Effect of Collimator-Wake in BC-Bypass

Martin Dohlus and Igor Zagorodnov

BC-chicane with collimator in bypass

vertical correlation and emittance

estimation for collimator wake

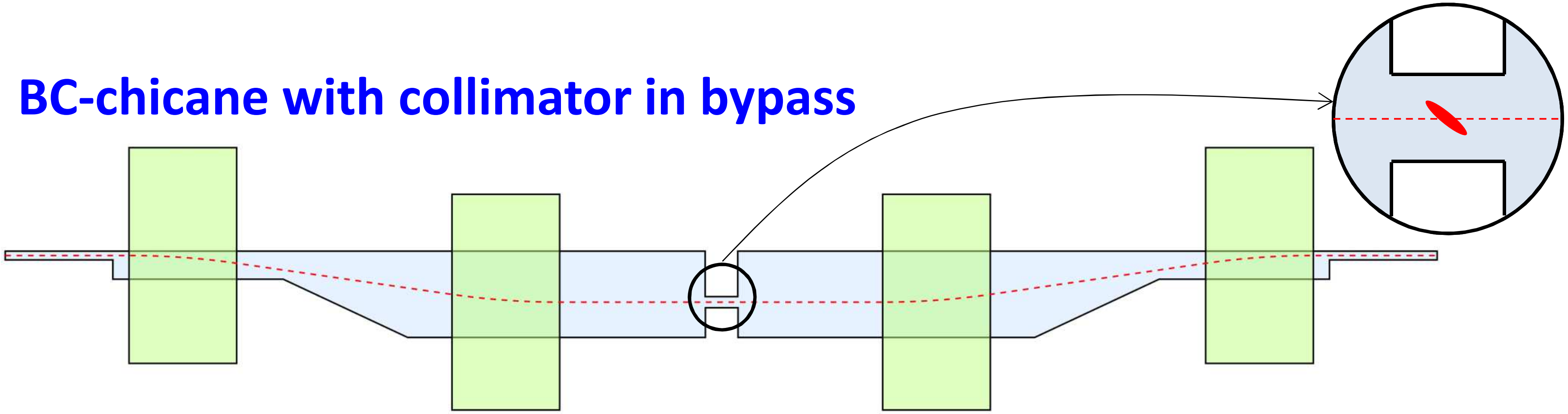
XFEL bunch compressors

relative emittance growth

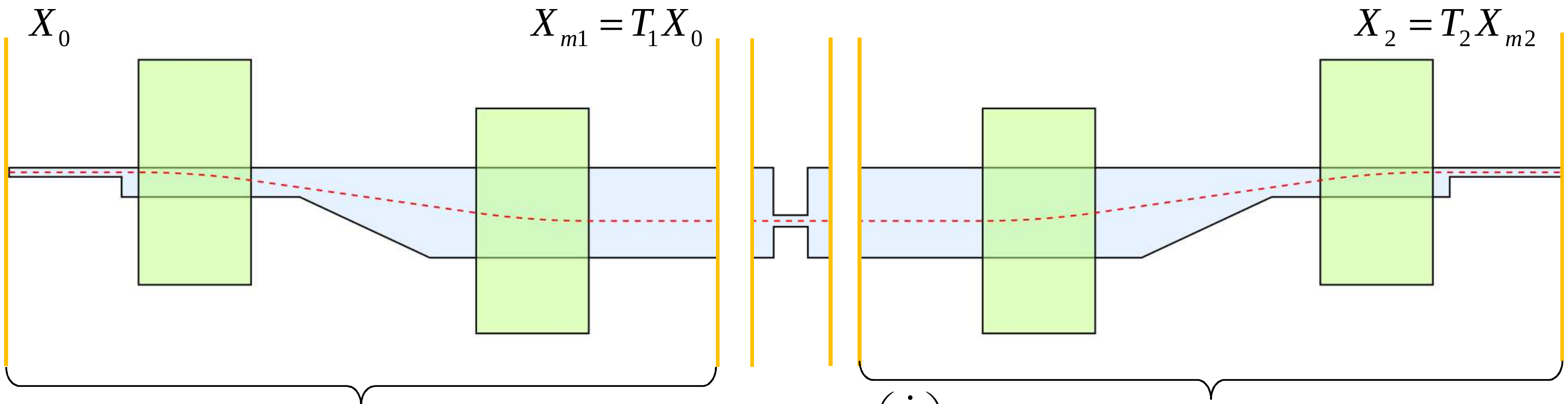
comparison with free-space-steady-state CSR

