Convergence Study for FEM- and FIT-Based Eigenvalue Solvers

Applied to a TESLA 1.3 GHz Test Structure

W. Ackermann, H. De Gersem, W. F. O. Müller
Institut Theorie Elektromagnetischer Felder, Technische Universität Darmstadt

Status Meeting
June 8, 2018
TEMF, Darmstadt

Outline

- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

Outline

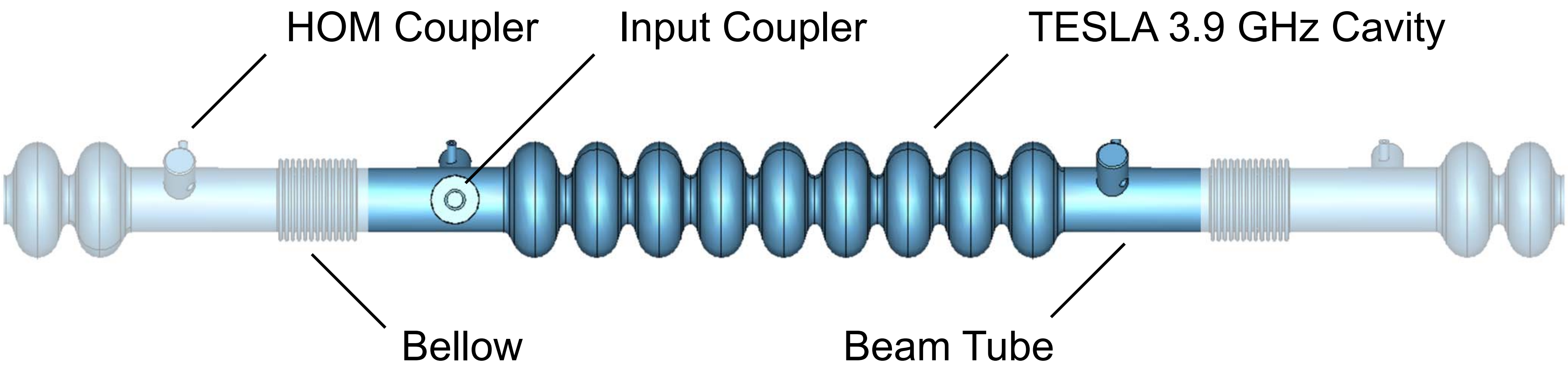
- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

Motivation

- Eigenanalysis of the TESLA Cavities
  - Frequencies below cutoff frequency of the beam tube

$$f_{\text{cut},\ \varnothing=39\text{mm}} = 4.505\ \text{GHz}$$ 1.3 GHz structure

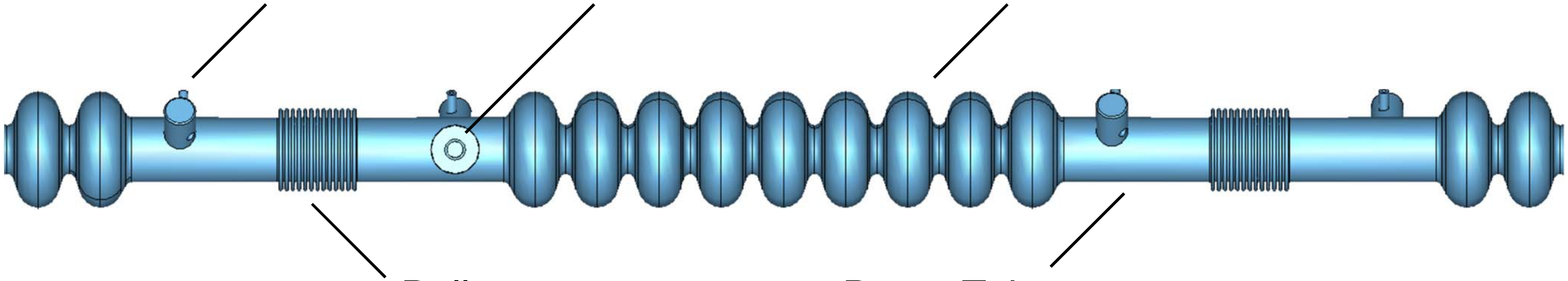
$$f_{\text{cut},\ \varnothing=40\text{mm}} = 4.392\ \text{GHz}$$ 3.9 GHz structure



➡ Concentration on a single cavity if no interaction with adjacent elements

Motivation

- Eigenanalysis of the TESLA 3.9 GHz Cavities
  - Chain of cavities



- FLASH: 4 cavities per module + remaining beam line
- X-FEL: 8 cavities per module + remaining beam line

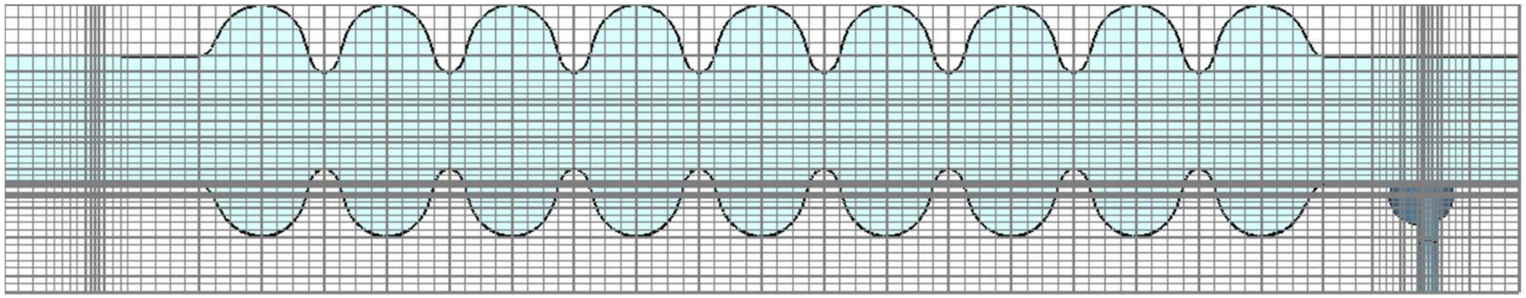
➡ Select a proper numerical method to accurately solve this problem

Outline

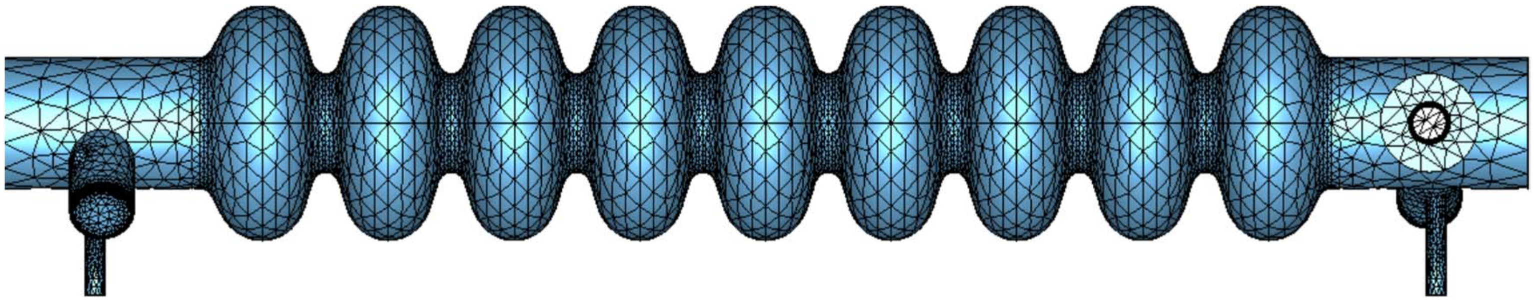
- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

