Comparison of CSR Wake Fields with CSRDG and Other Methods

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Outline of Talk

- Present brief overview of a few CSR methods:
  - CSRDG Code (D. Bizzozero)
  - Paraxial Method (R. Warnock and D. Bizzozero)
  - Eigenmode Method (G. Stupakov and I. Kotelnikov)
  - CSRtrack Code (M. Dohlus and T. Limberg)
- Compare methods for a single bend and DESY BC0
  - Comparison between paraxial and full geometry CSRDG
  - Examine effects of a collimator with CSRDG
- Examine CSRtrack simulations and other issues
- Summarize and discuss outlook

Brief Overview of CSRDG Code - 1

- CSRDG – MATLAB GPU-enabled Maxwell field solver for modeling CSR with a Discontinuous Galerkin (DG) finite element method
- Approximations and Limitations of CSRDG:
  - Ultra-relativistic electron bunch on a curved planar 2D orbit
  - Rectangular cross-section vacuum chambers (extruded 2D domain into 3D)
  - Modal Fourier series-decomposition in extruded coordinate
  - PEC boundary conditions only (as of current version)
  - No collective effects and rigid bunch approximation (known source terms and orbit)

Brief Overview of CSRDG Code - 2

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- CSRDG Capabilities:
  - Compute electromagnetic fields generated by CSR in a given domain such as vacuum chambers
  - Compute wake functions and impedance (by FT of wake)
  - Visualize field and wake evolution throughout a simulation (CSRDG is a time-domain code)
- CSRDG Goals:
  - Compare and validate other CSR methods
  - Establish range of validity for paraxial methods
  - Estimate effect of CSR on wake fields

Brief Overview of Paraxial Method

- Frequency-domain method developed by R. Warnock and D. Bizzozero [1]
- Key idea of method: From Maxwell's Eqs. in (s, x, y)
  - apply a Fourier transform to $s - ct$, with wavenumber k
  - Apply Fourier-series decomposition in y-coordinate
  - Drop $\partial^2/\partial s^2$ terms (paraxial approximation)
  - Evolve $\hat{E}_{yp}(s, x; k)$ and $\hat{H}_{yp}(s, x; k)$ Schrödinger-type 1D PDEs in s for each k
  - Longitudinal impedance $\hat{E}_{sp}(s, x; k)$ obtained from $\hat{E}_{yp}, \hat{H}_{yp}$
  - Wake field obtained by inverse Fourier transform

[1] R. L. Warnock and D. A. Bizzozero , "Efficient computation of coherent synchrotron radiation in a rectangular chamber", Phys. Rev. Accel. Beams **19**, 090705, September 2016.

Brief Overview of Eigenmode Method

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- Frequency-domain method presented by G. Stupakov and I. Kotelnikov [2]
- Key idea of method: From Maxwell's Eqs. in (s, x, y)
  - apply a Fourier transform to $s - ct$, with wavenumber k
  - Drop $\partial^2/\partial s^2$ terms (paraxial approximation)
  - Decompose transverse (x, y) coordinates modally as (m, p)
  - Couple modes in straight-bend-straight regions (m, m', m'')
  - Sum expression for $\int_0^\infty \hat{E}_s(s, 0, 0; k)ds$ over (m, m', m'', p)
  - Wake field obtained by inverse Fourier transform
  - Note: a lot of details omitted here!

[2] G. V. Stupakov and I. A. Kotelnikov, "Calculation of coherent synchrotron radiation impedance using the mode expansion method", Phys. Rev. ST Accel. Beams **12**, 104401, October 2009.

Brief Overview of CSRtrack Code

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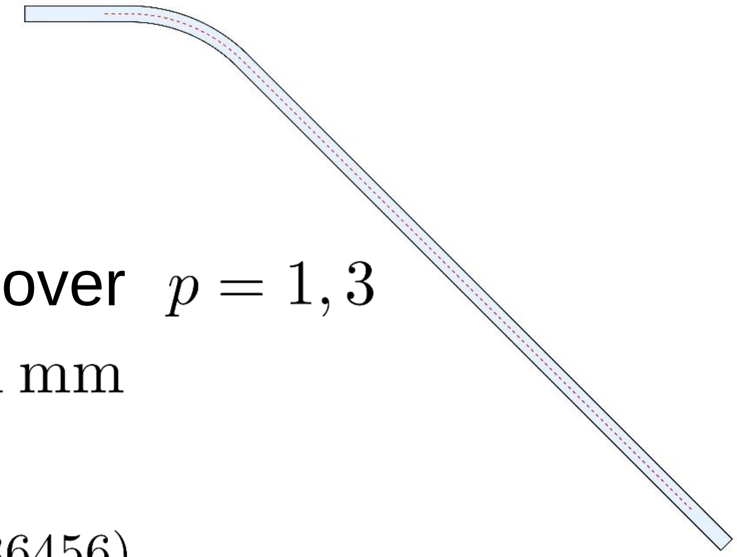
- Particle tracking code to estimate CSR effects on a bunch written by M. Dohlus and T. Limberg [3]
- Key idea of code: Start with list of macro-particles
  - Define lattice structure (e.g. dipoles, quadrupoles, etc.)
  - Track particles through structure by computing forces:
    - Projected 1D: ignores transverse beam dimensions (fast)
    - Grid-to-Particle: uses pseudo-Green's function sub-bunches (slow)
    - Particle-to-Particle: 3D direct particle tracking (very slow)
  - Outputs particle positions, momenta, and forces
  - Only free-space or parallel-plate domains available

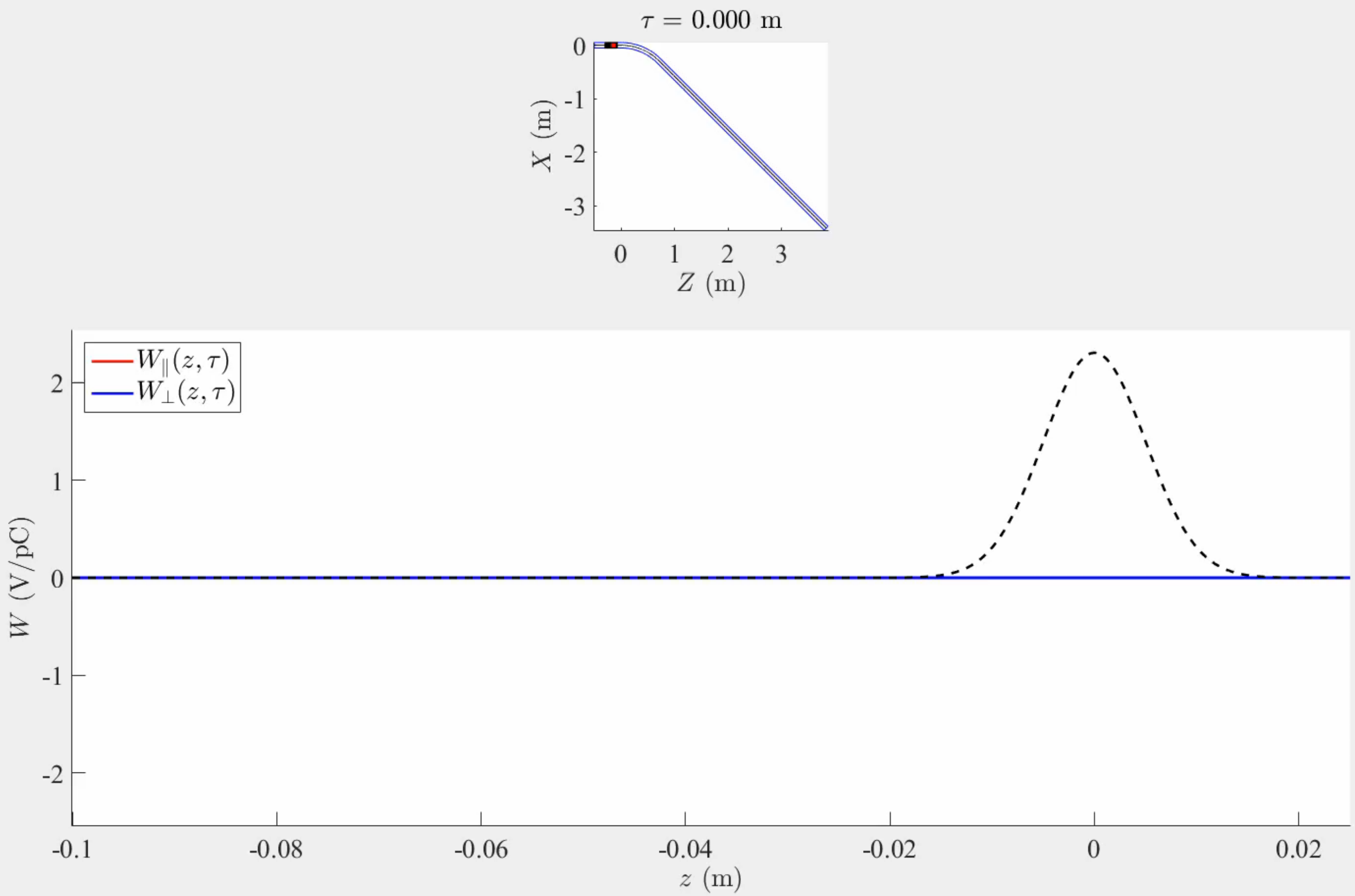
[3] M. Dohlus and T. Limberg, "CSRtrack: Faster Calculation of 3D CSR effects", FEL 2004 Conference, September 2004.