summary/conclusion

BC-chicane with collimator in bypass



→ BC-chicane with (resistive) wake in bypass



$$T_1 = \begin{pmatrix} 1 & L & 0 & r_{36} \\ 0 & 1 & 0 & 0 \\ 0 & r_{36} & 1 & r_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$X_{m2} = X_{m1} + \begin{pmatrix} \vdots \\ w \end{pmatrix}$$

$$w = q W^{(\sigma)}(z) \frac{e}{\mathcal{E}}$$

$$T_2 = \begin{pmatrix} 1 & L & 0 & -r_{36} \\ 0 & 1 & 0 & 0 \\ 0 & -r_{36} & 1 & r_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

initial particles with linear chirp, without uncorrelated energy spread $X_0 = \begin{pmatrix} y_0 \\ y_0' \\ z_0 \\ c_h z_0 \end{pmatrix}$

initial current $i(z_0) = c\lambda(z_0)$

transport without wake $T = T_2 T_1 = \begin{pmatrix} 1 & 2L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & R_{56} = 2r_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $TX_0 = \begin{pmatrix} y_0 + 2Ly_0' \\ y_0' \\ \boxed{(1+c_h R_{56})} z_0 \\ c_h z_0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_1' \\ z_1 \\ c_h z_0 \end{pmatrix}$

compression factor $C_t = \dfrac{1}{\boxed{1+R_{56}c_h}}$

compression to middle of chicane $C_m = \dfrac{1}{1+r_{56}c_h} = \dfrac{2}{1+1/C_t} < 2$

longitudinal wake $w = RC_m i(z_0)\dfrac{e}{\mathcal{E}}$

transport with wake $X_1 = T_2\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix} + T_1 \begin{pmatrix} y_0 \\ y_0' \\ z_0 \\ c_h z_0 \end{pmatrix}\right) = \begin{pmatrix} y_1 \\ y_1' \\ z_1 \\ c_h z_0 \end{pmatrix} + \begin{pmatrix} r_{36} \\ 0 \\ r_{56} \\ 1 \end{pmatrix} w(z_0)$

vertical correlation and emittance

at entrance $$C_0 = \begin{pmatrix} \langle y_0 y_0 \rangle & \langle y_0 y_0' \rangle \\ \langle y_0 y_0' \rangle & \langle y_0' y_0' \rangle \end{pmatrix} \qquad \varepsilon_{y0} = \sqrt{|C_0|}$$

at exit, without wake $$C_1 = \begin{pmatrix} \langle y_1 y_1 \rangle & \langle y_1 y_1' \rangle \\ \langle y_1 y_1' \rangle & \langle y_1' y_1' \rangle \end{pmatrix} = \varepsilon_{y1} \begin{pmatrix} \beta_{y1} & -\alpha_{y1} \\ -\alpha_{y1} & \gamma_{y1} \end{pmatrix} \qquad \varepsilon_{y1} = \sqrt{|C_1|} = \sqrt{|C_0|}$$

effect of wake $$\tilde{y}_1 = y_1 r_{36} w$$

$$\tilde{C}_1 = \begin{pmatrix} \langle \tilde{y}_1 \tilde{y}_1 \rangle & \langle \tilde{y}_1 y_1' \rangle \\ \langle \tilde{y}_1 y_1' \rangle & \langle y_1' y_1' \rangle \end{pmatrix} = C_1 + r_{36}^2 \begin{pmatrix} \langle ww \rangle & 0 \\ 0 & 0 \end{pmatrix} \qquad \text{if} \quad \begin{matrix} \langle y_1 w \rangle = 0 \\ \langle y_1' w \rangle = 0 \end{matrix}$$