Computational Model

- Compare two Widespread Numerical Methods
  - FIT on hexahedral mesh

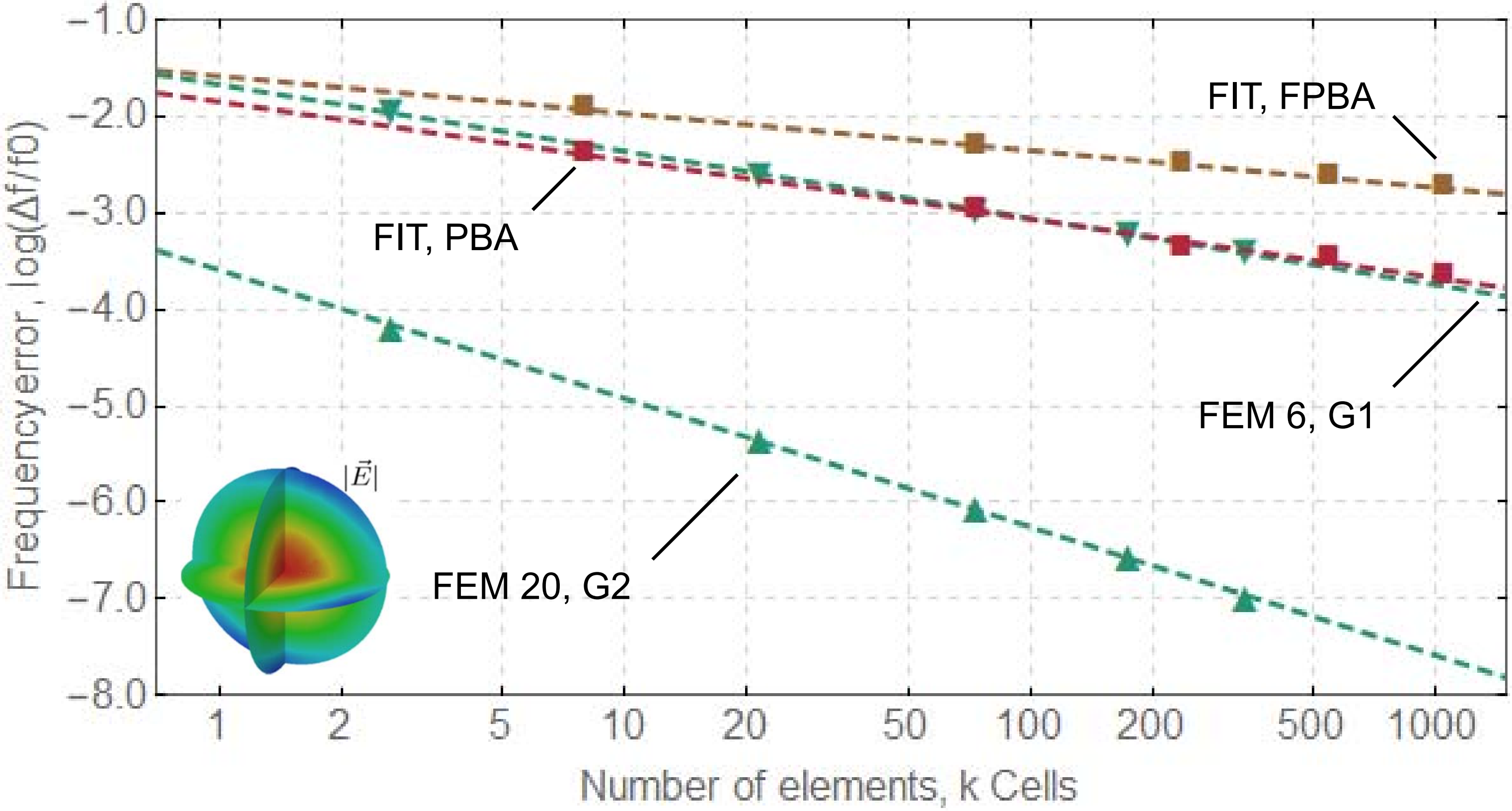


  - FEM on tetrahedral mesh



Computational Model

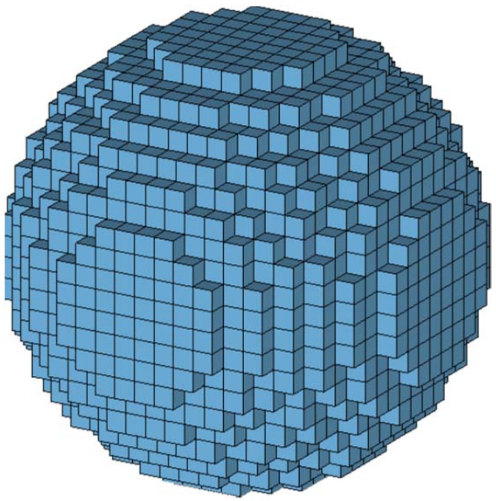
- Compare two Widespread Numerical Methods
  - Convergence study based on a spherical resonator



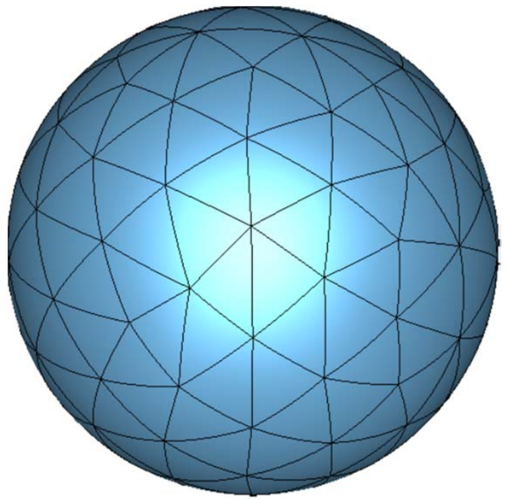
Computational Model

- Compare two Widespread Numerical Methods
  - Survey of advantages/disadvantages

FIT



FEM

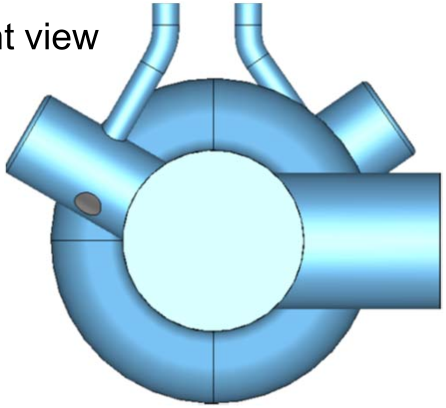


FIT	FEM
• Structured grid simple to evaluate	• Unstructured grid nicely fits to geometry
• Port faces aligned to the mesh	• Arbitrary port face orientations
• Field components are decoupled	• Field components are coupled
• Standard eigenvalue problem	• Generalized eigenvalue problem

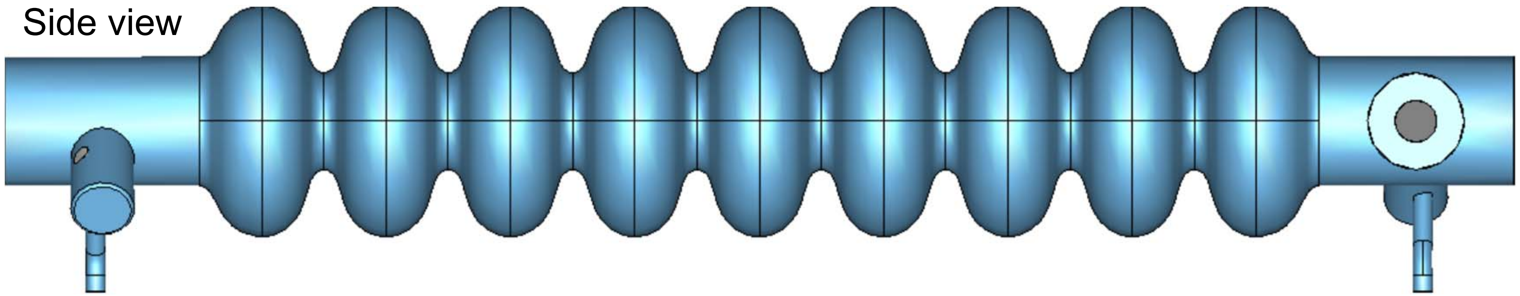
Computational Model

- Eigenanalysis of a TESLA 3.9 GHz Test Structure
  - Simplifications suggested by DESY, URO, and TEMF
    - Single zero-loss cavity (ports replaced by PMC)
    - Short beam pipes
    - Port faces aligned to HEX mesh
    - Frequency range from 2 to 6 GHz

