Two Poisson Approaches

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Hamiltonian and equation of motion

Poisson approach 1 & 2

few results from our last meeting

comparison with old gun-benchmark

summary/conclusions

… Beam-Dynamics with 3D-Gun

Hamiltonian and Equation of Motion

$$\mathcal{H}(\mathbf{r},\mathbf{P},t) = c\sqrt{(\mathbf{P}-q\mathbf{A})^2 + (m_0 c)^2} + qV$$

Hamiltonian with scalar and vector potential

$$\mathbf{p} = \mathbf{P} - q\mathbf{A}$$

momentum and canonical momentum

$$\frac{d\mathbf{r}}{dt} = \nabla_{\mathbf{p}}\mathcal{H} = c\frac{\mathbf{P}-q\mathbf{A}}{\sqrt{(\mathbf{P}-q\mathbf{A})^2+(m_0c)^2}} = c\frac{\mathbf{p}}{\sqrt{\mathbf{p}^2+(m_0c)^2}} = \mathbf{v}(\mathbf{p})$$

$$\frac{d\mathbf{P}}{dt} = -\nabla_{\mathbf{r}}\mathcal{H} = -c\frac{\nabla_{\mathbf{r}}(\mathbf{P}-q\mathbf{A})\cdot\left(-q\overset{\downarrow}{\mathbf{A}}\right)}{\sqrt{(\mathbf{P}-q\mathbf{A})^2+(m_0c)^2}} - q\nabla_{\mathbf{r}}V = \nabla_{\mathbf{r}}\mathbf{v}\cdot\left(q\overset{\downarrow}{\mathbf{A}}\right) - q\nabla_{\mathbf{r}}V = -q\nabla_{\mathbf{r}}(\mathbf{v}\mathbf{A}-V)$$

$$\frac{d\mathbf{p}}{dt} = -q\nabla_{\mathbf{r}}(\mathbf{v}\mathbf{A}-V) - q\frac{d\mathbf{A}}{dt} = q\underbrace{\left(-\frac{\partial\mathbf{A}}{\partial t} - \nabla_{\mathbf{r}}V\right)}_{\mathbf{E}} + q\underbrace{\left(\mathbf{v}\times\nabla_{\mathbf{r}}\times\mathbf{A}\right)}_{\mathbf{B}}$$

equation of motion with Lorentz force (E,B)

existence of Hamiltonian + Liouville's theorem $\rightarrow$ conservation of phase space

Poisson Approach 1

Field calculation assumes collective motion with velocity $\mathbf{v}_c$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\right)V = -\frac{\rho(\mathbf{r}-\mathbf{v}_c\tau,t)}{\varepsilon} \qquad \rightarrow V(\mathbf{r},t) = \Big|_{\tau=0} V(\mathbf{r}-\mathbf{v}_c\tau,t)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\right)\mathbf{A} = -\mu\mathbf{v}_c\rho(\mathbf{r}-\mathbf{v}_c\tau,t) \qquad \rightarrow \mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}_c}{c^2}V(\mathbf{r},t)$$

phase space is conserved: $\frac{d\mathbf{r}}{dt} = \nabla_{\mathbf{p}}\mathcal{H}$ or: $\frac{d\mathbf{r}}{dt} = \mathbf{v}(\mathbf{p})$

$$\frac{d\mathbf{P}}{dt} = -\nabla_{\mathbf{r}}\mathcal{H}$$

$$\frac{d\mathbf{p}}{dt} = q\left(-\frac{\partial\mathbf{A}}{\partial t} - \nabla_{\mathbf{r}}V\right) + q(\mathbf{v}\times\nabla_{\mathbf{r}}\times\mathbf{A})$$
$$= q\left(-\frac{\mathbf{v}_c}{c^2}\frac{\partial V}{\partial t} - \nabla_{\mathbf{r}}V\right) + q\left(\mathbf{v}\times\nabla_{\mathbf{r}}\times\frac{\mathbf{v}_c}{c^2}V\right)$$
$$= q\left(-\frac{\mathbf{v}_c}{c^2}\frac{\partial V}{\partial t} - \nabla_{\mathbf{r}}V + \frac{\mathbf{v}}{c}\times\nabla_{\mathbf{r}}V\times\frac{\mathbf{v}_c}{c}\right)$$

either with use canonical variables, or we need the potential and its time derivative

Poisson Approach 2

Field calculation assumes collective motion with velocity

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\right)V = -\frac{\rho\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)}{\varepsilon} \qquad \rightarrow V\left(\mathbf{r},t\right) = \Big|_{\tau=0} V\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)$$

second approximation

$$\frac{\partial V\left(\mathbf{r},t\right)}{\partial t} \approx \Big|_{\tau=0} \frac{\partial V\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)}{\partial\tau}$$

$$\approx -\mathbf{v}_c \nabla V\left(\mathbf{r},t\right)$$

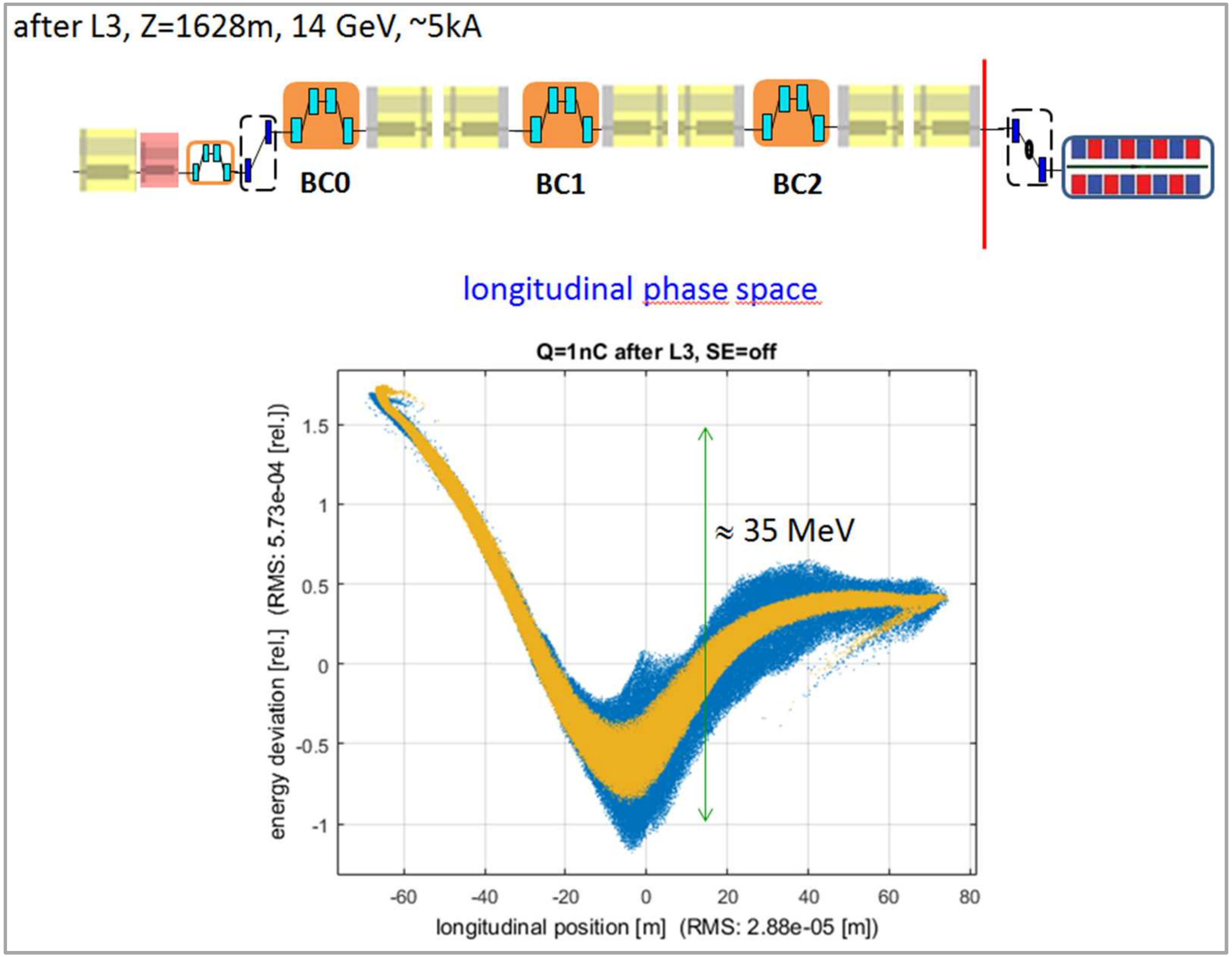
therefore $\frac{d\mathbf{r}}{dt} = \mathbf{v}$

