Updates on CSR Wake Fields with a Discontinuous Galerkin Time Domain Method

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Outline of Talk

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- Motivation for the Study of Coherent Synchrotron Radiation (CSR)
- Maxwell’s Equations + Transformations
- Simulations of Wake Fields and Impedances
  - Test Case 1: Tapered Beam Pipe
  - Test Case 2: Simple Bend
  - Test Case 3: Model of DESY BC0
- Conclusions and Future Work

Motivation

- Study the generation and propagation of CSR…
- With some approximations!
  - Ultra-relativistic electron bunch ($\beta = 1$) along a curved planar (2D orbit)
  - Rectangular cross-section vacuum chambers such as in a bunch compressor (2D domain)
  - PEC boundary conditions (simple boundary conditions)
  - Ignore collective effects in this work (known source terms)
- Goals:
  - Compute electromagnetic fields in a given domain
  - Compute wake functions and impedance

Maxwell's Equations and Coordinates

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- Maxwell's Equations
  - Starting with Cartesian coordinates: $\mathbf{R} = (Z, X, Y), \quad \tau = ct$

$$\nabla \times \mathbf{E} = -Z_0 \frac{\partial \mathbf{H}}{\partial \tau}, \qquad \nabla \times \mathbf{H} = \frac{1}{Z_0} \frac{\partial \mathbf{E}}{\partial \tau} + \mathbf{j}$$

  - Next consider a planar reference orbit (along $Y = 0$):

    $\mathbf{R}_{\text{ref}}(s) = (Z_{\text{ref}}(s), X_{\text{ref}}(s), 0)$ parameterized by arc length s
  - Define curvilinear coordinate transformation by:

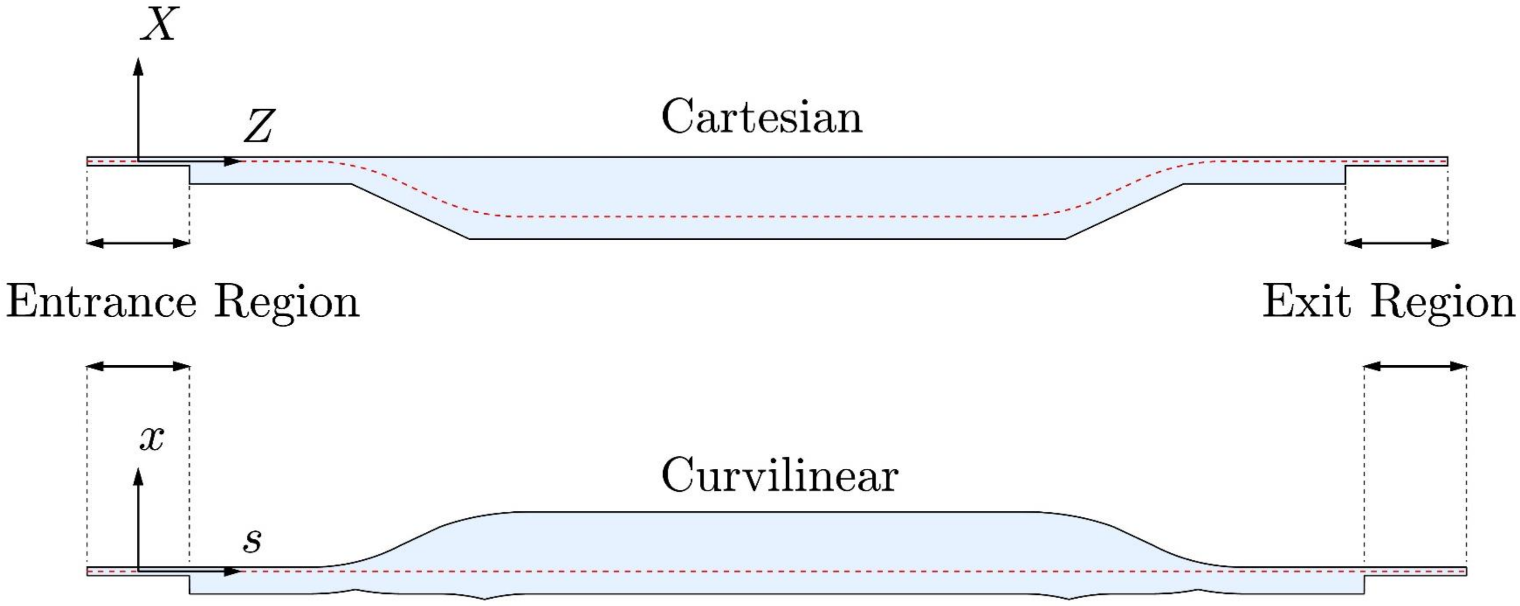
$$\mathbf{e}_s = (Z'_{\text{ref}}(s), X'_{\text{ref}}(s), 0), \quad \mathbf{e}_x = (-X'_{\text{ref}}(s), Z'_{\text{ref}}(s), 0), \quad \mathbf{e}_y = (0, 0, 1)$$

  - Also, define signed curvature κ and scale factor η by:

$$\kappa(s) = Z''_{\text{ref}}(s) X'_{\text{ref}}(s) - Z'_{\text{ref}}(s) X''_{\text{ref}}(s), \qquad \eta(s, x) = 1 + \kappa(s) x$$

Example of Geometry and Coordinates

- Example of mapping to curvilinear coordinates:



- Advantage: source orbit is straight: longitudinal and transverse field components easily obtained
- Disadvantage: only works if $\eta > 0$, problems with large κ

Source Term Definitions

- Charge and current model for ultra-relativistic bunch
  - In (s, x, y) coordinates: $\rho(s, x, y, \tau) = q\lambda(s - \tau)\delta(x)G(y)$

    $$\mathbf{j}(s, x, y, \tau) = qc\lambda(s - \tau)\delta(x)G(y)\mathbf{e}_s$$

    with Gaussian distributions: $\lambda(s), \ G(y)$

    and Dirac distribution: $\delta(x)$
  - Note: $\sigma_s, \ \sigma_y$ for $\lambda(s), \ G(y)$ chosen such that source terms are supported only in the entrance region at $\tau = 0$
  - Other distributions can be used for $\lambda(s), \ G(y)$

Fourier Series Decomposition

- Domain with parallel planar walls: $y = \pm h/2$
  - Assuming PEC boundaries: use a Fourier decomposition

$$f(s,x,y,\tau) = \sum_{p=1}^{\infty} f_p(s,x,\tau)\phi\left(\alpha_p(y+h/2)\right),$$

$$f_p(s,x,\tau) = \frac{2}{h}\int_{-h/2}^{h/2} f(s,x,y,\tau)\phi\left(\alpha_p(y+h/2)\right)dy,$$

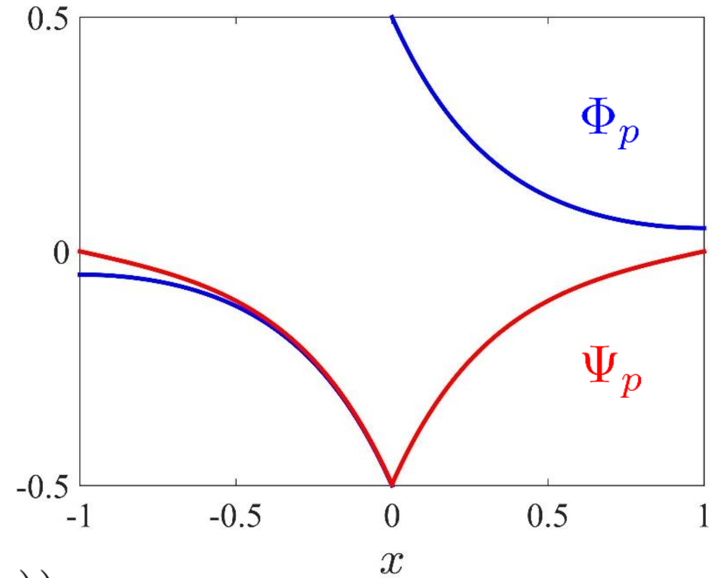
$$\alpha_p = \pi p/h, \qquad \phi(\cdot) = \sin(\cdot) \text{ or } \cos(\cdot)$$

  - E_s, E_x, H_y, j_s, j_x use sine and E_y, H_s, H_x, j_y use cosine
  - If source is symmetric about $y = 0$ then even modes vanish
  - Few modes needed to compute wake function

Initial Conditions

- With PEC boundary conditions for $a \leq x \leq b$

$$E_{sp}(s,x,0) = 0$$
$$E_{xp}(s,x,0) = -qZ_0 c G_p \lambda(s)\Phi_p(x)$$
$$E_{yp}(s,x,0) = -qZ_0 c G_p \lambda(s)\Psi_p(x)$$
$$H_{sp}(s,x,0) = 0$$
$$H_{xp}(s,x,0) = qcG_p\lambda(s)\Psi_p(x)$$
$$H_{yp}(s,x,0) = -qcG_p\lambda(s)\Phi_p(x)$$



$$\Phi_p(x) = \sinh(\alpha_p b)\frac{\cosh(\alpha_p(x-a))}{\sinh(\alpha_p(b-a))} - \cosh(\alpha_p x)\Theta(x)$$
$$\Psi_p(x) = \sinh(\alpha_p b)\frac{\sinh(\alpha_p(x-a))}{\sinh(\alpha_p(b-a))} - \sinh(\alpha_p x)\Theta(x)$$

Combining All Transformations

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- Issue: how to evaluate $\delta(x)$ in $\partial E_{sp}/\partial\tau$ equation?
- Fix: replace H_{yp} by $\widetilde{H}_{yp} = H_{yp} - qcGp\lambda(s-\tau)\Theta(x)$
- Result:
  - ✓ Maxwell's Eqs.
  - ✓ C. Transform
  - ✓ Source Def.
  - ✓ F. Decomp.
  - ✓ Smoother Src.
  - ✓ Initial Conds.