CSRDG Comparison for a Bend - 1

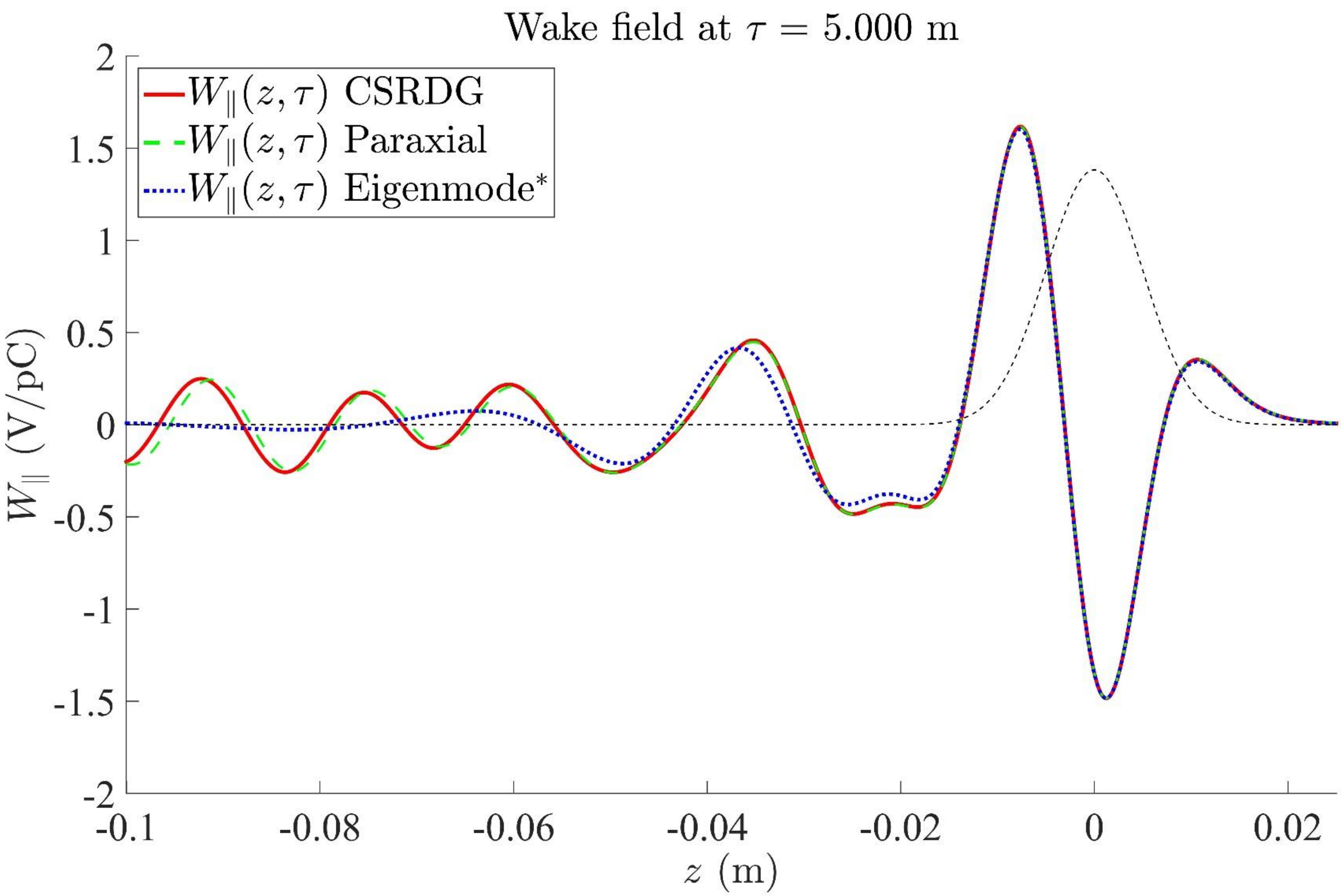
- Rectangular pipe with bend
  - Straight-bend-straight wave guide
  - CSR only, no geometry variation
  - Fields sampled along $x = 0$, sum over $p = 1, 3$
  - Source size: $\sigma_s = 5\,\mathrm{mm},\ \sigma_y = 0.1\,\mathrm{mm}$
  - Additional parameters:
    - DG order and elements: $(N, K) = (8, 36456)$
    - Total chamber width: $d = 100\,\mathrm{mm}$
    - Chamber height: $h = 50\,\mathrm{mm}$
    - Radius of curvature: $R = 1\,\mathrm{m}$
    - Bend angle: $\Theta = 45^\circ$