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) \quad \text{with} \quad \mathbf{E} = \left(\frac{\mathbf{v}_c}{c}\otimes\frac{\mathbf{v}_c}{c} - I\right)\nabla_{\mathbf{r}}V$$

$$\mathbf{B} = \nabla_{\mathbf{r}}\times\mathbf{A}$$

this is done in usual E,B tracking codes

from our last meeting:



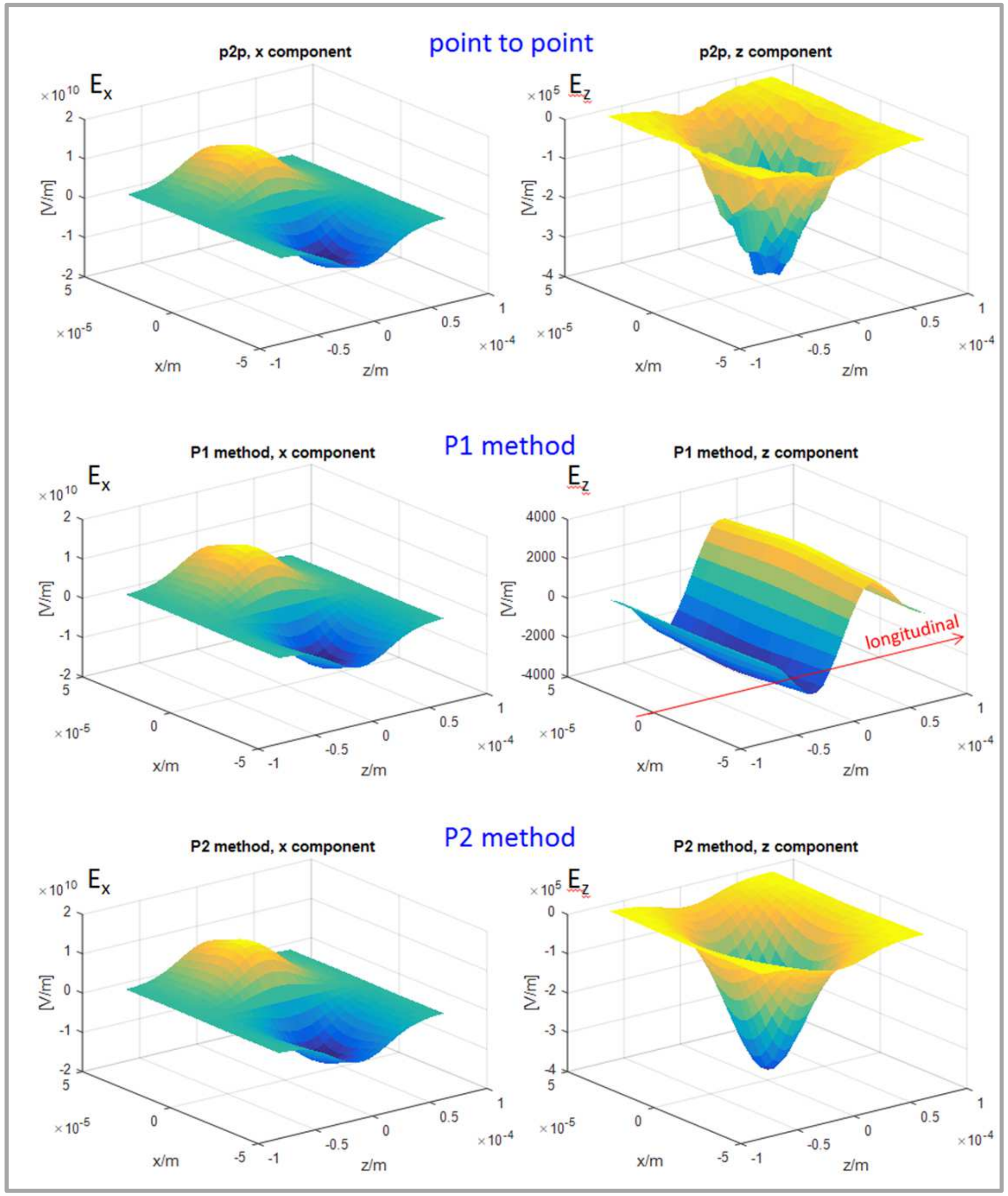
tracking over long distance → growth of phase space volume

from our last meeting:

```
Q=1e-9;
% LONGITUDINAL
pz=2.4e9;
sigz=24E-6;
emitz=0;
% HORIZONTAL
emitx=1e-6/gam;
alphax= 0.2;
betax = 1.0;
% VERTICAL
emity=emitx;
alphay= 1.0;
betay = 0.322;
```

$$\frac{\alpha_y}{\beta_y} = 3.1$$

$$\left|\frac{\alpha_x}{\beta_x}\right| \ll \left|\frac{\alpha_y}{\beta_y}\right|$$



approach 1 is inaccurate for longitudinal fields of beams with high divergence the examples from our last meetings are high-energy examples

it was asked how the approaches behave for low energy, in particular for a gun-calculation