$$\frac{1}{Z_0}\frac{\partial E_{sp}}{\partial\tau} = \frac{\partial \widetilde{H}_{yp}}{\partial x} + \alpha_p H_{xp}$$

$$\frac{1}{Z_0}\frac{\partial E_{xp}}{\partial\tau} = -\alpha_p H_{sp} - \frac{1}{\eta}\frac{\partial \widetilde{H}_{yp}}{\partial s} - \frac{1}{\eta} qcG_p\lambda'(s-\tau)\Theta(x)$$

$$\frac{1}{Z_0}\frac{\partial E_{yp}}{\partial\tau} = \frac{1}{\eta}\frac{\partial H_{xp}}{\partial s} - \frac{\partial H_{sp}}{\partial x} - \frac{\kappa}{\eta}H_{sp}$$

$$Z_0\frac{\partial H_{sp}}{\partial\tau} = \alpha_p E_{xp} - \frac{\partial E_{yp}}{\partial x}$$

$$Z_0\frac{\partial H_{xp}}{\partial\tau} = \frac{1}{\eta}\frac{\partial E_{yp}}{\partial s} - \alpha_p E_{sp}$$

$$Z_0\frac{\partial \widetilde{H}_{yp}}{\partial\tau} = \frac{\partial E_{sp}}{\partial x} + \frac{\kappa}{\eta}E_{sp} - \frac{1}{\eta}\frac{\partial E_{xp}}{\partial s} + qZ_0cG_p\lambda'(s-\tau)\Theta(x)$$

Final Steps for the Numerical Method

- Boundary conditions?
  - Impose PEC on all boundaries including end pipes (simulation ends before reflections occur in exit region)
- Evolve fields with 4th order low-storage RK
- Additional Notes:
  - Important: align elements along $x = 0$ and where κ is discontinuous (i.e. when using piecewise-defined orbits)

Wake and Impedance Definitions

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- Define longitudinal and transverse wake functions:

$$w_s(z) = \frac{-1}{q} \int_0^T E_s(\tau - z, 0, 0, \tau) d\tau$$
$$w_x(z) = \frac{1}{q} \int_0^T E_x(\tau - z, 0, 0, \tau) - Z_0 H_y(\tau - z, 0, 0, \tau) d\tau$$

- Panofsky-Wenzel theorem: $\partial w_x / \partial z = \partial w_s / \partial x$

- Bunch impedance:

$$Z_s^{\mathrm{b}}(\omega) = \frac{1}{c} \int_{-\infty}^{\infty} w_s(-z) e^{i\omega z/c} dz$$
$$Z_t^{\mathrm{b}}(\omega) = \frac{-i}{c} \int_{-\infty}^{\infty} w_x(-z) e^{i\omega z/c} dz$$

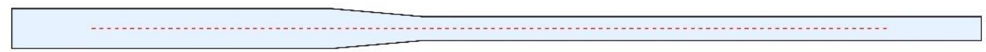
- Single particle impedance (only valid for $\omega \lesssim c/\sigma_s$):

$$Z_s(\omega) = Z_s^{\mathrm{b}}(\omega)/\hat{\lambda}(\omega)$$
$$Z_t(\omega) = Z_t^{\mathrm{b}}(\omega)/\hat{\lambda}(\omega)$$

$$\hat{\lambda}(\omega) = e^{-\omega^2 \sigma_s^2 / 2c^2}$$

Tapered Beam Pipe Simulation

- Tapered Beam Pipe



  - Straight wave guide with taper
  - No CSR, only geometry generates wake
  - Fields sampled along $x = 0$, sum over $p = 1, ..., 9$
  - Source size: $\sigma_s = 10\,\mathrm{mm},\ \sigma_y = 1\,\mathrm{mm}$
  - Additional parameters:
    - DG order and elements: $(N, K) = (8, 27544)$
    - Initial chamber width: $d_I = 50\,\mathrm{mm}$
    - Final chamber width: $d_F = 30\,\mathrm{mm}$
    - Chamber height: $h = 50\,\mathrm{mm}$
    - Taper angle: $\Theta = 5^\circ$

Tapered Beam Pipe Wake Function

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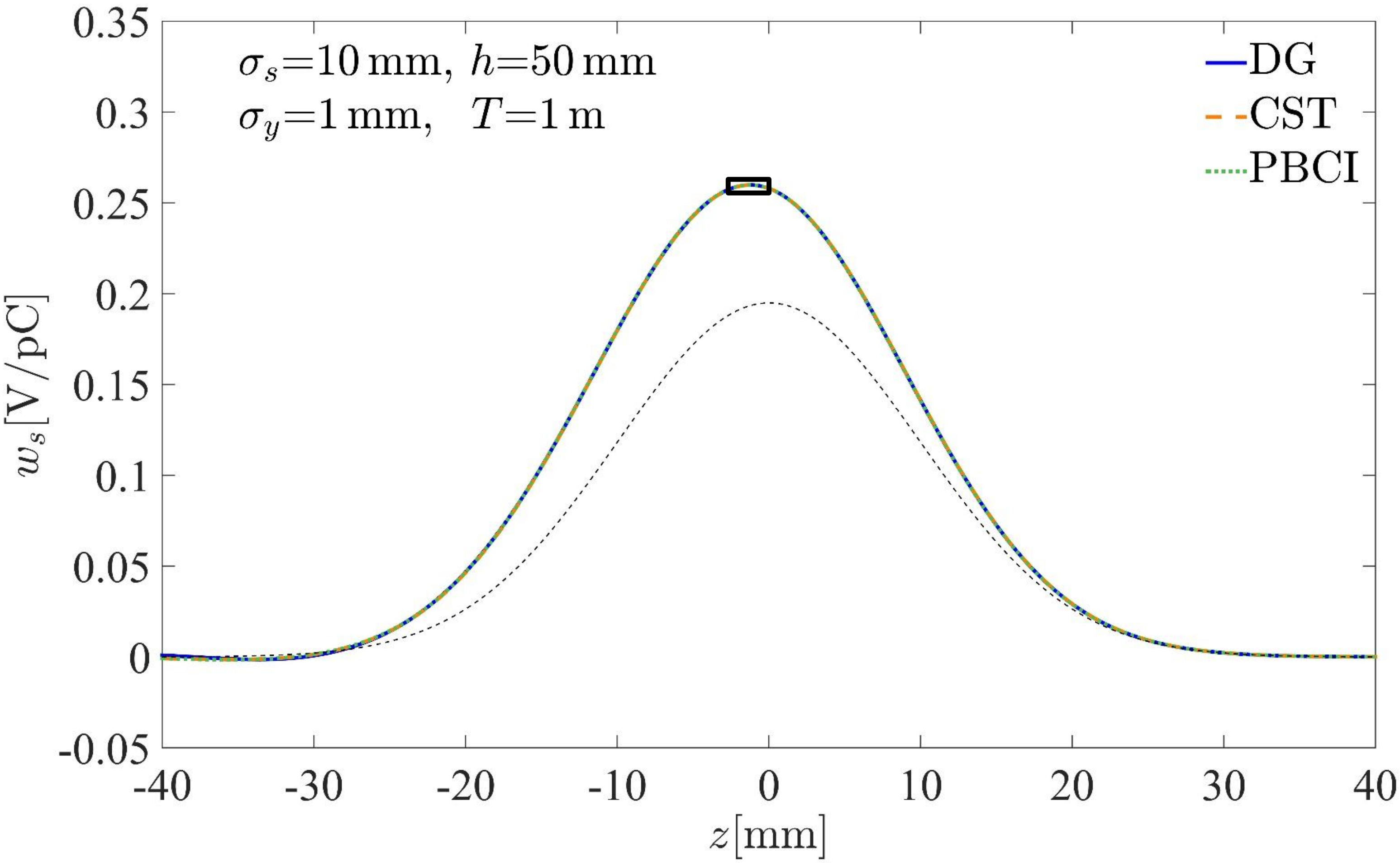