Front view



Side view

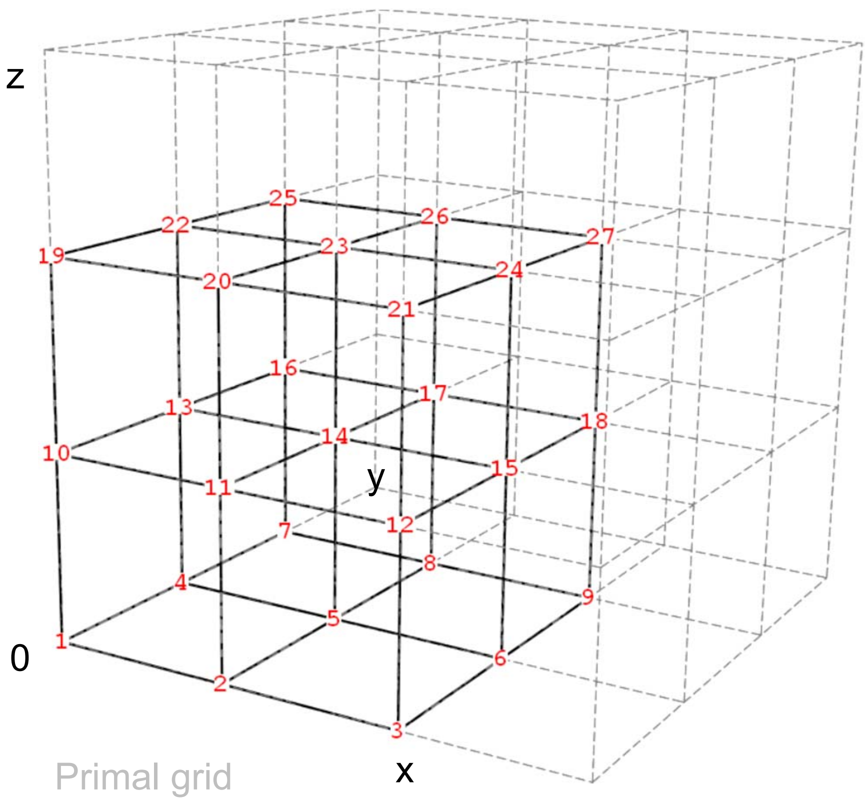


Outline

- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

Computational Model

- **Canonical Indexing**
  - **Classical global ordering**



Example:
$N_x=3$
$N_y=3$
$N_z=3$

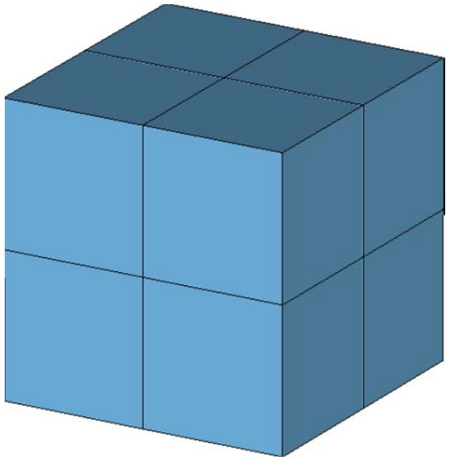
$$\hat{e} = \begin{pmatrix} \hat{e}_1^x \\ \hat{e}_2^x \\ \boxed{\hat{e}_3^x} \\ \vdots \\ \hat{e}_1^y \\ \hat{e}_2^y \\ \hat{e}_3^y \\ \vdots \\ \hat{e}_1^z \\ \hat{e}_2^z \\ \hat{e}_3^z \\ \vdots \end{pmatrix}$$

eliminate

$$\hat{b} = \begin{pmatrix} \hat{b}_1^x \\ \hat{b}_2^x \\ \hat{b}_3^x \\ \vdots \\ \hat{b}_1^y \\ \hat{b}_2^y \\ \boxed{\hat{b}_3^y} \\ \vdots \\ \hat{b}_1^z \\ \hat{b}_2^z \\ \boxed{\hat{b}_3^z} \\ \vdots \end{pmatrix}$$

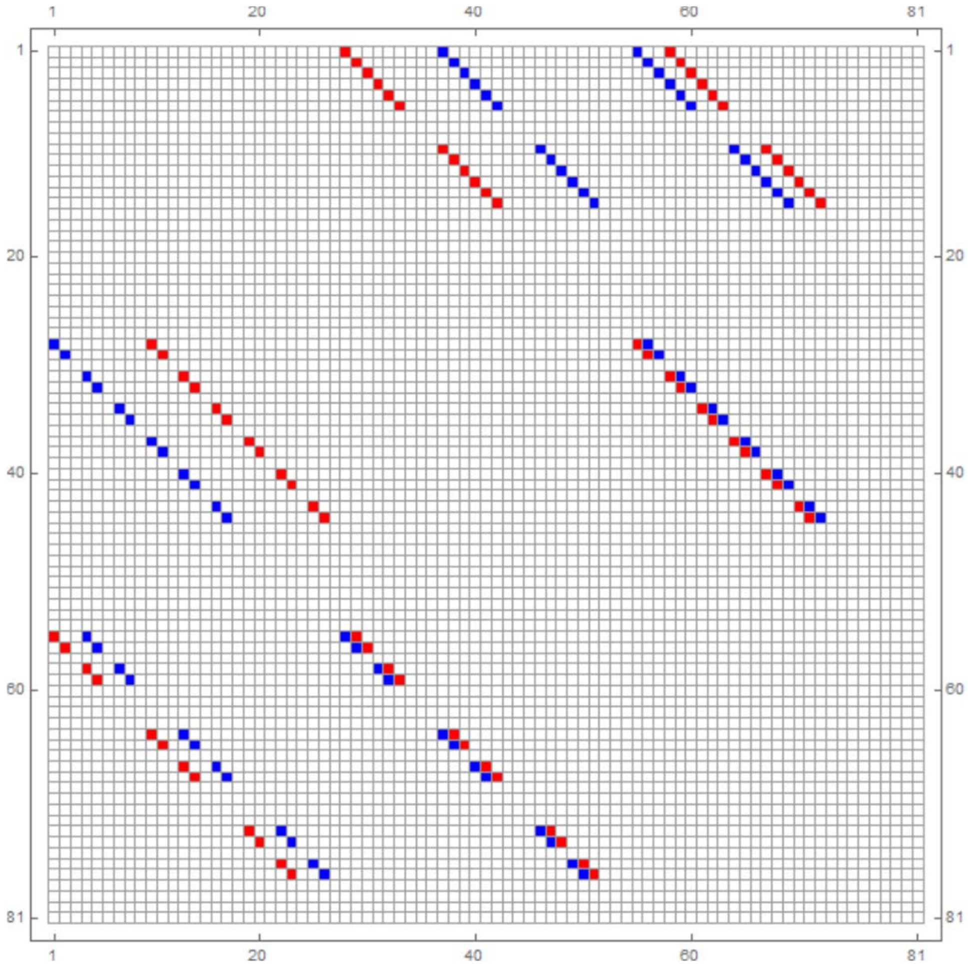
Computational Model

- **Structure of the Matrix C**
  for the simple example



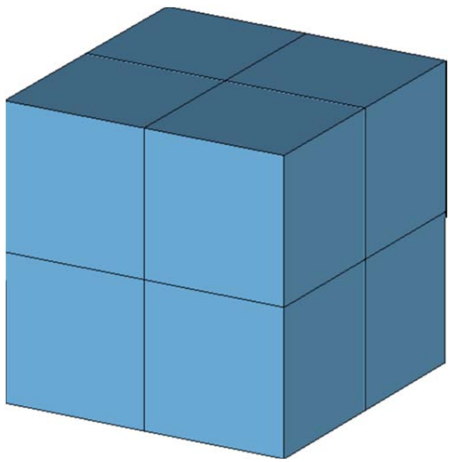
N_x=3
N_y=3
N_z=3

N_P=27
$3*N_P$=81



Computational Model

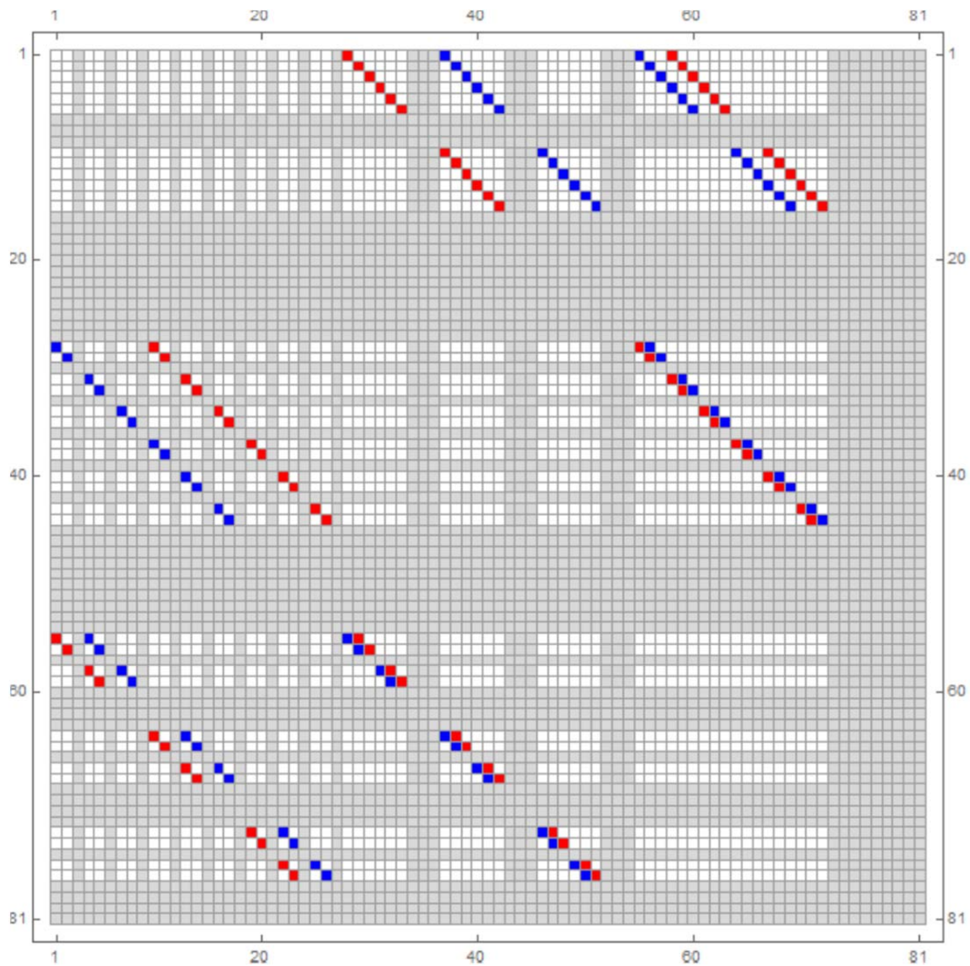
- **Structure of the Matrix C**
  for the simple example