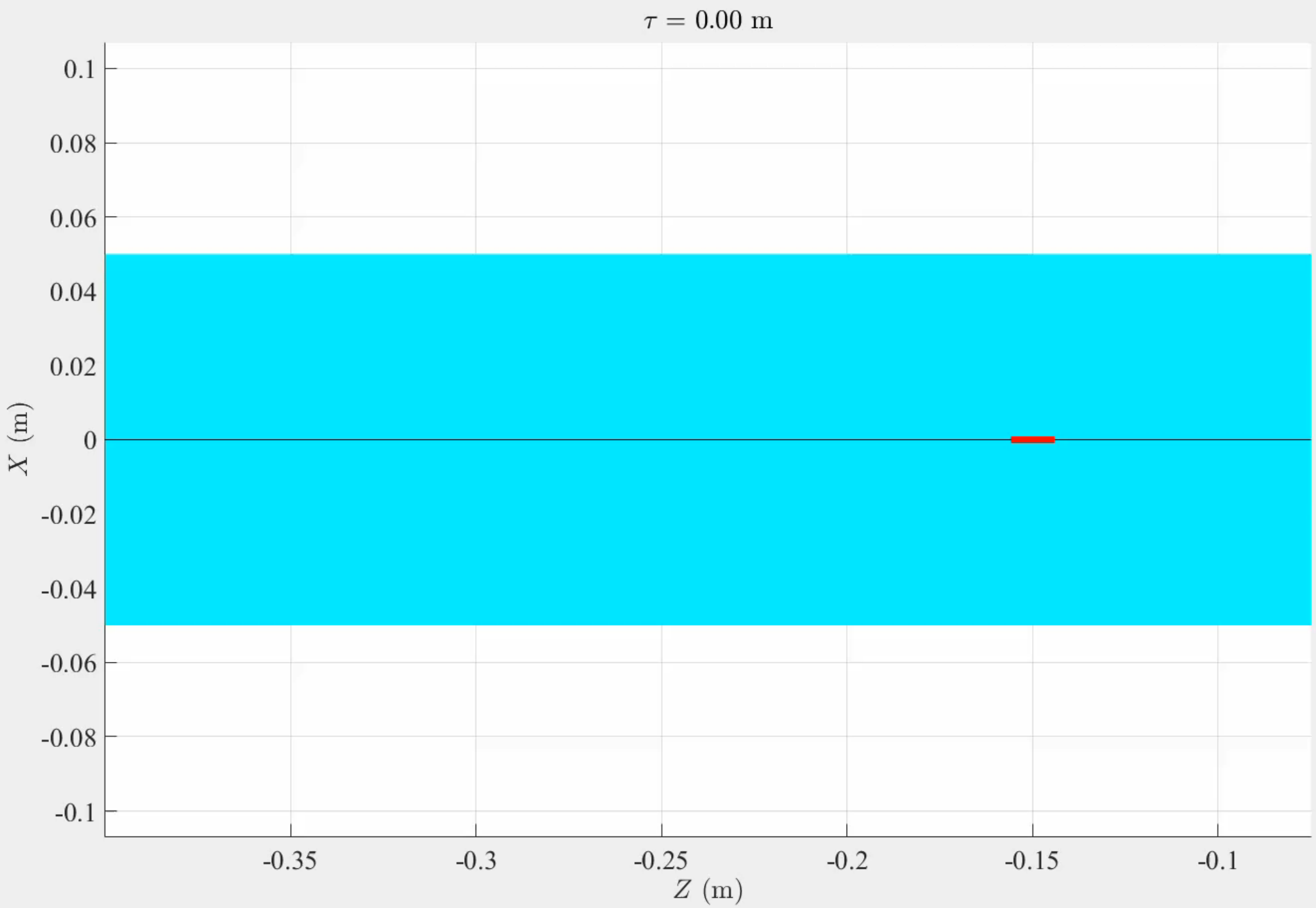
CSRDG Comparison for a Bend - 2



CSRDG Comparison for a Bend - 3



CSRDG Comparison for a Bend - 4



CSRDG Comparison for DESY BC0 - 1

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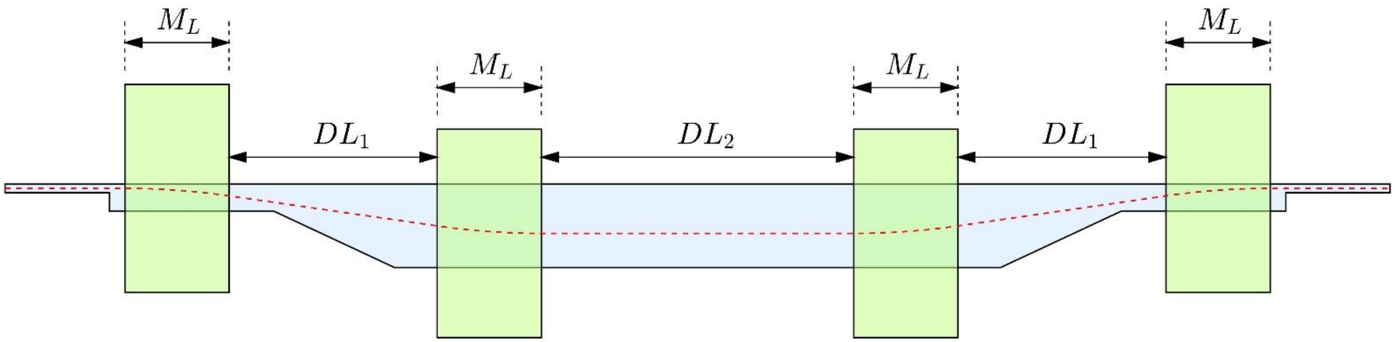
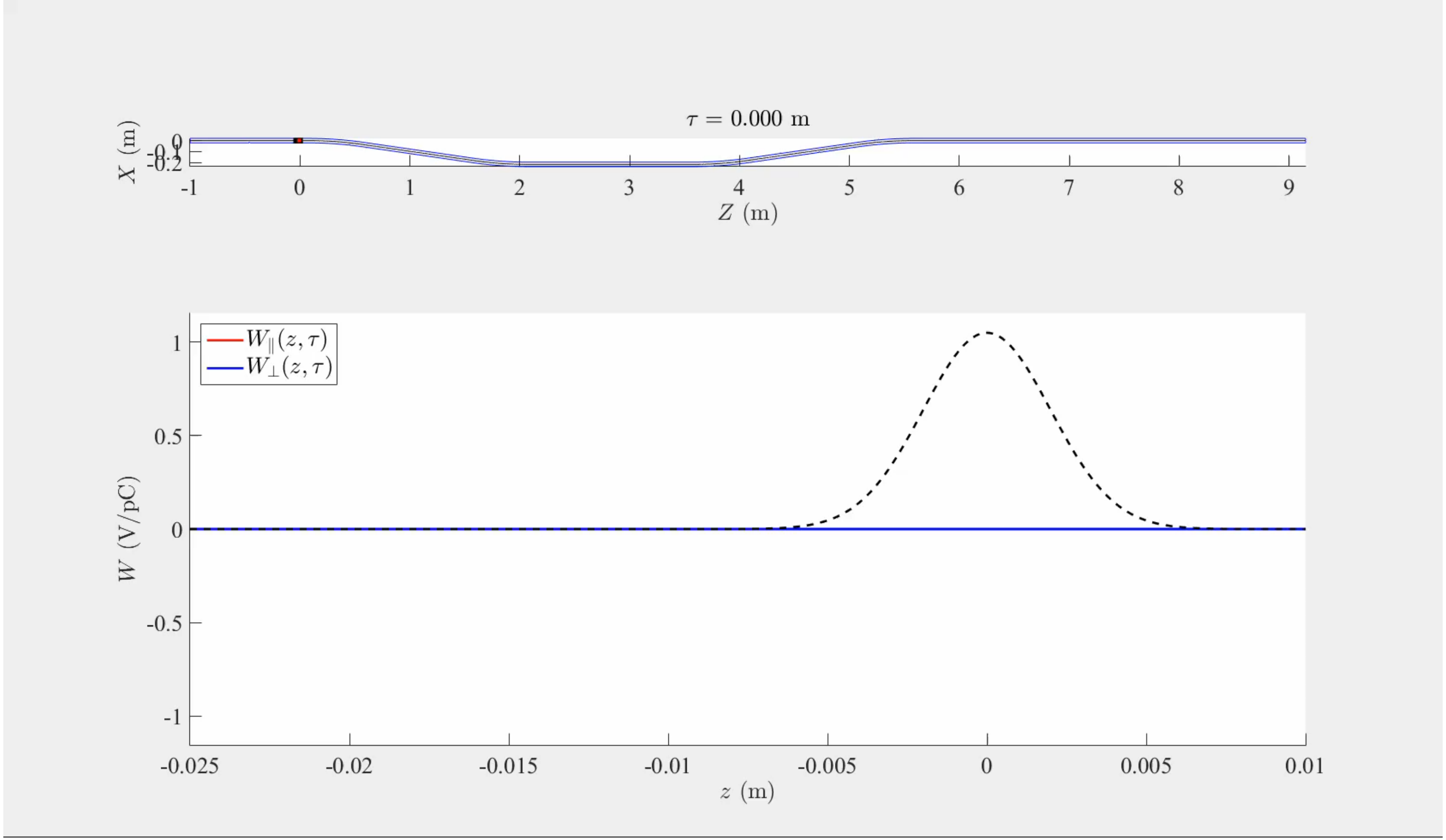
- DESY BC0 geometry
  - CSR and geometry generates wake
  - Fields sampled along $x = 0$, sum over $p = 1, 3$
  - Source size: $\sigma_s = 2\,\mathrm{mm},\ \sigma_y = 0.1\,\mathrm{mm}$