$$\tilde{\varepsilon}_{y1} = \sqrt{|\tilde{C}_1|} = \sqrt{|C_1| + r_{36}^2 \langle ww \rangle \langle y_1' y_1' \rangle} = \sqrt{|C_1| + r_{36}^2 \langle ww \rangle \langle y_1' y_1' \rangle} = \varepsilon_{y1} \sqrt{1 + r_{36}^2 \frac{\langle ww \rangle}{\langle y_1 y_1 \rangle} \boxed{\beta_{y1} \gamma_{y1}}}$$

typical BC optics with waist at exit $\rightarrow$ $\alpha_{y1} \approx 0 \rightarrow \beta_{y1} \gamma_{y1} \approx 1$

rms wake

$$w = qW^{(C_m\sigma)}(C_m z_0)\frac{e}{\mathcal{E}}$$

$$\sqrt{\langle ww\rangle} = w_{rms} = qW_{rms}^{(C_m\sigma)}\frac{e}{\mathcal{E}}$$

$$\tilde{\varepsilon}_{y1} = \varepsilon_{y1}\sqrt{1+\left(r_{36}\frac{w_{rms}}{y_{1,rms}}\right)^2\beta_{y1}\gamma_{y1}}$$

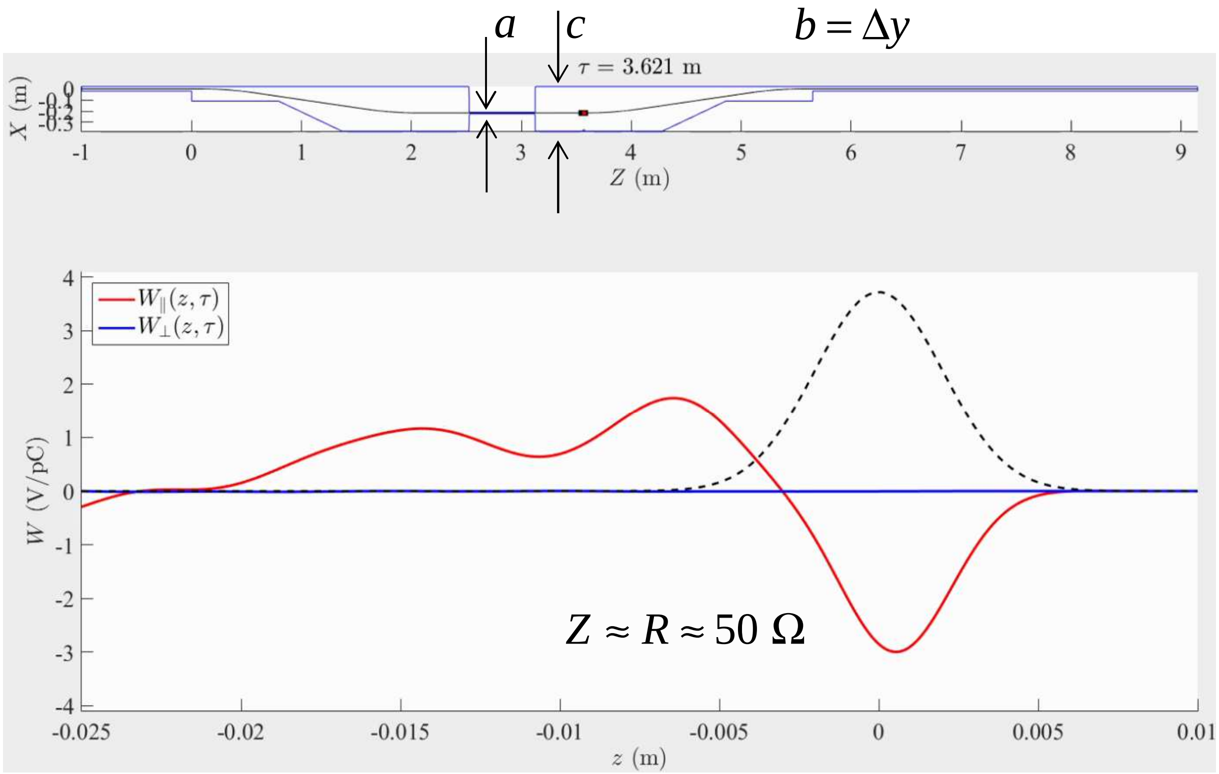
$W^{(\sigma)}(z)$	is the longitudinal wake potential for a bunch with given shape, f.i. a Gaussian bunch with rms length σ
$q,\mathcal{E}$	bunch charge and reference energy
$\sigma, C_m\sigma$	bunch length before chicane/in bypass

$$W_{av} = \int W\lambda dz \qquad W_{rms} = \sqrt{\int (W - W_{av})^2 \lambda dz}$$

