spectral widths $\sigma_+ = \sigma_-$ one obtains the intuitive formula

$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 + \sigma_{-,decon}^2}{2s^2}} = \frac{\sigma_{decon}}{s}$$

, from which one obtains for the **temporal resolution** τ_{res} of the THz transient field streak camera with the unstreaked width σ_0 (as for every streak camera)

$$\tau_{res} = \frac{\sigma_0}{s}$$

Streaking theory – Gaussian pulse with linear chirp

In case of asymmetric streaking with two different streaking speeds $|s_{\pm}|$, the situation becomes more complex.

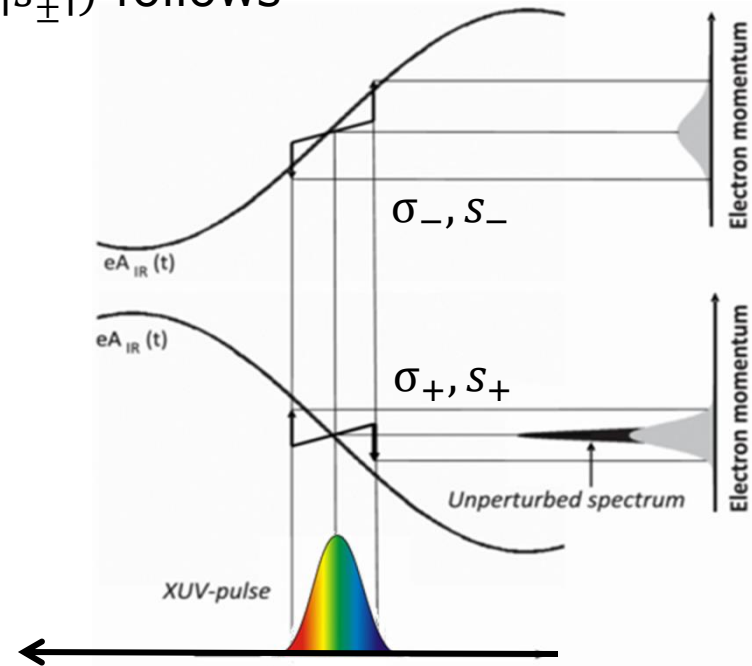
From the two formula $\sigma_{\pm} = \sqrt{\sigma_0^2 + \tau_{XUV}^2 (s_{\pm}^2 \pm 4c|s_{\pm}|)}$ follows

$$\tau_{XUV} = \sqrt{\frac{s_-(\sigma_{+,decon}^2) + s_+(\sigma_{-,decon}^2)}{(s_-s_+)(s_- + s_+)}}$$

$$c = \frac{(\sigma_{+,decon}^2)s_- - (\sigma_{-,decon}^2)s_+}{4(s_-s_+)(s_- + s_+)\tau_{XUV}^2}$$

$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 + \sigma_{-,decon}^2}{2s^2}}$$

$$c = \frac{(\sigma_{+,decon}^2) - (\sigma_{-,decon}^2)}{8s\tau_{XUV}^2}$$



Streaking theory – Gaussian pulse with linear chirp

For the simple case of equal streaking speed $s_+ = s_-$ one obtains from the denominator $(s_-s_+)(s_- + s_+) = 2s^3$ and thus again

$$\tau_{XUV} = \sqrt{\frac{\sigma_{+,decon}^2 + \sigma_{-,decon}^2}{2s^2}}$$

$$c = \frac{(\sigma_{+,decon}^2) - (\sigma_{-,decon}^2)}{8s\tau_{XUV}^2}$$