$\rightarrow$ comparison (2013) with the benchmark case from 2010

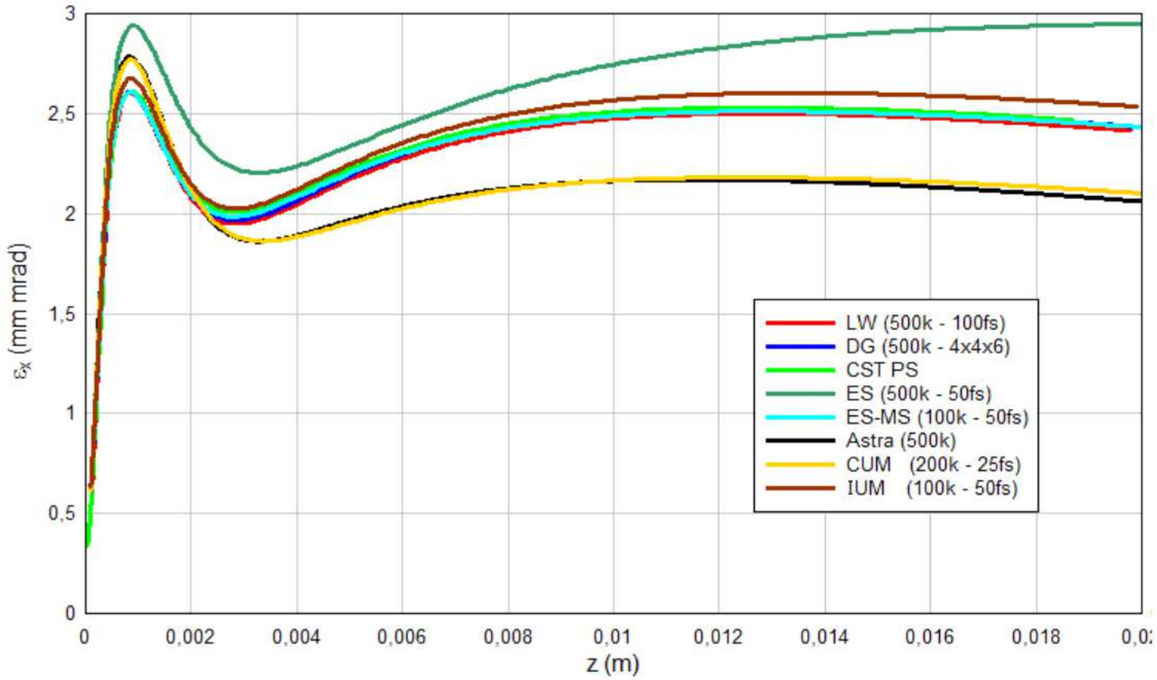
Emission Modeling for PITZ

TECHNISCHE UNIVERSITÄT DARMSTADT

Erion Gjonaj
TU Darmstadt, TEMF

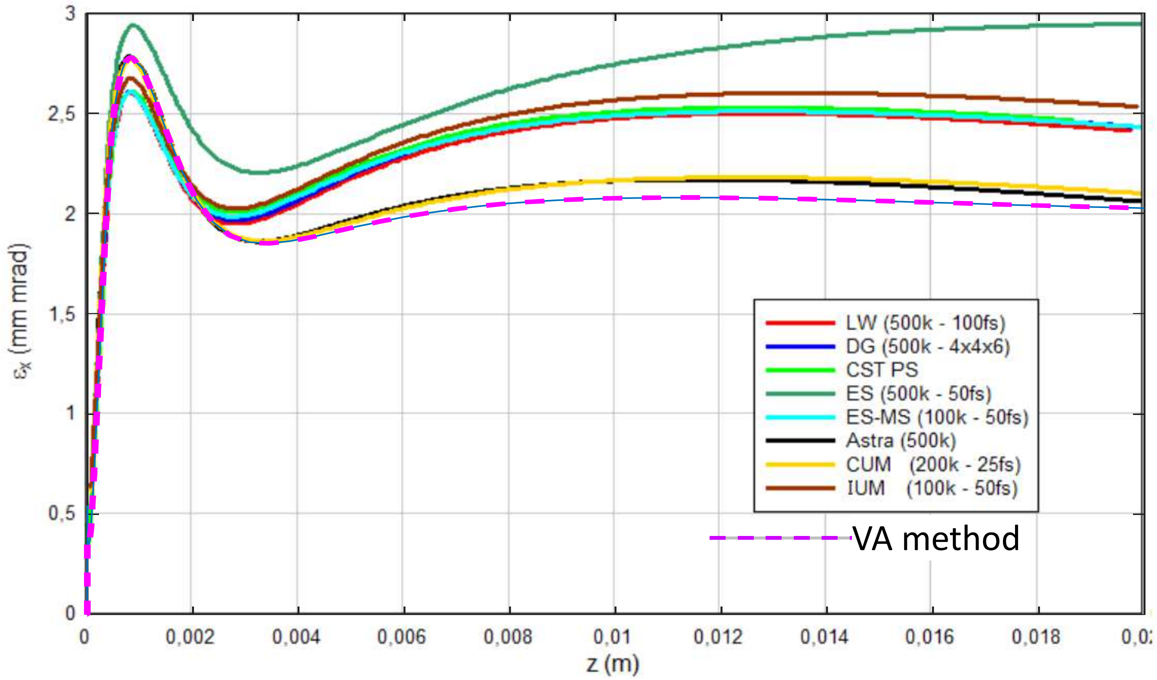
TEMF-DESY Collaboration Meeting
December 19th, 2013

tracking with different types of self fields



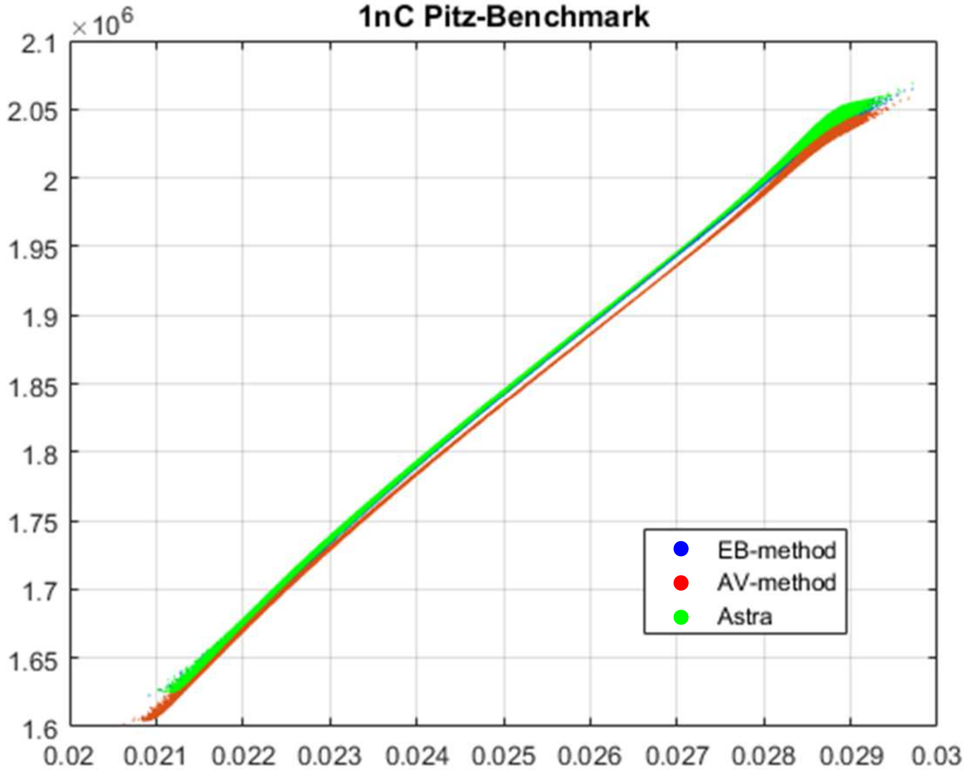
LW	Lienert Wiechert	full Maxwell
DG		
CST	CST particle studio	
ES	electro static approximation	static
ESMS	electro-static and magneto-static approximation	
Astra	collective uniform motion approach	uniform motion
CUM	collective uniform motion approach	
IUM	individual (per particle) uniform motion approach	

Poisson approach 2 = “VA method”