Dave's Wakes

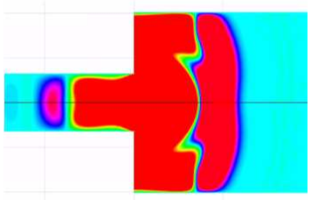
collimator in BC0

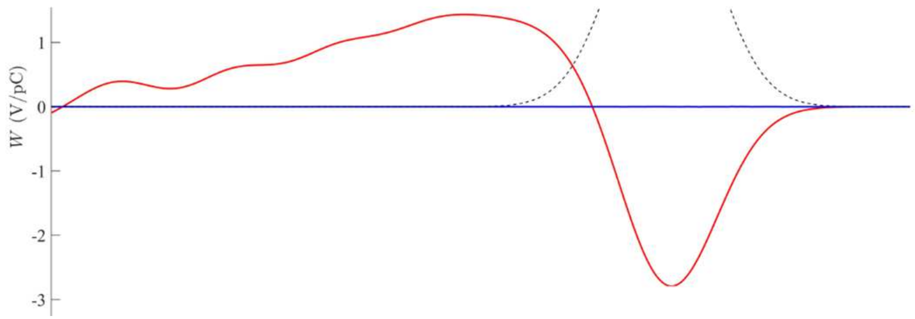
a = 1.3 cm
b = 4 cm
c= 40 cm
σ= 2mm



collimator only

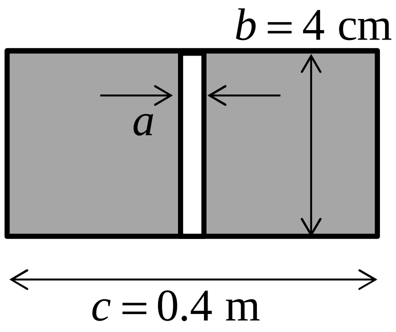
a = 1.3 cm
b = c= 4 cm
σ= 2mm

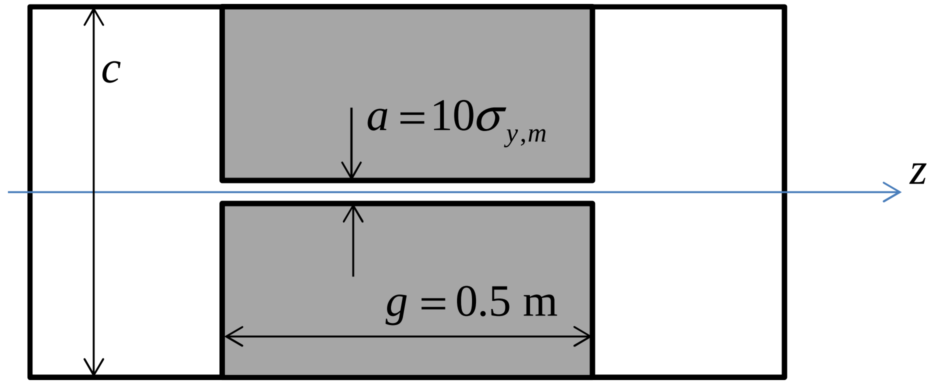




wakes are similar and (in very rough estimation) resistive

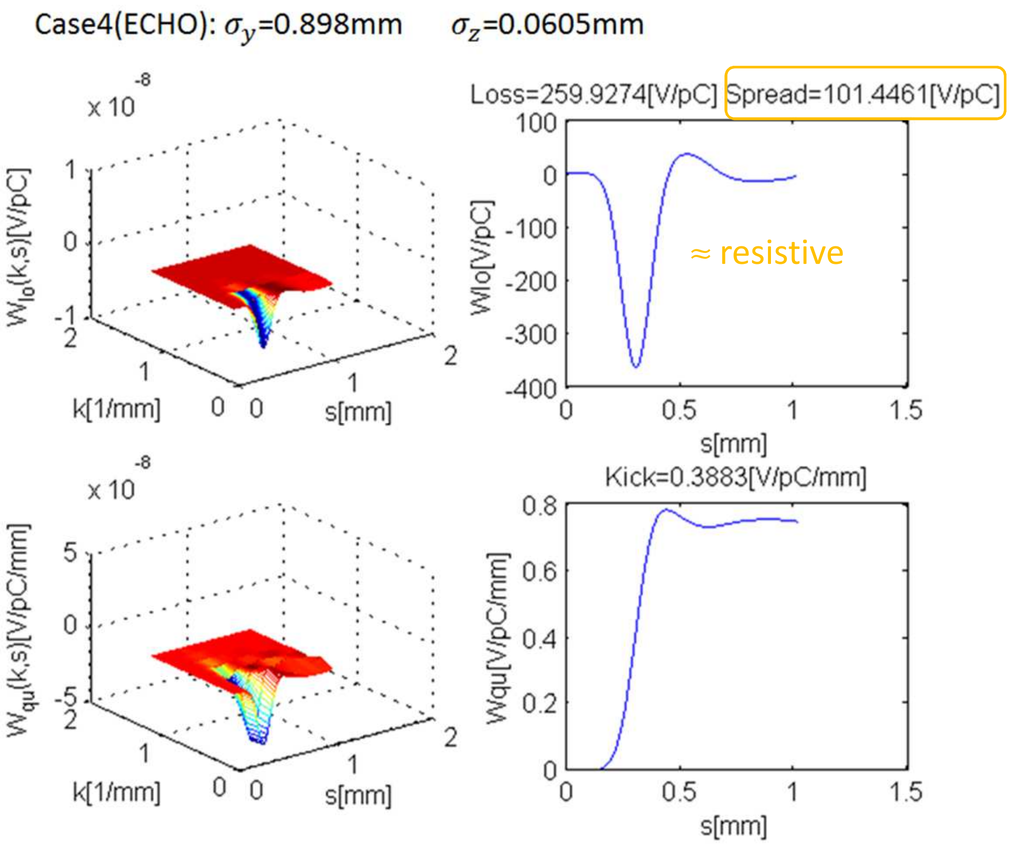
Igor's Wakes