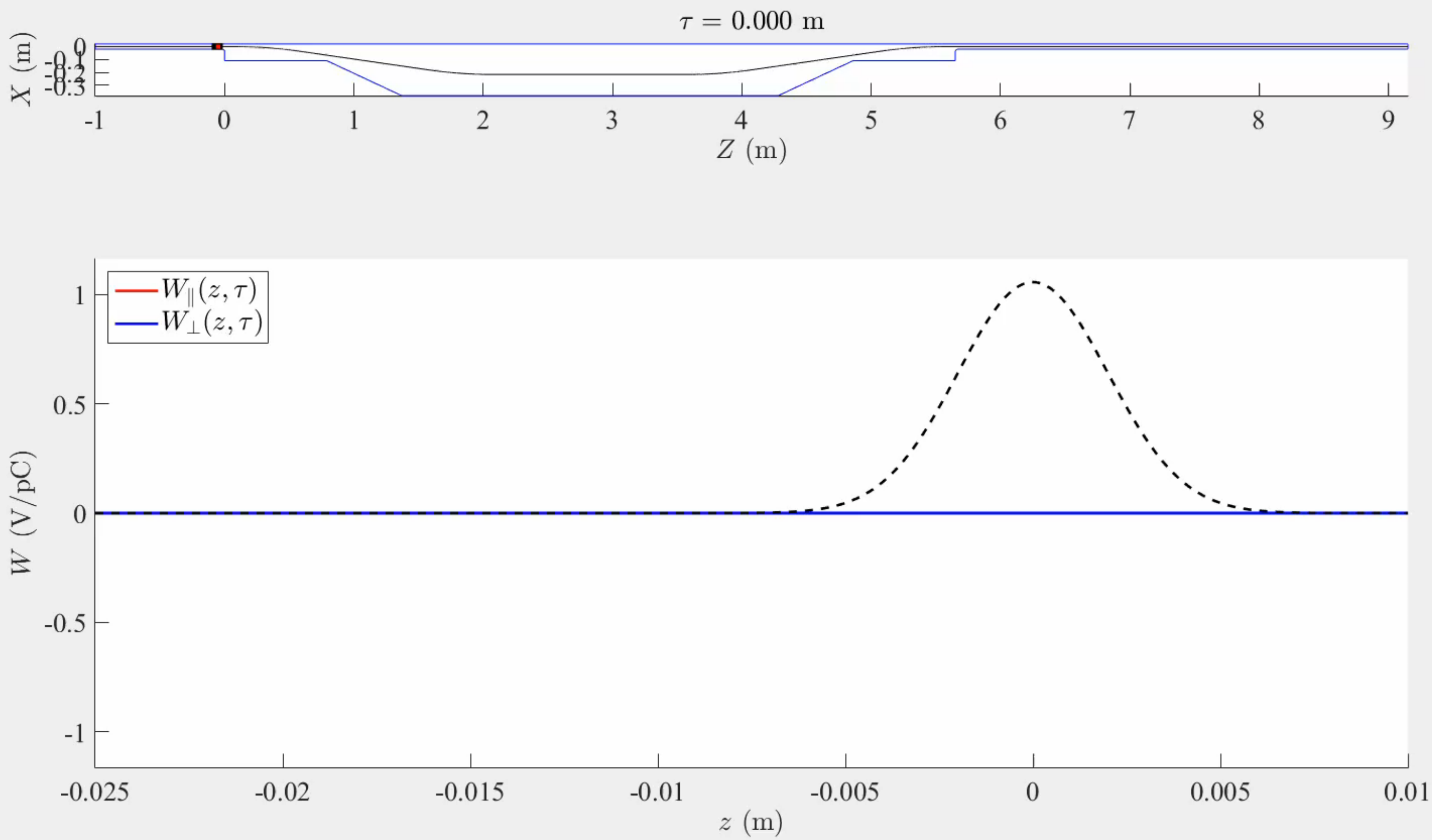


- DESY BC0 test geometry (constant-width pipe)