Tapered Beam Pipe Wake Function

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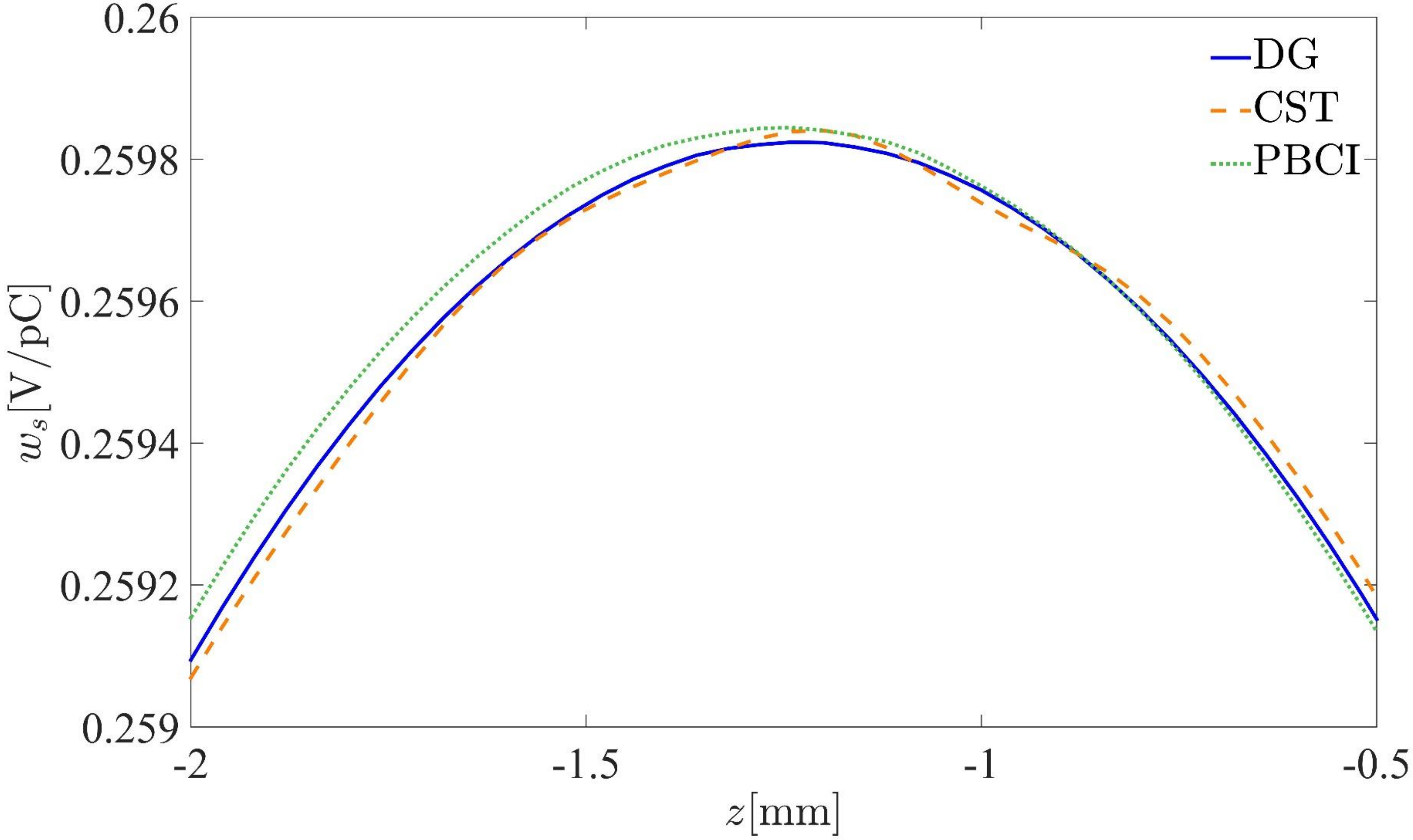
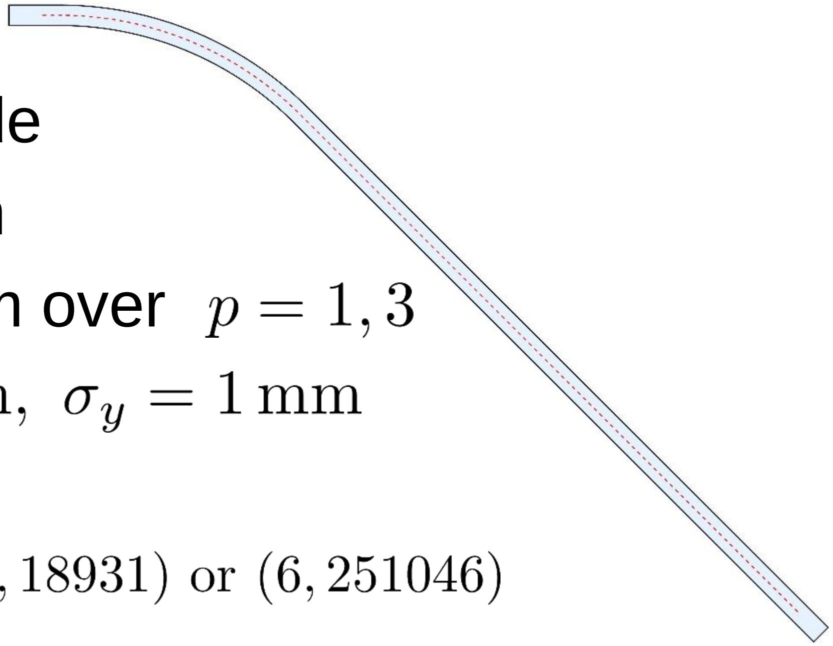
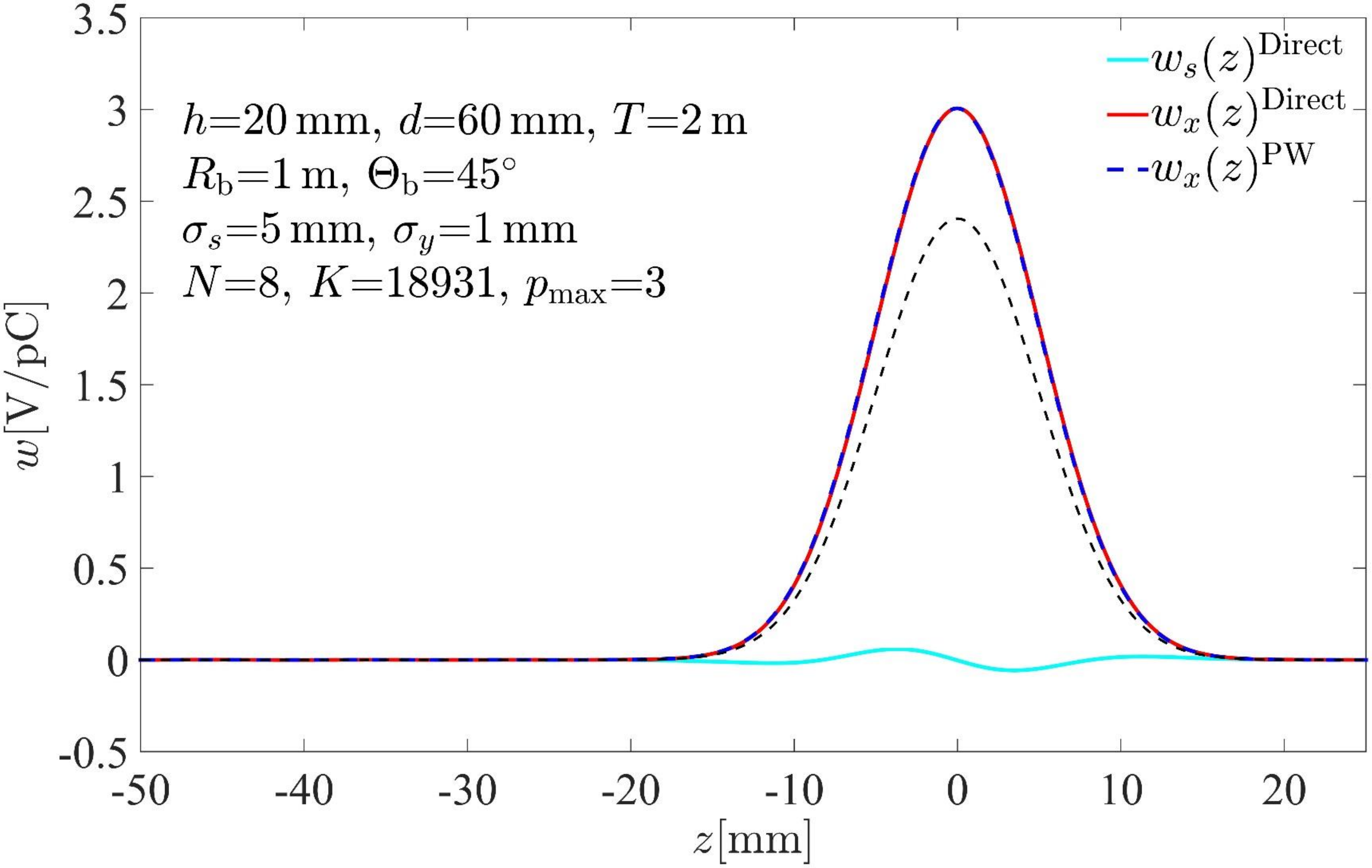


Simple Bend Simulation

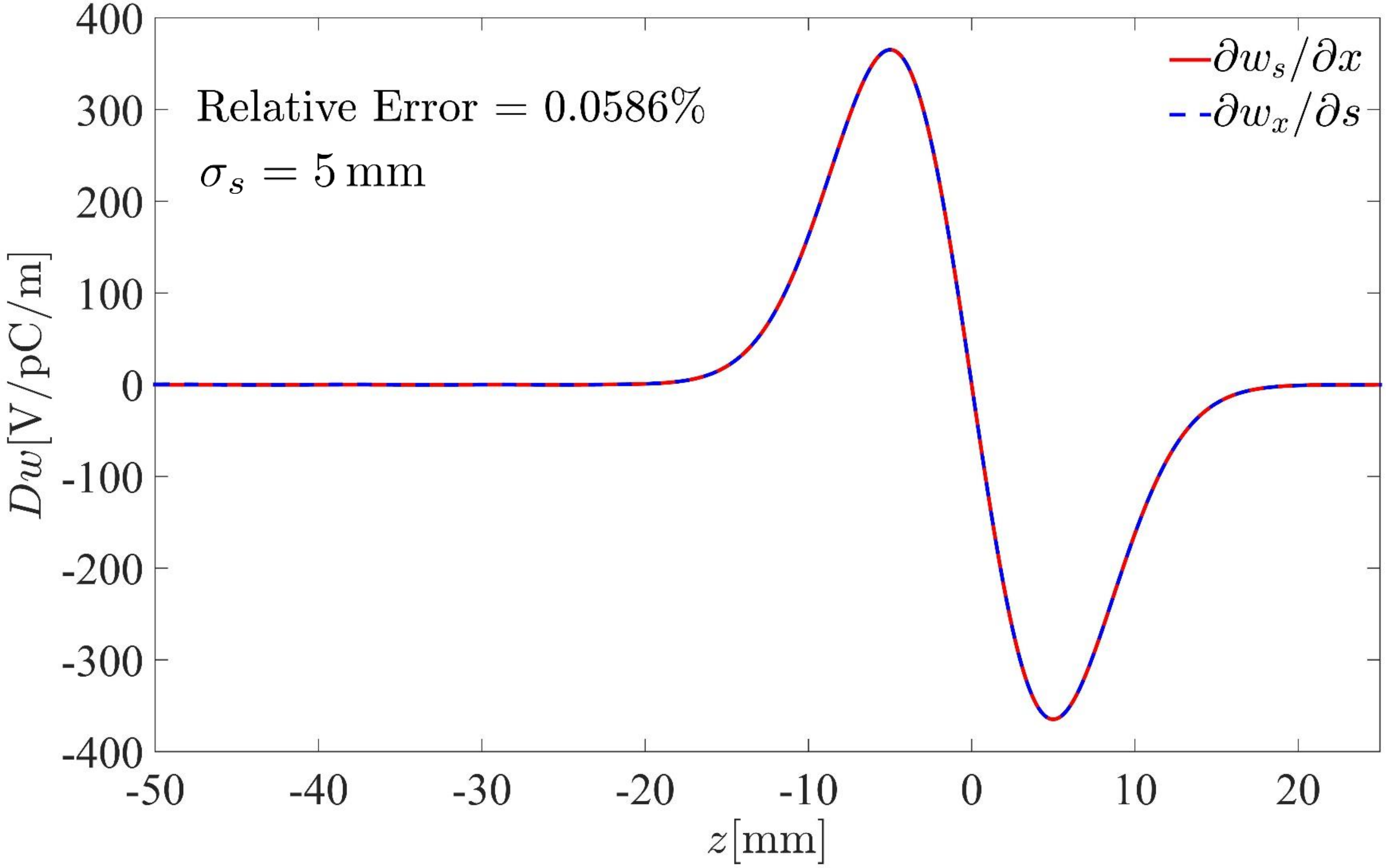
- Rectangular Pipe with Bend
  - Straight-bend-straight wave guide
  - CSR only, no geometry variation
  - Fields sampled along $x = 0$, sum over $p = 1, 3$
  - Source size: $\sigma_s = 5\,\mathrm{mm}$ or $1\,\mathrm{mm}$, $\sigma_y = 1\,\mathrm{mm}$
  - Additional parameters:
    - DG order and elements: $(N, K) = (8, 18931)$ or $(6, 251046)$
    - Total chamber width: $d = 60\,\mathrm{mm}$
    - Chamber height: $h = 20\,\mathrm{mm}$
    - Radius of curvature: $R = 1\,\mathrm{m}$
    - Bend angle: $\Theta = 45°$