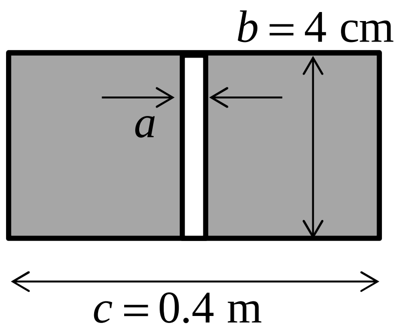


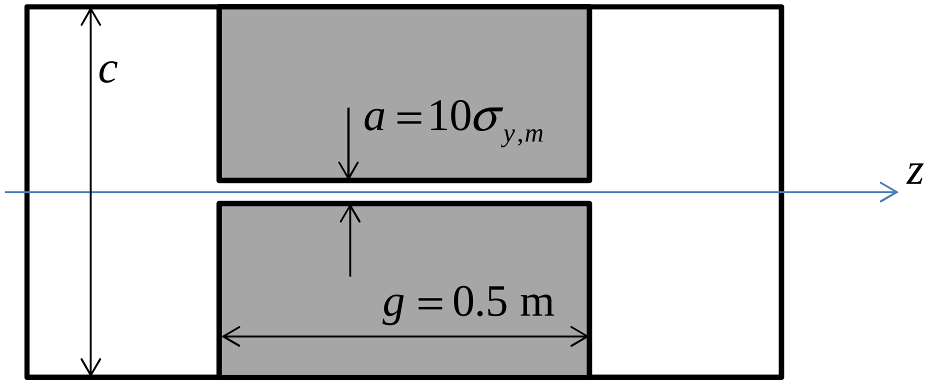
calculation with Echo

case = 1nC, BC2



Igor's Wakes





optical model

$$Z = \frac{Z_0}{2\pi}\left(2\ln\left(\frac{c}{a}\right) + \sum_{n=1}^{\infty}\frac{4\left(\cos\left(n\pi\right)-1\right)}{\left(1+\exp\left(n\pi b/c\right)\right)n} - \sum_{n=1}^{\infty}\frac{4\left(\cos\left(n\pi\right)-1\right)}{\left(1+\exp\left(n\pi b/a\right)\right)n}\right)$$