LW	Lienert Wiechert	full Maxwell
DG		
CST	CST particle studio	
ES	electro static approximation	static
ESMS	electro-static and magneto-static approximation	
Astra	collective uniform motion approach	uniform motion with E&B
CUM	collective uniform motion approach	
IUM	individual (per particle) uniform motion approach	
VA	collective uniform motion with scalar- & vector potential	

longitudinal phase space



my implementation of approach 1 is in good agreement with Astra
approach 2 vs. approach 1:
the bunch is longer,
energy is few keV lower,
energy spread is smaller

summary/conclusions

conventional Poisson approach “EB-method” does not conserve phase space

approach 2 needs canonical variables or time derivative of V

integration with canonical variables needs scalar & vector potential of external fields and their spatial derivatives

in principle: time derivative of V can be calculated, one has to solve two poisson problems:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial \tau^2}\right)V = -\frac{\rho\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)}{\varepsilon}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial \tau^2}\right)\frac{\partial}{\partial t}V = -\frac{\partial}{\partial t}\frac{\rho\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)}{\varepsilon} = \frac{\mathrm{div}\left(J\left(\mathbf{r}-\mathbf{v}_c\tau, t\right)\right)}{\varepsilon}$$

tracking at moderate and high energy behaves better with approach 2
(slice energy spread, divergent beams)

comparison with old gun-benchmark: small differences in projected emittance, methods are anyhow not too precise (comparison with “full Maxwell”); different bunch length/energy (unfortunately no “full Maxwell” data available)

Beam-Dynamics with 3D-Gun

Martin Dohlus

field-maps from Wolfgang Ackermann

what no man has seen before

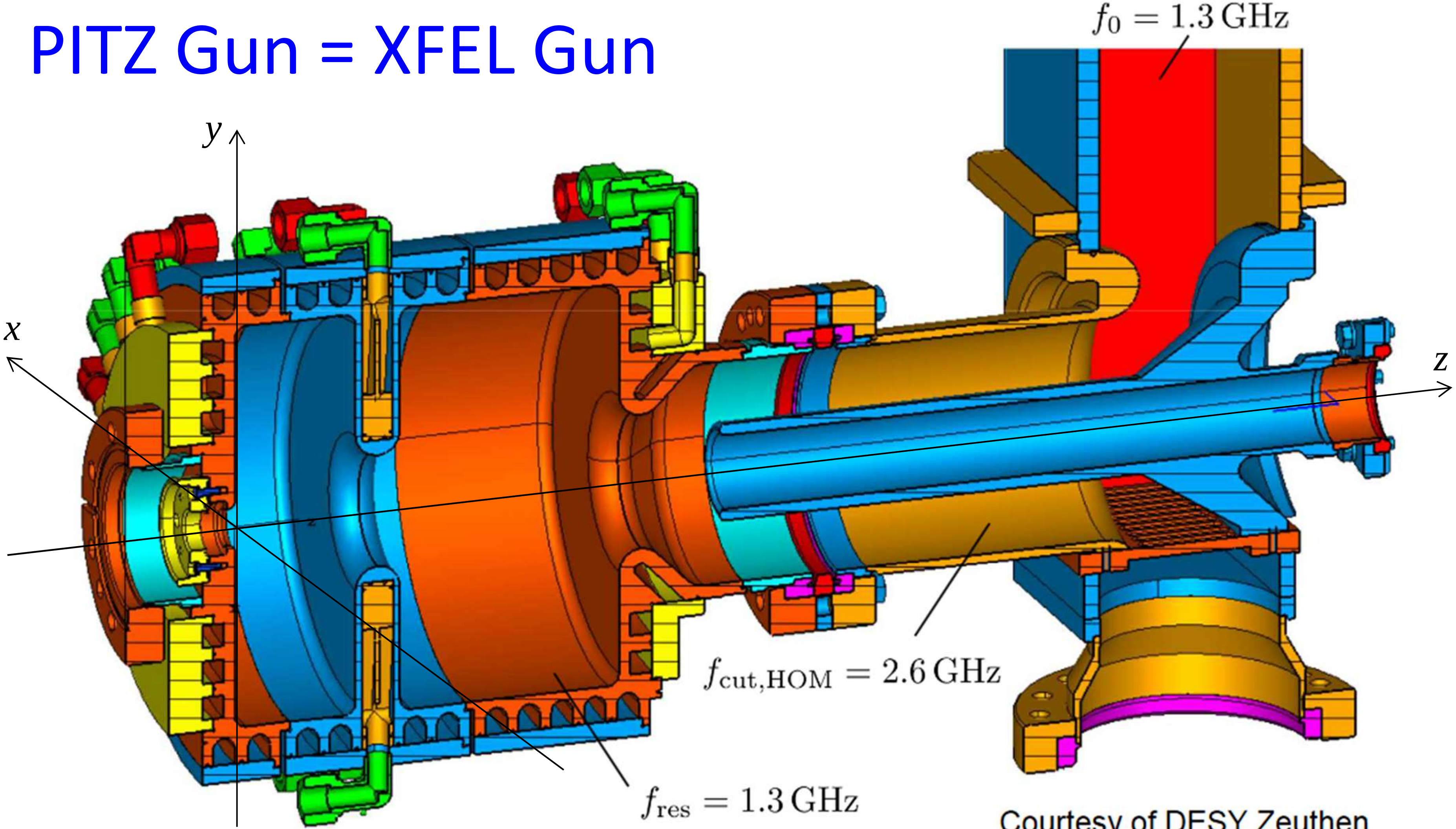
simulation with cathode distribution “BSA”, 250 pC

transverse phase-space

trajectory in ACC1

summary/conclusion

PITZ Gun = XFEL Gun

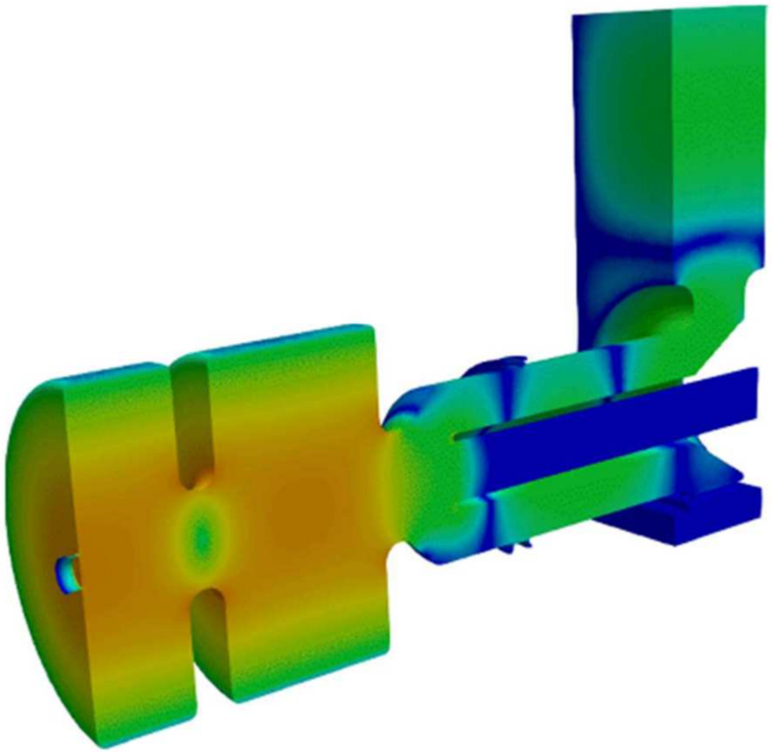


Courtesy of DESY Zeuthen

Field-Maps from Wolfgang Ackermann

- Electric Field Strength $\vec{E}(t) = \mathrm{Re}(\vec{E} \cdot e^{i\omega t})$