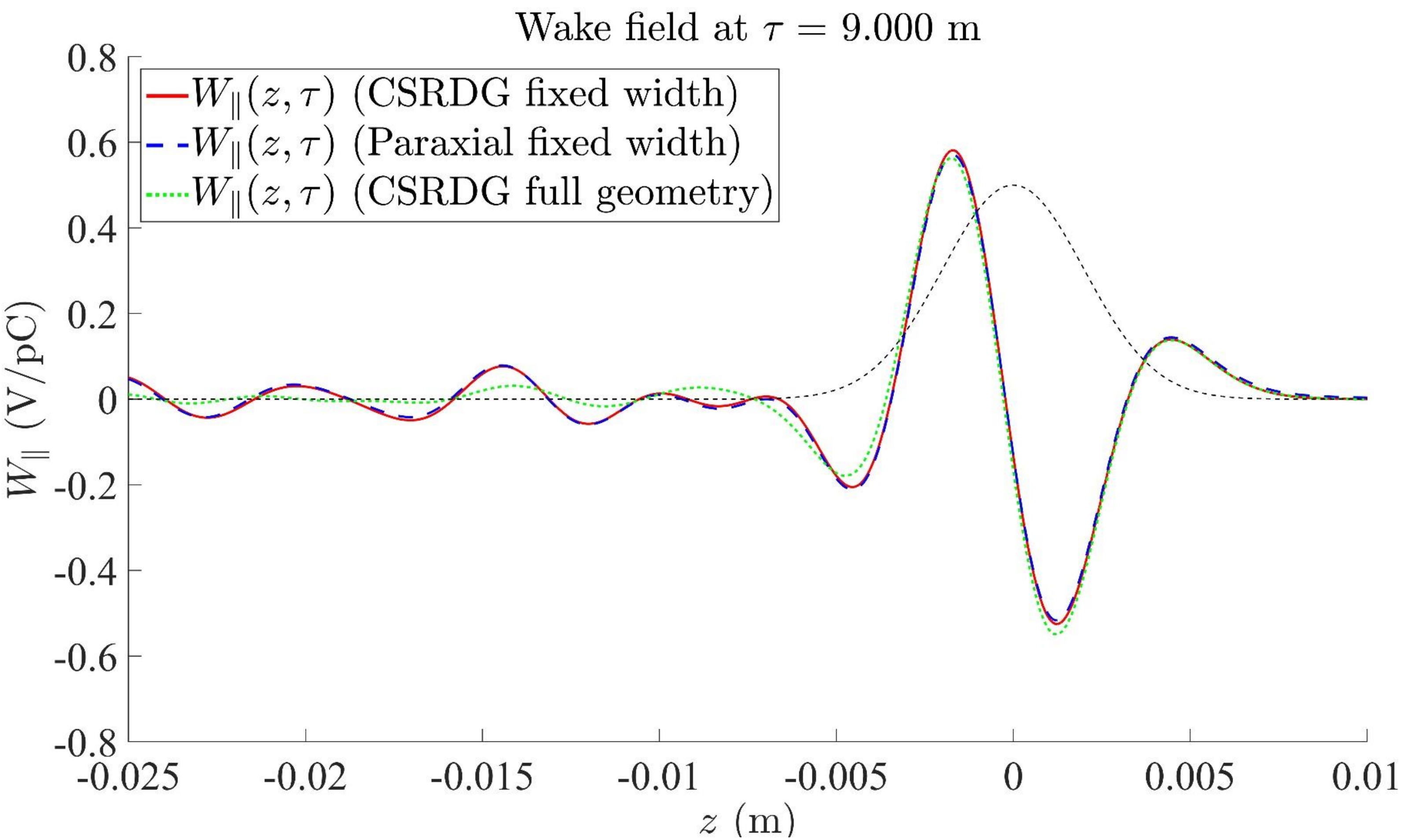
CSRDG Comparison for DESY BC0 - 2



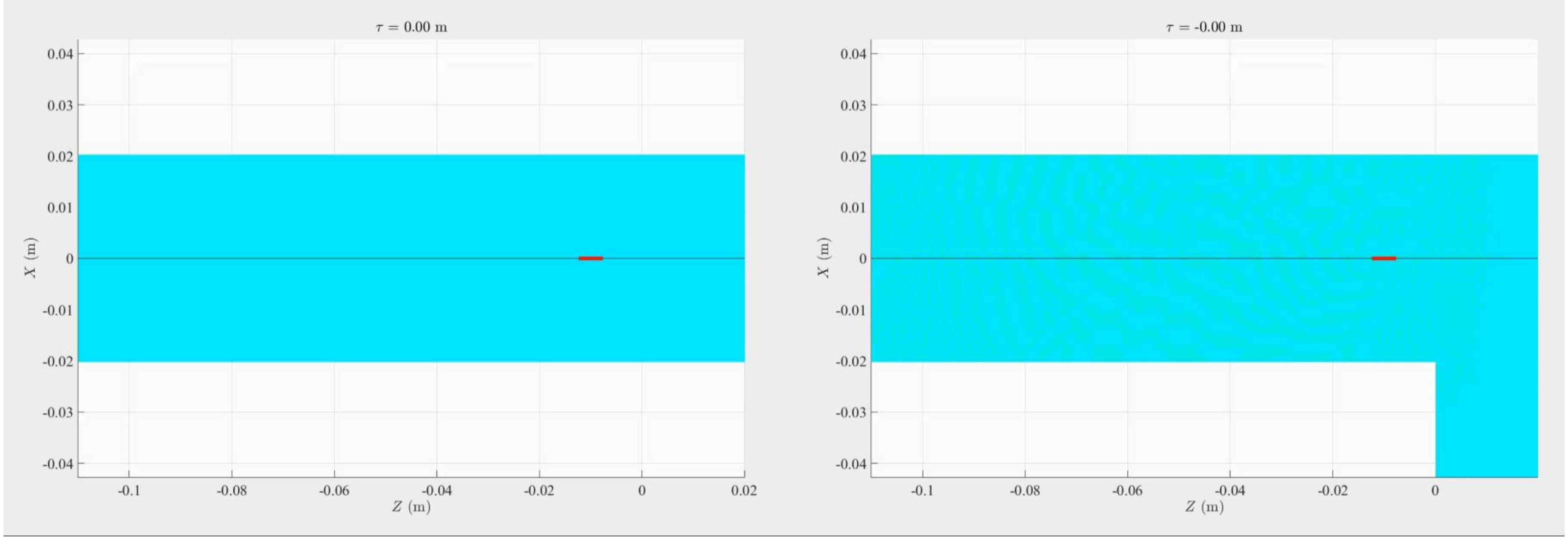
CSRDG Comparison for DESY BC0 - 3



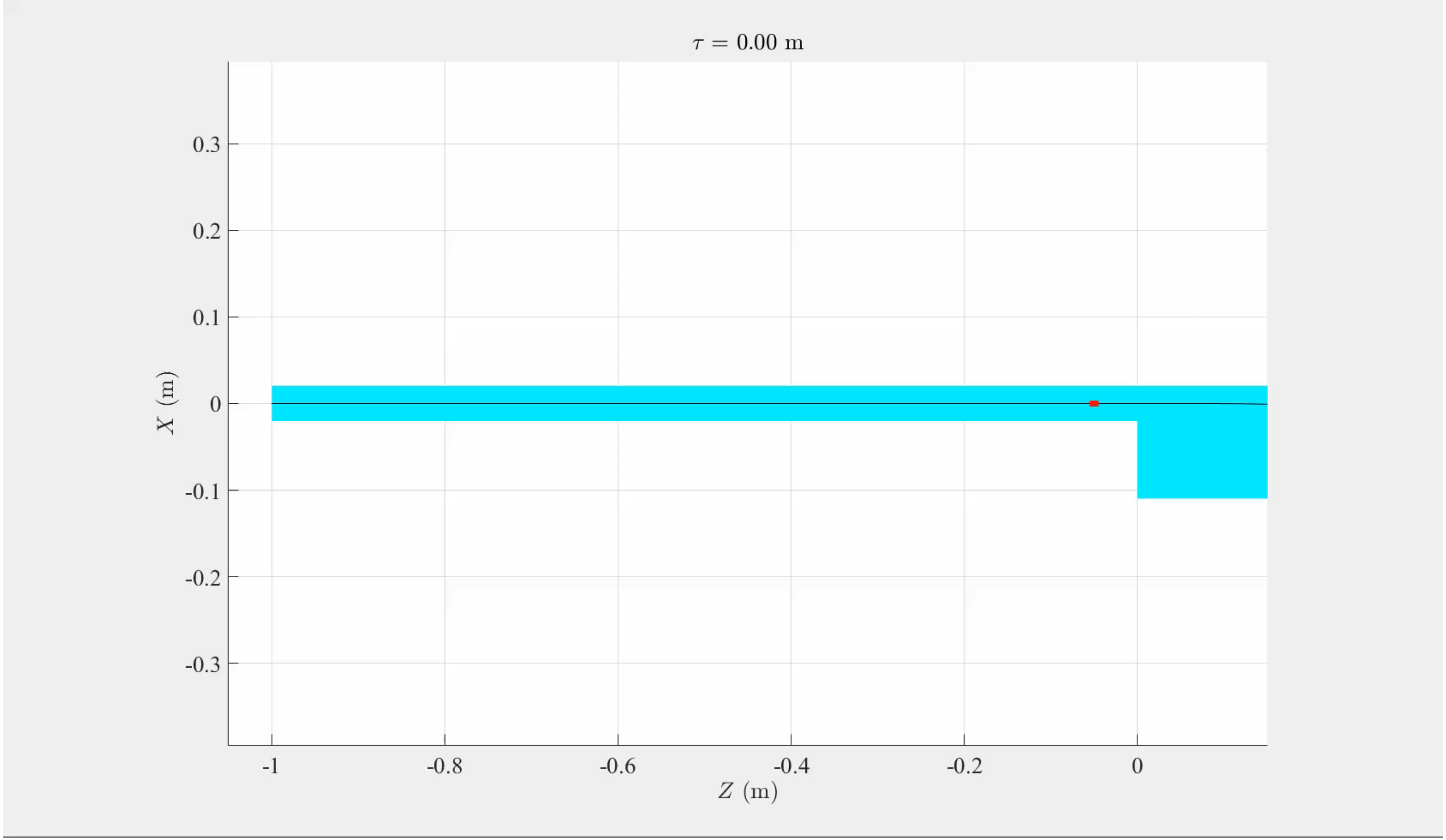
CSRDG Comparison for DESY BC0 - 4



CSRDG Comparison for DESY BC0 - 5

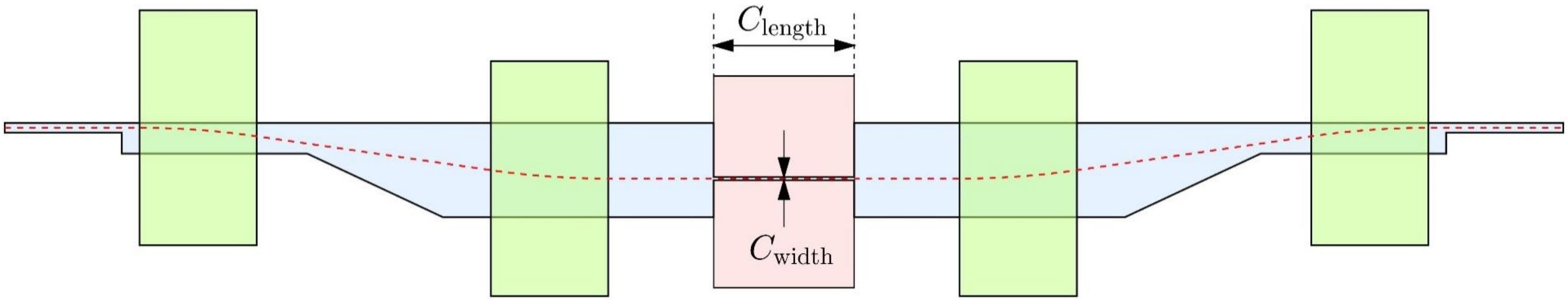


CSRDG Comparison for DESY BC0 - 6

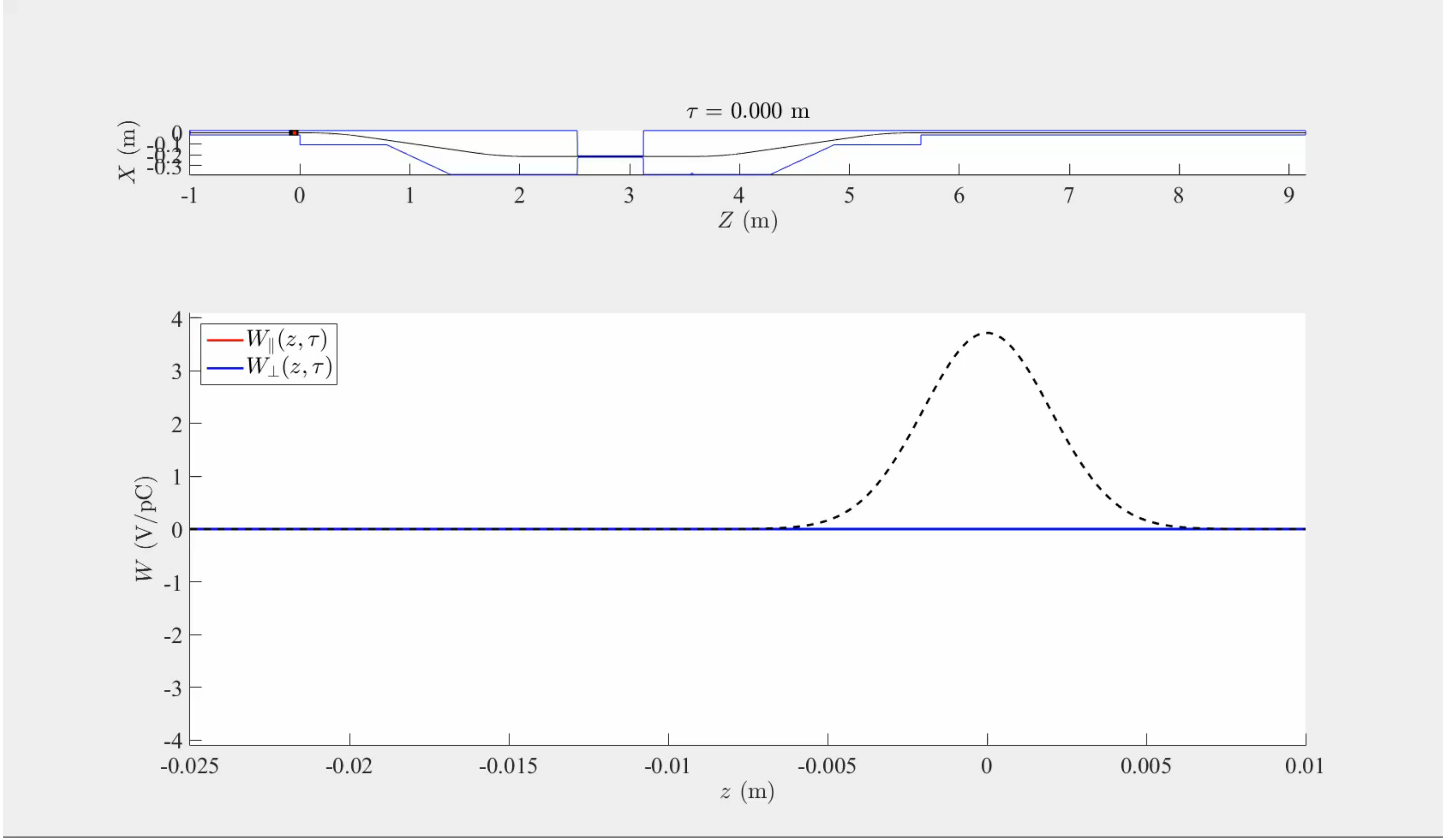