$$k_{av} = \frac{c}{2\sqrt{\pi}\sigma_z}Z$$

$$k_{rms} = k_{av}\sqrt{\frac{2}{\sqrt{3}}-1}$$

case = 1nC, BC2 $k_{av} = 250\ \text{V/pC}$ echo: 260

$k_{rms} = 100\ \text{V/pC}$ 101

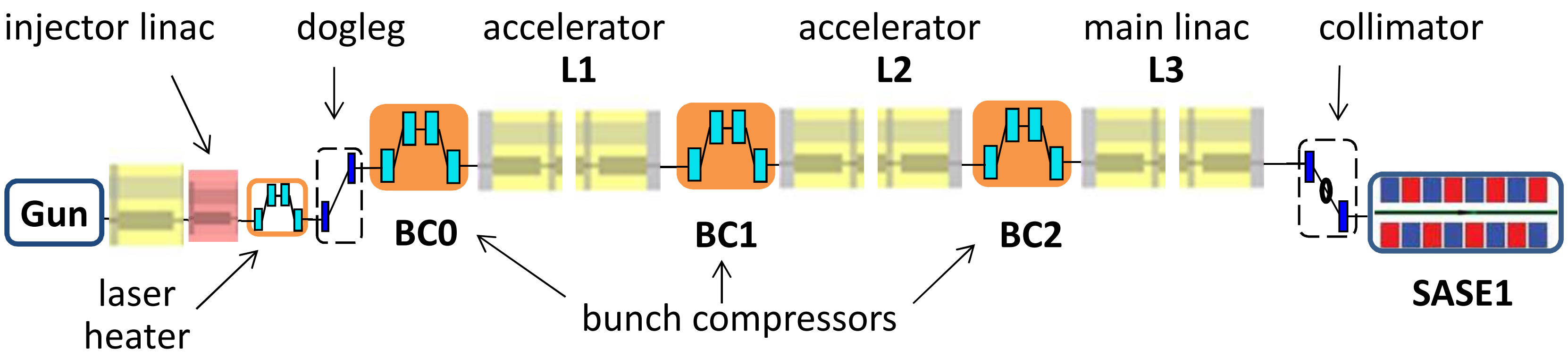
XFEL Bunch Compressors

BC0 $R_{56} = 39$ mm $r_{36} = 182$ mm $\mathcal{E} = 130$ MeV	$\phi = 4.95$ deg $L = 2.78$ m $\alpha_{y1} = 0.49$ $\beta_{y1} = 4.85$ m
BC1 $R_{56} = 44$ mm $r_{36} = 451$ mm $\mathcal{E} = 700$ MeV	$\phi = 2.07$ deg $L = 10.28$ m $\alpha_{y1} = -0.16$ $\beta_{y1} = 8.72$ m
BC2 $R_{56} = 31$ mm $r_{36} = 374$ mm $\mathcal{E} = 2.4$ GeV	$\phi = 1.72$ deg $L = 10.27$ m $\alpha_{y1} = -0.02$ $\beta_{y1} = 5.00$ m