N_x=3
N_y=3
N_z=3

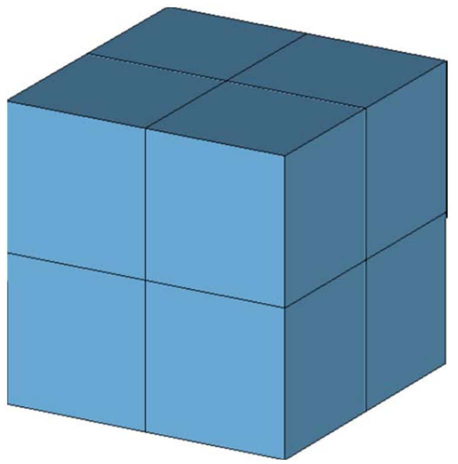
N_P=27
3*N_P=81

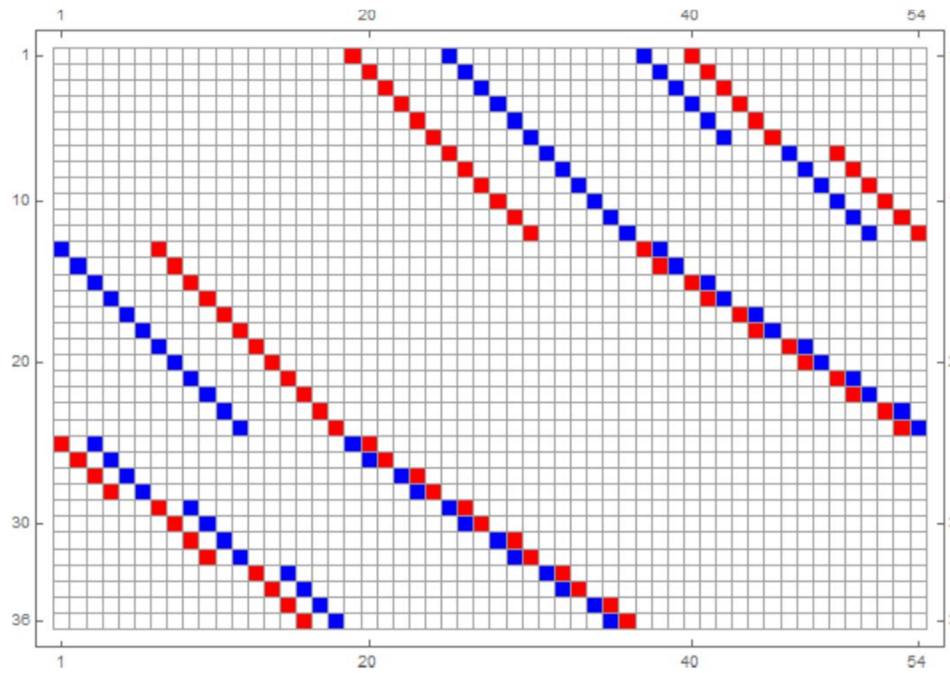
eliminate



Computational Model

- **Structure of the Matrix C**
  for the simple example





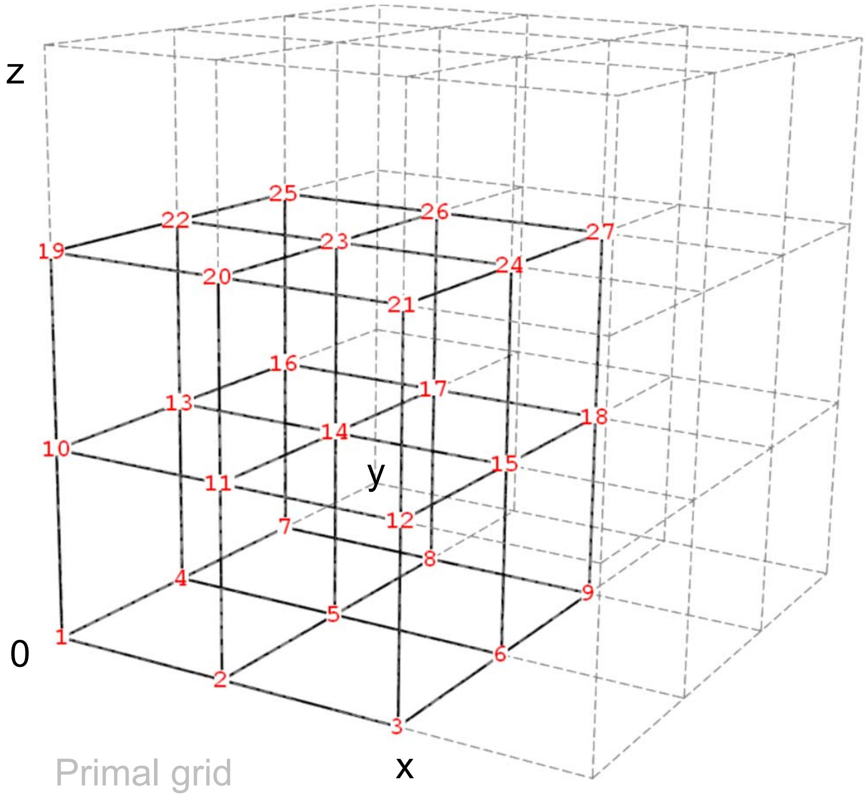
N_x=3
N_y=3
N_z=3

N_P=27
$3*N_P$=81

Reduced matrix

Computational Model

- **Canonical Indexing**
  - **Classical local ordering**



Primal grid

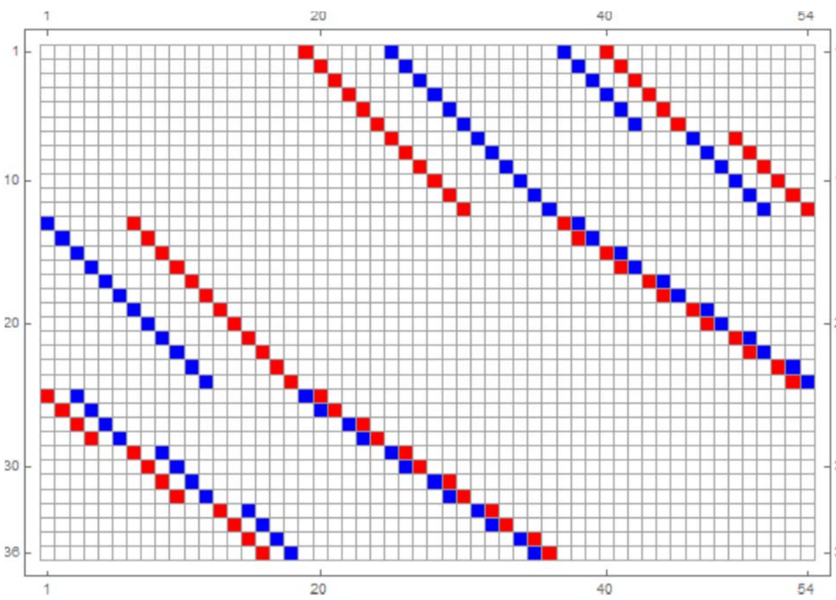
Beispiel:
$N_x=3$
$N_y=3$
$N_z=3$

$$\vec{e} = \begin{pmatrix} \vec{e}_1^{\,x} \\ \vec{e}_1^{\,y} \\ \vec{e}_1^{\,z} \\ \vec{e}_2^{\,x} \\ \vec{e}_2^{\,y} \\ \vec{e}_2^{\,z} \\ \boxed{\vec{e}_3^{\,x}} \\ \vec{e}_3^{\,y} \\ \vec{e}_3^{\,z} \\ \vec{e}_4^{\,x} \\ \vec{e}_4^{\,y} \\ \vdots \end{pmatrix} \qquad \text{eliminate} \qquad \vec{b} = \begin{pmatrix} \vec{b}_1^{\,x} \\ \vec{b}_1^{\,y} \\ \vec{b}_1^{\,z} \\ \vec{b}_2^{\,x} \\ \vec{b}_2^{\,y} \\ \vec{b}_2^{\,z} \\ \vec{b}_3^{\,x} \\ \boxed{\vec{b}_3^{\,y}} \\ \boxed{\vec{b}_3^{\,z}} \\ \vec{b}_4^{\,x} \\ \vec{b}_4^{\,y} \\ \vdots \end{pmatrix}$$