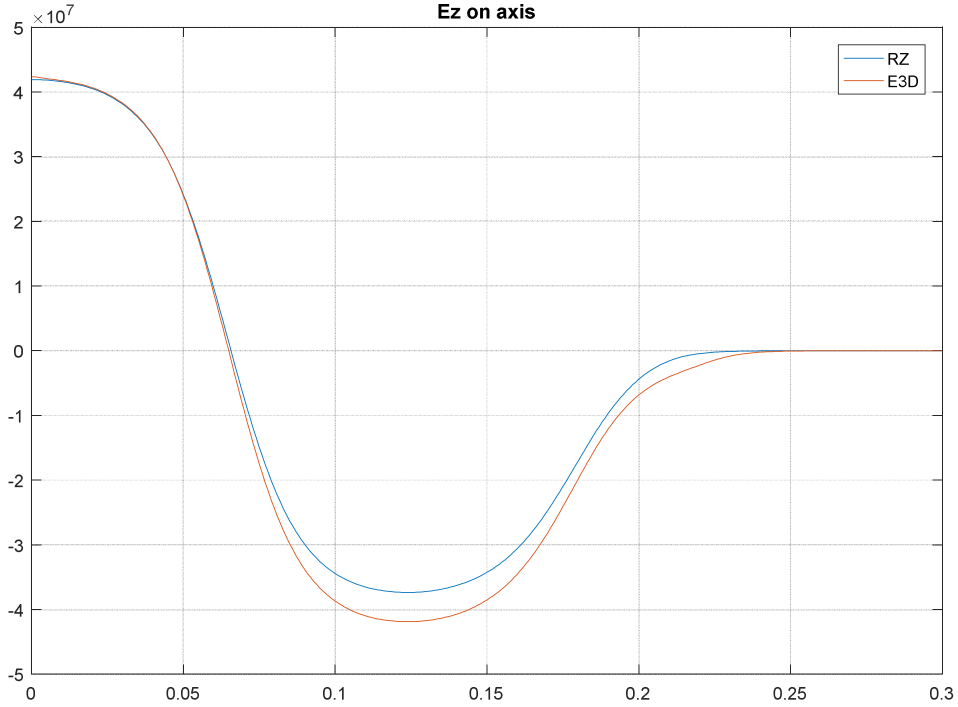
$\mathrm{Log}(|\vec{E}(t)|)$

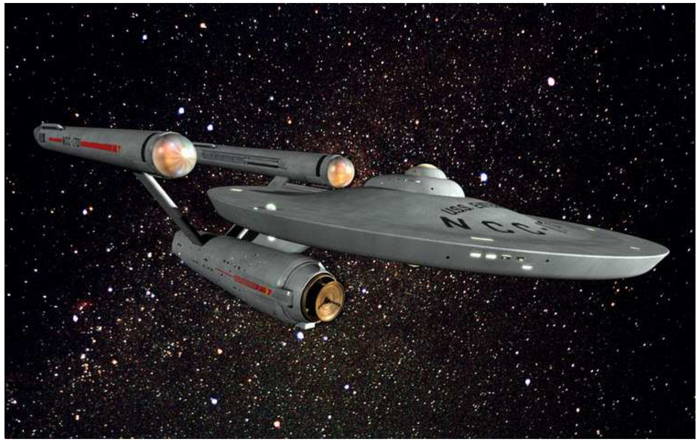


on the internet: http://www.desy.de/xfel-beam/s2e/codes.html

- Gun cavity field maps 2018 (ackermann@temf.tu-darmstadt.de & Martin.Dohlus@desy.de)
- TESLA field maps 2018 (ackermann@temf.tu-darmstadt.de & Martin.Dohlus@desy.de)
- TESLA field maps 2014 (ackermann@temf.tu-darmstadt.de & Martin.Dohlus@desy.de)
- 3rd harmonic field maps 2017 (ackermann@temf.tu-darmstadt.de & Martin.Dohlus@desy.de)
- 3rd harmonic field maps 2014 (gjonaj@temf.tu-darmstadt.de & Martin.Dohlus@desy.de)
- Steady-state resistive wake with oxid layer and roughness (Martin.Dohlus@desy.de & Igor.Zagorodnov@desy.de)

what no man has seen before



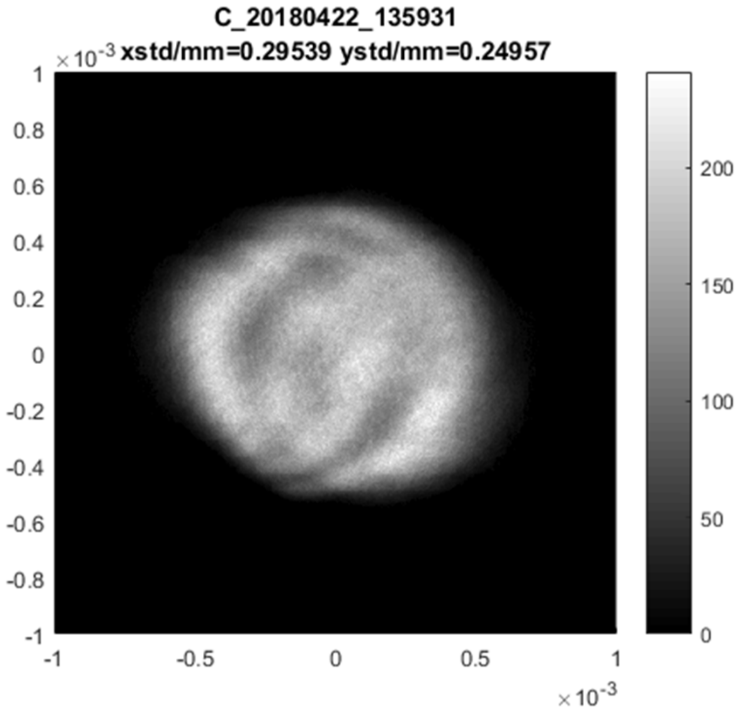


most of our simulations (see beam-dynamics homepage) have been done with an asymmetric RZ-field; therefore the optimal working point (amplitude, phase and solenoid) was different;

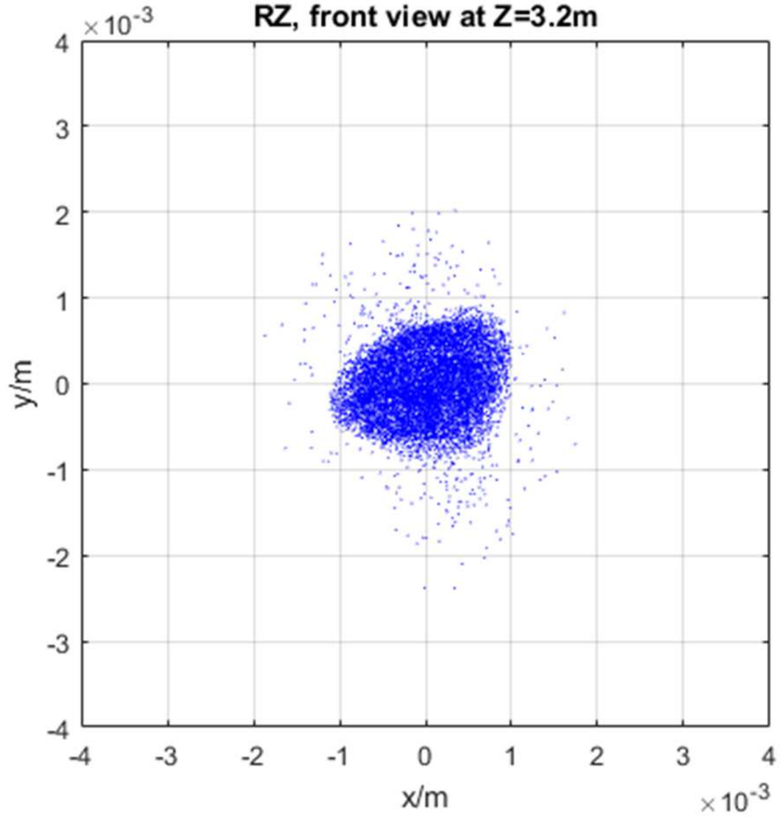
but the results (for same energy + optimal solenoid and phase) are quite close;
but the optimal solenoid strength is different …