scaling $r_{36} \sim \phi$

$r_{56} \sim \phi^2$

$$\gamma\varepsilon_{y0} \approx 1\mu\text{m}\sqrt{\frac{Q}{1\text{ nC}}}$$



	250 pC, 14 A	500 pC, 22 A	1 nC, 44 A
BC0 $R_{56} = 39$ mm $r_{36} = 182$ mm $\mathcal{E} = 130$ MeV	20.3 A $\rightarrow$ 49 A $\sigma_{y,m} = 4.06$ mm $\sigma_{z,m} = 1.04$ mm	31.9 A $\rightarrow$ 77 A $\sigma_{y,m} = 5.16$ mm $\sigma_{z,m} = 1.33$ mm	63.8 A $\rightarrow$ 154 A $\sigma_{y,m} = 5.16$ mm $\sigma_{z,m} = 1.33$ mm
BC1 $R_{56} = 44$ mm $r_{36} = 451$ mm $\mathcal{E} = 700$ MeV	49 A $\rightarrow$ 392 A $\sigma_{y,m} = 5.44$ mm $\sigma_{z,m} = 0.343$ mm	77 A $\rightarrow$ 616 A $\sigma_{y,m} = 6.93$ mm $\sigma_{z,m} = 0.437$ mm	154 A $\rightarrow$ 1.23 kA $\sigma_{y,m} = 6.93$ mm $\sigma_{z,m} = 0.437$ mm
BC2 $R_{56} = 31$ mm $r_{36} = 374$ mm $\mathcal{E} = 2.4$ GeV	392 A $\rightarrow$ 5 kA $\sigma_{y,m} = 0.860$ mm $\sigma_{z,m} = 41.2$ μm	616 A $\rightarrow$ 5 kA $\sigma_{y,m} = 1.04$ mm $\sigma_{z,m} = 54.6$ μm	1.23 A $\rightarrow$ 5 kA $\sigma_{y,m} = 0.898$ mm $\sigma_{z,m} = 60.5$ μm

$$\sigma_{y,m} = r_{36}\left|c_h\right|\sigma_{z,0} = \left(1 - \frac{1}{C_t}\right)\frac{r_{36}}{R_{56}}\sigma_{z,0} = \left(1 - \frac{1}{C_t}\right)\frac{r_{36}}{R_{56}}\frac{cQ}{\sqrt{2\pi}\hat{I}}$$

$$\sigma_{z,m} = C_m\sigma_{z,0} = \frac{2}{1 + 1/C_t}\sigma_{z,0} = \frac{2}{1 + 1/C_t}\frac{cQ}{\sqrt{2\pi}\hat{I}}$$

Relative Emittance Growth

	250 pC, 14 A	500 pC, 22 A	1 nC, 44 A
BC0 $R_{56} = 39$ mm $r_{36} = 182$ mm $\mathcal{E} = 130$ MeV	20.3 A $\rightarrow$ 49 A $W_{rms}^{(opt)} = 0.61$ V/pC	31.9 A $\rightarrow$ 77 A $W_{rms}^{(opt)} = 0.21$ V/pC $W_{rms}^{(echo)} = 0.23$ V/pC	63.8 A $\rightarrow$ 154 A $W_{rms}^{(opt)} = 0.21$ V/pC $W_{rms}^{(echo)} = 0.23$ V/pC
BC1 $R_{56} = 44$ mm $r_{36} = 451$ mm $\mathcal{E} = 700$ MeV	49 A $\rightarrow$ 392 A $W_{rms}^{(opt)} = 0.65$ V/pC	77 A $\rightarrow$ 616 A $W_{rms}^{(opt)} = 0.14$ V/pC $W_{rms}^{(echo)} = 0.16$ V/pC	154 A $\rightarrow$ 1.23 kA $W_{rms}^{(opt)} = 0.14$ V/pC $W_{rms}^{(echo)} = 0.16$ V/pC
BC2 $R_{56} = 31$ mm $r_{36} = 374$ mm $\mathcal{E} = 2.4$ GeV	392 A $\rightarrow$ 5 kA $W_{rms}^{(opt)} = 151$ V/pC	616 A $\rightarrow$ 5 kA $W_{rms}^{(opt)} = 94$ V/pC $W_{rms}^{(echo)} = 100$ V/pC	1.23 A $\rightarrow$ 5 kA $W_{rms}^{(opt)} = 101$ V/pC $W_{rms}^{(echo)} = 100$ V/pC

$$\left.\begin{aligned} qW_{rms} &= 100 \text{ kV} \\ \mathcal{E}/e &= 2.4 \text{ GV} \end{aligned}\right\} \rightarrow w_{rms} = 4.2\cdot 10^{-5}$$