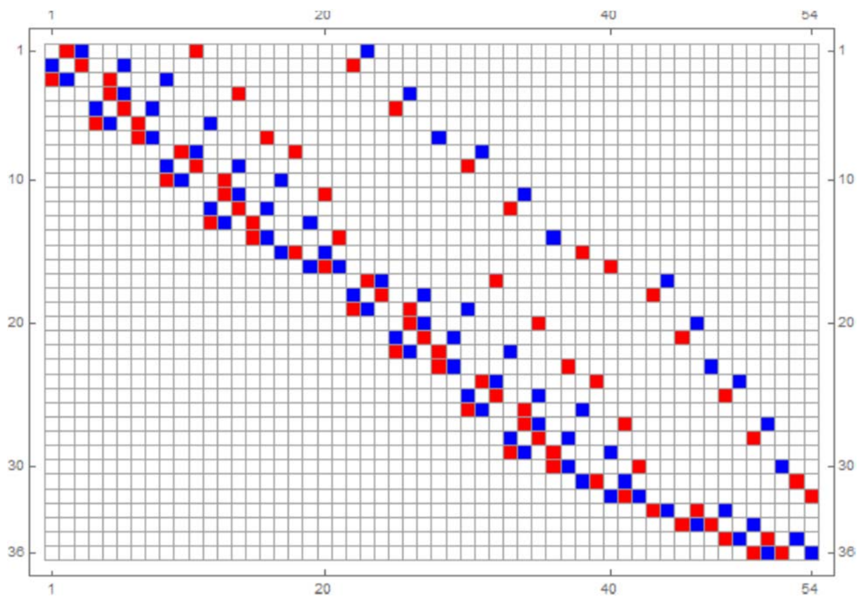
Computational Model

- **Structure of the Matrix C**
  for the simple example

Reduced Matrices



global canonical indexing



local canonical indexing

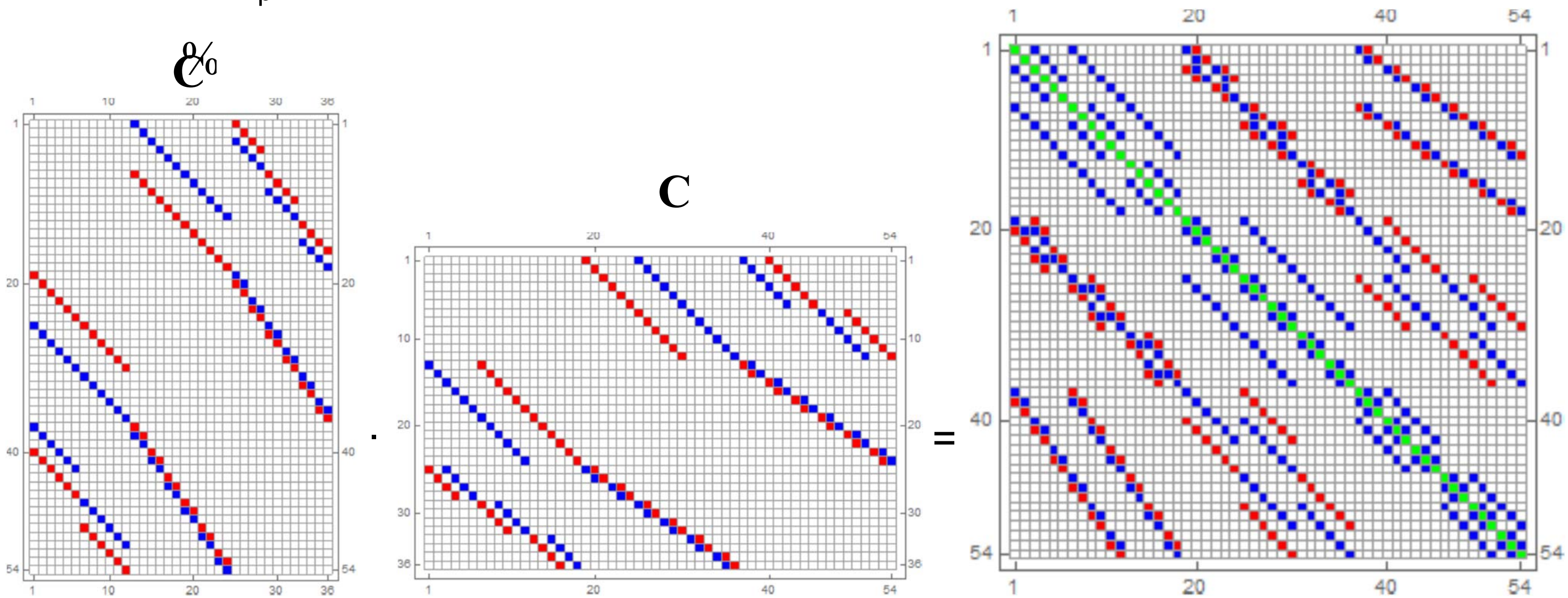
Computational Model

TECHNISCHE UNIVERSITÄT DARMSTADT

- **Layout of the System Matrix:**

$$\mathbf{A} = \mathbf{M}_{\varepsilon}^{-1/2} \tilde{\mathbf{C}} \mathbf{M}_{\mu}^{-1} \mathbf{C} \mathbf{M}_{\varepsilon}^{-1/2}$$

Example: N_p=27



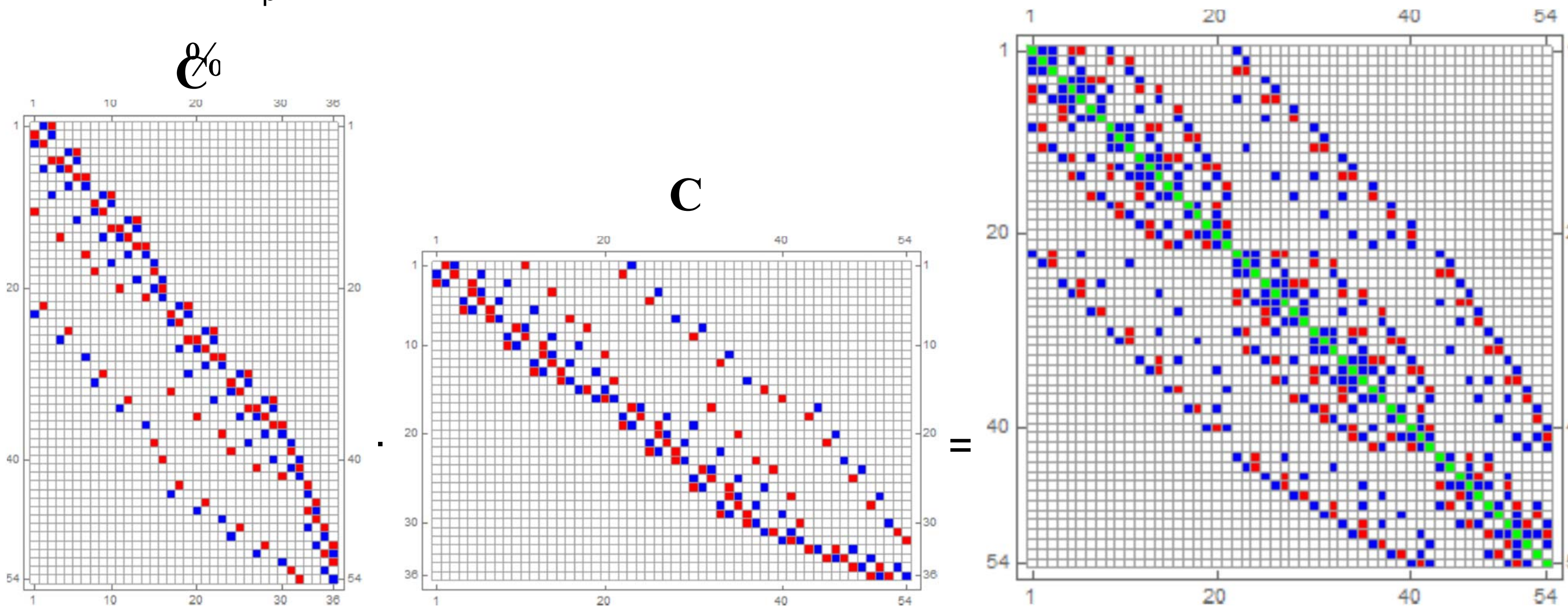
Computational Model

TECHNISCHE UNIVERSITÄT DARMSTADT

- **Layout of the System Matrix:**

$$\mathbf{A} = \mathbf{M}_{\varepsilon}^{-1/2}\tilde{\mathbf{C}}\mathbf{M}_{\mu}^{-1}\mathbf{C}\mathbf{M}_{\varepsilon}^{-1/2}$$