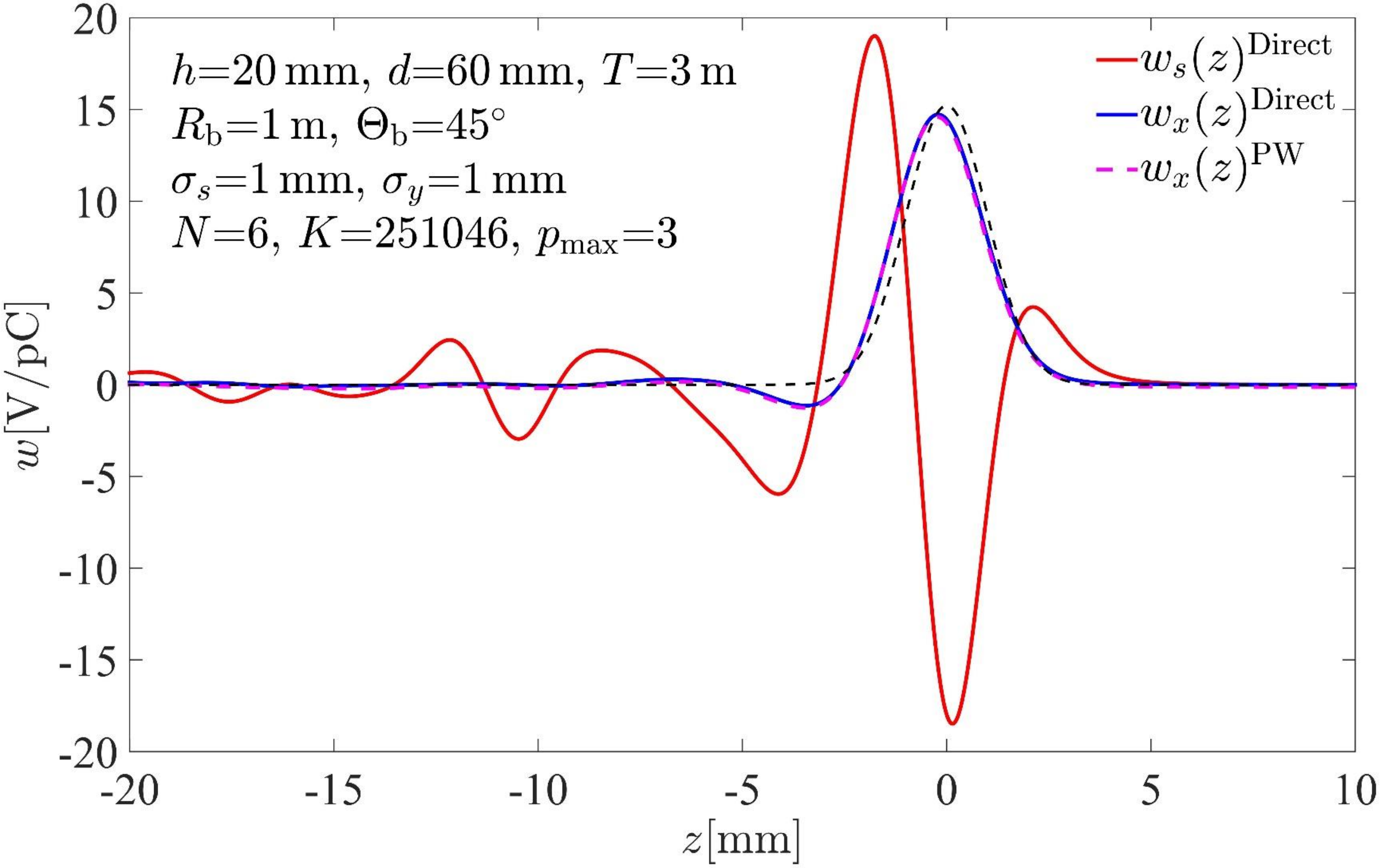
Simple Bend Wake Function (5mm)



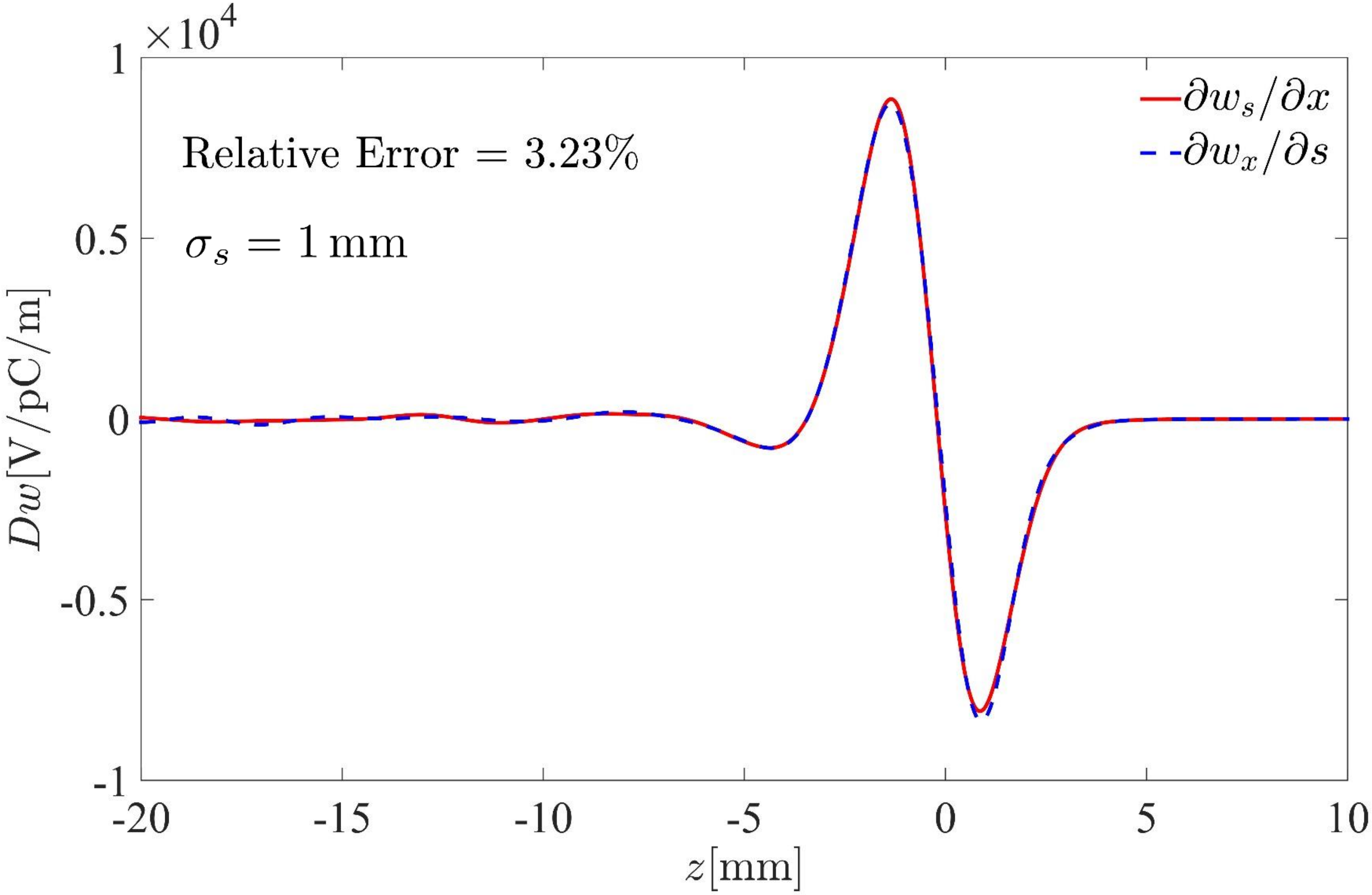
Panofsky-Wenzel Validation (5mm)



Simple Bend Wake Function (1mm)



Panofsky-Wenzel Validation (1mm)