CSRDG for DESY BC0 w/ Collimator- 1

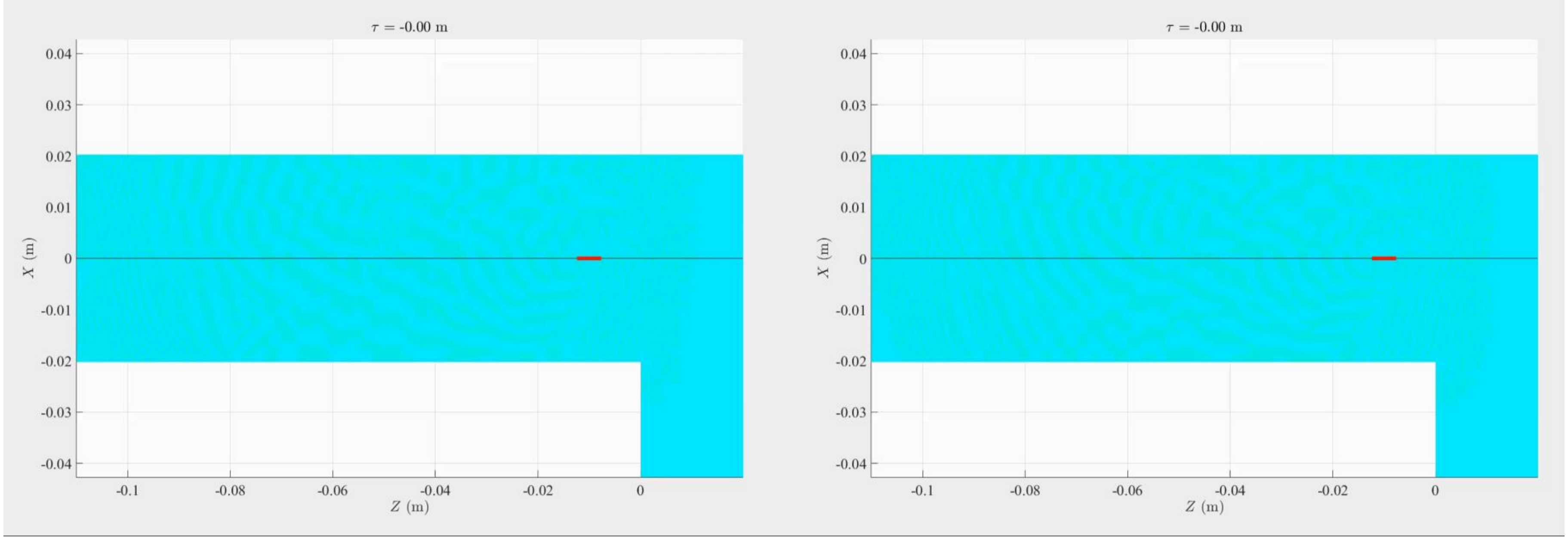
- DESY BC0 geometry with collimator
  - Fields sampled along $x = 0$, sum over $p = 1, 3$
  - Source size: $\sigma_s = 2\,\mathrm{mm},\ \sigma_y = 0.1\,\mathrm{mm}$
  - Collimator parameters: $C_{\mathrm{length}} = 60\,\mathrm{cm},\ C_{\mathrm{width}} = 13\,\mathrm{mm}$