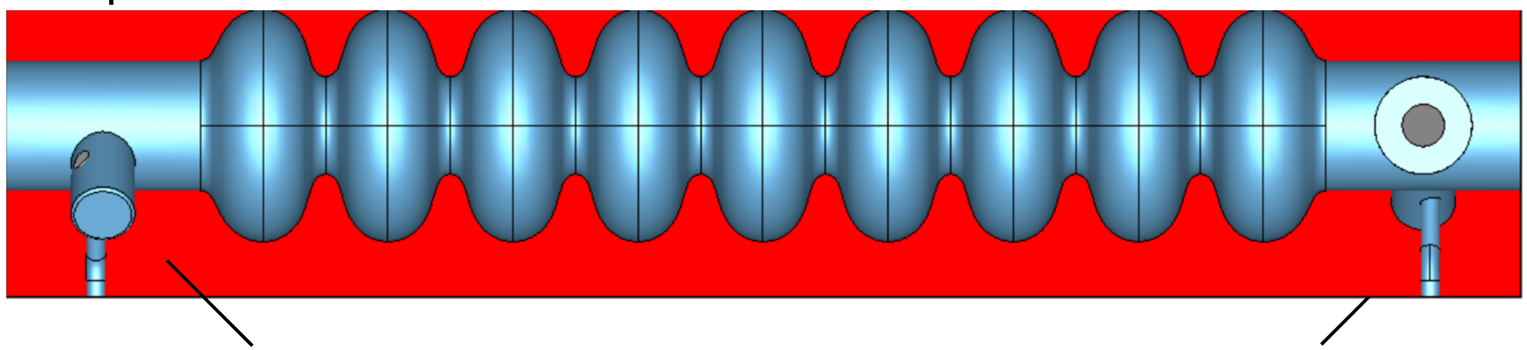
Example: N_p=27



Computational Model

- **Cem3D Implementation for the FIT Solver**
  - Parallel layout of the matrices and vectors using the PETSc library
  - Renumbering of DOF
    - Enables to eliminate unnecessary memory allocations
    - Less zero eigenvalues
    - Length of the attached coaxial lines no longer significant for the memory consumption

Computational domain



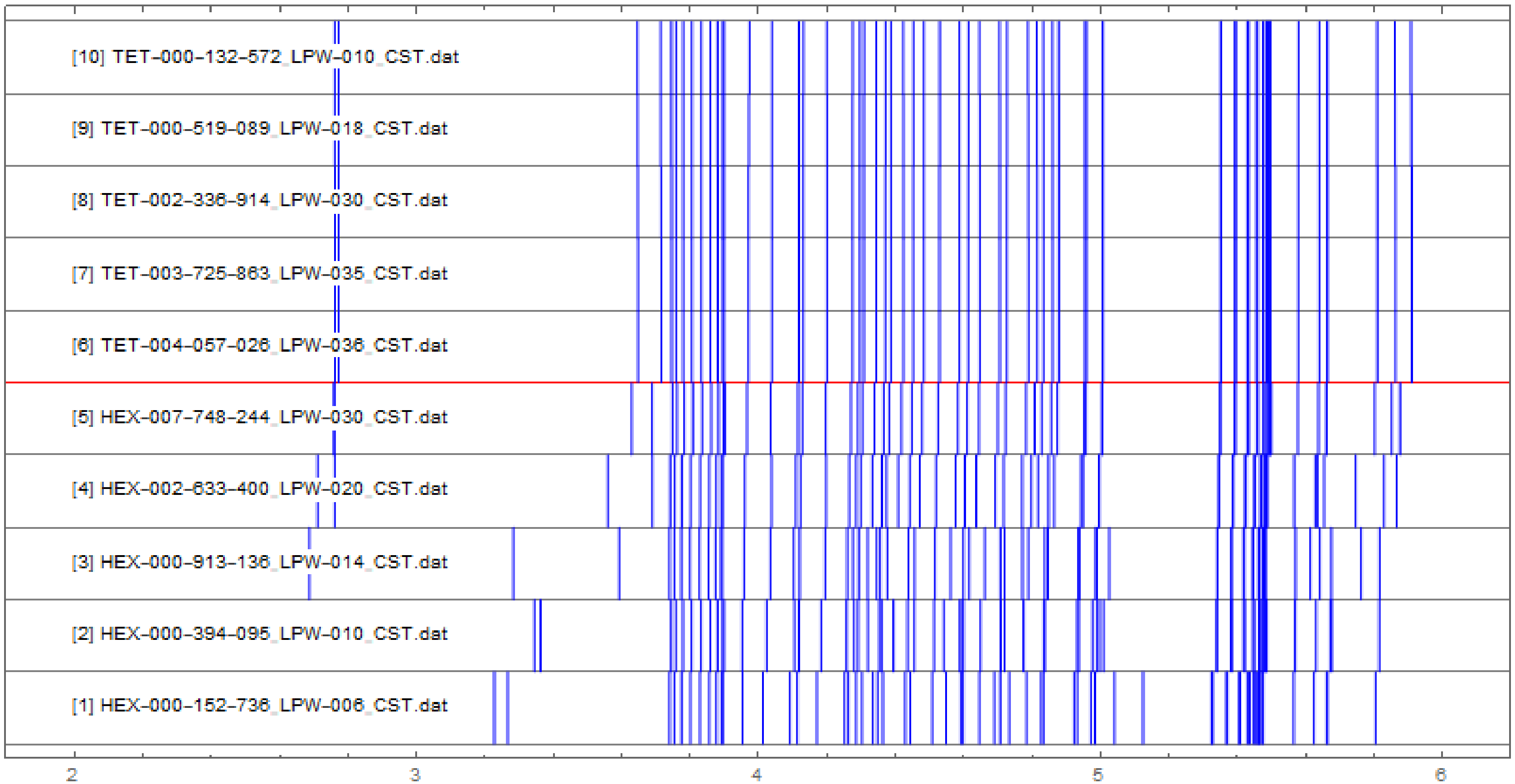
Eliminate unnecessary DOF   Bounding box