Simple Bend Bunch Impedance

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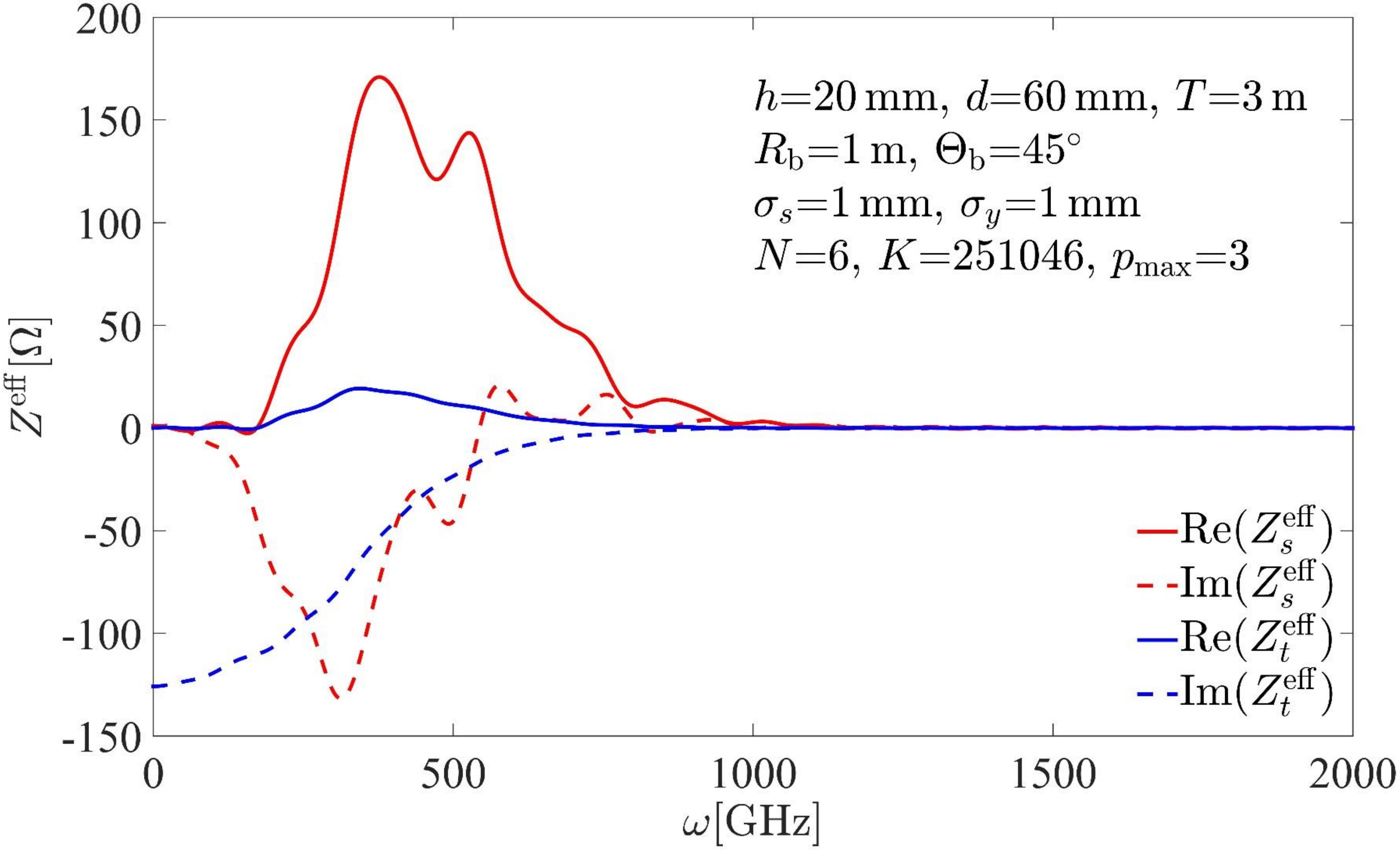

Simple Bend Single Particle Impedance

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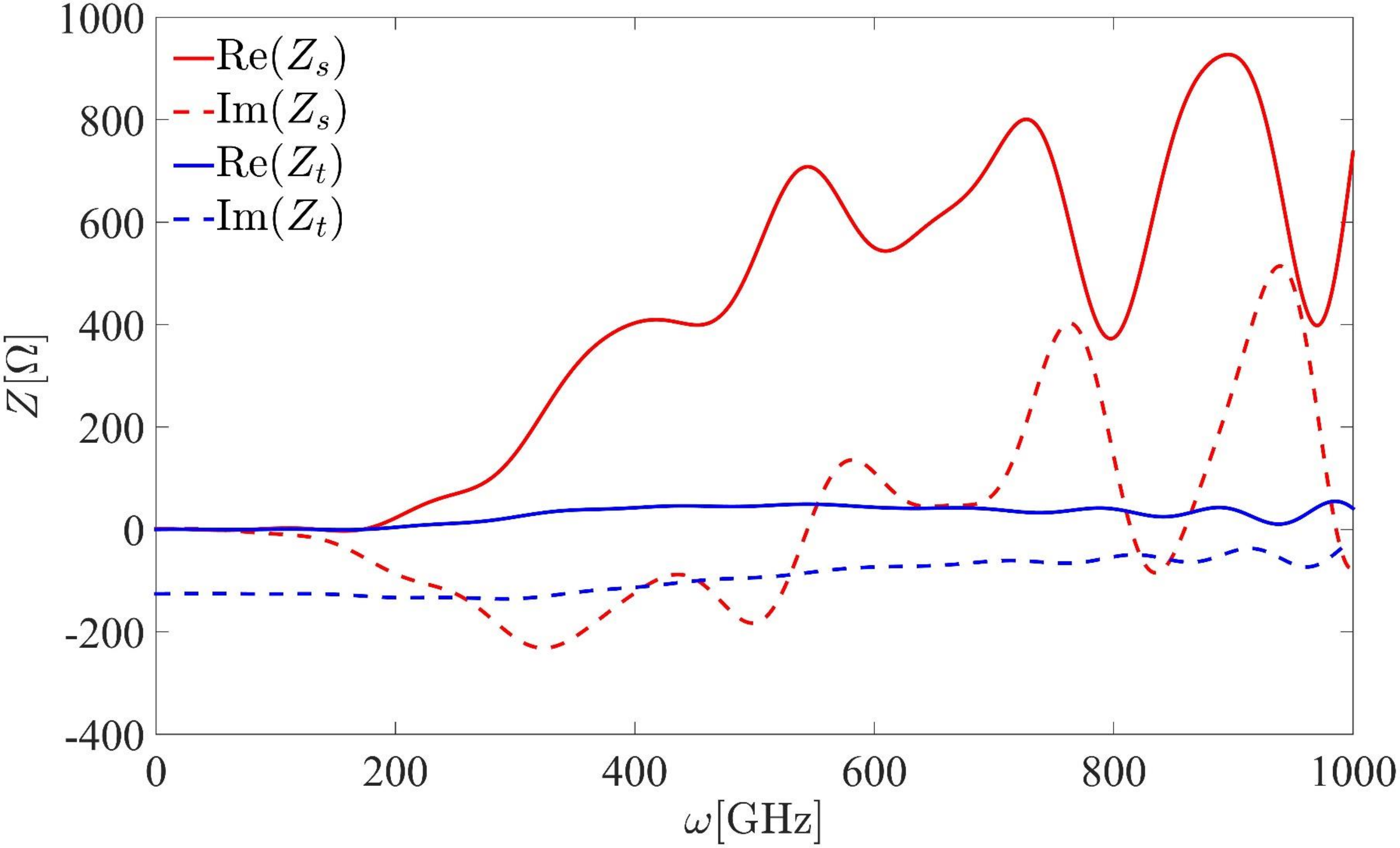
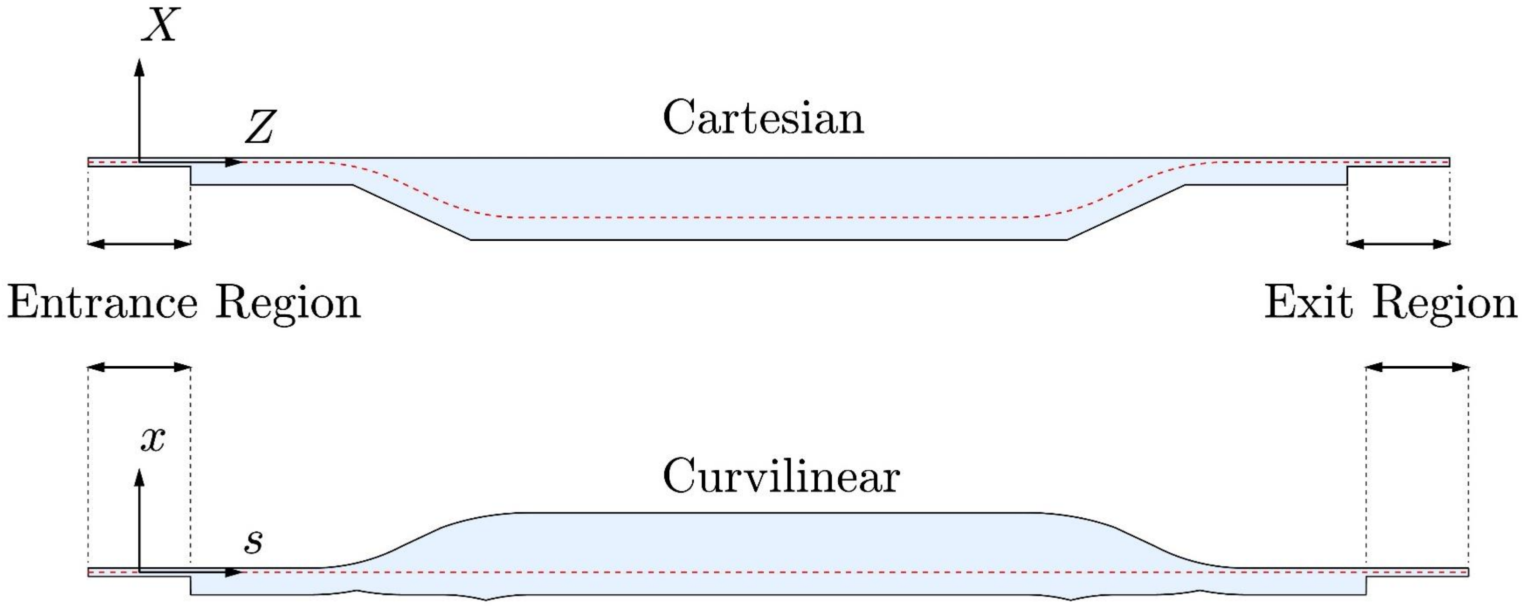
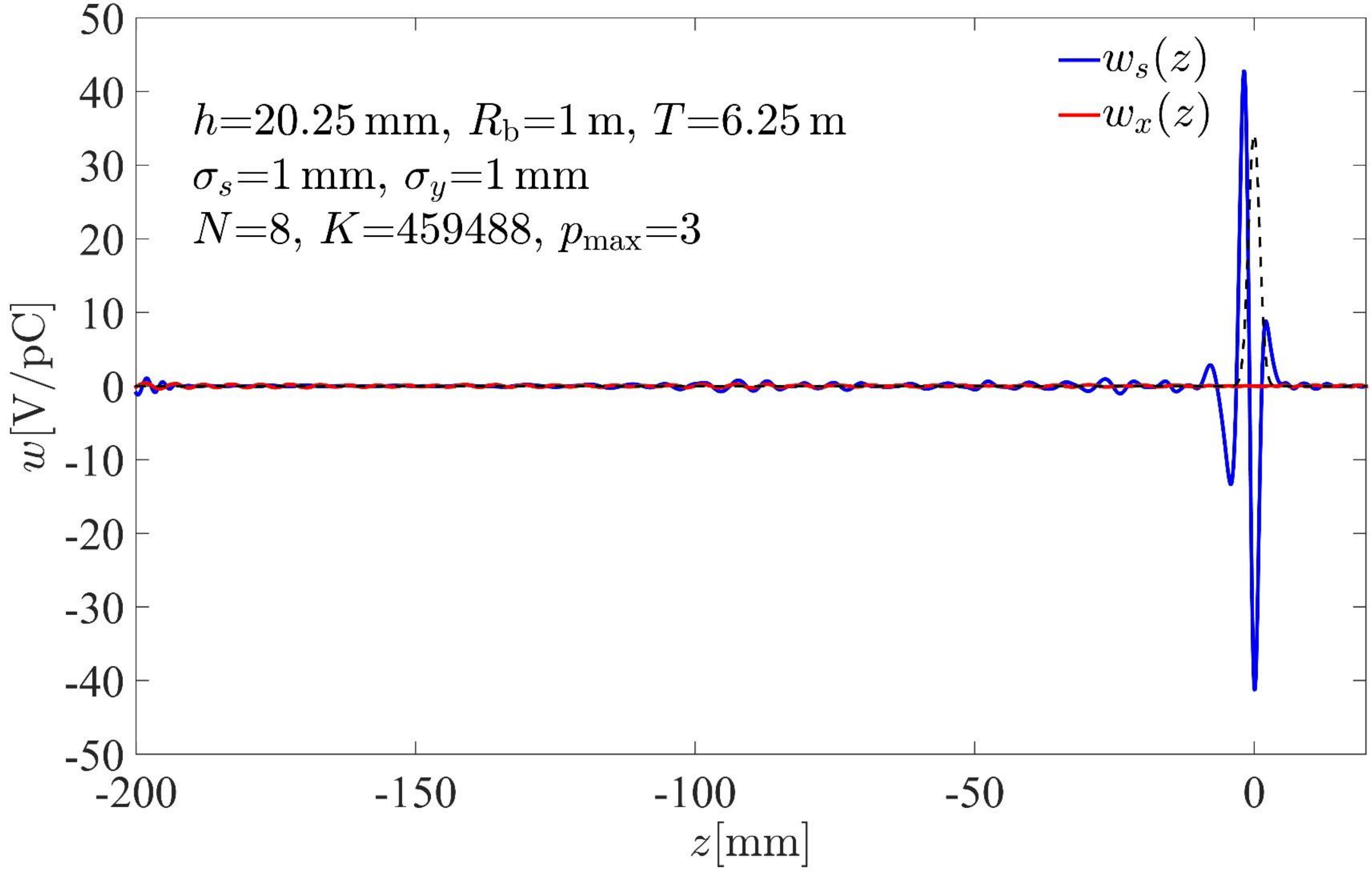