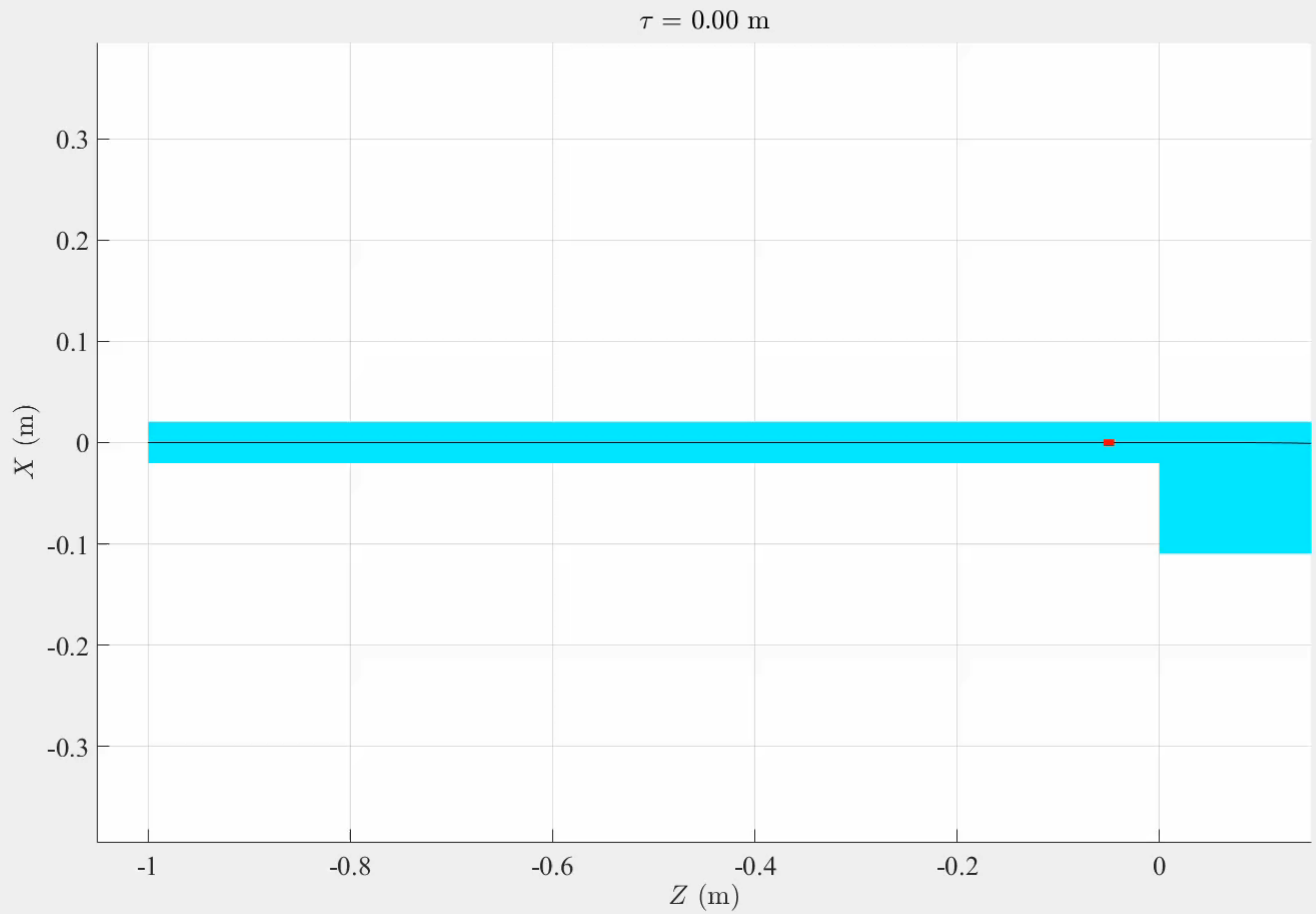
CSRDG for DESY BC0 w/ Collimator- 2



CSRDG for DESY BC0 w/ Collimator- 3



CSRDG for DESY BC0 w/ Collimator- 4



CSRtrack Studies for Comparison - 1

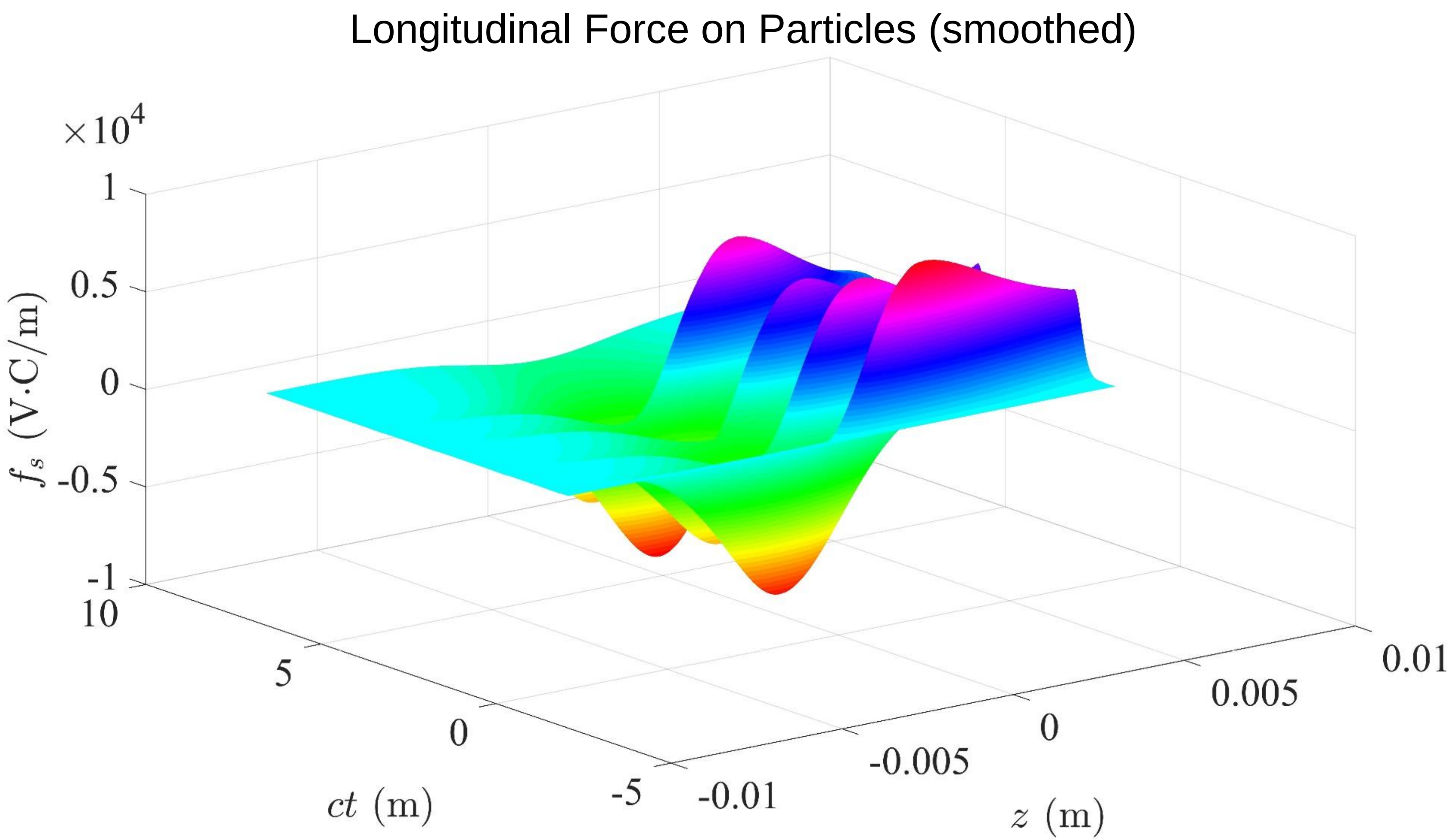
- CSRtrack with DESY BC0 additional parameters:
  - Method: Projected
  - Particle Number: $N_{\text{part}} = 10^5$
  - Bunch Profile: Gaussian
  - No chirp, no shielding

$$\sigma_z = 2\,\text{mm}, \quad \sigma_{pz} = 0$$
$$\sigma_x = 1\,\mu\text{m}, \quad \sigma_{px} = 0$$
$$\sigma_y = 1\,\mu\text{m}, \quad \sigma_{py} = 0$$
$$q = 1\,\text{nC}, \quad E = 13\text{TeV}$$

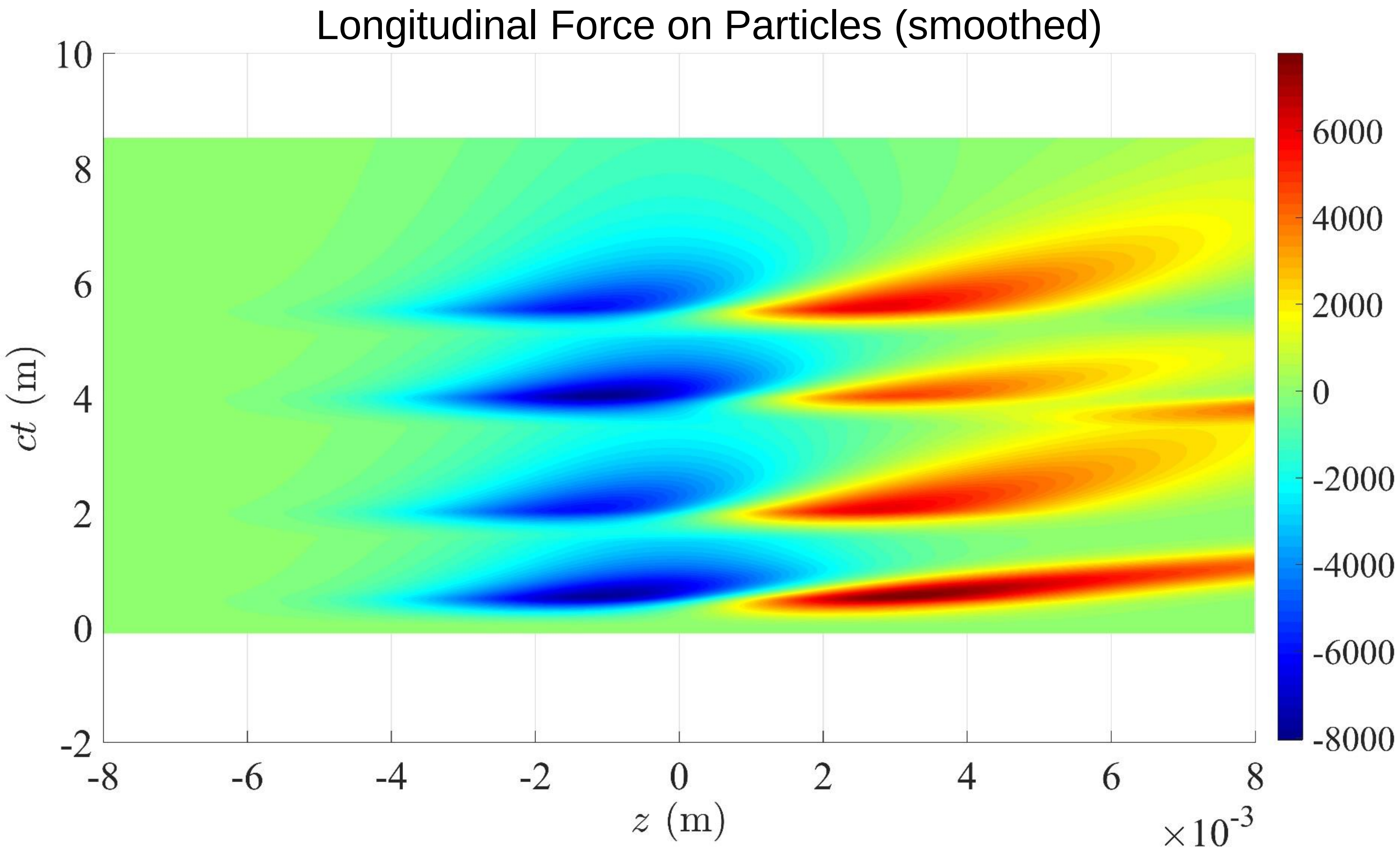
To ignore bunch compression effects
for rigid beam approximation!