Outline

- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

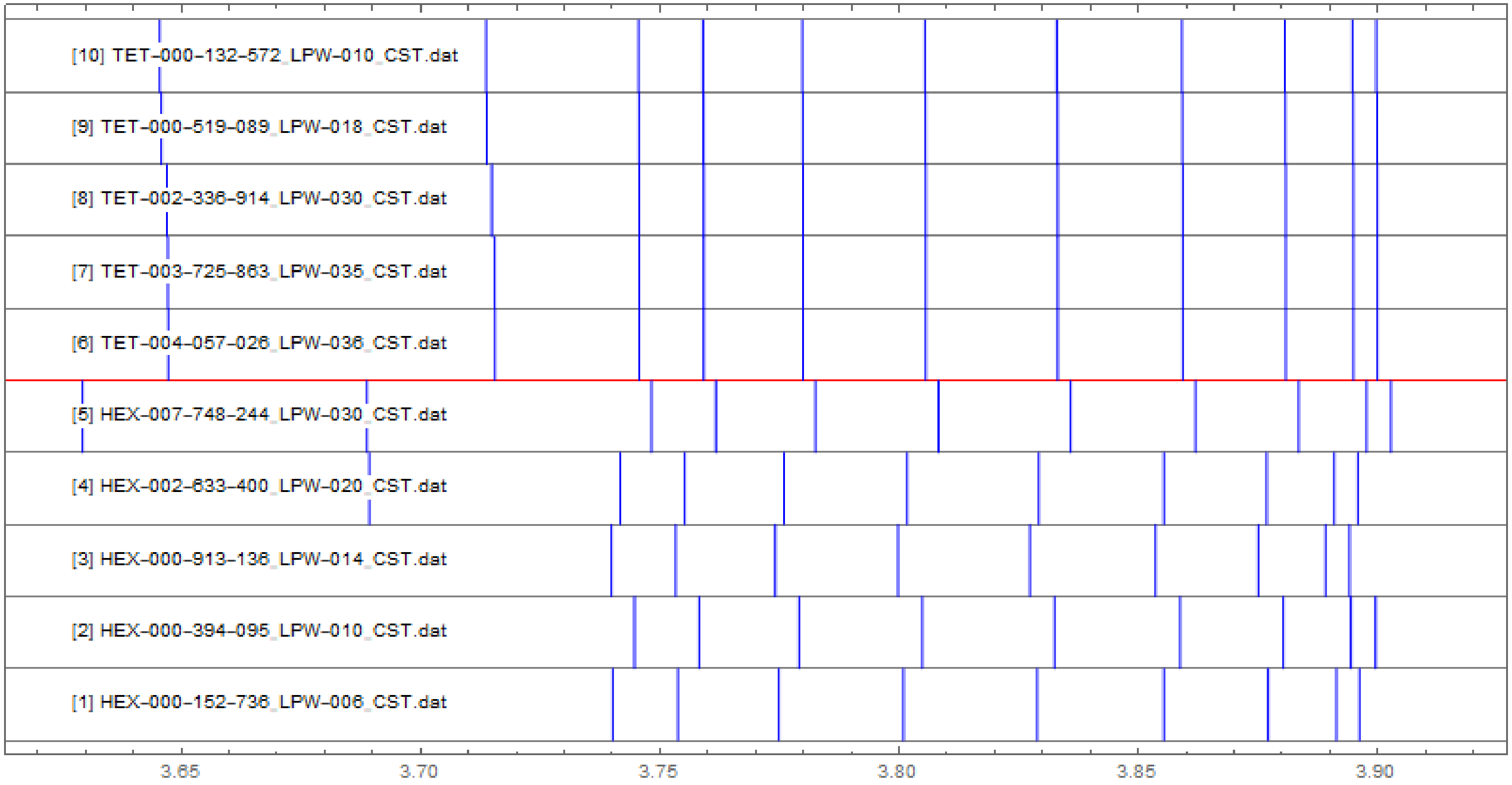
Simulation Results

- CST Implementation for the FIT and FEM Solver



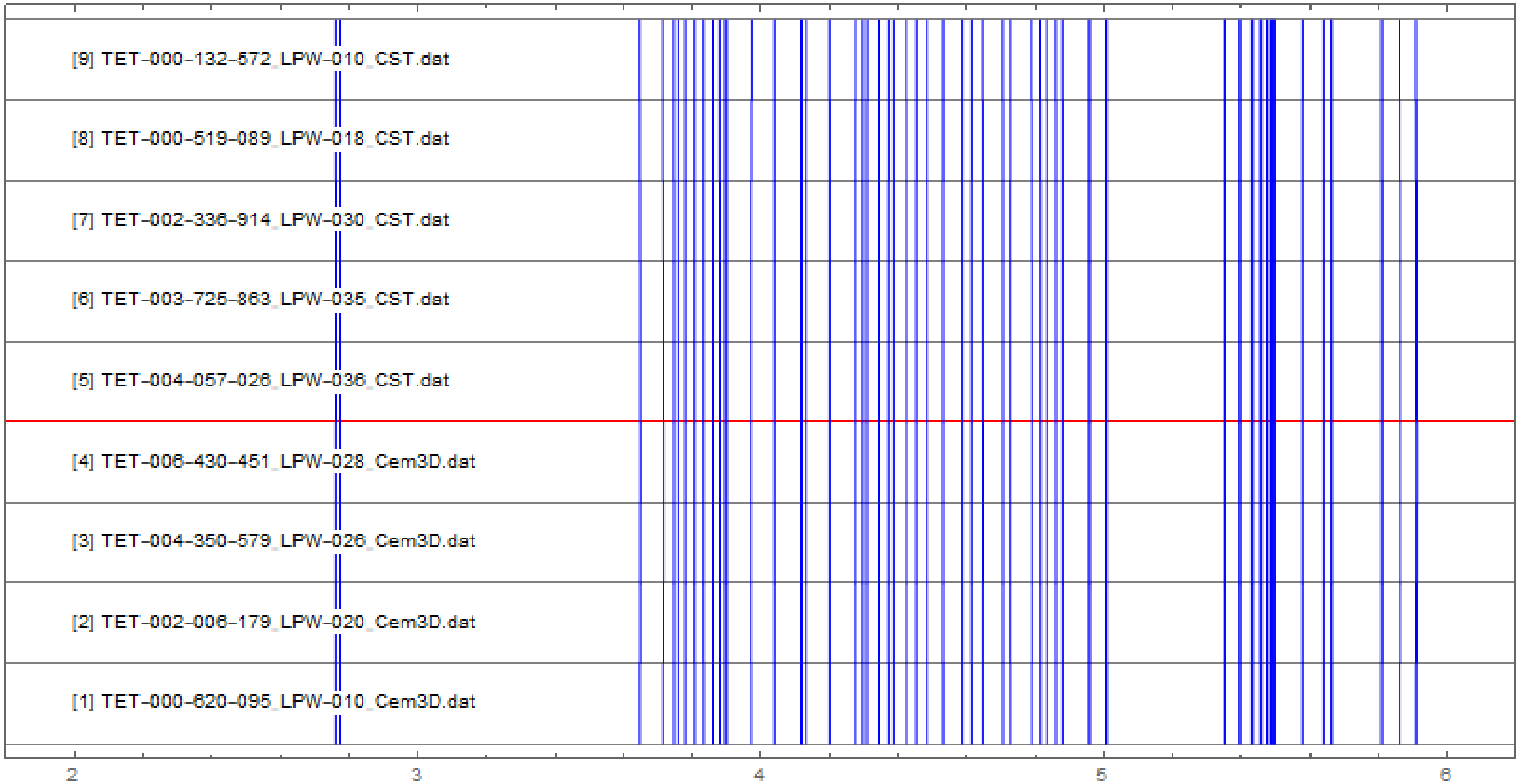
Simulation Results

- CST Implementation for the FIT and FEM Solver



Simulation Results

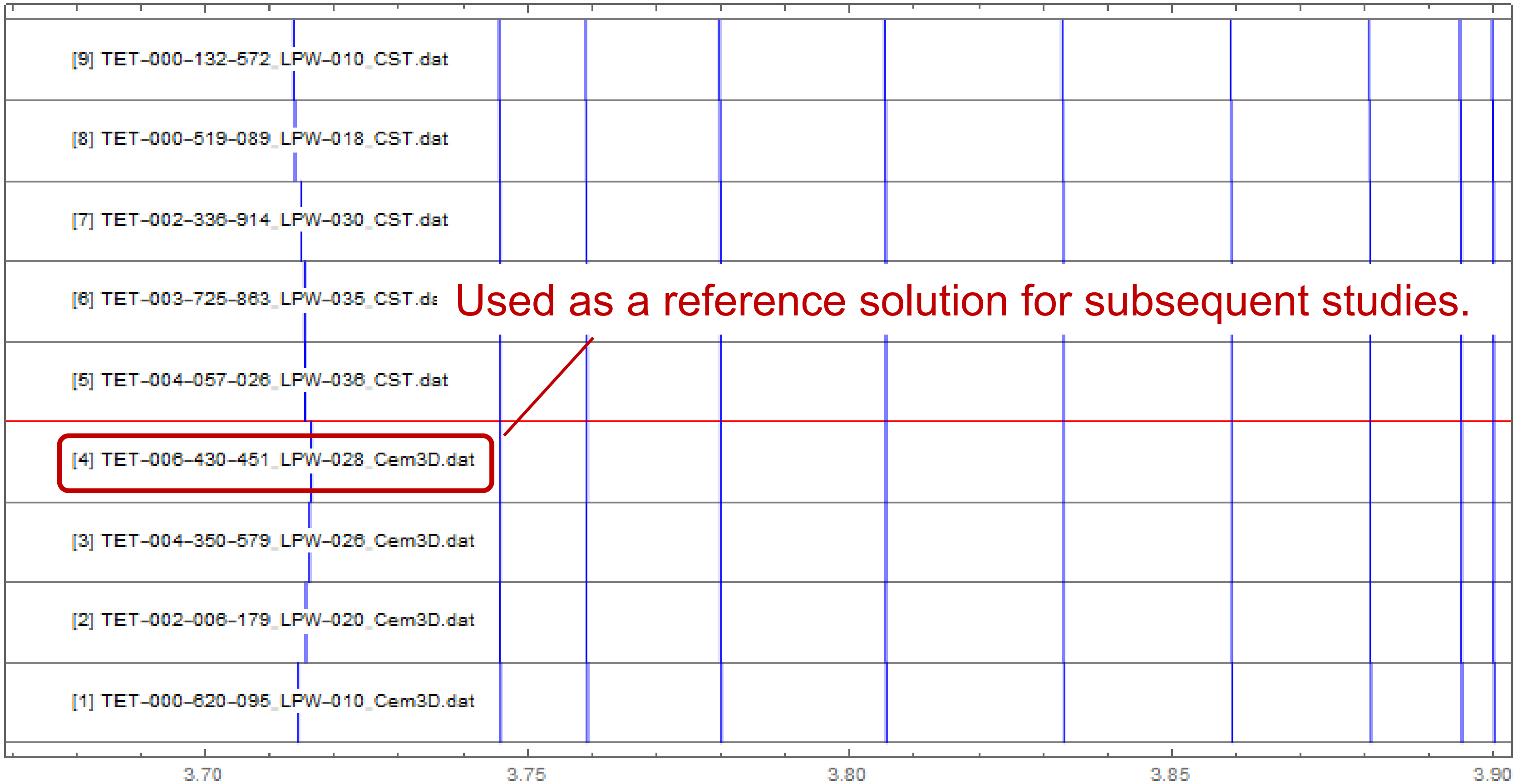
- CST and Cem3D Implementation for the FEM Solver