$$y_{1,rms} = \sqrt{\varepsilon_{y1}\beta_{y1}} = 3.3\cdot 10^{-5}\text{m}$$

$$r_{36} = 0.374 \text{ m}$$

$$\beta_{y1}\gamma_{y1} = 1+\alpha_{y1}^2 = 1.00$$

$$\frac{\tilde{\varepsilon}_{y1}}{\varepsilon_{y1}} = \sqrt{1+\left(r_{36}\frac{w_{rms}}{y_{1,rms}}\right)^2 \beta_{y1}\gamma_{y1}} = 1.085 \quad \text{(1 nC)}$$

$$500 \text{ pC} \rightarrow 1.040$$

$$250 \text{ pC} \rightarrow 1.032$$

Comparison with Free-Space-Steady-State CSR

	250 pC, 14 A	500 pC, 22 A	1 nC, 44 A
BC0 $R_{56} = 39\ \text{mm}$ $r_{36} = 182\ \text{mm}$ $\mathcal{E} = 130\ \text{MeV}$	20.3 A $\rightarrow$ 49 A $qW_{rms} = 153\ \text{V}$ $V_{rms}^{CSR} = 4.1\ \text{kV}$	31.9 A $\rightarrow$ 77 A $qW_{rms} = 115\ \text{V}$ $V_{rms}^{CSR} = 6.0\ \text{kV}$	63.8 A $\rightarrow$ 154 A $qW_{rms} = 210\ \text{V}$ $V_{rms}^{CSR} = 12\ \text{kV}$
BC1 $R_{56} = 44\ \text{mm}$ $r_{36} = 451\ \text{mm}$ $\mathcal{E} = 700\ \text{MeV}$	49 A $\rightarrow$ 392 A $qW_{rms} = 238\ \text{V}$ $V_{rms}^{CSR} = 37\ \text{kV}$	77 A $\rightarrow$ 616 A $qW_{rms} = 80\ \text{V}$ $V_{rms}^{CSR} = 54\ \text{kV}$	154 A $\rightarrow$ 1.23 kA $qW_{rms} = 160\ \text{V}$ $V_{rms}^{CSR} = 107\ \text{kV}$
BC2 $R_{56} = 31\ \text{mm}$ $r_{36} = 374\ \text{mm}$ $\mathcal{E} = 2.4\ \text{GeV}$	392 A $\rightarrow$ 5 kA $qW_{rms} = 38\ \text{kV}$ $V_{rms}^{CSR} = 980\ \text{kV}$	616 A $\rightarrow$ 5 kA $qW_{rms} = 50\ \text{kV}$ $V_{rms}^{CSR} = 780\ \text{kV}$	1.23 A $\rightarrow$ 5 kA $qW_{rms} = 100\ \text{kV}$ $V_{rms}^{CSR} = 620\ \text{kV}$

$$E^{CSR} \approx \frac{1}{\sqrt[3]{3}\,(2\pi)^{(3/2)}} \frac{q}{\varepsilon} \sqrt[3]{\frac{1}{R_0^2 \sigma^4}} H(z/\sigma)$$

with

$$H(x) = \sqrt{2\pi} \int_0^\infty \frac{g'(x+\xi)}{\sqrt[3]{\xi}} d\xi$$

$$V_{rms}^{CSR} \approx \frac{L_m}{\sqrt[3]{3}\,(2\pi)^{(3/2)}} \frac{q}{\varepsilon} \sqrt[3]{\frac{1}{R_0^2 \sigma^4}} H_{rms}$$

$H_{rms} \approx 1.115$

$L_m = 0.5\ \text{m}$ magnet length

R_0 bending radius

σ rms length after compression

Summary/Conclusion

simple wake model for collimator: discrete, offset-independent longitudinal kick
$\rightarrow$ analytic equation for transverse emittance growth

growth depends on $r_{36} \dfrac{w_{rms}}{y_{1,rms}}$

estimation of the rms-wake with Echo and optical model

the particle beam in BC0 and BC1 is very wide, therefore the collimator opening is wide and wake effects are nearly negligible

the growth of projected emittance in BC2 is 3-9 %

free space CSR fields for compressed bunches are much stronger; effects have been estimated earlier; see s2e-simulations on beam-dynamics home page

an improved model for s2e-simualtions should be prepared: we need the Taylor-coefficients of the wake function to consider offset-dependent longitudinal and transverse kicks (in Xtrack or Ocelot)