DESY BC0 Simulation

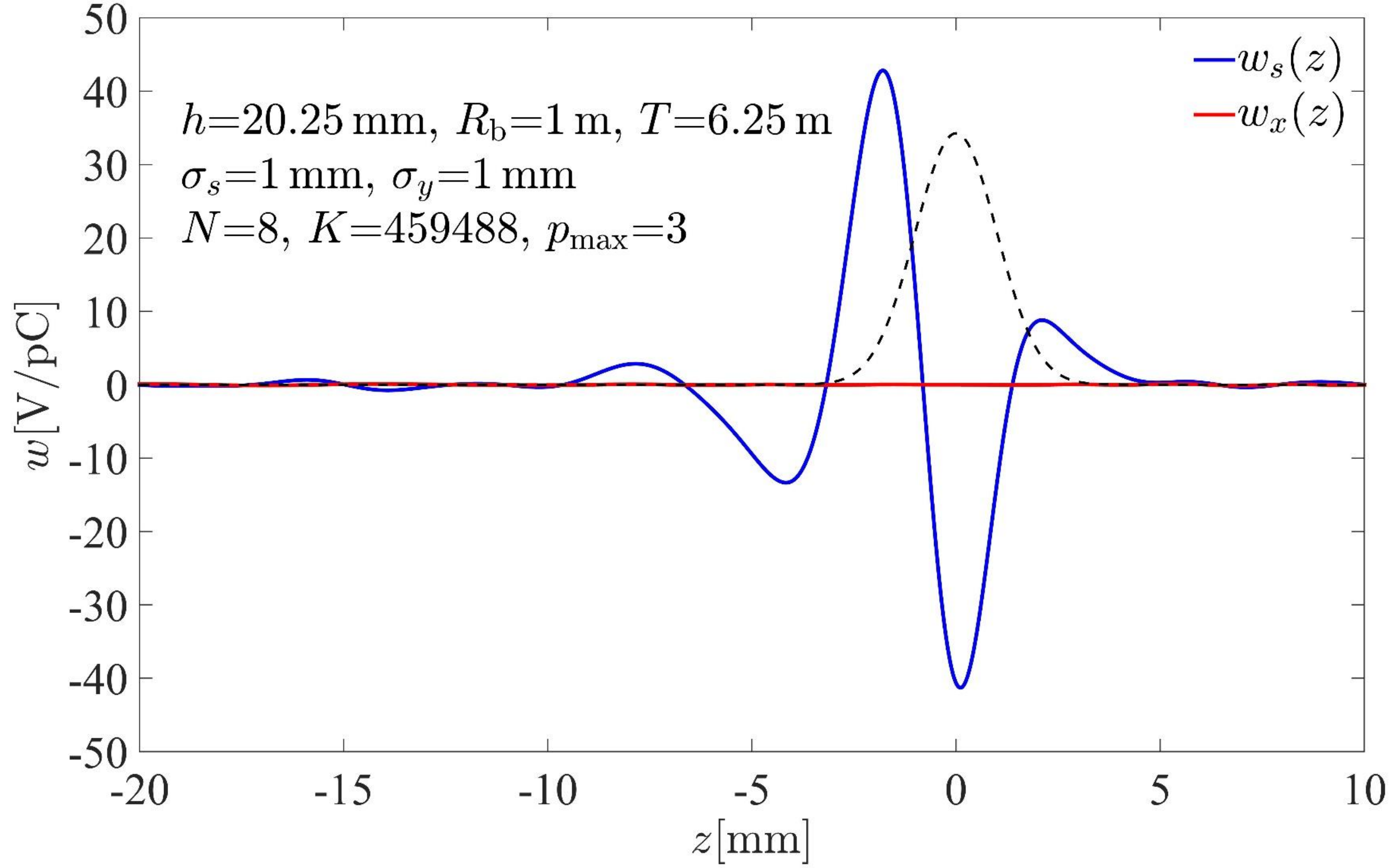
- Bunch Compressor Case
  - DESY BC0 – assume piecewise constant curvature
  - CSR and geometry generates wake
  - Fields sampled along $x = 0$, sum over $p = 1, 3$