Simulation Results

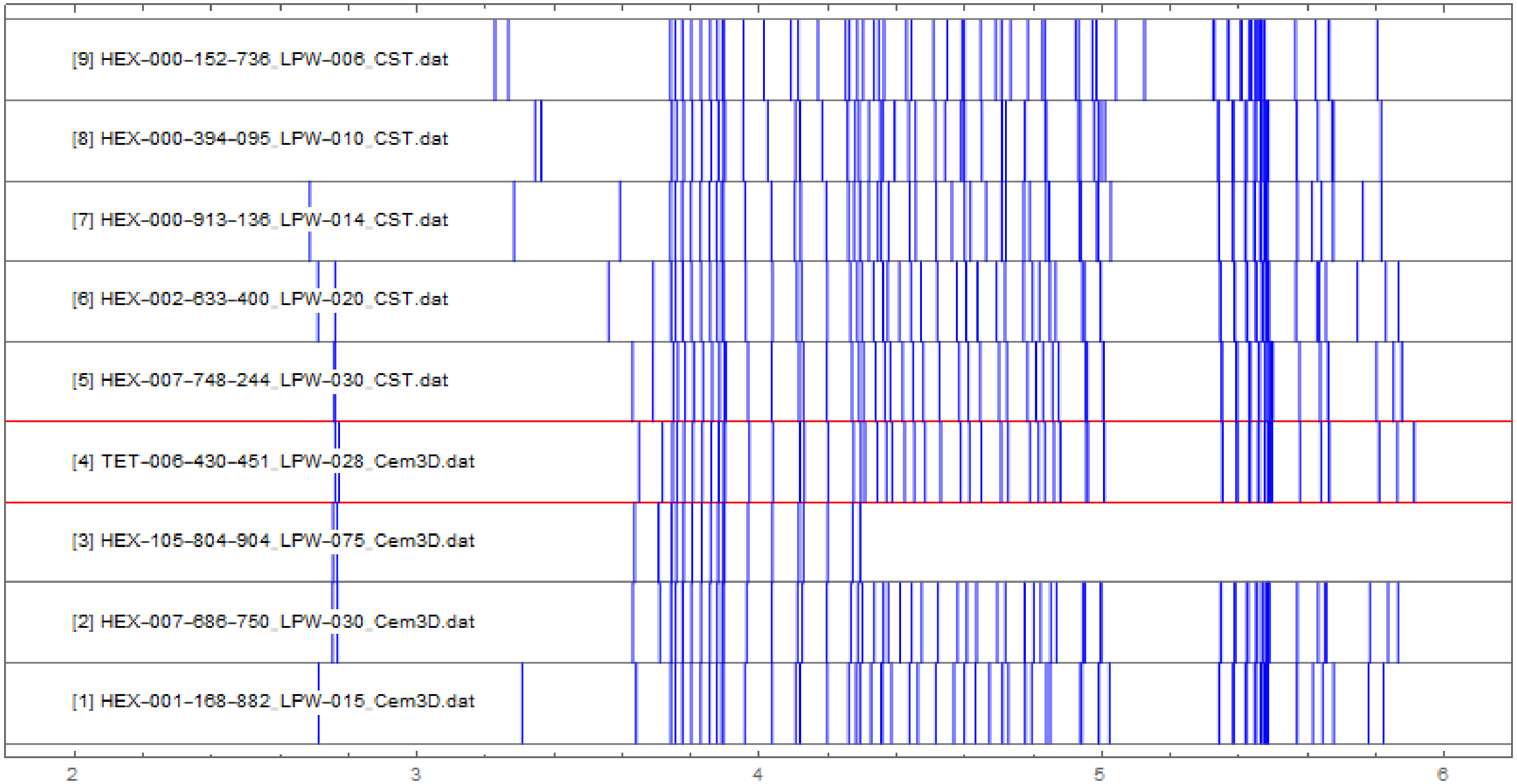
TECHNISCHE UNIVERSITÄT DARMSTADT

- CST and Cem3D Implementation for the FEM Solver



Simulation Results

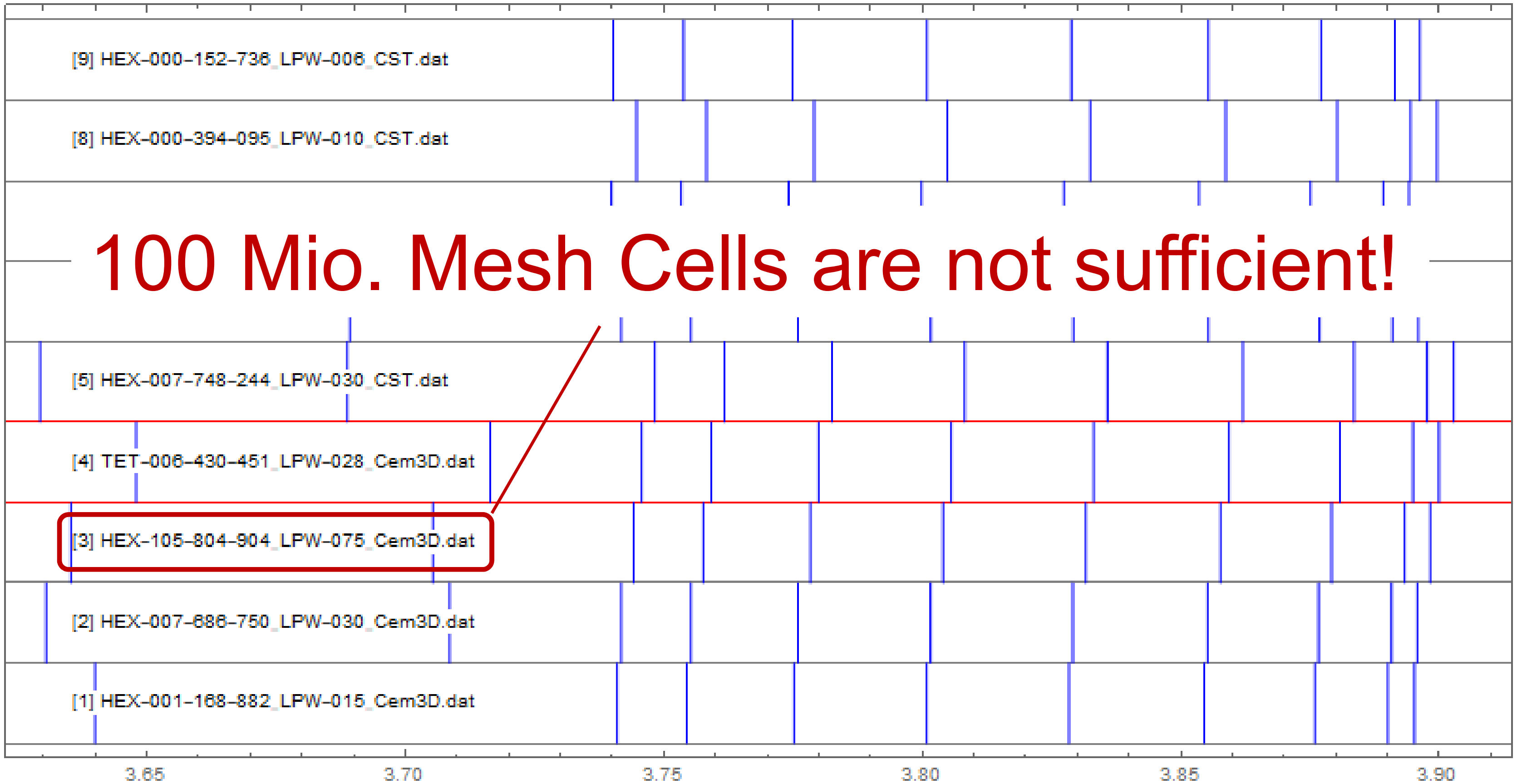
- CST and Cem3D Implementation for the FIT Solver



Simulation Results

TECHNISCHE UNIVERSITÄT DARMSTADT

- CST and Cem3D Implementation for the FIT Solver



Outline

- Motivation
- Computational Model
  - FIT on HEX and FEM on TET
  - Cem3D parallel implementation of FIT on HEX
- Simulation Results
  - CST implementation for the FIT and FEM solver
  - CST and Cem3D Implementation for the FEM Solver
  - CST and Cem3D Implementation for the FIT Solver
- Summary / Outlook

Summary / Outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

- Summary:
  - Precise modeling of a 3.9 GHz TESLA test structure including the input and higher-order mode couplers
  - Concentration on a single zero-loss cavity with coordinate-aligned port faces, ports are “closed” with PMC
  - Electromagnetic eigenmode analysis based on FIT on HEX and FEM on TET meshes
  - 100 Mio. Mesh cells for FIT on HEX not yet sufficient
- Outlook:
  - Investigate field-map data for the FIT and FEM approaches