simulation with cathode distribution “BSA”, 250 pC

max E = 60 MV/m
solenoid = 0.2050 T



fields with symmetry of revolution



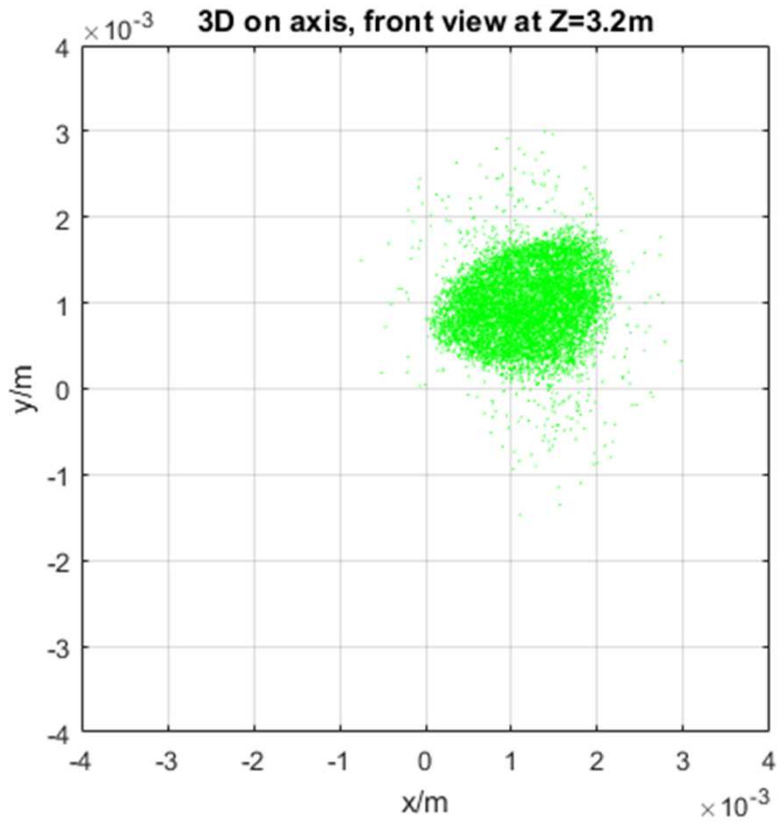
<x> ≈ 0

<y> ≈ 0

<x'> ≈ 0

<y'> ≈ 0

3d gun (with coupler), solenoid on axis



<x> ≈ 1.2 mm

<y> ≈ 1.0 mm

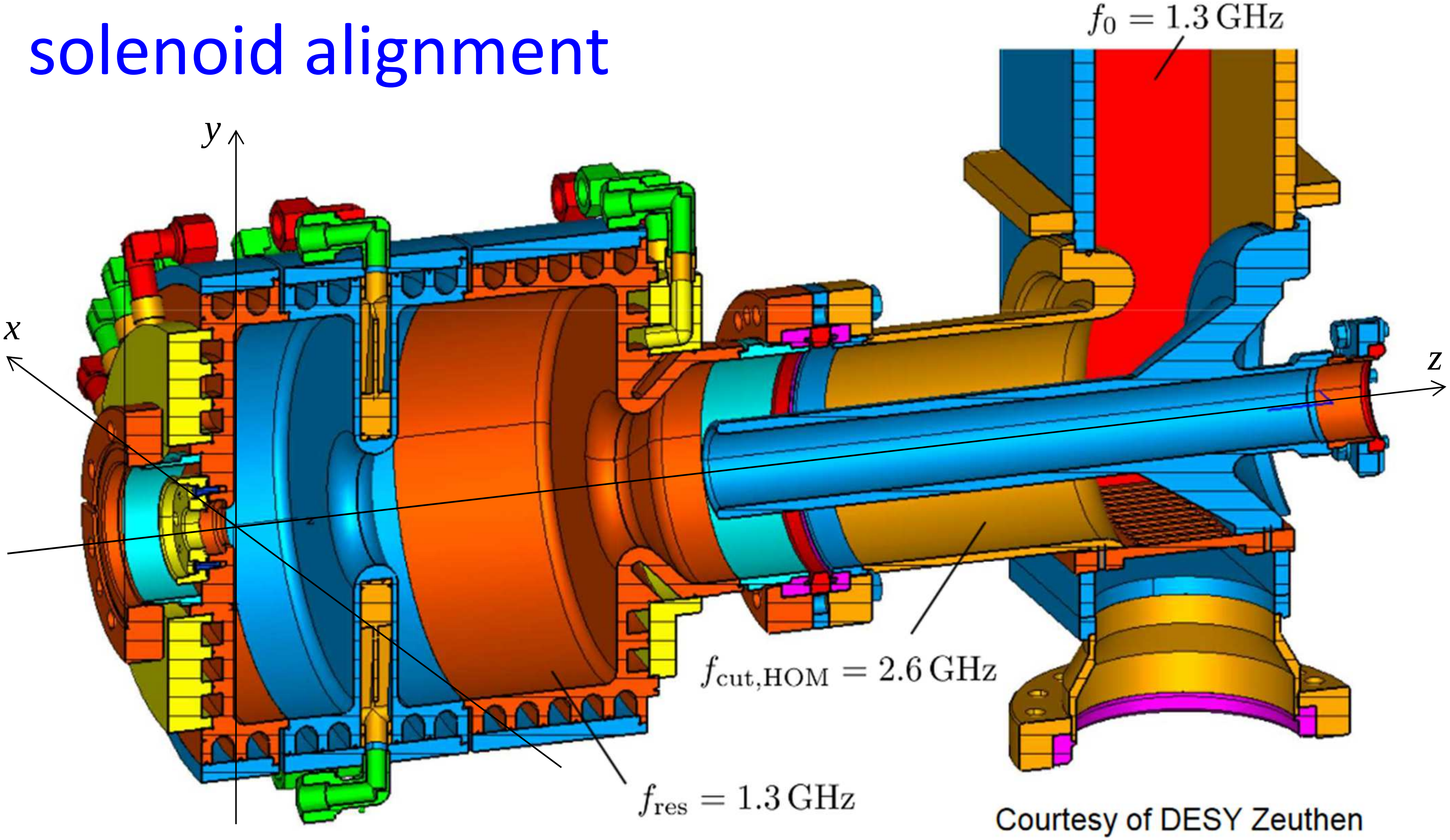
<x'> ≈ 0.39 mrad

<y'> ≈ 0.33 mrad

<x> ≈ <x'> (Z-0.2m)

<y> ≈ <y'> (Z-0.2m)