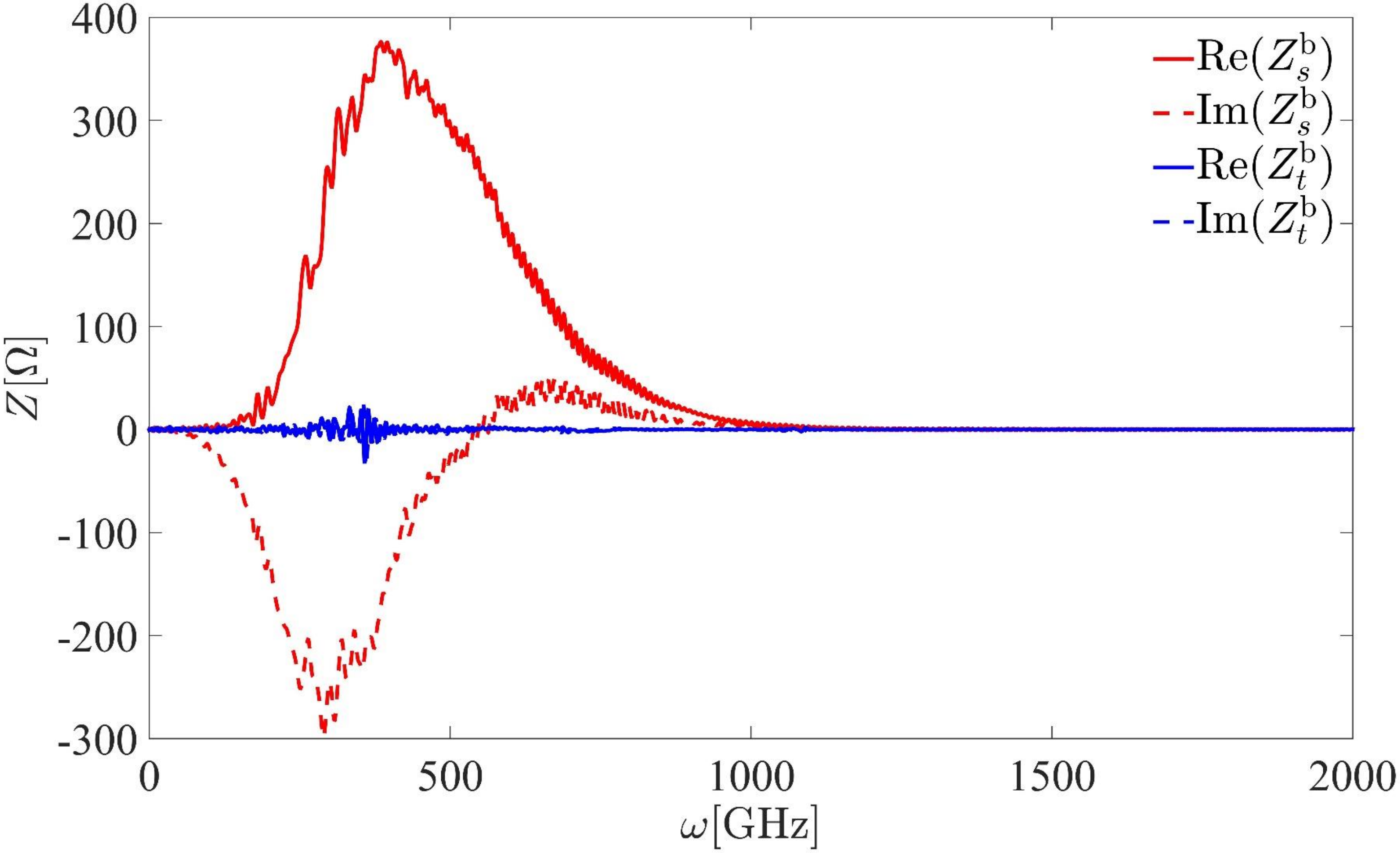
DESY BC0 Wake Function



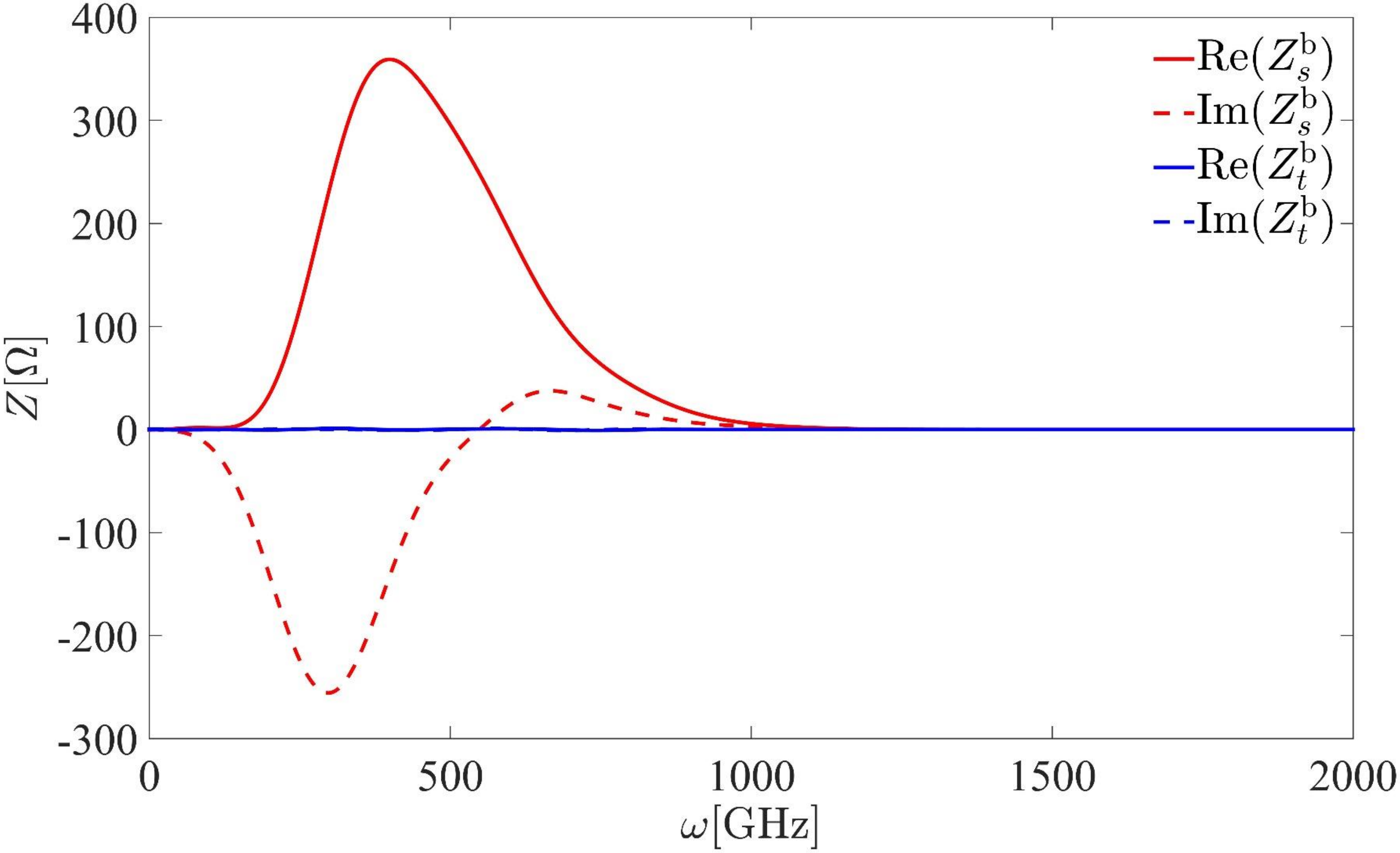
DESY BC0 Wake Function



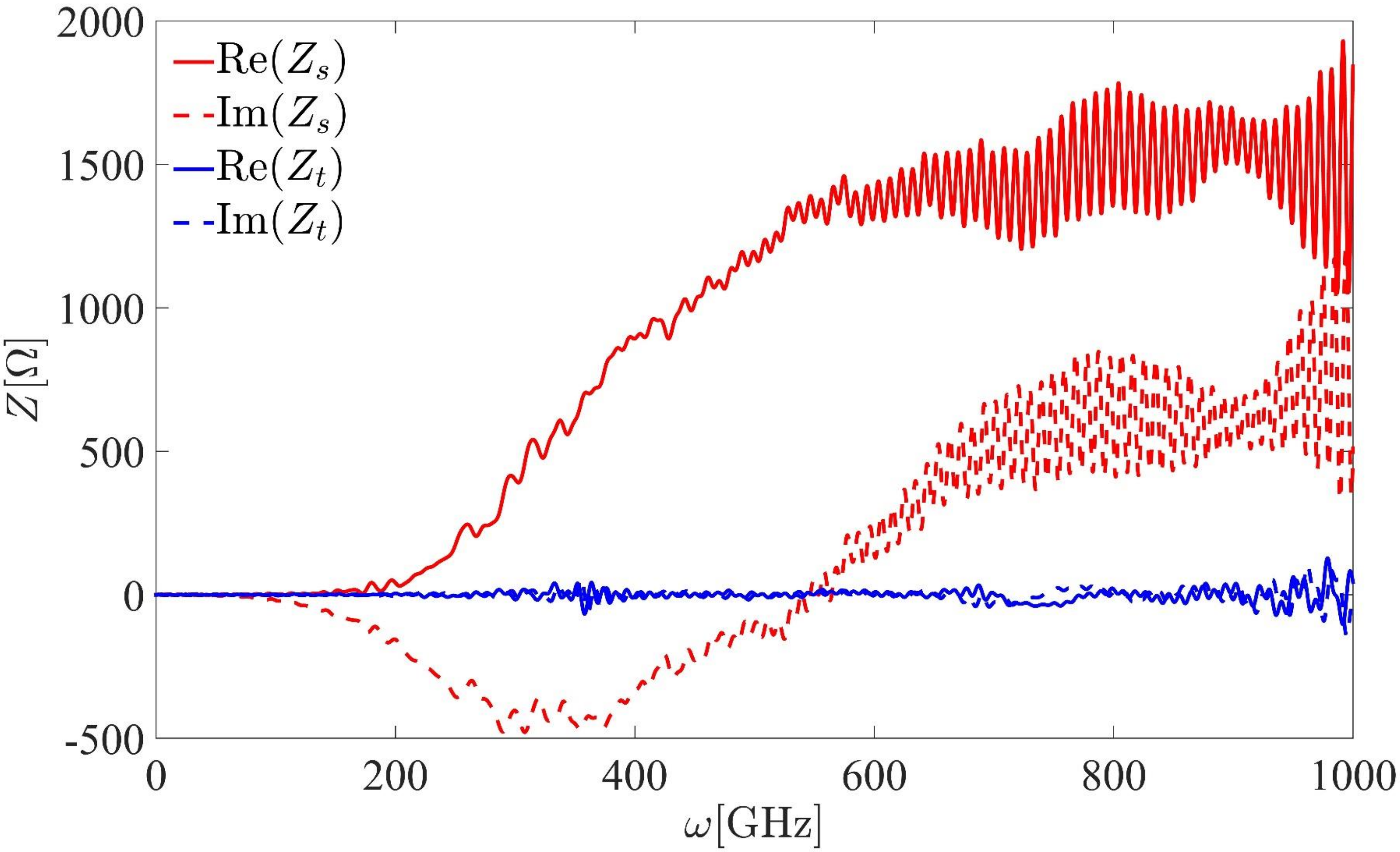
DESY BC0 Bunch Impedance



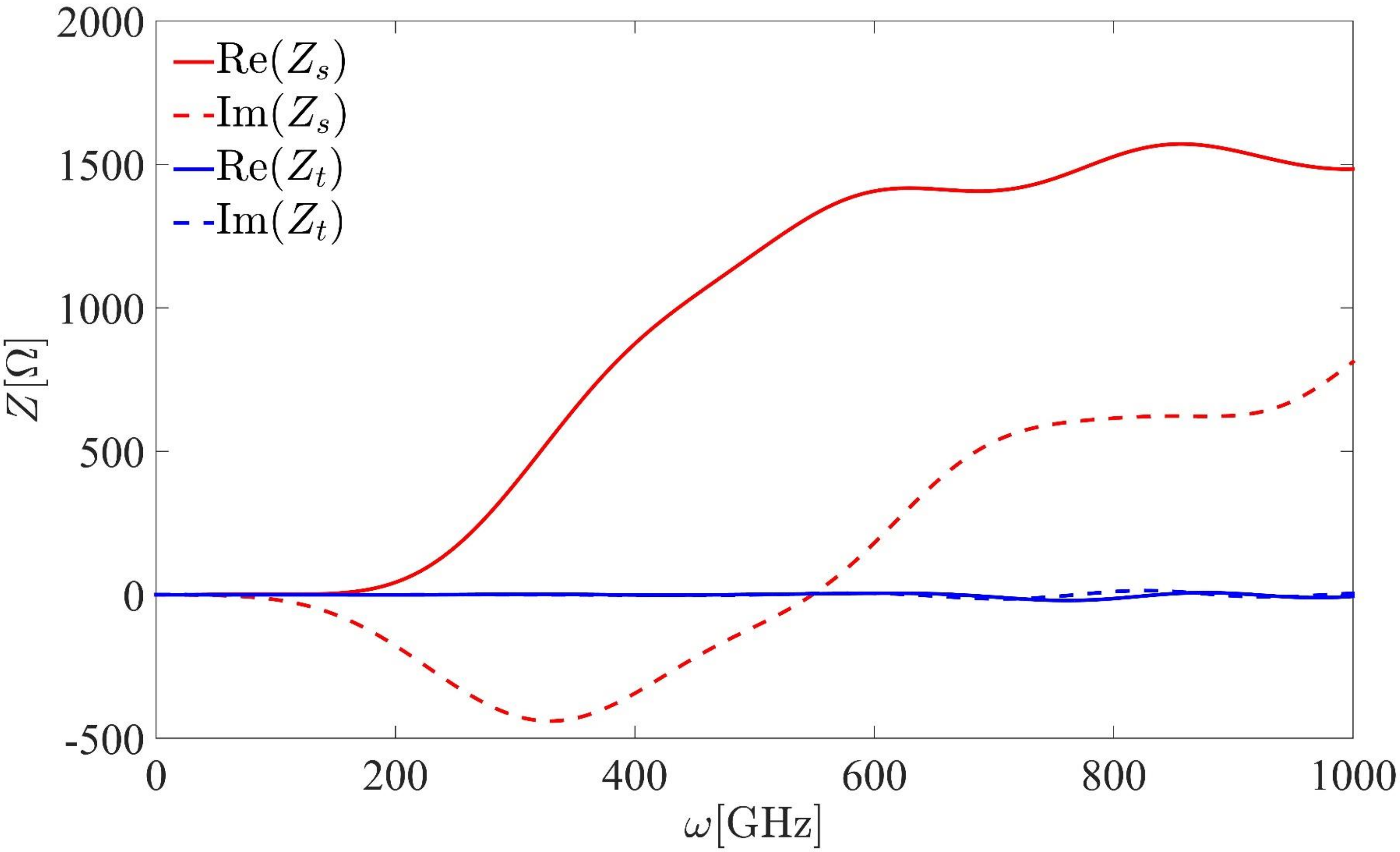
DESY BC0 Bunch Impedance (filtered)