- Positions and forces monitored for all particles
- Wake function obtained by integrating smoothed longitudinal force distribution over the trajectory

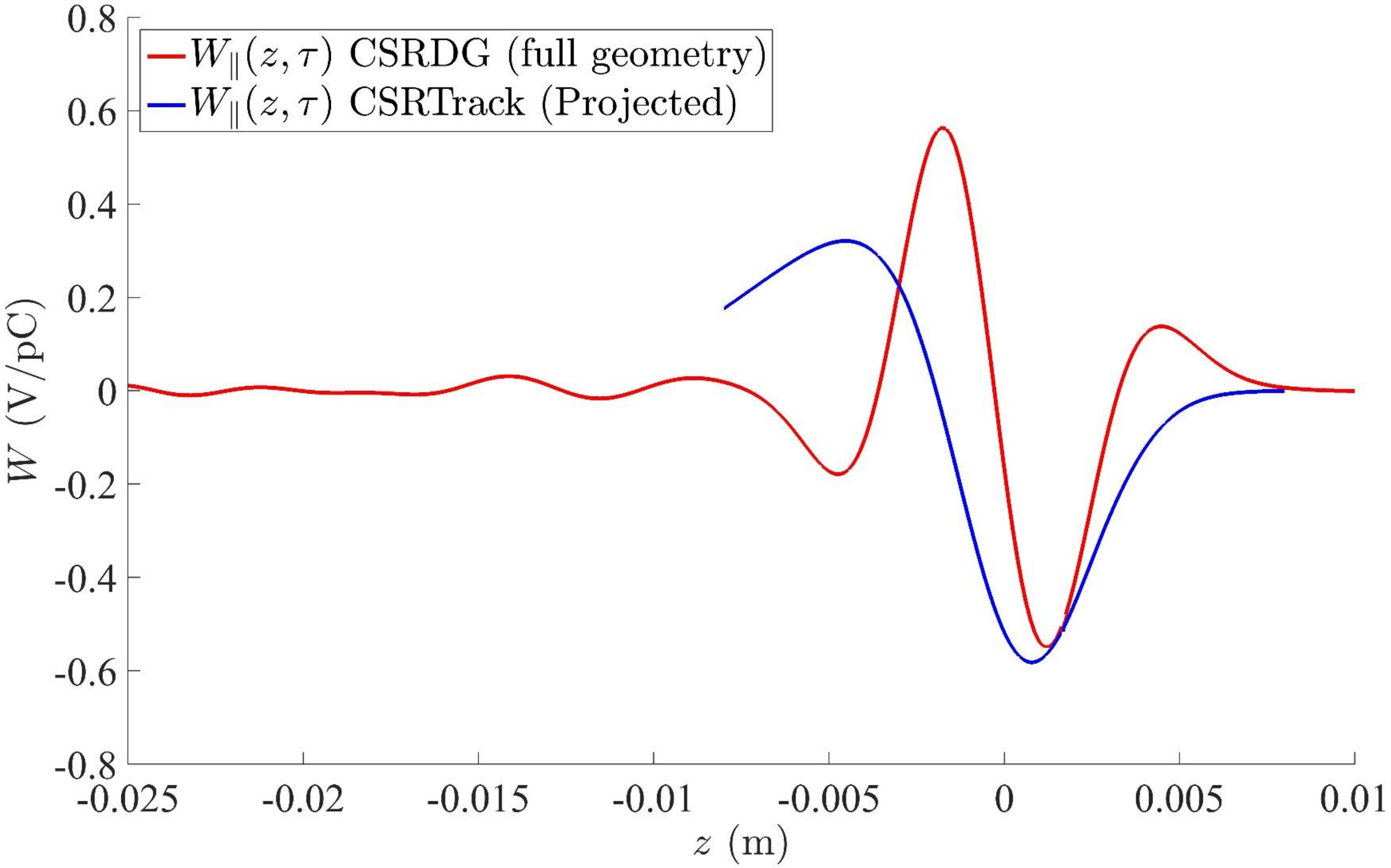
CSRtrack Studies for Comparison - 2



CSRtrack Studies for Comparison - 3



CSRtrack Studies for Comparison - 4



CSRtrack Studies for Comparison - 5

- Issues which were investigated:
  - Increased particle number (no significant change)
  - Increased energy to ensure rigid bunch (no issues)
  - Checked post-processing method for computing forces
  - Attempted other methods for CSRtrack (e.g. p-to-p)
  - Encountered issues with shielding parameter in CSRtrack
- Possible issues to investigate:
  - Transverse shielding strongly affects wake
  - Initial transverse fields influence longitudinal wake as coordinate system rotates, is this treated in CSRtrack?

Summary and Outlook

- Performed comparisons between CSRDG, a paraxial method, and the eigenmode method
- Observed wake differences when geometry was extended or pinched with a collimator
- Wake function is strongly dependent on geometry near bunch trajectory (CSR effect might be minor)
- Ongoing work:
  - Further examining validity range of paraxial methods
  - Testing CSRDG with wide beam pipe to emulate free space
  - Running new simulations with different parameters

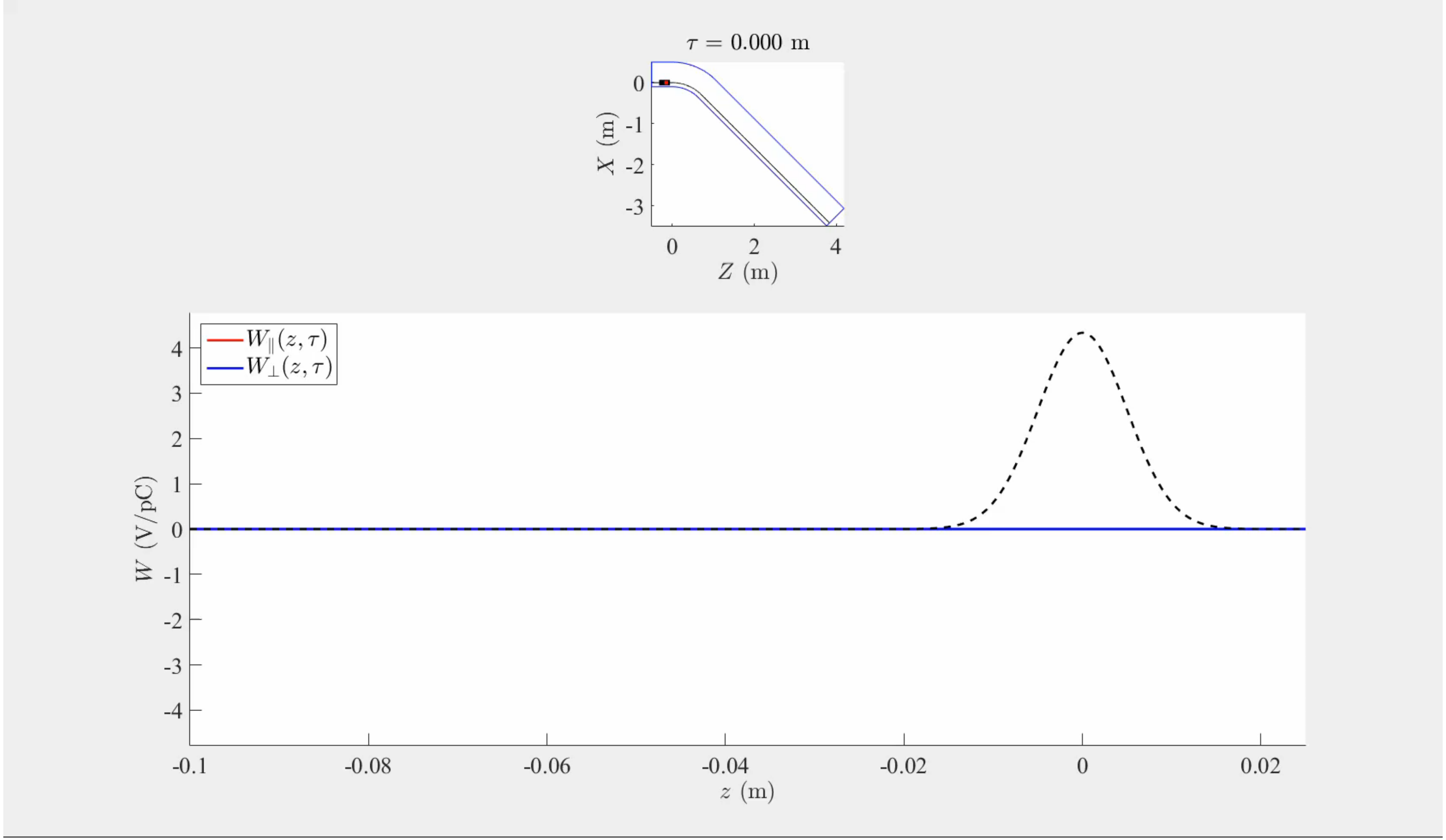
Thank you for your attention!

Work supported by:

TEMF, TU Darmstadt

DESY, Hamburg

Extra: CSRDG with a Wide Bend - 1



Extra: CSRDG with a Wide Bend - 2