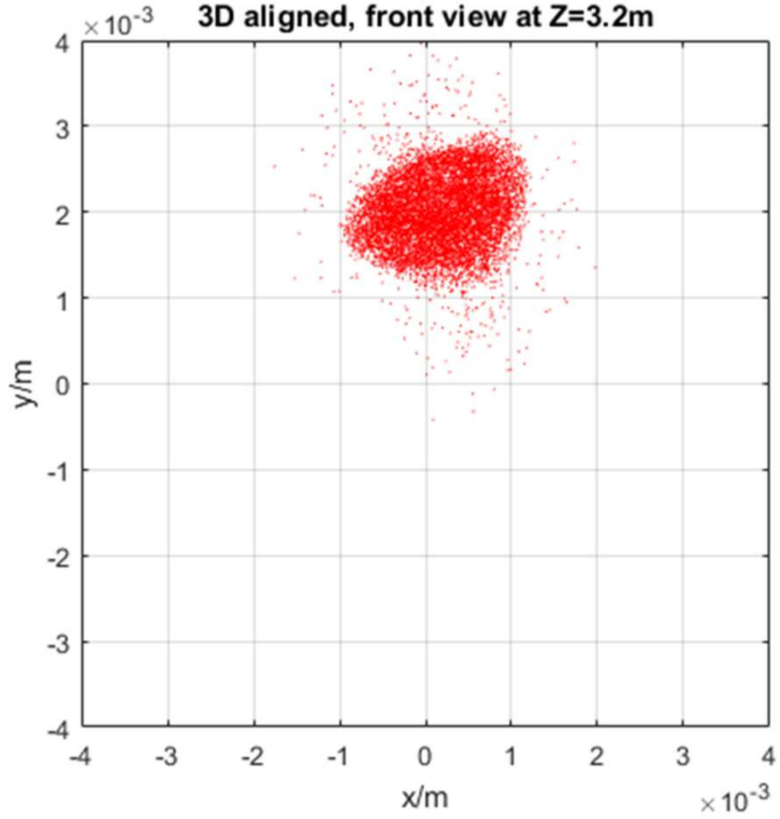
solenoid alignment



criterion for solenoid alignment: trajectory (offset at Z = 3.2 m) insensitive to solenoid current

asymmetry with respect to y=0 plane
$\rightarrow$ solenoid alignment needs y_shift =-0.1 mm and x_rot =-0.5 mrad

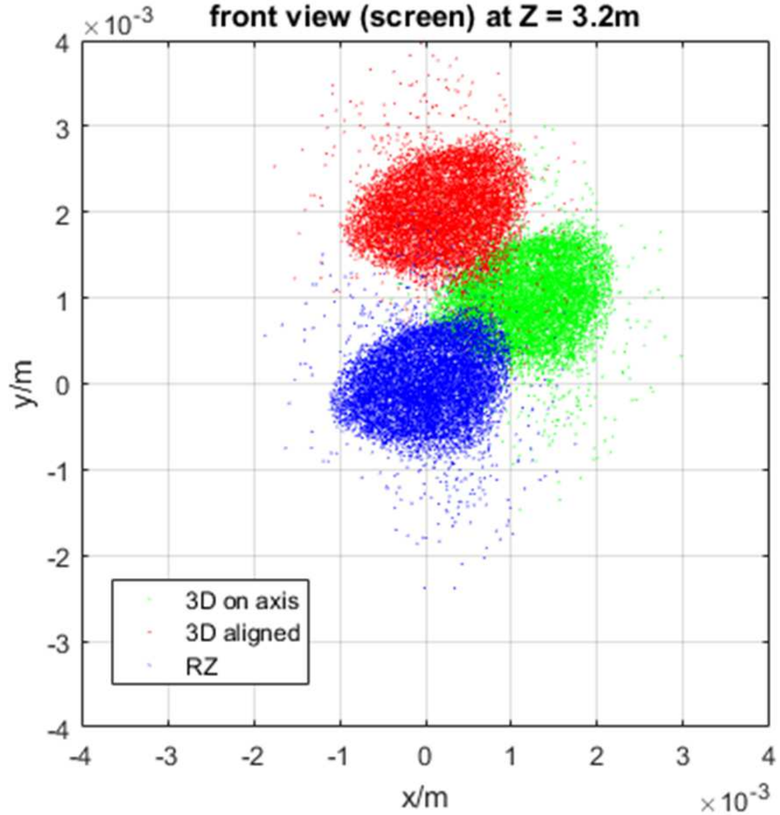
3d gun (with coupler), solenoid aligned



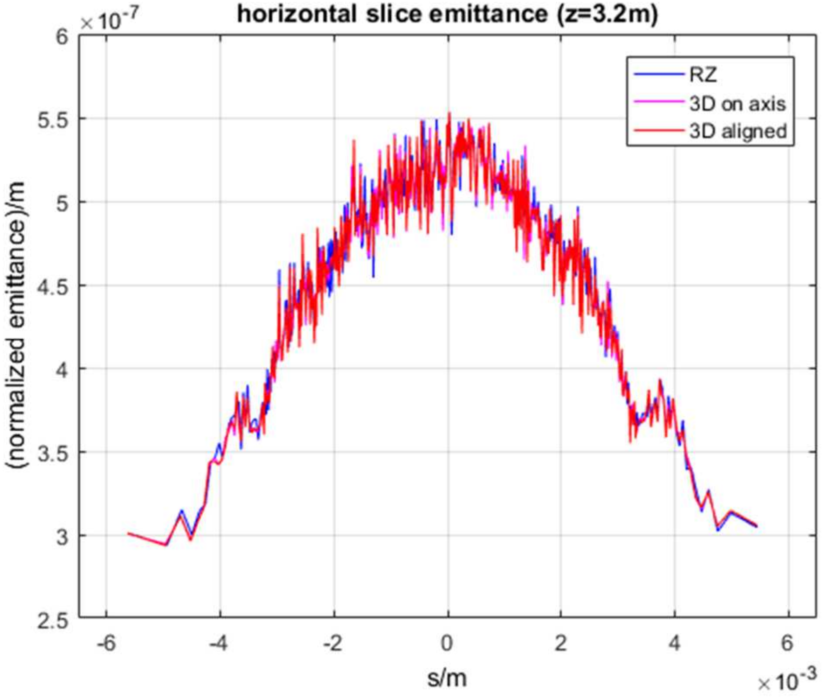
<x> ≈ 0 (0.2 mm)
<y> ≈ 2.0 mm
<x’> ≈ 0 (0.06 mrad)
<y’> ≈ 0.67 mrad

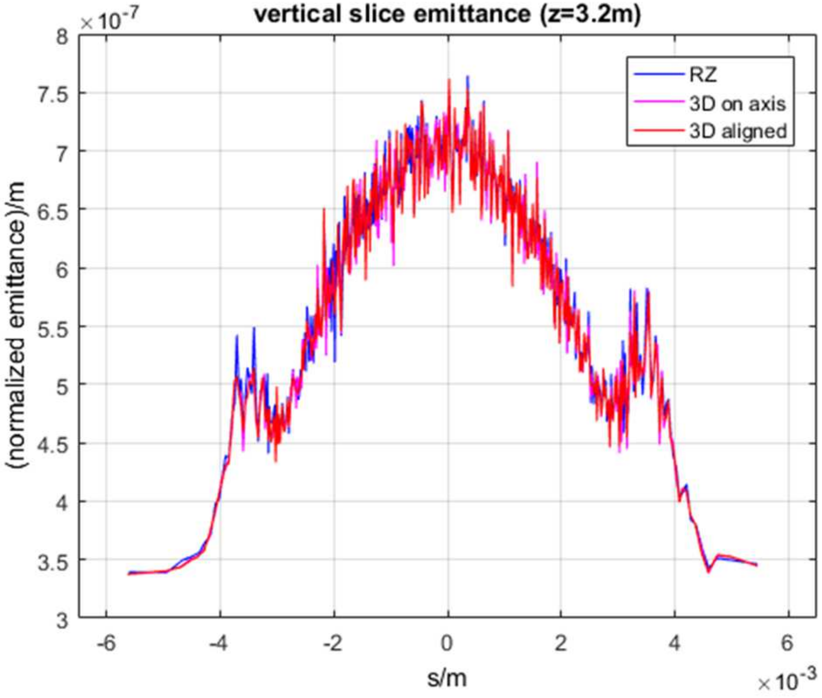
<x> ≈ <x’> (Z-0.2m)
<y> ≈ <y’> (Z-0.2m)

all together



transverse phase-space





C_20180422 (BSA)
Z = 3.2 m
B_{sol} = 0.2050 T $\varphi = \varphi_0 - 2.0$ deg
Q = 250 pC I_{peak} = 14.20 A