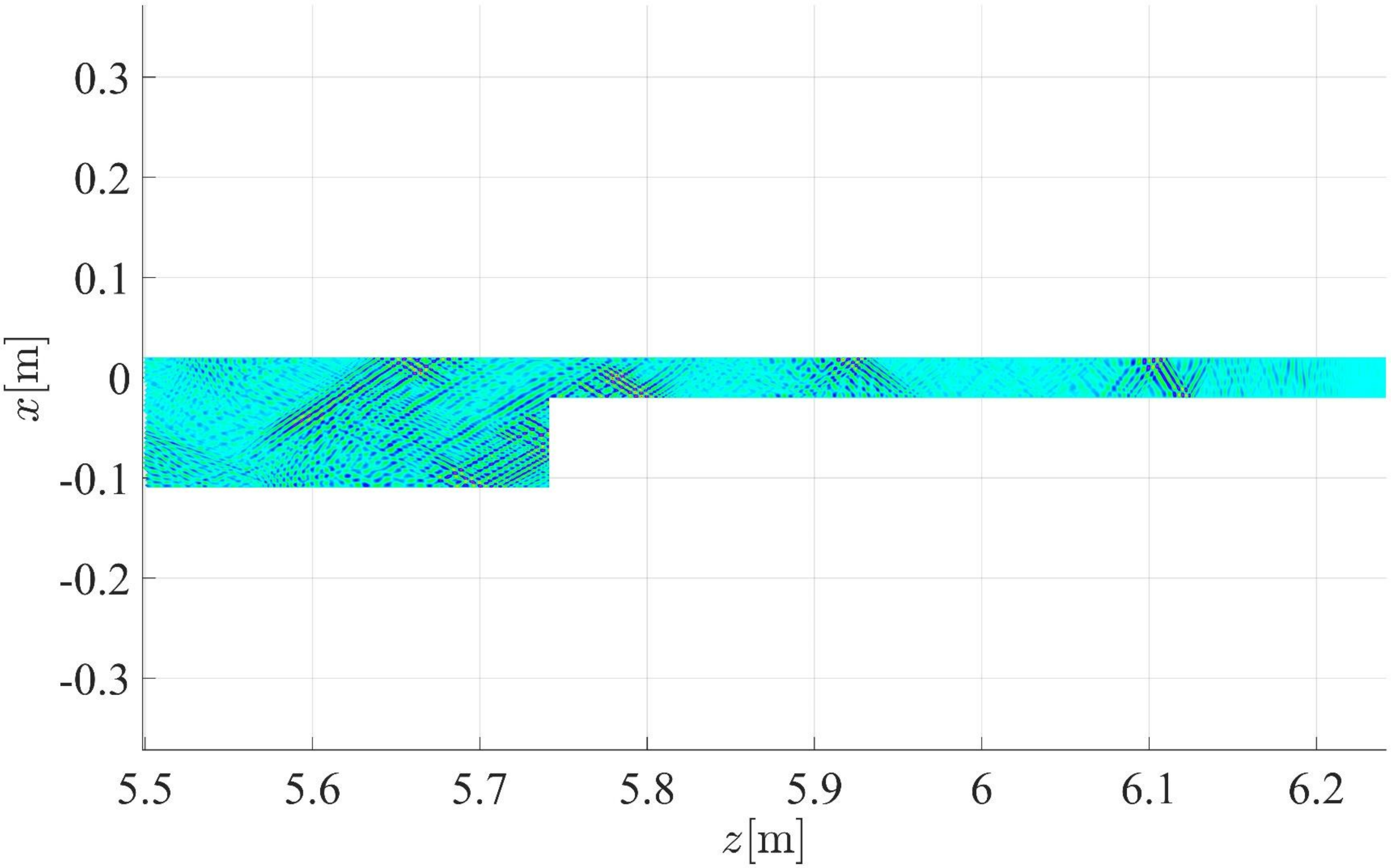
DESY BC0 Single Particle Impedance



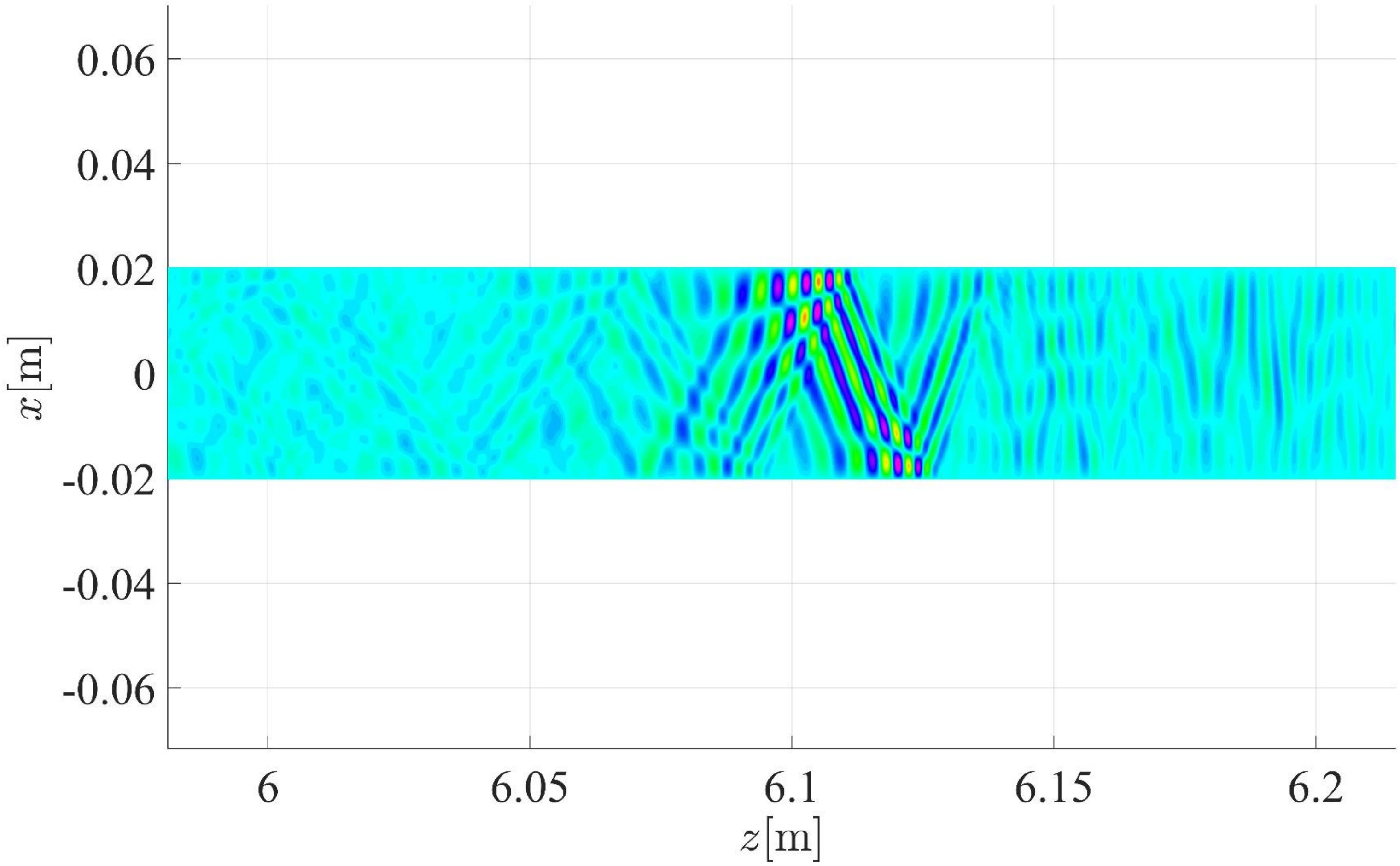
DESY BC0 Single Part. Imp. (filtered)



DESY BC0 Longitudinal Field Map



DESY BC0 Longitudinal Field Map



Summary and Future Outlook

- Compared 2D DG CSR code with CST Microwave Studio and PBCI for tapered beam pipe.
- Updated 2D DG CSR code to compute longitudinal and transverse wake fields and impedances
- Improved code performance for higher resolution
- Checked validity of Panofsky-Wenzel theorem (investigating issue with smaller bunch length)
- Current and Future Work:
  - Export wake fields to CSR Track (work in progress)
  - Examine validity of paraxial-approximation codes

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Thank you for your attention!

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