RZ
α_x = 0.641 α_y = 0.533
β_x = 6.64 m β_y = 2.91 m
$\varepsilon_{x,p}$ = 1.03 µm $\varepsilon_{y,p}$ = 1.19 µm
$\varepsilon_{x,s}$ = 0.53 µm $\varepsilon_{y,s}$ = 0.70 µm

3D solenoid on axis
α_x = 0.665 α_y = 0.573
β_x = 6.88 m β_y = 3.14 m
$\varepsilon_{x,p}$ = 1.05 µm $\varepsilon_{y,p}$ = 1.20 µm
$\varepsilon_{x,s}$ = 0.53 µm $\varepsilon_{y,s}$ = 0.70 µm

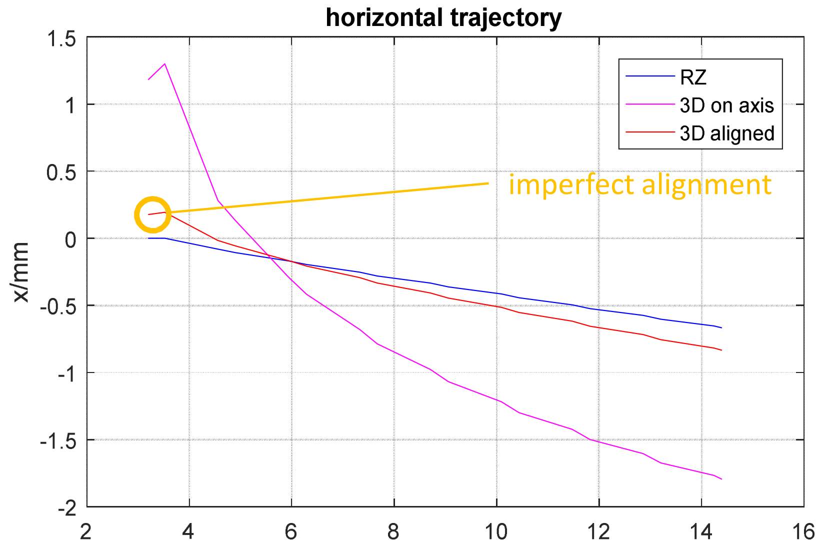
3D solenoid aligned
α_x = 0.674 α_y = 0.577
β_x = 7.00 m β_y = 3.21 m
$\varepsilon_{x,p}$ = 1.05 µm $\varepsilon_{y,p}$ = 1.20 µm
$\varepsilon_{x,s}$ = 0.53 µm $\varepsilon_{y,s}$ = 0.70 µm

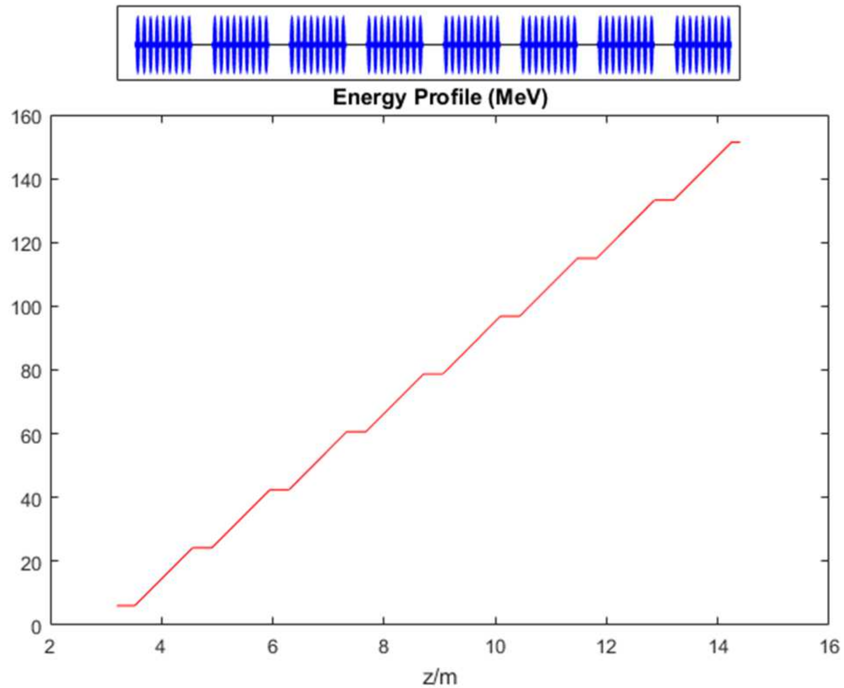
projected/slice emittance

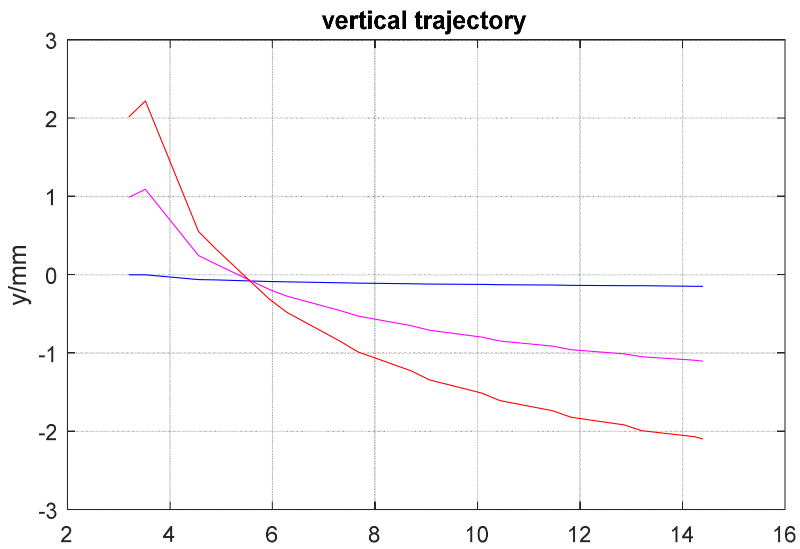
trajectory in ACC1

effect of Tesla coupler kicks and rf-focussing

all correctors off!







summary/conclusion

3D-field-maps for gun with coupler are available; there is also a rz-field-map, derived from the 3D map

do not use old rz-file (Feng's simulations)

transverse slice properties are not affected by gun asymmetry

effect of gun asymmetry is equivalent to a collective kick
further kicks by Tesla couplers in modules
this is compensated by correctors, but (due to limited space)
there are only few correctors between gun and ACC1

what is the purpose of solenoid alignment?
sensitivity of trajectory to solenoid strength $\rightarrow$ yes
optimal trajectory $\rightarrow$ perhaps no